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Digital Payments and Bank Competition

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Abstract

This article examines how competition between banks and a digital PSP impacts the lending rate and the consumers' use of payment instruments. The digital PSP offers a digital wallet and payment services, but does not offer credit. In contrast, banks invest their deposits in lending activities, which implies that they may incur some costs of adjusting their liquidity needs when consumers make payments. I show that the adoption of the digital wallet for payments may sometimes increase the volume of payments by bank deposit transfers and the lending rate. This results from banks' trade-off between lowering their costs of liquidity when consumers pay from their digital wallet and reducing the revenues they receive from bank transfer fees.

JEL Codes: G21, L31, L42.

Keywords: Payment systems, cash, digital currencies, CBDC, money demand, banking regulation.

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1 Introduction

Banks are defined in both the economic literature and by the legislation as institutions that engage in credit and deposit-taking activities. On the liability side of their balance sheet, banks bundle two different types of services with deposit accounts offered to retail consumers: money storage and payments. Maturity mismatches between assets and liabilities imply that banks incur regular costs of adjusting their liquidity needs if there is an outflow of bank deposits when consumers make payments.

In the recent years, several payment service providers have started to compete with banks in the retail payments industry: Venmo in the United States, Wise in the United Kingdom, FinTech Klarna from Sweden, Vipps in Norway, Apple Pay in several countries, or Alipay in China. Some of these non-bank intermediaries offer payment services only, while other also compete with banks for deposits. In developed countries, they often also require consumers have a valid bank account before accepting them as clients. Their business model differs from that of banks, because they often do not offer long-term credit.[1] The provision of payment services without long-term credit activities has revived the debate on the narrow banking business model of financial intermediation.[2]

In this paper, I study competition between banks and a payment service provider offering a digital wallet (hereafter, the digital PSP).[3] The main objective of my paper is to understand how the costs and benefits of liquidity may impact competition between banks and PSPs, because both types of payment intermediaries do not incur the same costs of adjusting their liquidity needs when consumers make payments. So far, the literature on retail payments has focused on other aspects of competition between banks and PSPs, such as the disruption of banks' information spillover when banks use payment data to learn about consumers' credit quality (see Parlour, Rajan and Zhu, 2022).

The paper studies outcomes in an environment where consumers need a bank account to pay with payment solutions offered by PSPs (hereafter, digital currency). Unlike most PSPs, banks are active in the long-term credit market. Both banks and PSPs may offer two different services to consumers: money storage and payments. The price of both services impacts consumer withdrawal decisions, and therefore the banks' costs of liquidity, because there are maturity mismatches between long-term loans and short-term deposits. Deposit accounts pay interests and payment instruments offer differentiated benefits to consumers, that depend on the transaction size. Consumer prefer to use their bank account for large value payments, their digital wallet for intermediate value payments and cash for small value payments. I determine whether banks strategically choose prices such that consumers use their bank accounts both to store money and make payments. The banks' choice depends on their marginal costs of adjusting their liquidity needs. I show that banks may sometimes prefer that consumers renounce using their bank account to make payments, to extract higher additional value from deposits and other services. I also analyze the impact of competition with the digital PSP on the equilibrium lending rate.

In the baseline model, banks choose the prices of payment transactions and the fixed deposit fees paid by consumers to open an account. All consumers can join the PSP and open a digital

[1] Bigtech companies often offer consumers short-term credit (e.g., Alipay in China). Several FinTechs which do not have a banking license partner with banks to offer short-term consumer credit, such as buy-now-pay-later options (e.g., Floa with BNP Paribas in France). This trend has accelerated recently in several countries (see Bian, Cong and Ji, 2023).

[2] During the Great Depression, several economists proposed to dissociate the distribution of credit from the provision of means of payment and to back deposits with reserves held at the Central Bank (see Knight et al., 1933, Fisher, 1936 or later Friedman, 1965, and Tobin, 1987).

[3] My model is not specifically focused on CBDCs. A paper by the ECB (2020 a) explains that the introduction of a Central Bank digital currency would have a similar impact on financial accounts if it is issued by a financial vehicule that manages a stablecoin or by a narrow bank.

wallet at no cost to make payments. Consumers split their funds between their digital wallet and their bank account, respectively. When they need to pay, they choose their payment instruments according to the foregone interest rates on deposits and the transaction fees. The transaction fee for transfers of funds from the digital wallet is set at the PSP's marginal cost (either by regulation or by competition between PSPs). The banks and the PSP incur different costs when consumers pay from their accounts. Since banks invest in illiquid loans, they incur additional costs of adjusting their liquidity needs when consumers pay from their bank account. In contrast, the digital PSP, which is organized as a narrow bank, keeps all the consumers' funds as reserves, and does not incur any additional cost of liquidity when consumers pay from their digital wallet.

I obtain the following results. For intermediary values of the marginal cost of the digital PSP, with respect to the endogenous marginal cost of bank payments, consumers pay from their bank account transactions of higher values and from their digital wallet transactions of intermediary values. Thus, both payment instruments coexist in the economy. However, competition with the PSP sometimes implies a reduction of the bank transfer fees. This results in a higher use of bank deposits for payments compared to a market without a PSP. The use of bank deposits for payments is increased if and only if the marginal cost of the digital PSP is higher than the marginal cost of liquidity generated by the variation of the equilibrium amount of reserves for banks.

If the marginal cost of the digital PSP is very low compared to the marginal cost of bank payments, competition with the digital PSP implies that consumers prefer to use their bank account to store value and their digital wallet to make payments. If the marginal cost of the digital PSP is too high, consumers do not adopt the digital currency for payments. The marginal cost of bank payments depends on the competitive conditions in the lending and deposit market. It includes the marginal operational cost of the payments infrastructure, the marginal cost of adjusting the bank's liquidity needs when consumers pay from their bank account, and the marginal cost implied by the consumer's decision to substitute a payment via digital currency for a payment by bank transfer.

The mechanism of the results rests on two-part tariff competition. Since they choose fixed deposit fees and transaction fees, banks price payment transactions at their marginal cost and are able to extract the consumer's surplus of opening a deposit account through the fixed deposit fee. Because having a bank account is required to open a digital wallet, the consumer's surplus of opening a bank account also includes the option value of using a digital wallet. Raising the bank transfer fee has two opposite impacts on banks' marginal cost of liquidity. First, consumers pay less frequently from their bank account, which reduces banks' marginal cost of liquidity, because banks need to borrow additional liquidity less frequently. However, on average, consumers transfer a higher amount from their account, which increases the marginal cost of liquidity. The magnitude of this last effect is all the more important since the cost of liquidity is high and the number of banks is small, as banks collect a higher share of deposits. The market share of the digital currency in equilibrium reflects both effects.

The lending rate depends on the use of the digital currency for payments if the banks' funding cost is not separable in the volume of deposits and loans. With a higher fee for digital currency transactions or a lower bank transfer fee, consumers tend to pay more from their bank account, which increases banks' cost of liquidity. This implies that banks pass through higher costs of adjusting their liquidity needs to the borrowers, resulting in an increase in the lending rate. Thus, in equilibrium, if banks react to competition with the digital PSP by reducing the fee for bank transfers, the equilibrium lending rate increases.

Then, I discuss some of the regulatory options that impact the adoption of the digital currency offered by the PSP. Designing a digital currency from an operational perspective is a complex task and involves several choices (see Duffie, Mathieson and Pilav, 2021, Bank of England, 2020, Fung and Halaburda, 2016). Regulators have to choose: i) which intermediary is allowed to distribute a digital payment solution (i.e., banks, the Central Bank in the form of a CBDC, a private operator operating as a narrow bank), ii) whether non-bank PSPs may hold reserves in the Central Bank (Bank of England, 2018), iii) whether the digital currency relies on a different unit of account (i.e., tokens), iv) whether digital wallets should bear interest, v) whether the fee for digital currency transactions should be regulated.

In my paper, I focus on a specific regulatory framework for the digital currency. First, I examine how the private operator is allowed to keep its clients' funds as central bank reserves, and I study the alternative option in the extension section.[4] Second, I assume that the digital currency relies on the same unit of account as cash and bank deposits. Therefore, I do not study whether it is optimal to use a different unit of account for the digital currency in the form of tokens.[5] In the last section of the paper, I analyze how alternative design choices for the digital currency impact its adoption for payments (distribution by banks, no access of the digital PSP to central bank reserves, different regulation of transaction fees).

The rest of the article is organized as follows. In section 2, I position my paper in the literature on financial intermediation and competition between payment media. In section 3, I present the model and the assumptions. In section 4, I solve for the equilibrium of the game. In section 5, I discuss the robustness of the results obtained in section 4. Finally, I conclude. All proofs are in the Appendix and in the online Appendix.

2 The literature

My paper contributes to the recent literature on the disruption of financial intermediation. The central research question of this literature is whether competition between banks and alternative models of financial intermediation impacts lending, payment instrument pricing, monetary policy and financial stability (see for instance Parlour, Rajan and Zhu, 2022, or Biancini and Verdier, 2023). To the best of my knowledge, so far, no research work has analyzed whether the costs and benefits of liquidity may impact competition between banks and Fintechs for retail payments.[6]

My paper is connected to a literature that analyzes the efficiency of financial intermediation (see Diamond and Dybvig, 1983, Diamond, 1984), and the role of narrow banks (see Shy and Stenbacka, 2000, Kashyap and Stein, 2002, Piazzesi and Schneider, 2020, and for a survey, Pennachi, 2012). Unlike in this literature, I am interested in understanding which payment instruments may be used by consumers given competition between different types of financial intermediaries.

My work belongs to a strand of the literature analyzing how consumer demand for money depends on the fees charged by financial intermediaries (see Towey, 1974, Saving, 1979, Santomero, 1979, Whitesell, 1989 and 1992, Shy and Tarkka, 2002).[7] I extend the setting of Whitesell (1992) by modelling banks' decision to hold reserves and competition with a digital PSP. In Whitesell's (1992) framework, consumers choose their payment method according to the costs of holding money and transferring value, which depend on the size of the transaction. Another branch of the literature

[4]If this is the case, consumers are able to transact indirectly in a central bank liability. Therefore, the digital currency can be qualified as a synthetic CBDC (Adrian and Mancini-Griffoli, 2021).

[5]Distinction between account-based and token-based depends on who is liable in the event of a fraudulent transaction In token-based payment systems (such as cash), the receiver of the payment is liable for fraud. In account-based, the provider of the account should check the identity of the account holder and bear the cost of record-keeping by verifying the authenticity of transactions.

[6]Parlour, Rajan and Walden (2022) analyze how banks' wholesale payments impact their lending activities, and consider the impact of a reduction in the settlement costs.

[7]Unlike in the literature studying why money is valued by consumers for trading (in the New Monetarist models or in overlapping generation frameworks), I take the presence of money as given and study how differentiation between financial intermediaries impacts the use of money storage and payment services.

studies consumer demand for tokens and token pricing (Biais et al., 2018, Schilling and Uhlig, 2019, Pagnotta, 2022, Cong, Li and Wang, 2021, Rogoff and You, 2023). In contrast, my paper does not consider asset prices and analyzes instead how payment instrument pricing impacts competition for deposits.

In this literature, a specific branch focuses on competition between differentiated payment instruments. As in Whitesell (1992), I assume that the benefit of paying with a given payment instrument depends on the transaction size. This choice is motivated by several use cases (for P2P or cross-border payments, e-wallets or CBDCs in China) in which a digital currency is introduced as a substitute for cash and bank transfers.[8] Other modeling approaches include differentiation with respect to the degree of anonymity to users (Agur et al., 2022) or differentiation in terms of liquidity services (Benigno, Schilling and Uhlig, 2022). A strand of the literature analyzes more specifically the impact of digital currencies, cryptocurrencies and stablecoins on international competition between fiat currencies (Benigno et al., 2022, Cong and Mayer, 2022, Ferrari et al., 2022). Other works focus on technological improvements brought by the blockchain for payments (Auer, Monnet and Shin, 2021) or on the role of network externalities in the dynamics of token adoption (Cong, Li and Wang, 2021).[9]

The disruption of financial intermediation could be caused by competition with the central bank for payments by CBDCs (see Kahn et al., 2018, and the survey by Auer, Frost et al., 2022).[10] Several authors analyze in general equilibrium models whether CBDCs could increase banks' funding costs, thereby reducing bank lending (e.g., Keister and Sanches, 2019, and Chiu et al., 2023) or crowd out banks' deposits (Andolfatto, 2021, Fernandez-Villaverde et al., 2020). As in Chiu et al. (2023), I find that competition with a non-bank intermediary may reduce the lending rate if consumers pay less from their bank account following the introduction of the digital currency. In Chiu et al. (2023), the mechanism rests on the assumption that the quantity of deposits is elastic to the deposit rate. In my paper, the result relies on the interaction between payment choice and transaction fees, and banks' cost of liquidity. I also contribute to the literature on the welfare effects of CBDCs (Huynh et al., 2020, Kwon, Lee and Park, 2022) by showing that the marginal cost pricing of digital currency transactions may not maximize the surplus of depositors if banks are imperfectly competitive.

Even if they do not act as operators in payment systems, central banks may impact the outcome of competition between payment intermediaries by regulating payment systems (see Agur et al., 2021, Ferrari et al., 2022, Green, 2008), such as the interoperability of payment systems, both at the national and the international level, the accessibility of central bank accounts to non-banks, or the regulation of interchange fees used in payment card systems (Verdier, 2011). My framework allows for a discussion of some of these important dimensions of regulation: the access of non-banks to central bank accounts, the distribution of digital currencies by banks, the regulation of the fee for transactions by digital currency. I show that the market share of the digital currency is lower when banks distribute it themselves and explain why the market share of the digital currency may be higher if the digital PSP does not have a dedicated central bank account.

[8]Using data from a leading e-wallet provider in China, Bian, Cong and Ji (2023) show that consumers pay with their e-wallets transactions of small values and with bank payment instruments transactions of high values.

[9]In particular, the blockchain could add value to specific types of payments characterized by credit or liquidity risk by implementing smart contracts. I do not discuss in this paper the risks associated with the payment transaction itself. This issue could become relevant in the future for retail payments. It is already very relevant for large value payment systems and wholesale CBDCs, that I do not discuss in this paper.

[10]Kahn et al. (2018) define a token-based system as relying on the identification of the object being transferred as a means of payment rather than relying on identification of the individual whose account is being debited.

3 The model

In this section, I construct a theoretical model to analyze how competition with a non-bank digital PSP impacts the prices charged by banks for deposits, payment transactions and loans.

There are three dates in the economy ($t = 0, 1, 2$) and four types of risk-neutral agents: $n \geq 2$ banks, one digital currency provider, a continuum of depositors, and numerous entrepreneurs. At date 0, banks choose the price of deposits, payment transactions, and loans. At date 1, they collect deposits from the public, keep a share of deposits in reserves and invest the rest in loans that mature at date 2.

If there is no digital currency, at date 1, depositors choose the bank from which to open an account. Between date 1 and date 2, they need to make a payment transaction and choose between paying in cash or with a transfer of bank deposits. Their choice of a payment instrument depends on the price of bank transfers, the foregone interest rate on deposits, and the value of the transaction that needs to be settled. If the transfer of deposits exceeds the bank's reserves, the bank incurs some costs of adjusting its liquidity needs. If there is a digital PSP, consumers deposit funds in both a bank account and a digital wallet. Between date 1 and date 2, they choose between paying in cash or by a transfer of deposits, from either their bank account or their digital wallet. To simplify the model, all interest and fees are paid out at date 2. Payment instruments are denominated in the same unit of account.

The lending market: At date 1, banks make risk-free term loans, which mature and are paid off at date 2. Banks offer differentiated lending contracts and compete in prices. Following Shubik and Levitan (1980) and Carletti, Hartmann and Spagnolo (2007), I assume that each bank $i \in \{1..n\}$ faces a linear demand for loans such that

$$L_i = L - \gamma(r_L^i - \frac{1}{n}\sum_{k=1}^{n} r_L^k), \tag{1}$$

where r_L^i represents the interest rate charged by bank i, r_L^{-i} represents the vector of interest rates charged by bank i's competitors, and $\gamma > 0$ measures the degree of substituability between loans. With this specification, the total demand for loans is constant and equal to nL. The mechanism of the model does not rely on this specific demand function, which gives simple expressions of the lending rate in equilibrium.[11] Borrowers are distinct from depositors.[12]

The deposit market: In the deposit market, banks compete for consumers in the Salop (1979) circle by offering money storage and payment services. Consumers incur a transportation cost $t_b > 0$ when they travel to open a bank account. All banks are located at a distance $1/n$ of each other. Consumers pay a fixed fee F_i to open a bank account. In addition, consumers also open a digital currency wallet with a digital PSP at no cost. The digital PSP is not located in the circle.[13]

[11] However, since the total demand for loans is inelastic, I abstract from studying the effect of a digital currency on the aggregate demand for loans, unlike Chiu et al. (2023) who use a Cournot model of competition. The model also works with Cournot competition.

[12] This assumption differs from Andolfatto (2021) who assumes that banks create money in the act of lending. Therefore, I do not model in this paper the fact that e-wallet providers may expand consumer short-term credit (as in Bian, Cong and Ji, 2023).

[13] This assumption is motivated by the fact that if the digital currency is offered by a private provider, consumers already have a relationship with most Internet giants that would be able to offer a digital currency. If the digital currency is supplied by the central bank, it seems unrealistic that the central bank would start to compete with banks for deposits by charging fixed fees for the deposits.

Consumers make two decisions. They choose their home bank at date $t = 1$ and a payment instrument at date $t = 1.5$ when they need to pay. When they choose their home bank, consumers consider the linear transportation cost $t_b x > 0$ of opening an account in a bank branch located at a distance x from their location in the circle, the benefits and costs of money storage, and payment services, respectively.

All consumers derive fixed utilities for opening a bank account and a digital wallet because payment intermediaries often bundle the provision of deposit accounts with other services that bring value to consumers (such as safety, see Bijlsma et al., 2021).[14] To simplify the model, I assume that the fixed utility $u_b > 0$ of opening a bank account is sufficiently large such that the market for bank deposits is covered in equilibrium (see Assumption A0 hereafter). In most countries, the citizens need to have a bank account to pay their taxes and receive money from the government. In addition, the fixed utility of opening a digital currency wallet $u_d > 0$ is also larger than the cost of making payments by digital currency (see Assumption A1 hereafter). Such an assumption is a first approximation, as the model does not focus on studying financial inclusion.

Money storage services: Consumers obtain benefits of storing money in their deposit account and their digital wallet. In the rest of the paper, these benefits are referred to as interest rate payments. However, for the digital PSP, they could also represent the value offered to the consumers who store a high amount of deposits in their accounts (with rewards or discounts on other services). What matters in the model is that consumers incur some opportunity costs of not storing money when they make payments from their deposit account or their digital wallet.

Banks pay the same exogenous interest rate r_b on bank deposits and the digital PSP pays the interest rate r_d on the funds left in the digital wallet.[15] The difference between the interest rate on deposits in the bank account and the digital wallet is given by

$$\Delta r = r_b - r_d.$$

I do not make any assumption on the sign of the interest rate r_d.[16] If the digital PSP is a public operator, several authors argue that the interest rate of the digital wallet could not exceed the interest rate on deposits (i.e., $\Delta r > 0$) to avoid a crowding-out of bank deposits. Private PSPs may also offer consumers interest-bearing accounts (e.g., Alipay in China) or discounts on other services.

All consumers are endowed with 1 dollar, which they split as a deposit between their bank account in share $\alpha \in (1/2, 1)$ and their digital wallet in share $(1 - \alpha)$. This decision is considered to be exogenous. They also hold an additional amount of cash which is sufficient to cover small payments. If consumers do not make any deposit transfers (either because they are not paying or because they are paying in cash), they obtain the benefit b_c of storing money in their deposit accounts, where

$$b_c \equiv \alpha r_b + (1 - \alpha) r_d. \tag{2}$$

[14] A November 2021 report by the Monetary Authority of Singapore acknowledges that "payments are increasingly bundled with other value-added services."

[15] As shown later in the paper, banks' profits can be expressed as a function of the minimum transaction size such that consumers pay by a transfer of bank deposits. This minimum transaction size is a function of the transaction fee and the interest rate on deposits (see Lemma 1 hereafter). Therefore, maximizing profits with respect to transaction fees or interest rates on deposits is equivalent in the base model. This explains the choice to set exogenous interest rates.

[16] I consider the digital currency deposit rate as being exogenous. If the digital currency is a CBDC, the interest rate on CBDC deposits is likely to be regulated in the future. In a speech at the Bruegel online seminar, F.Panetta from the ECB argued that a negative rate on the CBDC would foster its use as a payment instrument (February 10, 2021).

Transactions and payments: Between date 1 and date 2, all consumers of bank i face a consumption shock and need to make a payment of size $s^i \in [0,1]$. All agents anticipate that s^i will follow a uniform distribution on $[0,1]$.[17] The results of the paper do not rest on the use of this specific distribution. The consumption shocks of all banks are mutually independent.

The consumption shocks faced by consumers imply that each bank may face an outflow of deposits that depends on the consumers' choice of payment instrument. In practice, a bank's depositors do not transfer money at the same time, and banks net the positions of their consumers before clearing and settling transactions at a later point in time. It is not uncommon for a bank to issue more payments than it receives, which implies that it incurs some additional costs of liquidity when its consumers make payments. Therefore, though simplistic, the assumption that the consumers of bank i may face the same consumption shock captures the bank's liquidity needs for its retail payment transactions at some point in time.

Consumers may use three payment instruments denoted by $k \in \{c, d, b\}$ to pay:

- i) cash ($k = c$),
- ii) a bank deposit transfer with the bank payment instrument ($k = b$),
- iii) a transfer of funds from their digital wallet with the digital currency ($k = d$).

As in Whitesell (1992), and consistently with the empirical study of Wang and Wolman (2016), I construct a model such that the consumers of bank i trade off between a pair of payment instruments based on the transaction size s^i, and choose the instrument that brings them the highest net benefit $b_k(s^i)$.

The consumers' net benefit $b_k(s^i)$ depends on their convenience benefits, their cost of foregoing interest rates when they transfer deposits, and the transaction fees, which are given by f_d and f_b^i for a digital wallet and bank i, respectively. If a consumer does not have enough funds in his bank account to pay, an instant transfer of value from his digital wallet to his bank account can be made at no cost, or the reverse.

To obtain threshold transaction sizes such that consumers choose a given payment method, convenience benefits increase linearly with the transaction size at different rates. Thus, when a consumer pays with payment instrument k a transaction of size s^i, he obtains $v_k(s^i) = v_k s^i$ for $k \in \{c, b, d\}$. I normalize v_c to $v_c = 0$ without loss of generality and denote by $\Delta v = v_b - v_d > 0$. The convenience benefits of payment instruments are ranked such that if the value of the transaction is small, consumers prefer to pay in cash, if it is higher, they transfer funds from their digital wallet, and if it is very high, they transfer funds from their bank account.[18] All payment instruments are accepted everywhere. Partial acceptance of payment instruments can be easily added to the model without modifying its mechanism.

When a consumer pays in cash, he receives the money storage benefit b_c because cash payments are free.[19] The net benefit $b_d(s^i)$ of paying a transaction of size s^i with digital currency (including

[17] The framework of analysis is a discrete-time model, that simplifies the consumer choice of a payment method to one payment decision. With a higher number of payments, consumers would need to make inventory-management decisions as in Baumol, 1952, Tobin, 1956, Santomero, and Seater, 1996, Whitesell, 1989, or Alvarez and Lippi, 2009.

[18] Without any knowledge of what the demand for digital currency would be, I model the demand for digital currency as a substitute for cash transactions that impacts the volume of transactions paid by bank transfer. I motivate this assumption with the argument mentioned in several papers (e.g., Adrian and Mancini-Griffoli, 2021) that access to a digital currency will be more convenient than travelling to an ATM. In the ECB report (2020), it is also argued that ideally, a digital euro should allow citizens to make payments such as they do today with cash. Moreover, the People's Bank of China has started to run an experiment on CBDCs for small retail transactions.

[19] The consumer also receives the maximum money storage benefit when he does not pay, because in both cases, there is no outflow of deposits from his bank account.

storage benefits) is given by

$$b_d(s^i) = b_c + (v_d - r_d)s^i - f_d. \tag{3}$$

The net benefit $b_b(s^i)$ of paying a transaction of size s^i by bank deposit transfer (including storage benefits) is given by

$$b_b(s^i) = b_c + (v_b - r_b)s^i - f_b^i. \tag{4}$$

The threshold transaction size such that the consumers of bank i prefer to pay by bank deposit transfer rather than with digital currency is denoted by s^i_{dc}. The threshold transaction size such that the consumers of bank i prefer to pay by a transfer of deposits with payment instrument $k = b, d$ rather than cash is s^i_k. To obtain equilibria in which cash, the digital currency, and the bank payment instrument may be used by consumers, I assume that $\Delta v > \max(\Delta r, 0)$ and $v_k > r_k$ for $k = d, b$.

- *Transfers of deposits:*

If consumers pay in cash or with digital currency, there is no outflow of bank deposits.[20] I assume that the receivers of cash payments keep the funds in cash until date 2 and do not deposit them in a bank account. The receivers of a payment in digital currency do not transfer them instantly to their bank account. If consumers pay with the bank payment instrument, there may be a variation of the quantity of deposits in bank i. When a consumer initiates a payment from his bank account, his deposit account is instantly debited. The receiver of the funds only obtains them at date 2. Such a delay between the initiation of a payment and the reception of the funds is common in most retail payment systems (e.g., Visa, MasterCard, the Danish retail payment system the Sumclearing).[21] With probability φ, the receiver holds an account in another bank. In that case, the quantity of deposits in bank i is reduced between date 1 and date 2. With probability $1 - \varphi$ (and independently from the size of the transaction), the receiver holds an account at the same bank i. In that case, there is no outflow of deposits from bank i between date 1 and date 2 and the amount of bank i's reserves does not vary.[22]

Bank profits: A bank's profits are the sum of the profits from deposits, loans and the expected revenues and costs of liquidity management.

The profits from deposits: Each bank i obtains a margin per depositor that is the sum of the fixed deposit fee F_i, the revenues from bank transfers, from which are subtracted the interest rates paid to depositors IR_b^i and the operational cost of bank transfers C_b, which is increasing and convex in the volume of payments.[23] Therefore, bank i's margin per depositor is given by

$$\mu_i \equiv F_i + f_b^i \beta_b^i - IR_b^i - C_b(\beta_b^i). \tag{5}$$

Each bank i has a share D_i of deposits and makes a profit $\mu_i D_i$ from deposits. Since each depositor leaves α in a bank account, the total volume of deposits in bank i is given by

$$\alpha D_i. \tag{6}$$

[20]In the paper, I focus on a case in which consumers always have enough funds in their digital currency account to pay with digital currency.

[21]In my paper, I do not look at the mechanisms that reduce liquidity needs (see Copeland and Garratt, 2019). This assumption simplifies the model.

[22]Recall that in case the consumer does not have enough funds in his bank account to pay, he can transfer funds instantly and at no cost to himself.

[23]I do not consider network effects in this model, which also explain the importance of scale economies in some retail payment systems (see Hancock, Humphrey and Wilcox, 1999, for a survey of this literature and Humphrey, 2009).

To give simple expressions of the market shares of each payment instrument in equilibrium, I assume that the bank's operational cost of bank transfers C_b is convex in the volume of payments, and it is given by

$$C_b(\beta_b^i) = c_b\beta_b^i + k_b(\beta_b^i)^2/2,$$

where β_b^i is the expected market share of bank transfers for the consumers of bank i given the transaction fees for payments.

The profits from lending: Each bank i makes a profit $r_L^i L_i$ from lending.[24]

The revenues and costs of liquidity management:

- *Liquidity management:*

Each bank i obtains a benefit of keeping reserves in quantity R_i and incurs some costs of adjusting its liquidity needs when there is an outflow of bank deposits, which happens if consumers need to make payments to another bank. As in Klein (1971), from (6), the amount of reserves R_i of each bank i satisfies the balance sheet identity given by

$$L_i + R_i = \alpha D_i. \tag{7}$$

If the outflow of bank deposits is lower than R_i, the bank has enough reserves to make a payment to another bank on behalf of its depositors. The excess reserves remaining after the transfer of deposits, which are denoted by $\underline{a_i}$, are remunerated at the interest-on-reserves (IOR) τ.[25] If the outflow of bank deposits is higher than R_i, the bank borrows additional funding sources at a rate ρ, where $\rho > \tau$. The cost ρ reflects all the additional costs of liquidity adjustments including transaction costs.[26] The additional amount of funding required to meet the consumers' demand for payments is denoted by $\overline{a_i}$ and it is obtained by subtracting the outflow of bank deposits from the initial amount of reserves.

The bank's expected net benefit of liquidity EL_i is the expected sum of the benefit of excess reserves and the costs of liquidity adjustments, which depend on both the demand for loans and payments. It is then given by:

$$EL_i = \tau\underline{a_i} - \rho\overline{a_i}, \tag{8}$$

where the expressions of $\underline{a_i}$ and $\overline{a_i}$ are given in Appendix C for the sake of simplicity. A detailed motivation for this model of the banking firm, the possible extensions and the alternatives is given in Baltensperger (1980).[27]

Total bank profits and choice variables: Each bank i maximizes its expected profit π_i with respect to the deposit fee F_i, the fee for bank transfers f_b^i and the lending rate r_L^i. Bank i's profit is the sum of the profit on loans and deposits, and the expected net benefit of liquidity:

$$\pi_i(r_L^i, F_i, f_b^i) = r_L^i L_i + \mu_i D_i + EL_i, \tag{9}$$

[24] The banks' marginal cost of lending (excluding funding costs) is normalized to zero without loss of generality.

[25] This excess amount of reserves is obtained by subtracting the outflow of bank deposits (if any) from the initial amount of reserves.

[26] As argued by Baltensperger (1980), the rate ρ cannot strictly be identified as the discount rate, because some banks "cannot borrow freely from the Central Bank or (have) an aversion against borrowing from it." In this simple setting, I do not model whether the bank borrows from the interbank market or from the Central Bank.

[27] The bank could also fund its loans with equity and be forced to respect a leverage ratio. In that case, if the quantity of deposits is reduced in equilibrium because of the presence of the digital currency, the bank could be constrained to hold more equity to respect the leverage ratio.

where the interest rate on loans r_L^i is given by (1), the amount of loans L_i is given by (7), the margin per depositor μ_i is given by (5) and the share of deposits D_i is given by competition between banks. Banks implicitly set their level of liquidity reserves by choosing how much to lend when setting the lending rate given the level of deposits. The amount of reserves R_i, the share of deposits D_i and the amount of lending L_i are linked by the balance sheet identity given in Eq. (7).

The Digital PSP: The digital PSP is a private entity that differs from banks along three dimensions. First, it operates online, without any bank branch. Therefore, consumers can reach it without incurring any transportation costs. Second, it does not distribute credit to consumers and keeps all the consumers' funds as reserves. This implies that the digital PSP does not incur any additional cost of liquidity when consumers pay from their digital wallet. Third, it offers consumers different storage benefits and convenience payment benefits. The price of payments by digital currency is set at its marginal cost, that is, $f_d = c_d$. This assumption simplifies the model, and could result either from perfect competition between payment service providers, or from the regulation of the transaction fee for the digital currency.[28] I denote by $\Delta c = c_b - c_d$.

In the regulatory framework that I consider, the digital PSP is allowed to store all the deposits of its consumers in a central bank account, remunerated at the interest rate on reserves in digital currency τ_d (IOR-DC).[29] The case in which the digital currency provider does not have access to central bank accounts is discussed in section 5.2.2.[30]

Assumptions: I make the following assumptions:

(A0) $u_b \geq c_d + \frac{3t_b}{2n} + \frac{\varphi\rho}{2}$.

Assumption (A0) implies that all consumers prefer to open a bank account even if they do not leave any money in it.[31]

(A1) $u_d \geq c_d$.

Assumption (A1) implies that all consumers prefer to open both a bank account and a digital wallet.

(A2) $\alpha > Ln$.

Assumption (A2) ensures that banks hold a positive amount of reserves in equilibrium.

[28] For example, in 2021 the Bank of Russia's director of financial technologies department said the fees for Russia's central bank digital currency (CBDC) transactions will be lower than those of wire transfers (source: coindesk.com). A November 2021 report by the Monetary Authority of Singapore on retail CBDCs mentions the example of Whatsapp which charges a 4 percent merchant fee in Brazil and acknowledges "that it may be difficult to compare fees charged by current and emerging payment services providers on a like-for-like basis" because of bundling with other services.

[29] As discussed by Cukierman (2020), allowing a central bank to compete with banks both in the credit and in the deposit market would be politically challenging, though this proposal has been discussed before (see Benes and Kumhof, 2012). In China, Alipay and Wechat are obliged by the People's Bank of China to hold a 100% reserve ratio on their assets. The designers of the Diem project of a stablecoin distributed by Facebook (abandoned later) argue that 80% of Diem reserves are invested in short-term treasury bonds.

[30] Adrian and Mancini-Griffoli (2021) argue that e-money providers could have access to central bank reserves under strict conditions. In that case, consumers transact and hold a synthetic CBDC. Several central banks have recently adopted reforms which allow non-banks to access the Central Bank payment system (in China, Hong-Kong, Switzerland, United Kingdom).

[31] In the online Appendix (O-7) of the paper, I demonstrate that such a condition is sufficient at the equilibrium of the game for all consumers to open a bank account.

(A3) $k_b + r_d \geq \varphi\rho$.

Assumption (A3) is a necessary condition for the second-order conditions to hold when a bank maximizes its profit. The second-order conditions hold if k_b is sufficiently large and are given in the Appendix.

(A4) $c_d \leq c_b + k_b$.

Assumption (A4) ensures that some consumers pay from their digital wallet.

Timing: The timing of the game is as follows:

0. At date 0, each bank $i \in \{1..n\}$ decides on the deposit fee F_i, the price of bank transfers f_b^i and the price of loans r_L^i.

1. At date 1, consumers choose in which bank to deposit money. They split their funds between their bank account and their digital wallet, respectively. Banks lend to borrowers.

1.5. Between date 1 and date 2, consumption shocks are revealed and depositors decide how to pay for their expenses. With probability φ, each bank i initiates a transfer of deposits to another bank if its depositors pay from their bank accounts.

2. At date 2, the depositors receive interest rate payments from their bank and from the digital PSP. Banks incur the cost of liquidity or receive the interest rate on overnight deposits. All banks receive the transfers of deposits. Loans mature and are paid off.

In the following section, I look for the subgame perfect equilibrium and solve the game by backward induction. All the variables of the model are summarized in Appendix A. All proofs are in the Appendix and the online Appendix.

4 Bank competition with a digital PSP

In this section, I study whether consumers have incentives to pay with a digital currency given the interest rates on deposits and the number of banks that compete for deposits.

4.1 The choice of a payment instrument

Between date 1 and date 2, each consumer of bank i chooses whether to pay using cash, digital currency or bank transfer. A consumer is indifferent between paying by digital currency and by a transfer of bank deposits if and only if he obtains the same net benefit of using either payment method given the transaction size, or else

$$b_d(s^i) = b_b(s^i),$$

where b_d and b_b are given in Eq. (3) and (4), respectively.

In Lemma 1, I give the threshold value of the consumption shock that maximizes the consumer surplus at the payments stage. For this purpose, I define

$$\underline{f_b} \equiv c_d \frac{v_b - r_b}{v_d - r_d}$$

and

$$\overline{f_b} \equiv c_d + \Delta v - \Delta r,$$

the minimum and the maximum values of the bank transfer fee such that consumers trade off between the three payment instruments. If the bank transfer fee is lower than $\underline{f_b}$, consumers always prefer to pay by a transfer of bank deposits. This happens if the interest rate for digital wallets is sufficiently large relative to the interest rate on bank deposits or if the marginal cost of the digital PSP is relatively large. If the bank transfer fee is higher than $\overline{f_b}$, consumers always prefer to pay with digital currency instead of transferring bank deposits. This happens if the marginal cost of the digital PSP is very low and if the bank offers a much higher interest rate on deposits than the digital PSP.

Lemma 1 *Let $\hat{s}_d \equiv c_d/(v_d - r_d)$, $\hat{s}_b \equiv f_b^i/(v_b - r_b)$ and $\hat{s}_{dc} \equiv (f_b^i - c_d)/(\Delta v - \Delta r)$.*
If $f_b^i \in (\underline{f_b}, \overline{f_b})$, the consumer pays in cash if $s^i \leq \hat{s}_d$, with digital currency if $s^i \in (\hat{s}_d, \hat{s}_{dc})$ and by bank transfer if $s^i \geq \hat{s}_{dc}$. The share of payments by bank transfer is $\beta_b^i = 1 - \hat{s}_{dc}$ and the share of payments with digital currency is $\beta_d^i = \hat{s}_{dc} - \hat{s}_d$.
If $f_b^i < \underline{f_b}$, the consumer pays in cash if $s^i < \hat{s}_b$ and by bank transfer if $s^i \geq \hat{s}_b$. The share of payments by bank transfer is $\beta_b^i = 1 - \hat{s}_b$ and the share of payments with digital currency is $\beta_d^i = 0$.
If $f_b^i > \overline{f_b}$, the consumer pays in cash if $s^i < \hat{s}_d$ and with digital currency if $s^i > \hat{s}_d$. The share of payments by bank transfer is $\beta_b^i = 0$ and the share of payments with digital currency is $\beta_d^i = 1 - \hat{s}_d$.

Proof. See Appendix B-1. ■

If bank i increases the fee for bank transfers (compared to the fee for the digital currency), the market share of the digital currency for consumers of bank i increases. The same outcome occurs if the interest rate revenues from the bank account increase with respect to the digital wallet. This implies that consumers prefer to use their bank account as a means of storing value and their digital wallet as a means of initiating payments.

Figure 1 below illustrates the adoption of the digital currency (i.e., the threshold s_{dc}) as a function of the bank transfer fee f_b for the following values of the parameters: $v_b = 4$, $v_d = 2.5$, $r_b = 1$, $c_d = 0.5$, and thus, $\underline{f_b} = 0.75$ and $\overline{f_b} = 2.5$. The solid lines delimit the three different regions of use of each payment instrument (cash, digital currency and bank transfer, respectively) if $r_d = 0.5$. The dashed lines also illustrate Lemma 1 for the same values of the parameters except that the interest rate for the digital currency is set to zero (i.e., $r_d = 0$). We see that a lower interest rate for the digital currency implies a higher adoption of the digital currency for payments, which is a substitute for both card and cash payments.

4.2 Competition for deposits

At date 1, prior to making transactions, consumers have to choose a bank in which to open an account. A consumer's expected surplus of opening an account in bank i is the sum of the utilities u_b and u_d, from which are subtracted the expected transaction costs TC_i, the interest rates on deposits IR_b^i and IR_d^i, the fixed deposit fee F_i, and the transportation costs. Therefore, the total expected surplus of a consumer is given by:

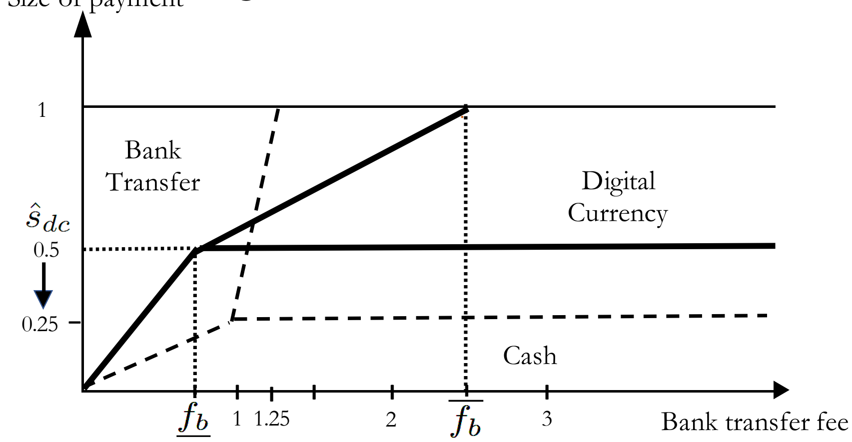
$$CS_i = u_b + u_d + IR_b^i + IR_d^i - TC_i - F_i, \tag{10}$$

where the expected transaction costs are

$$TC_i = f_b^i \beta_b^i + c_d \beta_d^i, \tag{11}$$

and the expected revenues from interest rates IR_b^i and IR_d^i are given in Appendix B, respectively.

Figure 1: Illustration of Lemma 1



Banks compete on the total surplus that they are able to deliver to their consumers. Thus, each bank i obtains a share of deposits given by

$$D_i = \frac{1}{n} + \frac{CS_i}{t_b} - \frac{(CS_k + CS_l)}{2t_b}. \tag{12}$$

4.3 The profit-maximizing prices

At date 0, banks compete for loans and deposits. In this section, I analyze how the presence of a digital currency impacts banks' prices for loans, deposits, and payment transactions.

4.3.1 A benchmark: No digital PSP

If there is no digital PSP, all consumers deposit all their funds in their bank account ($\alpha = 1$). Consumers trade off between paying in cash and by bank transfer. From the perspective of consumers, cash is equivalent to a free digital currency ($c_d = 0$) that brings neither storage benefits ($r_d = 0$) nor any additional service with respect to bank transfers ($v_d = 0$). Therefore, the benchmark setting in which the digital currency is not available can be deduced from the general analysis of competition between banks and the digital currency provider.

4.3.2 Competition with a digital PSP

If consumers trade off between payments with cash, digital currency, and bank transfer, each bank i chooses the fixed fee for deposits F_i, the fee for bank transfers $f_b^i \in (\underline{f_b}, \overline{f_b})$ and the interest rate on loans r_L^i that maximize its expected profit π_i given by Eq. (9) for $i = 1..n$, subject to the balance sheet identity given by Eq. (7) and to the constraint that the bank is able to fund term loans after depositors' withdrawals at date $t = 1.5$. I focus on symmetric best-responses to the fee c_d for digital currency payments and denote the vector of profit-maximizing prices by $P^* = (r_L^*, F^*, f_b^*)$. In a symmetric equilibrium, each bank i obtains an amount of deposits given by $D^* = \alpha/n$, lends $L^* = L$ and keeps a quantity $R^* = \alpha/n - L$ of reserves in a central bank account for $i = 1..n$.

From Lemma 1, in equilibrium, the optimal threshold transaction size s_{dc}^* such that consumers pay by bank transfer is given by

$$s_{dc}^* \equiv \hat{s}_{dc}(f_b^*, c_d) = \frac{f_b^* - c_d}{\Delta v - \Delta r}. \tag{13}$$

Consumers' decision to pay by digital currency may impact banks' margin and the marginal cost of liquidity. Thus, in equilibrium, the vector of profit-maximizing prices $P^* = (F^*, f_b^*, r_L^*)$ depends on the design of the digital currency (i.e., the transaction fee c_d and the interest rate on digital currency accounts r_d).

- **No costs of liquidity:**

Proposition 1 gives the profit-maximizing prices in a symmetric equilibrium if there are neither costs nor benefits of holding liquidity (i.e., if $\rho = \tau = 0$).

Proposition 1 *Each bank i chooses an interest rate on loans given by*

$$r_L^* = \frac{nL}{\gamma(n-1)}. \tag{14}$$

If $\Delta c \leq r_d$, the threshold transaction value s_{dc}^* such that consumers pay by bank transfer is given by

$$s_{dc}^* = \frac{\Delta c + k_b}{r_d + k_b}. \tag{15}$$

If $\Delta c > r_d$, banks set a transaction fee for bank transfers such that consumers pay for all their expenses with digital currency, that is, we have $f_b^* = \overline{f_b} = c_d + \Delta v - \Delta r$.
Each bank i charges a deposit fee F^* such that it makes a margin $\mu^* = t_b/n$. In equilibrium, each bank i makes a profit given by

$$\pi^* = \frac{t_b}{n^2} + \frac{nL^2}{\gamma(n-1)}. \tag{16}$$

Proof. See Appendix D-1. ■

Each bank i chooses an interest rate on loans given by the sum of its marginal cost of lending (which is null) and a mark-up that depends on the number of banks and the degree of substituability between bank loans. Hence, if there are neither costs nor benefits of holding liquidity, the design of the digital currency has no impact on the equilibrium lending rate, because banks' cost function is separable in the lending rate and the fee for payments by bank transfer. Andolfatto (2021) obtains the same result in a monopolistic environment: if banks lend to and borrow from the central bank at the same rate, the digital currency does not affect the lending rate.

Banks choose the bank transfer fee such that their marginal benefits are equal to their marginal costs of processing payments and extract the consumer surplus of storing money and making payments through the fixed deposit fee. The marginal benefit of increasing the bank transfer fee is $C_b'(1 - s_{dc}^*) = c_b + k_b(1 - s_{dc}^*)$, because the bank saves the marginal cost of bank transfers when consumers pay from their digital wallet instead of their deposit account. The marginal cost of increasing the bank transfer fee is the reduction of the surplus that the bank extracts from consumers when they pay more from their digital wallet by $c_d + r_d s^*_{dc}$. Indeed, consumers incur the cost c_d of paying with digital currency and forego an amount $r_d s^*_{dc}$ of interest rates revenues from their digital wallet. Therefore, banks choose s^*_{dc} such that

$$c_b + k_b(1 - s^*_{dc}) = c_d + r_d s^*_{dc},$$

which gives the result of Eq.(15), with $\Delta c = c_b - c_d$.

It is noteworthy that banks' profits only depend on the fee for bank transfers (resp., the interest rate on deposits) via the threshold transaction value $\hat{s}_{dc}$ (see Lemma 1). Therefore, the optimum can be reached with a different combination of the bank transfer fee and the deposit rate. Indeed, banks are indifferent between any combination of the fee f_b and the deposit rate r_b such that consumers pay with a transfer of bank deposits when the transaction size exceeds s^*_{dc}.[32]

Competition in two-part tariffs implies that banks choose the deposit fee so as to make a constant margin $\mu^* = t_b/n$ in equilibrium. Because banks trade off between the profits from deposits and loans, since the deposit market is covered, there exists several combinations of bank transfer fees, fixed deposit fees and interest rates on loans that yield the same profit for banks given in Eq. (16).

- **Costly liquidity:**

If liquidity is costly, banks' expected net benefit of liquidity management EL_i may depend on the depositors' trade-off between paying by bank transfer or with digital currency.[33] Proposition 2 establishes a link between banks' profit-maximizing prices and the marginal cost of liquidity. I use this result in the following subsections to characterize banks' profit-maximizing prices $P^* = (r^*_L, F^*, f^*_b)$ according to their equilibrium amount of reserves.

Proposition 2 *In equilibrium, each bank i chooses an interest rate on loans given by*

$$r^*_L = \frac{nL}{\gamma(n-1)} - \left.\frac{\partial EL_i}{\partial L_i}\right|_{P^*}, \tag{17}$$

makes a margin per depositor given by

$$\mu^* = \frac{t_b}{n} - \left.\frac{\partial EL_i}{\partial D_i}\right|_{P^*}, \tag{18}$$

and charges a fee for deposits given by

$$F^* = \frac{t_b}{n} - \left.\frac{\partial EL_i}{\partial D_i}\right|_{P^*} - (1 - s^*_{dc})f^*_b + IR^i_b(f^*_b, c_d) + C_b(1 - s^*_{dc}). \tag{19}$$

If consumers use the three payment instruments to pay, banks choose the transaction size s^*_{dc} such that:

$$c_b + k_b(1 - s^*_{dc}) + \left.\frac{\partial EL_i}{\partial \hat{s}_{dc}}\right|_{P^*} = c_d + r_d s^*_{dc}. \tag{20}$$

[32] This can be seen by noting that bank i's margin given in Eq.(26) of Appendix D-1 and the amount of reserves of bank i respectively depend on r_b and f^i_b via the minimum transaction size $\hat{s}_{dc}$.

[33] As can be seen from Appendix C, the banks' expected marginal benefit of liquidity is defined by parts according to the quantity of reserves held in a central bank account. First, banks may choose to hold a low amount of reserves that never meets consumer demand for payments from their bank account. In that case, banks' profit-maximizing quantity of reserves does not cover the minimum share of deposits that is transferred by consumers from their bank account. Second, banks may hold enough reserves to meet some but not all consumer demand for payments by bank transfer.

Proof. See Appendix D-1. ∎

In a symmetric equilibrium, the lending rate is the sum of the interest rate found in Proposition 1 when there are no costs of liquidity, from which the marginal benefit of liquidity is subtracted. If the benefit of liquidity EL_i for banks is not separable in the bank transfer fee f_b^i and the lending rate r_L^i, the depositors' trade-off between paying by bank transfer and with digital currency impacts banks' marginal cost of liquidity.[34]

As in Proposition 1, banks choose the bank transfer fee such that their marginal benefits are equal to their marginal costs, which now include the marginal benefits and costs of liquidity. The latter are detailed in the following subsections according to the amount of reserves held by banks.

A higher quantity of bank deposits increases the marginal benefit of liquidity. Thus, competition for depositors becomes more intense when the marginal benefit of liquidity increases, which implies that banks' margin per depositor is decreasing with the marginal benefit of liquidity (see Eq. (18)). If consumers substitute deposits in their digital wallet for bank deposits, the marginal benefit of liquidity becomes less sensitive to the quantity of deposits attracted by the bank. This implies that banks' margin per depositor increases. Thus, the presence of the digital PSP softens bank competition for depositors.

4.3.3 The use of the digital currency if banks hold a low amount of reserves

Suppose that each bank i holds a low amount of additional reserves that never meets consumer demand for bank transfers. In that case, each bank i's benefit of liquidity management EL_i is not separable in the lending rate r_L^i and the fee for bank transfers f_b^i (See Appendix C, case 1). Proposition 3 gives the profit-maximizing prices in equilibrium if banks hold a low amount of additional reserves.

For this purpose, I define the bank transfer fee f_b^R such that the size of the transaction above which consumers pay from their bank account equals the average amount of reserves per unit of deposits (in quantity $1/n$), that is, $\hat{s}_{dc}(f_b^R, c_d) = nR^*$. I also define the bank transfer fee f_b^D such that consumers always pay with digital currency when they have enough funds in their digital currency account, that is, $\hat{s}_{dc}(f_b^D, c_d) = 1 - \alpha$.[35]

Proposition 3 *Assume that banks never hold enough excess reserves to meet consumer demand for payments. Each bank i chooses an interest rate on loans given by*

$$r_L^* = \frac{nL}{\gamma(n-1)} + \varphi(\rho - (\rho - \tau)s_{dc}^*) + (1-\varphi)\tau,$$

makes a margin per depositor given by

$$\mu^* = \frac{t_b}{n} - \varphi\left(\tau\alpha s_{dc}^* + \rho\frac{(s_{dc}^* - \alpha)^2}{2}\right) - (1-\varphi)\alpha\tau,$$

and charges a deposit fee given by $F^* = \mu^* - (1 - s_{dc}^*)f_b^* + IR_b^i(f_b^*, c_d) + C_b(1 - s_{dc}^*)$.
There exists $c_d^{\min}$ *and* $c_d^{\max}$ *such that if* $c_d \in (c_d^{\min}, c_d^{\max})$*, each bank i sets the bank transfer fee* f_b^* *so that consumers pay from their bank account if the transaction size exceeds:*

$$s_{dc}^* = \frac{\Delta c + k_b - \varphi(\rho - \tau)(\alpha - nL)}{k_b + r_d - \varphi\rho}.$$

[34]A standard result in microeconomics of banking is that if the bank's cost function is separable in the amount of deposits and the amount of loans, the prices in the lending market and in the deposit market are independent (see Monti, 1972 and Klein, 1971).

[35]Since I chose to focus on determining an equilibrium in which $\hat{s}_{dc} \leq 1 - \alpha$ (i.e., a digital currency for small retail payments), if $f_b^i \geq c_d$, Lemma 1 implies that $f_b^R = c_d + (\Delta v - \Delta r)nR^*$, and $f_b^D = c_d + (1-\alpha)(\Delta v - \Delta r)$.

If $c_d \geq c_d^{\max}$, each bank i sets $f_b^* = f_b^R$ and $s_{dc}^* = nR^*$.
If $c_d \leq c_d^{\min}$, each bank i chooses $f_b^* = f_b^D$. Consumers always pay by digital currency when they have money in their digital currency account, that is, $s_{dc}^* = 1 - \alpha$.

Proof. See Appendix D-2 and Proposition 2. Substituting $\partial EL_i/\partial L_i$ given in Appendix D-2 into Eq. (18) of Proposition 2 and $\partial EL_i/\partial \hat{s}_{dc}$ given in Appendix D-2 into Eq. (20) of Proposition 2 gives the profit-maximizing prices chosen by banks in a symmetric equilibrium. ■

If banks incur additional costs of liquidity when consumers pay by bank transfer, the interest rate on loans depends on consumers' payment decisions. When consumers transfer funds less frequently from their bank account to another bank (i.e., the threshold s_{dc}^* increases or if φ decreases), banks' marginal cost of liquidity is reduced if $\rho > \tau$, and, therefore, the interest rate on loans becomes lower.[36] As the total demand for loans is constant, banks reduce the lending rate according to their expected liquidity needs, such that the volume of lending does not vary in equilibrium.[37]

The choice of the bank transfer fee reflects banks' marginal benefits and costs of liquidity. A higher bank transfer fee has two opposite effects on banks' marginal benefit of liquidity. First, if $\rho > \tau$, as consumers transfer funds less frequently from their bank account to make a payment, the marginal benefit of liquidity is reduced. The intensity of this first effect increases with the amount of reserves R^* held by banks. As the amount of reserves becomes lower when the digital currency crowds out a higher share of deposits, this effect tends to be smaller when the use of the digital currency as a means to store value increases. Second, when the bank transfer fee increases, consumers transfer a higher average amount from their bank account when they pay, which increases the marginal benefit of liquidity. The magnitude of this second effect is all the more important since the expected cost of liquidity $\varphi\rho$ is high and the number of banks is small. Indeed, the marginal benefit of liquidity is higher when banks attract a higher share of deposits. Therefore, banks have lower incentives to compete with the digital currency as a means of payment when the use of the digital currency as a store of value decreases.

The design of the digital currency impacts banks' trade-off between extracting surplus from depositors and reducing their cost of liquidity. If the fee for the digital currency is low enough ($c_d \leq c_d^{\min}$), the second effect dominates the first. Indeed, a higher bank transfer fee increases the bank's marginal benefit of liquidity. Thus, banks have incentives to set a high bank transfer fee such that consumers pay for all their transactions with digital currency when they have enough money in their digital currency account. If the fee for the digital currency is high enough ($c_d \geq c_d^{\max}$), the first effect is dominant. Banks set a low bank transfer fee such that consumers pay as much as possible from their bank account, given the constraint on reserves. For intermediary values of the fee for the digital currency, banks set a bank transfer fee such that consumers pay some transactions with digital currency and others by bank transfer.[38] The results of Proposition 3 can be easily extended to the case in which only a fraction of receivers accepts the digital currency.[39]

My framework shows that if the central bank's objective is to foster the adoption of digital currencies, it is necessary to apply coherent transaction fees given banks' marginal cost of liquidity and given competition between banks for deposits. When digital currencies are offered by central banks, this results echoes the analysis of the report written by the European Central Bank (2020-b). In-

[36] In equilibrium, the interest rate on loans is higher than the IOR. The lending market is more profitable for banks than investment in central bank reserves.

[37] With a model of Cournot competition in the lending market, the amount of lending would vary in equilibrium.

[38] The model does not say whether banks prefer to hold a high or a low amount of reserves in a symmetric equilibrium. The choice between case i) and case ii) in Proposition 2 could be driven by regulatory requirements.

[39] In that case, banks compete with the digital currency provider only with probability $\sigma_d > 0$. Thus, r_d and c_d are multiplied by σ_d in the equation that defines the threshold value of the consumption shock such that consumers pay by bank transfer. See online Appendix O-5.

deed, to issue a digital euro, the central bank may need to offer long-term lending to banks that lose deposits (e.g., via Long Term Refinancing Operations, LTROs). As a consequence, the differential between the remuneration of the digital euro and the interest rate applied to LTROs may impact the use of the digital euro.

A numerical illustration:

Policymakers may target a market share for digital currency payments if they are able to control the interest rate on digital wallets and the fee for payments with digital currency. For example, with the parameters $\alpha = 0.8$, $L = 0.1$, $\varphi = 0.5$, $c_b = 0.1$, $k_b = 0.1$, from Proposition 1, if there are no costs of liquidity, a market share of 20 percent for the digital currency is reached by combining a transaction fee and an interest rate for the digital wallet such that: $c_d = 0.18 - 0.2r_d$. For instance, if $r_d = 0.1$, this gives a fee equal to $c_d = 0.16$. If there are some costs and benefits of liquidity, with $\rho = \tau = 0.1$, the equation becomes: $c_d = 0.19 - 0.2r_d$. Then, if $r_d = 0.1$, the transaction fee should be equal to $c_d = 0.17$ to reach a market share of 20 percent. Otherwise, if c_d stays at 0.16, the market share of the digital currency will exceed 20 percent, because banks incur some costs and benefits of managing their liquidity needs. If $\rho = 0.1$, $\tau = 0$ and $n = 2$, the equation becomes $c_d = 0.187 - 0.2r_d$. Thus, if $r_d = 0.1$, then reaching a market share of 20 percent requires setting $c_d = 0.167$, and even a higher transaction fee c_d if the number of banks is strictly higher than $n = 2$.

Competition and the adoption of the digital currency: Even if there are positive transaction fees for the digital currency, consumers may decide to adopt it for payments because it is not a perfect substitute for cash transactions. In Corollary 1, I discuss how the market conditions (IOR, number of banks, cost of liquidity for banks) impact the share of payments made with digital currency when consumers trade off between the three payment instruments.

Corollary 1 *If banks hold a low amount of excess reserves, the market share of the digital currency for payments increases with the number of banks n, the IOR τ, the cost of liquidity ρ, and decreases with the interest rate on digital currency account r_d, and the share of deposits left in bank account α.*

Proof. See Appendix D-2 ∎

When the number of banks increases, each bank captures a lower share of deposits and holds a lower amount of reserves in equilibrium. Since each bank lends a constant amount (i.e., L), the probability that a bank needs to borrow additional liquidity when a consumer makes a transaction of high value increases with the number of banks. Hence, all else being equal, the bank transfer fee becomes higher when the number of banks increases, and therefore the market share of the digital currency increases.

A higher cost of liquidity for banks has an ambiguous impact on the market share of the digital currency. This is because it both increases the two opposite impacts of a higher bank transfer fee on banks' marginal benefit of liquidity. On the one hand, a higher cost of liquidity raises the marginal benefit of holding reserves. Thus, banks have higher incentives to increase the bank transfer fee, such that consumers use their digital wallet more often for payments. On the other hand, a higher cost of liquidity increases the cost incurred by banks when consumers pay from their bank account, because the optimal threshold for bank transfers increases. This provides banks with the countervailing incentive to decrease the bank transfer fee. The first effect is dominant when the market share of the digital currency is positive, which implies that the market share of the digital currency is increasing with banks' cost of liquidity.

A higher adoption of the digital currency as a store of value reduces its use as a means of payments when the interest rate on digital wallets increases. Depending on the design of the digital currency, the digital currency may be used as a store of value but not as a means of payment and vice versa. As argued by Brunnermeier, James and Landau (2019), the digital revolution may lead to an unbundling of the separate roles of money as a means of exchange and store of value. When switching costs are low, consumers may no longer have a strong incentive to use a single currency as both a store of value, medium of exchange, and unit of account.[40]

4.3.4 The use of digital currency if banks hold a high amount of reserves

If banks hold enough reserves to cover some but not all deposit transfers, the size of the transaction such that consumers pay from their bank account does not impact banks' management of reserves. The banks' expected net benefit of liquidity management is now separable in the lending rate and in the fee for bank transfers. Therefore, unlike in Proposition 3, the fee for digital currency transactions has no impact on the interest rate on loans. Below, I give the intuition of the results and refer the reader to the online Appendix for details of the calculations.[41]

As in Proposition 3, when they choose the bank transfer fee, banks trade off the benefits from lower costs of liquidity and the losses due to the reduction in consumer surplus of opening a digital currency account. However, a higher bank transfer fee always increases the marginal benefit of liquidity. In this case, banks have no incentives to decrease the bank transfer fee to reduce their marginal cost of liquidity, because the substitution between payment instruments has no impact on their management of reserves.

If the fee for the digital currency is high or if the interest rate on the digital currency account is high enough, banks set a low bank transfer fee such that consumers do not use digital currency to pay. This case is all the more likely to happen since the interest rate on the digital wallet is high with respect to the expected return offered by the central bank on bank deposits (i.e., if r_d is high with respect to τ). If the fee for digital currency transactions is low enough, banks may choose a bank transfer fee such that consumers always pay with digital currency when they have enough money in their digital currency account. For intermediary values of the fee for the digital currency, consumers trade off between making payments using bank transfer, cash and digital currency.

4.3.5 The impact of digital currency on loan interest rates

In Corollary 2, I analyze whether the presence of the digital currency impacts the interest on loans when consumers trade off between the three payment instruments.

Corollary 2 *Assume that banks hold a low amount of reserves before and after the introduction of the digital currency. If consumers make a lower use of their bank account for payments following the introduction of digital currency, the interest rate on loans is reduced, whereas the reverse is true otherwise.*

Proof. See Appendix E. ■

Since banks pass-through the cost of liquidity to their borrowers via higher lending rates, a lower use of the bank account for payments reduces banks' liquidity needs, and therefore reduces the lending rate.

If banks hold a low amount of reserves, the digital currency may either increase or decrease the use of bank deposits for payments. Without a digital currency, the use of bank deposits for payments depends on competition between bank transfers and cash. Since the use of cash is costless, the size

[40] This is indeed the case in my paper because value can be transferred instantly and costlessly from the bank account to the digital currency account.

[41] See online Appendix O-1.

of the transaction such that consumers pay from their bank account depends on the bank transfer fee, the cost of liquidity, and the benefits from reserves. The presence of digital currency implies two differences with respect to cash. First, transactions in digital currency are costly. Second, consumers incur an opportunity cost of paying with digital currency if $r_d > 0$. As a result, banks choose a different minimum transaction value for bank transfers (i.e., s^*_{dc}) when they compete with the digital PSP instead of cash. This choice is determined by the design of the digital currency.

4.4 A special case: A zero interest-bearing digital currency

In this subsection, I use the results of the previous part to analyze the special design of a digital currency with a zero interest rate on the digital wallet. From a policy perspective, several central banks are considering this option for a CBDC (e.g., the PBoC).

4.4.1 The crowding-out of bank deposits for payments

In Corollary 3, I analyze whether a zero interest-bearing digital currency crowds out the use of bank deposits for payments if the interest rate on digital currency accounts is set to $r_d = 0$ and if $c_b = 0$.

Corollary 3 *If banks hold a low amount of reserves, the digital currency crowds out the use of bank deposits for payments if* $c_d \leq \max(0, (\tau - \rho)n\varphi R^* + \rho\varphi(1-\alpha) + k_b\alpha)$.
If banks hold a high amount of reserves, the digital currency crowds out the use of bank deposits for payments if $c_d \leq (v_d - r_d)k_b/(\varphi\tau + k_b - v_d)$.

Proof. From Proposition 3, Appendix D-2-A and D-2-B. ■

One policy implication of this result is that, even if there are no interest rates on digital currency accounts, digital currency may not always crowd out the use of bank deposits for payments. This happens only if the marginal cost of digital currency payments is sufficiently low with respect to banks' marginal net costs of liquidity and the marginal cost of bank transfers.

Depending on the value of the parameters, there are also equilibria in which the digital currency is not adopted by consumers for payments. This may happen, for instance, if the marginal benefit of liquidity τ is low and if the marginal cost of digital currency transactions is high with respect to the marginal cost of payments by bank transfer.

4.4.2 The use of bank deposits for payments with and without a digital currency

In Corollary 4, I analyze whether digital currency reduces the use of bank deposits for payments, compared to the benchmark in which consumers trade off between paying in cash and by bank transfer.

Corollary 4 *Assume that* $c_d \in (c_d^{\min}, c_d^{\max})$ *and that banks hold a low amount of reserves. If* $c_b = r_d = 0$*, the digital currency reduces the use of bank deposits for payments if and only if*

$$c_d \leq \varphi(1-\alpha)(\rho - \tau).$$

Proof. From Proposition 3, we have

$$s^*_{dc} - s^*_b|_{\alpha=1} = \frac{\varphi(\rho-\tau)(1-\alpha) - c_d}{k_b - \varphi\rho},$$

where s^*_b is obtained by setting $r_d = c_d = 0$ in Proposition 3. Since $k_b > \varphi\rho$ from (A3) and $\rho \geq \tau$, we have $s^*_{dc} - s^*_b|_{\alpha=1} \geq 0$ iff $c_d \leq \varphi(1-\alpha)(\rho - \tau)$. ■

The use of bank deposits for payments is reduced with a digital currency if the marginal cost of the digital currency is lower than the marginal cost of liquidity generated by the variation of the equilibrium amount of reserves for banks.

5 Discussion on critical assumptions and extensions

In this section, I discuss alternative modeling possibilities and extensions of the model.

5.1 The behavior of depositors

Endogenous deposit decisions: So far, to simplify the analysis, I have considered a fixed exogenous amount of deposits ($d = 1$). However, the volume of deposits may be elastic to payment instrument prices. In that case, a higher bank transfer fee may imply that consumers substitute deposits in digital currency for bank deposits, because consumers take into account the cost of paying from their bank account in their deposit decisions. This increases the banks' cost of liquidity. Therefore, banks have incentives to choose a lower fee for bank transfers than in the model of section 4, in order to reduce their costs of adjusting their liquidity needs (s^*_{dc} increases).

To illustrate this result, suppose that a consumer decides to keep in his digital wallet the maximum amount that he may need to pay (that is, s^*_{dc}). With the numerical parameters used in section 4 ($\alpha = 0.8$, $L = 0.1$, $\varphi = 0.5$, $\rho = 0.1$, $\tau = 0$, $c_b = 0.1$, $k_b = 0.1$ and $r_d = 0.1$), a market share of 20 percent is reached for the digital currency with a lower value for the transaction fee c_d ($c_d = 0.14 < 0.167$) when the volume of deposits is elastic to transaction prices.[42] Since banks lower the bank transfer fee compared to the benchmark, the transaction fee for payments by digital currency needs to be lower to reach a market share of 20 percent.

The volume of deposits could also be elastic to the interest rates on bank accounts and digital currency accounts. In addition, the way consumers share their wealth between deposits and cash could be endogenous. Suppose, for instance, that bank accounts, digital currency accounts, and cash offer liquidity services which are imperfect substitutes. Then, the total quantity of deposits increases both with the interest rate on bank accounts and digital wallets. Moreover, higher opportunity costs of holding cash reduce the size of the transaction such that consumers pay with digital currency instead of paying in cash.

Differentiation between payment instruments: The outcome of competition between banks and the PSP depends on the modeling of the differentiation of their money storage and payments services, and their differentiation with respect to cash. Alternative models could be considered. Other dimensions of differentiation between deposit services include safety (Fernandez-Villaverde et al. 2022) and privacy considerations (Agur et al., 2022). These other aspects do not modify the mechanism of the model, as long as the volume of deposits is not elastic to payment prices. If the volume of deposits is elastic to payment prices, the additional effects described in the previous paragraph arise

To conclude this discussion, empirical research measuring how the volumes of bank deposits react to the various parameters of competition (safety, interest rates, payment instrument pricing) is needed more than ever to motivate theoretical research. Estimating whether the demand for deposits is elastic to payment instrument pricing is empirically challenging. A discussion of this issue can

[42] See online Appendix O-3 for the details of the calculations. The result of the numerical application with $c_d = 0.14$ is obtained by replacing the parameters in Eq. (**??**).

be found in Bolt et al. (2008), and in the more recent study on the instant payment system Pix in Sarkisyan (2023).

Is it essential to open a bank account to use a digital wallet? In the baseline model, I assume that the fixed value of opening a bank account is sufficiently high, such that all consumers prefer to open a bank account in equilibrium. Thus, no consumer renounces using a bank account only to open a digital wallet, and consumers prefer to open a bank account even if they do not leave any money in it. This assumption corresponds to the situation of a majority of developed countries, where the proportion of unbanked consumers is low, and consumers need to have a bank account to pay their taxes.[43] This first approximation describes a transition phase in the competitive environment. Admitting new consumers is costly for digital PSPs, because the latter have to comply with the existing KYC regulations. In emerging countries, payment service providers sometimes require that consumers have a bank account to open a digital wallet (as in Brazil). In the long run, it is possible that some digital PSPs will serve consumers who do not have bank accounts. This does not impact the results of the model if the digital PSPs improve the financial inclusion of consumers. In that case, digital PSPs will allow unbanked consumers to access deposit services. However, the conclusions of the model may change if some consumers choose not to open a bank account in order to open a digital wallet. Though this issue is beyond the scope of this paper, one can anticipate that this would limit banks' ability to extract surplus from depositors through the fixed deposit fee, thereby impacting their trade-off between expanding their profits from deposits and from payments (intensive vs. extensive margins).

5.2 The design of the digital currency

5.2.1 Banks as distributors of digital currency

An alternative possibility for the regulator consists of allowing the banks themselves to manage the digital wallets in dedicated accounts. Banks could hold reserves in two separate accounts, one for the digital currency (reserves of type d remunerated at the IOR-dc τ_d) and one for standard bank accounts (reserves of type b remunerated at the IOR-b τ_b). Digital wallets are still backed by a ratio of 100 percent of reserves and the fee for digital currency payments is set at the marginal cost c_d.[44]

In Proposition 4, I compare the adoption of digital currency if banks distribute it themselves and hold a low amount of excess reserves (i.e., s^*_{db}) and if it is distributed by the digital PSP.

Proposition 4 *If $\tau_d > r_d$, should banks hold a low amount of excess reserves of type b, adopting the digital currency for payments is lower when banks distribute it themselves than if it is distributed by a digital PSP, because the threshold transaction size such that consumers pay from their digital wallets is lower:*

$$s^*_{db} = \frac{\Delta c + k_b - \varphi(\rho - \tau_b)(\alpha - nL)}{k_b + \tau_d - \varphi\rho} < s^*_{dc}.$$

Proof. See online Appendix O-4. ■

If banks themselves distribute the digital currency, they are able to internalize the impact of the transaction fee for the digital currency and the interest rates on digital currency accounts on their profit. Therefore, the effects of r_d and c_d on the choice of the bank transfer fee are cancelled.

[43] In several countries where non-banks offer payment services, a bank account is required to register for non-bank account services (e.g., for Alipay and Wechat in China for Chinese citizens. For foreigners, an international payment card is required).

[44] Shy and Stenbacka (2007) show that adding a policy instrument that would allow to the regulator to control the fraction of perfectly liquid accounts offered by banks would enhance social welfare.

However, banks take into account the impact of the consumer's choice of a payment method on their marginal benefit of liquidity, which differs for reserves of type b and reserves of type d.

The adoption of digital currency may be higher if it is distributed by banks rather than a digital PSP. The comparison of both designs depends on the pass-through of the IOR-dc to the depositors when the latter leave some funds in their digital wallet. If the pass-through is perfect, that is, if $\tau_d = r_d$, the adoption of digital currency is exactly identical if banks distribute the digital currency themselves (see Proposition 3). However, if $\tau_d > r_d$, banks make profits on reserves in digital currency, which gives them the incentive to lower the bank transfer fee. Hence, when banks themselves distribute the digital currency, this reduces its adoption for payments, compared to a situation in which the digital currency is priced at marginal cost and distributed by a digital PSP.

5.2.2 The access to central bank reserves

So far, I have assumed in the model that the digital currency provider is allowed to deposit reserves in a central bank account. However, the access to central bank accounts may be restricted to banks. In that case, the digital currency provider may prefer to open an account in a bank to store the depositors' money, if banks are allowed to be the custodians of digital currency funds. It can be assumed that banks compete à la Bertrand to attract the deposits of the digital PSP, and store them in a dedicated central bank account. Each bank is able to offer the digital currency provider a deposit rate equal to its marginal benefit of storing funds in digital currency, that is, the IOR-dc, and does not make any profit on deposits in digital currency. In equilibrium, the digital currency provider splits his deposits between all banks, such that each bank holds a quantity $(1-\alpha)/n$ of deposits in digital currency. Therefore, the adoption of digital currency is identical to Proposition 4 with $\tau_d = 0$. It follows that if $r_d > 0$, the adoption of digital currency for payments is higher if the digital currency provider does not have access to central bank accounts. The practicing of splitting deposits between different banks was frequent in China before 2017, when the PBoC allowed Big Tech companies (Alipay and Wechat) to settle transactions directly and open central bank accounts.

5.2.3 Marginal cost pricing of digital currency payments?

Several papers argue that the Central Bank could regulate the fee for digital currency transactions at marginal cost or set a zero transaction fee for payments in digital currency. However, this may not maximize the surplus of depositors. The fee for digital currency payments that maximizes the surplus of depositors is denoted by f_d^* and may differ from c_d.[45]

Suppose that depositor surplus is concave in the transaction fee for payments. Depositor surplus is maximized when the fee for digital currency payments is chosen ex ante by the regulator such that the marginal benefits of opening a bank account for the depositor are equal to the marginal cost:

$$nL(\tau-\rho)\left.\frac{d\hat{s}_{dc}}{df_d}\right|_{f_d^*} + f_d^* \left.\frac{d\beta_d^i}{df_d}\right|_{f_d^*} = \beta_d^i\big|_{f_d^*}. \tag{21}$$

The marginal costs of increasing the fee for digital currency payments correspond to the higher transactions costs of digital currency payments (see the right-hand side of (21)). The marginal benefits of increasing the fee for digital currency payments are caused by the decrease in the share of digital currency payments with respect to cash, and with respect to bank transfers.

Banks do not choose a fee for bank transfers that maximizes depositor surplus, because they price payment transactions according to the liquidity needs generated by the lending market. However,

[45] See online Appendix O-6.

in our setting, depositors only obtain surplus from money storage and payments.[46] It follows that, given banks compete for loans and deposits, the marginal cost pricing of digital currency payments does not maximize depositor surplus.[47]

5.3 Bank payment systems and the costs of liquidity

The model could also be enriched to discuss the role of new technologies used by banks for their payment systems such as instant payments. This technology reduces the delay between the initiation of a payment transaction and reception of the funds. If payments occur almost instantly between the issuer of the bank transfer and the receiver, each bank takes into account the probability of receiving funds from the other banks when it manages its liquidity risk. If banks are sufficiently differentiated in the market for deposits (i.e., if t_b is high enough), this does not affect the results of the model. The results change if competition for deposits becomes more intense. A bank's incentives to attract a depositor are reduced if a bank expects to economize on liquidity costs when it receives funds from a competing bank, which softens competition for depositors. The result that competition between compatible networks softens competition is standard in the literature (see Foros and Hansen, 2001, Massoud and Bernhardt, 2002, for the case of ATMs). We could therefore expect a higher fee for bank transfers than in the model of section 4, and therefore, a higher adoption of the digital currency for payments.

6 Conclusion

Whether consumers will use a digital currency to pay essentially depends on the cost of liquidity in existing payment systems, on the possibility for non-banks to obtain revenues from central bank accounts, and the degree of competition in the deposit market. My paper identifies the conditions such that consumers pay from digital wallets offered by PSPs in a market where banks compete for loans and deposits, while incurring costs of managing liquidity. I also discuss how the distribution mode of the digital currency may impact its market share.

More research would be needed to understand the welfare effects of digital currencies. From a theoretical perspective, it would be valuable to construct a framework that takes into account not only their impact on price stability, financial stability, and efficient risk-sharing in crisis times, but also efficient use of payment instruments when there is no specific stress on liquidity. From an empirical perspective, more reseach is needed to understand how the volume of bank deposits responds to competition with alternative providers of innovative payment solutions, and the elasticity of substitution between payment instruments.

[46] Banks partly internalize the substitution effect for payments by bank transfer when they choose the bank transfer fee. Indeed, the bank transfer fee is such that the marginal cost of card payments (including the marginal cost of liquidity) is equal to the marginal surplus that the bank extracts from depositors thanks to their digital currency account. The marginal cost of liquidity includes the impact of the substitution between payment methods both on the deposit and the lending market.

[47] The first term on the left-hand side of Eq. (21) corresponds to the marginal impact of a higher fee for digital currency transactions on the lending market. The second term of Eq. (21) corresponds to the marginal impact of the substitution between payments in cash and with digital currency on depositor surplus (that is not internalized by banks at the next stage). The right-hand side of Eq. (21) corresponds to the transaction cost effect.

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Appendix A: Summary of the variables used in the model

The table below summarizes the parameters used in the model:

	Exogenous variables:	Endogenous variables:
Consumers	v_b variable benefit of bank transfer	$1-\alpha$ wealth in DC account
	v_d variable benefit for DC	α wealth in bank account
	r_b interest rate on deposits	CS_i expected surplus of deposits
	r_d interest rate on DC	
	s^i transaction size	
	f_d payment fee for DC	
Banks/DC	ρ cost of liq. φ probability of transfer	f_b^i bank transfer fee
	τ IOR and τ_d IOR-DC	$f_d = c_d$ payment fee for DC
	c_b and k_b cost of card payments	F_i deposit fee
	γ degree of substituability for loans	r_L^i interest rate on loans
	c_d marginal cost of DC payments	D_i market share on the circle

Appendix B: Depositor choices (payment instruments and bank accounts)

Appendix B-1: Proof of Lemma 1 - payment instrument choices

- **Benchmark: Choice of payment instrument without a digital currency**

If the digital currency is not available, consumers leave all their deposits in their bank account (i.e., $\alpha = 1$). Consumers trade off between paying in cash and through a bank deposit transfer. They pay by bank transfer if they obtain a higher benefit of doing so, that is, if $s^i \geq \hat{s}_b \equiv f_b^i/(v_b - r_b)$. Otherwise, they pay in cash. If $f_b^i \leq 0$, consumers pay for all their expenses by bank transfer, whereas if $f_b^i \geq v_b - r_b$, consumers pay for all their expenses in cash. Consumers obtain the interest rates from deposits given by $IR_b^i = (r_b \hat{s}_b)/2$.

- **Competition with the digital currency: Cash/digital currency/bank transfer:**

Choice of payment instrument:
I focus on a scenario in which the digital currency is used for low value retail payments (i.e., $\hat{s}_{dc} \leq 1-\alpha \leq \alpha$). If the consumer trades off between paying from his bank account and his digital currency account, he obtains the same benefit of paying by bank transfer and with digital currency if and only if $b_b(\hat{s}_{dc})$ given by Eq. (4) is equal to $b_d(\hat{s}_{dc})$ given in Eq. (3). Therefore, we have $\hat{s}_{dc} = (f_b^i - c_d)/(\Delta v - \Delta r)$.

If the consumer trades off between the digital currency and cash, he obtains the same benefit of paying in cash and with digital currency if and only if $b_c(\hat{s}_d)$ given by Eq. (2) is equal to $b_d(\hat{s}_d)$ given in Eq. (3). Therefore, we have $\hat{s}_d = f_d/(v_d - r_d)$.

If the consumer trades off between paying by bank transfer and cash, he obtains the same benefit of paying by bank transfer and in cash if and only if $b_b(\hat{s}_b)$ given by Eq. (4) is equal to $b_c(\hat{s}_d)$ given by Eq. (2). Therefore, we have $\hat{s}_b = f_b^i/(v_b - r_b)$.

If $\hat{s}_{dc} \leq \hat{s}_d$, the digital currency is not used to pay. We have $\hat{s}_{dc} \leq \hat{s}_d$ iff

$$f_b^i \leq \underline{f_b} \equiv c_d(v_b - r_b)/(v_d - r_d).$$

If $\hat{s}_{dc} \geq 1$, the digital currency is always used to pay. We have $\hat{s}_{dc} \geq 1$ iff $f_b^i \geq \overline{f_b} \equiv c_d + \Delta v - \Delta r$. Thus, for consumers to trade off between cash, payments with digital currency and payments by bank transfer, it must be that $f_b^i \in (\underline{f_b}, \overline{f_b})$.

- Interest rate revenues from deposits

a) If $f_b^i \in (\underline{f_b}, \overline{f_b})$, we have $\hat{s}_d \leq \hat{s}_{dc}$. The consumer pays by bank transfer if $s^i \geq \hat{s}_{dc}$, with digital currency if s belongs to $(\hat{s}_d, \hat{s}_{dc})$ and in cash if $s^i \leq \hat{s}_d$. As $\hat{s}_{dc} \leq 1-\alpha \leq \alpha$, the interest rate revenues from the bank account are given by

$$IR_b^i = r_b(\alpha^2 - \int_{\hat{s}_{dc}}^{\alpha} sds), \tag{22}$$

and the interest rate revenues from the digital currency account are given by

$$IR_d^i = r_d(1 - \alpha^2 - \int_{s_d}^{\hat{s}_{dc}} sds - \int_{\alpha}^{1} sds). \tag{23}$$

To obtain both equations, recall that a consumer makes a transfer of value from one account to the other at no cost if he does not have enough funds to pay.

b) If $f_b^i < \underline{f_b}$, the consumer pays by bank transfer if $s^i \geq \hat{s}_b$ and in cash if $s^i < \hat{s}_b$. The consumer obtains the interest rate revenues from his bank account given by

$$IR_b^i = r_b(\alpha^2 - \int_{\hat{s}_b}^{\alpha} sds),$$

and the interest rate revenues from his digital currency account given by

$$IR_d^i = r_d(1 - \alpha^2 - \int_{\alpha}^{1} sds).$$

c) A similar analysis applies if the consumer never uses his bank account to pay.

Appendix B-2: Competition for deposits A consumer located at point $x \in [0; 1/n]$ trades off between opening an account in bank k located at point 0 and bank i located at $1/n$. If he opens an account in bank i, he incurs a travelling cost $t_b(1/n - x)$, and obtains a net surplus CS_i. If he opens an account in bank k instead, he incurs a travelling cost $t_b x$ and obtains a net surplus CS_k. The consumer indifferent between bank i and bank k is given by

$$x_{ik} = \frac{1}{2n} + \frac{1}{2t_b}(CS_k - CS_i). \tag{24}$$

A consumer located at point $y \in [1/n; 2/n]$ trades off between opening an account in bank i located at $1/n$ and bank l located at $2/n$. The consumer indifferent between bank i and bank l is given by

$$y_{il} = \frac{3}{2n} + \frac{1}{2t_b}(CS_i - CS_l). \tag{25}$$

The total market share of bank i is $D_i = y_{il} - x_{ik}$. Replacing for y_{il} and x_{ik} given by (24) and (25), we obtain the total share of deposits attracted by bank i given in Eq. (12).

Appendix C: The expected net benefit of liquidity management

Recall that the expected net benefit of liquidity management is given by $EL_i = \tau \underline{a_i} - \rho \overline{a_i}$. I detail below the values of $\underline{a_i}$ and $\overline{a_i}$ according to the amount of reserves held by banks and the consumers' use of payment instruments.

Case 0 - Benchmark: No digital currency - trade-off cash/bank transfer. i) Suppose that bank i never has enough reserves to meet the demand of depositors when they need to pay by bank transfer ($R_i \in (0, \hat{s}_b D_i)$). With probability φ, bank i initiates a transfer to the other banks. If $s^i \in (0, \hat{s}_b)$, there is no transfer of deposits. Bank i lends R_i to the central bank at a rate τ, because consumers pay in cash. If $s^i \in (\hat{s}_b, 1)$, the consumer transfers funds from his bank account. Bank i needs to borrow $s^i D_i - R_i$ at a rate ρ. With probability $1 - \varphi$, bank i does not initiate any transfer to the other banks and lends R_i to the central bank until date 2. Therefore, we have

$$\underline{a_i} = (\varphi \hat{s}_b + (1 - \varphi)) R_i,$$

and

$$\overline{a_i} = \varphi \int_{\hat{s}_b}^{1} (sD_i - R_i) ds.$$

ii) Suppose that bank i has enough reserves to meet some (but not all) payments by bank transfer $R_i \in (\hat{s}_b D_i, D_i)$. Therefore, we have

$$\underline{a_i} = \varphi \hat{s}_b R_i + \varphi \int_{\hat{s}_b}^{R_i/D_i} (R_i - sD_i) ds + (1 - \varphi) R_i,$$

and

$$\overline{a_i} = \varphi \int_{R_i/D_i}^{1} (sD_i - R_i) ds.$$

Case 1 - digital currency - No reserves for payments by bank transfer. $R_i \in (0, \hat{s}_{dc} D_i)$. Bank i never has enough reserves to meet the demand of depositors when they need to pay by bank transfer (that is, if $s^i \geq \hat{s}_{dc}$). With probability φ, bank i initiates a transfer to the other banks. If $s^i \in (0, \hat{s}_{dc})$, there is no transfer of deposits. Bank i lends R_i to the central bank at a rate τ, because consumers pay with digital currency. If $s^i \in (\hat{s}_{dc}, \alpha)$, the consumer transfers funds from his bank account. Bank i needs to borrow $s^i D_i - R_i$ at a rate ρ. If $s^i \in (\alpha, 1)$, consumers transfer all their funds from their bank account and need to borrow $\alpha_i D_i - R_i$. With probability $1 - \varphi$, bank i does not initiate any transfer to the other banks and lends R_i to the central bank until date 2. Therefore, we have

$$\underline{a_i} = (\varphi \hat{s}_{dc} + (1 - \varphi)) R_i,$$

and

$$\overline{a_i} = \varphi \left(\int_{\hat{s}_{dc}}^{\alpha} (sD_i - R_i) ds + (1 - \alpha)(\alpha D_i - R_i) \right).$$

Case 2 - digital currency - reserves to cover some (but not all) payments by bank transfer. $R_i \in (\hat{s}_{dc}D_i, \alpha D_i)$. With probability φ, bank i initiates a transfer to the other banks. If s^i belongs to $(0, \hat{s}_{dc})$, bank i does not use any reserves from its central bank account because consumers pay with digital currency. If s^i belongs to $(\hat{s}_{dc}, R_i/D_i)$, bank i has enough reserves to cover the demand of depositors given by $s^i D_i$. If s^i belongs to $(R_i/D_i, \alpha)$, bank i does not have enough reserves to cover the demand of depositors and needs to borrow $s^i D_i - R_i$. If s^i belongs to $(\alpha, 1)$, a consumer of bank i transfers all his wealth α from his bank account and transfers funds from his digital currency account to his bank account to pay by bank transfer. Bank i needs to borrow all the funds lent to borrowers. Since bank i has a share D_i of deposits, it borrows $L_i = \alpha D_i - R_i$. With probability $1 - \varphi$, bank i does not initiate any transfer to the other banks and lends R_i to the central bank until date 2. Therefore, we have

$$\underline{a_i} = \varphi \hat{s}_{dc} R_i + \varphi \int_{\hat{s}_{dc}}^{R_i/D_i} (R_i - sD_i) ds + (1 - \varphi) R_i,$$

and

$$\overline{a_i} = \varphi \left(\int_{R_i/D_i}^{\alpha} (sD_i - R_i) ds + (1 - \alpha)(\alpha D_i - R_i) \right).$$

Appendix D: Proof of propositions 1, 2 and 3

Appendix D-1: Proof of propositions 1 and 2

Suppose that consumers trade off between paying in cash, by bank transfer, and with digital currency, that is, from Lemma 1, that $f_b^i \in (\underline{f_b}, \overline{f_b})$. Using Lemma 1, the choice of the fee for bank transfers f_b^i amounts to choosing a threshold $\hat{s}_{dc} \in (\hat{s}_d, 1 - \alpha)$ above which consumers prefer paying by bank transfer rather than digital currency. The choice of the deposit fee F_i amounts to choosing a level of consumer surplus CS_i given by bank i to its consumers.

If there is an interior solution, using Leibniz's rule and Eq. (9), the first-order conditions with respect to the (equivalent) choice variables CS_i, $\hat{s}_{dc}$ and r_L^i are given by

$$\frac{\partial \pi_i}{\partial CS_i} = \frac{\partial \pi_i}{\partial D_i} \frac{\partial D_i}{\partial CS_i} + \frac{\partial \mu_i}{\partial CS_i} D_i = 0, \tag{FOC-1}$$

$$\frac{\partial \pi_i}{\partial \hat{s}_{dc}} = D_i \frac{\partial \mu_i}{\partial \hat{s}_{dc}} + \frac{\partial EL_i}{\partial \hat{s}_{dc}} = 0, \tag{FOC-2}$$

and

$$\frac{\partial \pi_i}{\partial r_L^i} = L_i + \frac{\partial \pi_i}{\partial L_i} \frac{\partial L_i}{\partial r_L^i} = 0. \tag{FOC-3}$$

for all $i = 1..n$. In a symmetric equilibrium, banks' best-responses to the fee c_d chosen by the digital currency provider are identical. The vector of profit-maximizing prices is denoted by $P^* = (F^*, f_b^*, r_L^*)$. At P^*, we have $D_i^* = 1/n$ and $L_i^* = L$ for all $i = 1..n$. Since $\alpha/n - L \geq 0$ from (A2), we denote by $R^* = \alpha/n - L$ the amount of reserves held by each bank i in a symmetric equilibrium for all $i = 1..n$. Note that from Eq. (10) and (5), bank i's margin can be rewritten as

$$\mu_i = u_b + u_d + IR_d^i - f_d \beta_d^i - C_b(\beta_b^i) - CS_i. \tag{26}$$

In the online Appendix O-8 of the paper, I show that the second-order conditions hold if k_b is sufficiently high.

Replacing for $\partial D_i/\partial CS_i = 1/t_b$ given by Eq. (12), $\partial\mu_i/\partial CS_i = -1$ given by Eq. (5) and $D_i^* = 1/n$ into (FOC-1) gives

$$\frac{\partial\pi_i}{\partial D_i}\bigg|_{P^*} = \frac{t_b}{n}. \tag{FOC-1-B}$$

From Eq. (9), since $\partial\pi_i/\partial D_i = \mu_i + \partial EL_i/\partial D_i$, Eq. (FOC 1-B) implies that in a symmetric equilibrium, we have

$$\mu_i = \frac{t_b}{n} - \frac{\partial EL_i}{\partial D_i}\bigg|_{P^*}. \tag{C1}$$

Replacing at P^* for $D_i^* = 1/n$ into (FOC-2), we obtain that

$$\frac{\partial\mu_i}{\partial\hat{s}_{dc}}\bigg|_{P^*}\frac{1}{n} + \frac{\partial EL_i}{\partial\hat{s}_{dc}} = 0. \tag{FOC-2-B}$$

Since $d\beta_b^i/d\hat{s}_{dc} = -1$, $\partial IR_d^i/\partial\hat{s}_{dc} = -r_d\hat{s}_{dc}$, and $\beta_b^i = 1 - s_{dc}^*$, replacing for

$$\frac{\partial\mu_i}{\partial\hat{s}_{dc}} = (c_d - c_b - k_b\beta_b^i)\frac{\partial\beta_b^i}{\partial\hat{s}_{dc}} + \frac{\partial IR_d^i}{\partial\hat{s}_{dc}}$$

given by Eq. (5) into (FOC-2-B) gives

$$(k_b + \Delta c - (r_d + k_b)s_{dc}^*)\frac{1}{n} + \frac{\partial EL_i}{\partial\hat{s}_{dc}}\bigg|_{P^*} = 0. \tag{C2}$$

Replacing for $\partial L_i/\partial r_L^i = -\gamma(n-1)/n$ given by Eq. (1) and for $\partial\pi_i/\partial L_i = r_L^i + \partial EL_i/\partial L_i$ given by Eq. (9) into (FOC-3), we find that in a symmetric equilibrium

$$-\gamma\frac{(n-1)}{n}(r_L^i + \frac{\partial EL_i}{\partial L_i}\bigg|_{P^*}) + L = 0. \tag{FOC-3-B}$$

Eq. (FOC-3-B) implies that in a symmetric equilibrium, we have

$$r_L^* = \frac{nL}{\gamma(n-1)} - \frac{\partial EL_i}{\partial L_i}\bigg|_{P^*},. \tag{C3}$$

If there is a symmetric equilibrium, if there is an interior solution, the banks' best-responses to the fee c_d are given by (C1), (C2) and (C3). Proposition 1 is obtained in the special case in which $EL_i = 0$. This completes the proof of Proposition 1 and Proposition 2.

Appendix D-2: Proof of proposition 3 - low amount of reserves. $R_i \in (0, \hat{s}_{dc}D_i)$.

Suppose that consumers trade off between paying by bank transfer, with digital currency, and with cash (i.e., that $f_b^i \in (\underline{f_b}, \overline{f_b})$). For an interior solution to exist, it must be that at f_b^* given by condition (C2) the amount of reserves R^* held by bank i belongs to $(0, s_{dc}^*/n)$ (or equivalently $s_{dc}^* \geq nR^*$ and $\alpha - Ln \geq 0$ which is true from (A2)). Since $s_{dc}^* \leq 1-\alpha$, in an interior solution, it must be that $s_{dc}^* \in (nR^*, 1-\alpha)$. If $nR^* \geq \hat{s}_d$, there may be an interior solution such that consumers trade off between paying in cash, with digital currency and by bank transfer. If $nR^* < \hat{s}_d$. There is no interior solution in which consumers trade off between the three payment instruments, and consumers trade off between cash and bank transfer payments instead.

If $R_i \in (0, \hat{s}_{dc} D_i)$, from (8) and Appendix C case 1, we have

$$\left.\frac{\partial EL_i}{\partial \hat{s}_{dc}}\right|_{P^*} = \varphi\left((\tau - \rho)R^* + \rho\frac{s^*_{dc}}{n}\right), \tag{27}$$

$$\left.\frac{\partial EL_i}{\partial L_i}\right|_{P^*} = -\varphi\rho(1 - s^*_{dc}) - \tau(\varphi s^*_{dc} + 1 - \varphi), \tag{28}$$

and

$$\left.\frac{\partial EL_i}{\partial D_i}\right|_{P^*} = \varphi(\tau\alpha s^*_{dc} - \rho\int_{s^*_{dc}}^{\alpha}(s - \alpha)ds) + (1 - \varphi)\alpha\tau. \tag{29}$$

Replacing for Eq. (29) into Eq. (C1) gives bank i's margin per depositor as in Proposition 2. Replacing for Eq. (28) into Eq. (C3) gives the interest rate on loans of Proposition 2. If there is an interior solution, since $IR^i_d/\partial\hat{s}_{dc} = -r_d\hat{s}_{dc}$ if $\hat{s}_{dc} \leq 1 - \alpha$, replacing for Eq. (27) into Eq. (C2), s^*_{dc} (or equivalently f^*_b) is implicitly defined by

$$k_b + \Delta c - n\varphi(\rho - \tau)R^* - (r_d + k_b - \varphi\rho)\hat{s}^*_{dc} = 0. \tag{30}$$

We denote by

$$\lambda(f^i_b) \equiv k_b + \Delta c - n\varphi(\rho - \tau)R^* - (r_d + k_b - \varphi\rho)\hat{s}_{dc}.$$

From Eq. (30), f^*_b is implicitly defined by $\lambda(f^*_b) = 0$. We determine whether there exists a bank transfer fee $f^*_b \in (f^R_b, f^D_b)$ such that $\lambda_i(f^*_b) = 0$. For this purpose, we analyze the sign of $\lambda^{'}(f^i_b) = (\varphi\rho - r_d - k_b)\partial\hat{s}_{dc}/\partial f^i_b$. Since $\varphi\rho - r_d - k_b < 0$ from (A3) and since $\partial\hat{s}_{dc}/\partial f^i_b > 0$ from Lemma 1, we have $\lambda^{'}(f^i_b) < 0$.

If $\lambda(f^R_b) < 0$, bank i's profit is decreasing with f^i_b and there is a corner solution. Each bank i chooses $(f^i_b)^* = f^R_b$ such that $s^*_{dc} = nR^*$. The condition $\lambda(f^R_b) < 0$ is equivalent to

$$c_d > c^{\max}_d \equiv c_b + k_b - nR^*(r_d + k_b - \tau\varphi).$$

If $\lambda(f^D_b) > 0$, bank i's profit is increasing in f^i_b and there is a corner solution. Each bank i chooses $(f^i_b)^* = f^D_b$ such that $s^* = 1 - \alpha$. The condition $\lambda(f^D_b) > 0$ is equivalent to

$$c_d < c^{\min}_d \equiv c_b + (\varphi\rho - r_d)(1 - \alpha) + k_b\alpha - \varphi(\rho - \tau)(\alpha - Ln).$$

If $c_d \in (c^{\min}_d, c^{\max}_d)$, there is an interior solution, $\lambda(f^*_b) = 0$ and we have

$$s^*_{dc} = \frac{\Delta c + k_b - \varphi(\rho - \tau)(\alpha - Ln)}{k_b + r_d - \varphi\rho}.$$

This completes the proof of Proposition 3.

- **Proof of corollary 1**

From Proposition 1, we have $nR^* = \alpha - nL$. From Proposition 3, we have

$$s^*_{dc} = \frac{\Delta c + k_b - \varphi(\rho - \tau)(\alpha - nL)}{k_b + r_d - \varphi\rho}.$$

The market share of the digital currency is given by $s^*_{dc} - c_d/(v_d - r_d)$. Since s^*_{dc} is increasing with n (as $\rho - \tau > 0$) and with τ, the market share of the digital currency is increasing with n and τ. Moreover, s^*_{dc} is decreasing with r_d if $r_d > 0$ and with α. Finally, we have

$$\frac{\partial s^*_{dc}}{\partial \rho} = \frac{\varphi(c^{max}_d - c_d)}{(k_b + r_d - \varphi\rho)^2} > 0,$$

which implies that the market share of the digital currency is increasing with ρ.

Appendix E: The interest rate on loans with and without a digital currency

- **Low amount of reserves:** If there is no digital currency, we have

$$r_L^* = \frac{nL}{\gamma(n-1)} + \varphi(\rho - (\rho - \tau)s_b^*) + (1-\varphi)\tau,$$

where the transaction size such that consumers pay by bank transfer s_b^* is obtained by setting $r_d = c_d = 0$ in Proposition 3. If there is a digital currency (and banks still hold a low amount of reserves), we have

$$r_L^* = \frac{nL}{\gamma(n-1)} + \varphi(\rho - (\rho - \tau)s_{dc}^*) + (1-\varphi)\tau.$$

The interest rate increases after the introduction of the digital currency if and only if $\varphi(\rho - \tau)(s_b^* - s_{dc}^*) > 0$. If consumers make a lower use of their bank account to make payments following the introduction of the digital currency, we have $s_b^* \leq s_{dc}^*$. Hence, the interest rate on loans is reduced (and the reverse is true otherwise).

- **High amount of reserves:** If banks hold a high amount of reserves, the variation of interest rates is given by $\varphi(\rho - \tau)(1 - \alpha) \geq 0$ from Online Appendix O-1. Therefore, the interest rate on loans increases.