CRED WORKING PAPER n^o 2023-04

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February, 2022

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February 2022

"From the personalistic point of view, statistics proper can perhaps be defined as the art of dealing with vagueness and interpersonal difference in decision situations."
Leonard Savage, *The Foundations of Statistics*, Chapter 8 (1954)

Abstract: We consider product differentiation on competitive news markets, as determined by the characteristics of demand confronting basic informational non-convexities in the activities of news reporting. Non-manipulative profit-maximizing news media imperfectly report the information they draw from some normally distributed flow of source data. A natural measure of information loss due to the media is the Kullback-Leibler divergence between the normal distributions of news and raw data. We show that reporting distortions depend on: (i) *bias*, defined as the difference between the means of the probability distributions of news and raw data; and (ii) *noise*, defined as the difference between the standard deviations of these distributions. We show that expected utility maximizing consumers with concave Bernoulli utility functions are noise-averse. Distortion-averse consumers are both bias- and noise-averse. We show that the news products supplied at equilibrium are identical in terms of accuracy, as measured by their Kullback-Leibler divergence to raw data. These products make a one-dimensional locus in the mean-standard deviation space. This locus consists of horizontally differentiated products, ranging from conventional news products, characterized by large biases and by noise levels reduced to some incompressible minimum, to "noisy" news products, which set bias to zero at the expense of some maximum noise level. The frontier confronts distortion-averse consumers with a basic non-convexity. Non-convexity results in maximal product differentiation, the "conventional" and "noisy" extremes being the only news products actually demanded at equilibrium in some natural configurations of the latter. We moreover show that most types of noise-averse consumers choose their news providers in the close vicinity of the conventional end of the market. The model thus provides a rationale and partial explanation for the common distinction between mainstream and alternative news media.

Key Words: News media, competitive equilibrium, information accuracy, Kullback-Leibler divergence, distortion aversion, horizontal differentiation, fact reporting, fact checking

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JEL Codes: C1, D4, D8, L1

1-Introduction

This paper considers product differentiation on competitive news markets, as determined by the characteristics of demand confronting basic informational non-convexities in the activities of news reporting.

Industrial economics traditionally considers product differentiation under two complementary aspects: its driving forces and its types (Tirole (2015), 2.1). The driving force of differentiation may be located on the supply side or on the demand side. Supply-driven differentiation follows from the incentives of profit-maximizing firms to use product differentiation as a means of alleviating the downward pressure that competition exerts on profits (Hotelling (1929), Chamberlin (1951)). Demand-driven differentiation follows from the heterogeneity in product characteristics and in consumer preferences. More specifically, demand expresses consumers' individual preferences relative to the various characteristics of each commodity, such as type, quality, the time and location of delivery, and so on (Lancaster (1966)). The products, or some subsets of their characteristics, are vertically differentiated if all consumers have the same preference ranking relative to these characteristics; this happens, for instance, if, *ceteris paribus*, all prefer better quality to lesser quality. Otherwise the products are horizontally differentiated.

The vast literature relative to product differentiation in news media industries covers all the dimensions above, captured notably through the analysis of the relationship between market competition and the quality of news construed as the quality, in terms of completeness and accuracy, of the information conveyed by reported situations or events (Gentzkow and Shapiro (2008), Gentzkow, Shapiro and Stone (2015); see also Hu (2021) for recent extensions to the social media).

The theoretical literature reviewed in Gentzkow et al. (2015) typically assumes: (i) some unobserved state of the world attracting public interest; (ii) raw data relative to this state of the world, modelled as a random variable; and (iii) news reports of the latter, modelled in the same way. The main concern of the theory is the explanation of media bias, conceived, using the words of Gentzkow et al., as "systematic differences in the mapping of raw facts to news reports". These systematic differences are explained by strategic manipulations of raw facts by profit-maximizing news media operating on imperfectly competitive news markets. The authors distinguish two types of biases: outright distortion, involving, using their words again, "some integral measure of the distance between news reports and raw facts" (as random variables); and filtering of information, where bias consists of oriented summaries or of selective accounts of source data.

The present article studies the determination of *non-manipulative* informational distortion at competitive equilibrium in the canonical case of normally distributed source data.[2] We show that consumers' noise aversion is the main driver of product differentiation in this setup.

Accurate fact reporting is costly. We suppose that the cost-constrained reporting activities of each news provider introduce a specific ("idiosyncratic") distortion in source data, which is modelled as a random distortion term, normally distributed and statistically independent from the raw distribution. A natural measure of the information loss due to the media in this setup is the Kullback-Leibler divergence between the news provided by a media firm and the statistical distribution of raw information (Kullback (1959)). In the case of normal distributions, the divergence reduces to a simple function of the first and second moments of the distributions of news and raw data. This function introduces in turn a natural synthetic description of reporting distortion as a two-dimensional object consisting of (i) *bias*, construed as the difference between the means of the distributions of, respectively, news and raw data, and (ii) *noise*, construed as the difference between the variances (or standard deviations) of these distributions. We show that expected utility maximizing consumers with concave Bernoulli utility functions are noise-averse. They may or may not be bias-averse. A distortion-averse consumer, who, by definition, is both bias- and noise-averse, typically confronts, at equilibrium, a trade-off between the various combinations of bias and noise supplied by the market (in many respects analogous to the mean-variance arbitrage in portfolio choices, in the capital asset pricing model).

These notions of bias and noise match those used by Kahneman, Sibony, and Sunstein in *Noise* (2021), their comprehensive study of the informational basis of judgment. Their book reviews a considerable number of empirical studies relative to the cases of professional judgment involved in such diverse fields as crime punishment, medical diagnosis, and the management of human resources (among others). The authors convincingly argue that bias and

[2] The case of (manipulative) outright distortion of normally distributed raw data is studied notably by Mullainathan and Shleifer (2005). The comprehensive account of filtering bias by Gentzkow et al. (2015) also relies on the assumption that raw facts are normally distributed.

noise, as we define them in the present paper, should be treated on the same footing, as being, the both of them, major sources of flaws in human judgment.[3]

In contrast, the notion of bias that we use here sharply differs from the notion of media bias of Gentzkow et al. (2015). The latter notably refers to Blackwell's criterion for the comparison of information structures (1951; see also Laffont (1991), chap. 4). According to this criterion, an information structure (i.e. a mapping from the space of "signals" to the space of probability measures over signals) is better, or more informative, than another structure, if all Bayesian expected utility maximizers make better decisions (i.e. increase their expected utility) when they substitute the former for the latter in their Bayesian revision of probabilities conditional on observed signals. This criterion of first-order stochastic dominance yields a partial ranking of information structures, which provides a natural candidate for a notion of information accuracy, alternative to the Kullback-Leibler criterion mobilized in the present paper, and well suited to Gentzkow and Shapiro's basic object, namely, their discussion of the ability of media markets to "make *beliefs* converge toward the truth" (2008; my emphasis). We do not introduce any notion of Bayesian behavior or beliefs on behalf of consumers or media firms in the present paper. Our basic concern is, paraphrasing Gentzkow and Shapiro, the ability of media markets to make *fact reporting* converge toward *accurate reporting*. Nevertheless, it should be noted that the Kullback-Leibler divergence does fit the notion of outright distortion as "some integral measure of the distance between news reports and raw facts" of Gentzkow et al. (2015).[4]

In our setup, each news product is synthetically described as a mean-standard deviation pair. We show that, at competitive equilibrium, media firms provide news products that are identical in terms of accuracy, that is, in terms of their Kullback-Leibler divergence to raw data. These products make a one-dimensional locus in the mean-standard deviation space, hereafter called the frontier of equilibrium supply. This frontier provides an array of horizontally differentiated products, ranging from, at one end of the spectrum, "conventional" news products characterized by large biases and by noise levels reduced to some incompressible minimum, to, at the other end of the spectrum, "noisy" news products, setting bias to zero at the expense of some maximum noise level. The frontier confronts distortion-averse consumers with a basic non-convexity in their choice of a news outlet. Non-convexity results in maximal product

[3] They notably build on Gauss's classical formula equating the mean squared error with the sum of noise and squared bias as defined above (2021, chap. 5 ; see section 2 below).
[4] With only one minor qualification: the Kullback-Leibler divergence is not a distance in the strict, mathematical sense of the word (it does not verify the triangular inequality)

differentiation, the “conventional” and “noisy” extremes being the only news products actually demanded at equilibrium, at least in some natural configurations of the latter.

The type of horizontal differentiation implied by the model evokes some aspects of the distinction commonly made between (i) “mainstream” news media, on the one hand, such as, for example, in the USA, *ABC News*, *Fox News*, the *New York Times* or the *Wall Street Journal*, or, in France, *TF1*, *Le Figaro*, *Le Monde* or *Les Echos*, and (ii), on the other hand, “alternative” news media such as, for example again, *Vox* or the *HuffPost* in the USA, and *Mediapart* or *Atlantico* in France (e.g., for the French press, see Lyubareva et al. (2020)). In one possible reading of the model, the conventional news products would be mainly issued by mainstream outlets, and the noisy ones would be notably produced by alternative news media. We show that most types of noise-averse consumers choose their news providers in the close vicinity of the conventional end of the market. Noise aversion thus provides a rationale and partial explanation of why mainstream outlets are conventional, in our setup at least.

The paper is organized as follows. Section 2 details the information setup. Section 3 describes supply and characterizes competitive supply equilibrium. Section 4 describes demand and introduces distortion aversion. Section 5 characterizes the frontier of equilibrium supply, and describes the associate bias-noise arbitrage. Section 6 characterizes market equilibrium. Section 7 interprets product differentiation in terms of journalistic practice. Section 8 concludes.

2-Information setup

Information is construed as a flow of signals emitted from some underlying phenomenon of potential public interest, such as, for example, clusters of emerging viral epidemics, or rumors of inappropriate personal behavior of political leaders in conducting public or private affairs. The signals convey imperfect, incomplete[5] elements of description of the underlying “true” phenomenon (e.g. the true state of viral infection of the population, or the true personal behavior of political leaders). They are modelled below as a random variable d (the “raw data” source), normally distributed, with mean μ and variance σ^2.

[5] Information is necessarily incomplete if the purpose of description is defined comprehensively, as aiming to provide an exhaustive account of a state of the world (e.g. the circulation of a virus in a population; or the personal behavior of a political leader). It can be complete if the purpose of description is defined in a selective, tractable way, such as, for example, in terms of confirming or rejecting the hypothesis of the presence of antibodies of a definite type in a patient’s blood. Incompleteness is a fundamental form of information imperfection. Information can also be imperfect in a second sense, even when it is complete in the sense above. Namely, when it provides an erroneous (albeit complete) description of certain facts. For example, a “false negative” test rejecting the presence of antibodies of the relevant type in the blood of an infected patient.

There are J types of for-profit media firms, denoted by index $j=1,...,J$. Each firm collects imperfect, possibly incomplete pieces of the raw data flow, and converts them into a news flow, sold on the media market. The news produced by a media firm of type j is a random variable n^j, normally distributed, with mean μ_j and variance σ_j^2. We moreover assume that $n^j - d$, hereafter called "informational distortion" and denoted by ε^j, is uncorrelated to the data source (i.e. its variance is equal to $\sigma_j^2 - \sigma^2$; and of course ε^j is normally distributed and its mean is $\mu_j - \mu$). In other words, we suppose that the process of data collection and/or conversion into news is perturbed by phenomena that are not fully controlled by the firm and that are statistically unrelated to the data source. Media firms are not fully transparent in this respect. They are independent sources of imperfection or incompleteness of the information conveyed to consumers through the media market. Note that, by Gauss's classical formula, $\int_{-\infty}^{+\infty} f_j(s)(\varepsilon_j)^2 ds = \sigma_j^2 - \sigma^2 + (\mu_j - \mu)^2$, that is, the mean squared distortion is equal to the sum of "noise" $\sigma_j^2 - \sigma^2$ and squared "bias" $(\mu_j - \mu)^2$ (see Kahneman et al. (2021, chap. 5)).[6]

A natural measure of the mean loss of information incurred by substituting the news n^j for the raw data flow d is the Kullback-Leibler divergence between the associate densities, that is, $\int_{-\infty}^{+\infty} f_0(s)\log\frac{f_0(s)}{f_j(s)}ds$, where: s denotes an observation (a "signal" or "information", which may consist of a raw data d_e or a news n_e^j observed in a state of nature e); f_j denotes the probability density of n^j (i.e. $f_j : s \to \frac{1}{\sigma_j\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{(s-\mu_j)^2}{\sigma_j^2}\right)$); and $n^0 = d$. Following Kullback (1959), we denote the divergence of f_j relative to f_0 by $I(0:j)$. We refer to it as the K-L divergence below.

$I(0:j)$ can also be interpreted as a measure of the data-processing activity of a firm of type j, due to the natural connection between the K-L divergence and the maximum likelihood criterion (see Appendix I). We interpret $I(0:j)$ below as the outcome of some implicit process of constrained minimization of the K-L divergence, reflecting the practical characteristics of, and practical limitations on, type j's operations of collection and processing of the data. The

[6] Note also that Gauss's identity holds true for *any* statistical distribution of the distortion term, whether normal or not. It only depends on the assumption that distortion is uncorrelated to raw data.

constraints on K-L divergence minimization notably include the (monetary) cost of these operations, which is modelled explicitly in section 3 below.

As a standard fact of information theory, we have $I(0:j) = \log\left(\frac{\sigma_j}{\sigma}\right) + \frac{1}{2}\left(\frac{\sigma^2}{\sigma_j^2} + \frac{(\mu_j - \mu)^2}{\sigma_j^2} - 1\right)$.[7] The divergence is null if and only if $j = 0$. It is positive, and increasing in $|\mu_j - \mu|$, if $j \neq 0$. As measured by divergence $I(0:j)$, the distortion introduced by a media firm of type j in reporting the raw data thus involves two dimensions, which are respectively associated with the first and second moments of the statistical distribution of ε^j: (i) distance $|\mu_j - \mu|$, hereafter called, following Kahneman et al. (2021), j's *systematic bias*, which corresponds to the absolute value of the mean discrepancy between type j's news and the data ; and (ii) distance $|\sigma_j^2 - \sigma^2| = \sigma_j^2 - \sigma^2$, hereafter called, following again Kahneman et al. (2021), j's *idiosyncratic noise*, which corresponds to the variance that the news of type j add to the variance of raw data.[8]

Figure 1 maps the graph (Fig. 1a) and a set of contour lines (Fig. 1b) of $I(0:j)$ as a function of (σ_j, μ_j) when f_0 is the centered reduced law (i.e. when $(\sigma, \mu) = (1, 0)$).[9]

Propositions 1 and 2 below collect a number of basic properties of $I(0:j)$ and of function $\varphi : \mathbb{R}_{++} \times \mathbb{R} \to \mathbb{R}$ defined by $\varphi(x, y) = \log\left(\frac{x}{\sigma}\right) + \frac{1}{2}\left(\frac{\sigma^2}{x^2} + \frac{(y - \mu)^2}{x^2} - 1\right)$.[10]

Proposition 1: For all $j = 0, ..., J$, $I(0:j) \geq 0$, with equality if and only if $j = 0$.

Proposition 2: Function φ is: (i) C^∞; (ii) strictly increasing in its second argument; (iii) strictly increasing in its first argument over $\{(x, y) \in \mathbb{R}_{++} \times \mathbb{R} : x^2 - \sigma^2 > (y - \mu)^2\}$; (iv) strictly

[7] E.g. https://fr.wikipedia.org/wiki/Loi_normale, 5.5.3.

[8] Interestingly, the K-L divergence is increasing (resp. decreasing) in idiosyncratic noise $\sigma_j^2 - \sigma^2$ if and only if the latter is larger (resp. smaller) than squared systematic bias $(\mu_j - \mu)^2$ (see Proposition 2 below).

[9] We follow the standard practice established by the capital asset pricing model in finance, which consists of treating the variance (or standard deviation) as an abscissa and the expectation as an ordinate.

[10] The non-negativity and other properties below rely to a large extent on the convexity of function $t \to t.\log t$. Non-negativity, in particular, is closely related to Jensen's inequality. These properties should therefore be viewed as the expression, in the special case of normal distributions we consider here, of basic axiomatic properties of information measures.

decreasing in its first argument over $\left\{(x,y)\in\mathbb{R}_{++}\times\mathbb{R}: x^2-\sigma^2<(y-\mu)^2\right\}$. (v) Partial functions $y\to\varphi(x,y)$ are strictly convex for all $x\in\mathbb{R}_{++}$.

All proofs are collected in the appendix (Appendix III).

3-Supply setup

In order to spell out as neatly as possible the independent role of consumers' noise aversion in the determination of product differentiation on the market for news, we assume away any motive for product differentiation that may stem, on the supply side of the market, from situations of imperfect competition *à la* Hotelling or otherwise. This makes another sharp difference between the model examined in this paper and the literature reviewed in Gentzkow et al. (2015). The latter considers a mix of demand and supply determinants of product differentiation, which notably involves firms' strategic manipulation of both prices and information. In contrast, the model considered here presents a pure case of demand-driven differentiation *à la* Lancaster, where differentiation is solely determined by consumers' preferences relative to bias and noise as joint characteristics of news products. Accordingly, we suppose that the market for news is perfectly competitive, and we moreover assume, for simplicity, that firms' data-processing and broadcasting technologies are identical. We show below that, unsurprisingly, this implies that all firms provide the same quality of information at equilibrium. In particular, our setup leaves no room for vertical differentiation on news markets.

We argued above that divergence $I(0:j)$ could be interpreted as a measure of the quality, in the sense of the informational accuracy, of type j's information services: the smaller $I(0:j)$ is, the more accurately, on average, the news provided by a firm of type j match raw data, with full accuracy (i.e. exact match) obtained if and only if the divergence is null. In the sequel we let $q_j=\dfrac{1}{I(0:j)}$ measure the quality of the information services provided by the firms of type j to their consumers. The quality index q_j is monotonically decreasing in divergence $I(0:j)$. It runs over $]0,+\infty]$ as divergence $I(0:j)$ runs over $[0,+\infty[$. Full accuracy is obtained if and only if $q_j=+\infty$.

The representation of media behavior developed here combines two types of activities.

First, data collection and data processing activities aim at information accuracy. They are captured through the quality index q_j of the news provided by the media. The product of these activities can be viewed as information sheets, collecting facts appropriately arranged and interpreted. Their average informational content is measured, in units of information accuracy, by index q_j.

Second, broadcasting activities convey information sheets to news consumers. We let z_j denote the number of information sheets so dispatched by a firm of type j. Note that the "sheet" should not be interpreted literally, as a piece of paper. The informational content q_j can be conveyed in both verbal and written form, and via any technical support such as electronic or herzian channels, or traditional newspapers. The quantity z_j will be treated as a continuous variable below. It captures the mass dimension of the activities of the news industry.

Summing up, the product of a firm of type j consists of information sheets of average informational content q_j replicated z_j times. It is measured therefore by quantity $q_j.z_j$ corresponding to a number of units of (average) information accuracy broadcast to its customers. The media charges a price p for each replica. Here again, we need not be too specific about the financing details of these expenses. They are typically covered, in practice, by advertisement budgets and customer fees, in various proportions according to the media, but always narrowly related (by and large proportional) to its audience. We will suppose here, for simplicity, and without substantial loss of generality, that the price is paid by the customer. The firm's sales revenue is equal to $p.q_j.z_j$.

For the reasons developed at the beginning of this section, we make the following simple, conservative assumptions concerning the technology of the information industry, compatible in particular with a competitive partial equilibrium of the information market. All firms are endowed with the same decreasing (marginal) returns technology, described by cost function $(q_j, z_j) \to c(q_j, z_j)$, defined over $[0,+\infty] \times [0,+\infty]$, strictly increasing, strictly convex, C^2 in $\mathbb{R}^2_{++}$, and such that $c(0,0) = 0$ and $c(+\infty, z_j) = c(q_j, +\infty) = +\infty$ for all (q_j, z_j).

The specific structure of media products and activities, which combine, in a multiplicative way, a qualitative dimension (data-processing, captured through q_j) and a scale dimension (mass broadcasting, captured through z_j), may potentially be a source of production non-convexities, such as scale economies and a non-concave profit function

$(q_j, z_j) \to p.q_j.z_j - c(q_j, z_j)$.[11] This might occur even if the cost function is convex, as clearly shown by the following simple example:

Example 1-**Scale economies related to quality investments**: We suppose technically separable data-processing and broadcasting activities, and constant unit-costs equal to 1 for each, that is, $c(q_j, z_j) = q_j + z_j$. The average cost per dispatched unit $\frac{c(q_j, z_j)}{z_j} = \frac{q_j}{z_j} + 1$ is then decreasing in the scale factor z_j. That is, the quality of information plays the role of a fixed cost, generating scale economies. The corresponding profit function $(q_j, z_j) \to p.q_j.z_j - q_j - z_j$ is clearly not concave (it is even strictly convex for any positive fixed ratio $\frac{z_j}{q_j} = k$, that is, along any "ray" $\left\{(q_j, z_j) \in \mathbb{R}_+^2 : z_j = k.q_j; k > 0\right\}$, if $p > 0$).

The profit of a firm of type j reads $p.q_j.z_j - c(q_j, z_j)$. Competitive profit maximization consists of solving program $\max\left\{p.q_j.z_j - c(q_j, z_j) : (q_j, z_j) \in [0, +\infty[\times [0, +\infty[\right\}$, the same for all j, for any given price p.[12] Proposition 3 below illustrates, in the context of this model, the general issue of the compatibility of competitive pricing with profit-maximization in the presence of scale economies. Loosely speaking, the curvature of the cost function, as measured by $\partial_{qz}^2 c(q_j, z_j) + \sqrt{\partial_{qq}^2 c(q_j, z_j).\partial_{zz}^2 c(q_j, z_j)}$, puts an upper bound on the range of compatible competitive equilibrium prices. If, in particular, this curvature is null, as in Example 1 above, then there exists no competitive equilibrium. In other words, the cost function must be "convex enough" for a competitive equilibrium to exist. The examples 2 and 3 of the appendix (Appendix II) provide simple computable instances of such cost functions and of the corresponding equilibria.

[11] See Stromberg (2004) for a closely related representation of the cost function of media firms, and for an extensive discussion of the consequences of the particular type of scale economies involved. Reformulated in non-technical terms, a basic specificity of information industries, relative to other activities of mass-manufacturing, seems to lie in the larger importance, in relative terms, of the qualitative, immaterial features of its products. These characteristics are not embodied in physical objects as in most large-scale activities of the primary and secondary sectors. They are not even embodied in types of standardized services as in most large-scale tertiary activities of the financial or commercial branches. They "are" or "make" the product, so to speak.

[12] Note that we implicitly assume here that full accuracy (i.e. $q_j = +\infty$) is never achieved, due to prohibitive costs. This follows from the first-order conditions of Proposition 2-(i), for example, if marginal cost $\partial_q c(q_j, z_j)$ grows to infinity as $q_j \to +\infty$.

Proposition 3: Suppose that $p>0$, and let $(q^*, z^*) \in \mathbb{R}^2_{++}$ solve $\max\left\{p.q_j.z_j - c(q_j,z_j) : (q_j,z_j) \in \left[0,+\infty\right[\times \left[0,+\infty\right[\right\}$. Then: (i) $p = \dfrac{\partial_q c(q^*,z^*)}{z^*} = \dfrac{\partial_z c(q^*,z^*)}{q^*}$; and (ii) $\left|p - \partial^2_{qz} c(q^*,z^*)\right| \leq \sqrt{\partial^2_{qq} c(q^*,z^*).\partial^2_{zz} c(q^*,z^*)}$. If, moreover, $\left|p - \partial^2_{qz} c(q^*,z^*)\right| < \sqrt{\partial^2_{qq} c(q^*,z^*).\partial^2_{zz} c(q^*,z^*)}$, then (q^*,z^*) is the unique solution to $\max\left\{p.q_j.z_j - c(q_j,z_j) : (q_j,z_j) \in \left[0,+\infty\right[\times \left[0,+\infty\right[\right\}$.

In the remainder of this paper, we assume that a supply equilibrium (q^*, z^*) exists and is unique.

4-Demand setup

There are N types of individual consumers of information, denoted by index $i = 1,...,N$. Each of them chooses a single news provider in the set $\left\{n^1,...,n^J\right\}$ supplied by the market, and purchases the corresponding news at price p. Our basic behavioral assumption, in terms of consumption, is expected utility maximization. We also make a quasi-linearity assumption, well suited to this study's spirit of partial equilibrium analysis. That is, a consumer of type i selects the editorial product n^j that maximizes his quasi-linear expected utility $\int_{-\infty}^{+\infty} f_j(n_e^j) u_i(n_e^j) dn_e^j - p$ in the set of news products $\left\{n^1,...,n^J\right\}$ available on the market. Given our assumptions on probability densities, the expected utility derived by a consumer of type i from editorial product n^j is a function of the sole first and second moments of normal distribution f_j, of the type $U_i(\sigma_j, \mu_j, p) = V_i(\sigma_j, \mu_j) - p$.

The Bernoulli utility functions u_i are standardly assumed to be strictly increasing, twice continuously differentiable, and strictly concave. We retain the last two assumptions (twice continuous differentiability and strict concavity) but depart from the first one for the following reasons.

Recall that we assumed $n^j = d + \varepsilon^j$, where the term ε^j corresponds to the distortion that type j's fact reporting operations impose on raw data. It seems reasonable to assume that consumers are averse to distortion, at least under some circumstances. A strictly increasing Bernoulli utility function is clearly not compatible with distortion aversion, as it implies that the consumer's utility $u_i(n_e^j) = u_i(d_e + \varepsilon_e^j)$ is, *ceteris paribus*, always strictly increasing in the distortion term ε_e^j. Thus we complement the standard apparatus of expected utility maximization with the following specific assumption of distortion aversion.

Distortion aversion : U_i exhibits *bias aversion* over some subset S of its domain if it is decreasing in $|\mu_j - \mu|$ over S, that is, if $|\mu_j - \mu| > |\mu'_j - \mu|$ implies $U_i(\sigma_j, \mu_j, p) < U_i(\sigma_j, \mu'_j, p)$ for all pairs $((\sigma_j, \mu_j, p), (\sigma_j, \mu'_j, p))$ of elements of S. It exhibits *noise aversion* over some subset S of its domain if it is decreasing in σ_j over S. It exhibits *distortion aversion* over some subset S of its domain if it exhibits both bias and noise aversion over S.[13]

As a simple consequence of a standard fact of expected utility theory,[14] replicated in Proposition 4 below, the normal probability distribution, combined with the strict concavity of the Bernoulli utility function, together imply noise aversion.

Proposition 4: Suppose that u_i is C^2 and strictly concave. Then U_i exhibits noise aversion over its whole domain.

Note that a bias-averse expected utility can be decreasing in μ_j (see the proof of Proposition 4). We retain bias-aversion as one of our main behavioral assumptions below. In other words, we assume that news consumers agree with statisticians that "systematic" (i.e. average) misreporting should be reduced, *ceteris paribus*, whenever possible. Note, nevertheless, that expected utility maximization does *not* imply bias-aversion. Alternative behaviors, involving "prone-to-bias" preferences, are compatible with this framework. They might follow, for example, in relevant contexts, from the cases of slanting commonly discussed

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[13] Note that, as a consequence of Proposition 2, the inverse (or the opposite) of the K-L divergence $I(0:j) = \log\left(\frac{\sigma_j}{\sigma}\right) + \frac{1}{2}\left(\frac{\sigma^2}{\sigma_j^2} + \frac{(\mu_j - \mu)^2}{\sigma_j^2} - 1\right)$, viewed as a function of (σ_j, μ_j), displays bias aversion in the sense above everywhere over $[\sigma, +\infty[\times \mathbb{R}$, but displays noise aversion only over $\{(\sigma_j, \mu_j) \in [\sigma, +\infty[\times \mathbb{R} : \sigma_j^{\,2} - \sigma^2 > (\mu_j - \mu)^2\}$.

[14] See, for example, Laffont (1991), p. 229.

in the literature (e.g. Mullainathan and Shleifer (2005)), that is, biases expressing consumers' political preferences.[15]

5-The frontier of equilibrium supply and the bias-noise arbitrage

In this section and the next one, we let p^* denote a given, fixed (competitive) equilibrium price of the news market, and we assume that the sufficient condition of Proposition 3 for a unique equilibrium supply holds true. We let (q^*, z^*) denote the corresponding equilibrium quality-scale mix. We also assume that there exists a level of incompressible noise added by media firms to raw data, measured by $\bar{\sigma} - \sigma > 0$, that is, we suppose that $\sigma_j \geq \bar{\sigma} > \sigma$ for all j and all feasible n^j. And we define the *frontier of equilibrium supply* associated with p^* as $F(p^*) = \left\{ (\sigma_j, \mu_j) \in \left[\bar{\sigma}, +\infty\right[\times \mathbb{R} : \varphi(\sigma_j, \mu_j) = \frac{1}{q^*} \right\}$.[16]

We show below that the equilibrium condition embodied in the definition of the frontier maintains possibilities of horizontal product differentiation on the news market, on the basis of a trade-off between bias aversion and noise aversion.

Let $h : \mathbb{R}_{++} \to \mathbb{R}$ be defined by $h(x) = \left(1 + \frac{2}{q^*} - 2\log\left(\frac{x}{\sigma}\right)\right) x^2 - \sigma^2$. The frontier above may be conveniently characterized as $F(p^*) = \left\{ (\sigma_j, \mu_j) \in \left[\bar{\sigma}, +\infty\right[\times \mathbb{R} : \mu_j = \mu \pm \sqrt{h(\sigma_j)} \right\}$. Its main characteristics are detailed in Proposition 5 below and illustrated in Figure 3.

Proposition 5: Suppose that $\bar{\sigma} < \sigma.\exp\left(\frac{1}{q^*}\right)$. (i) There exists a unique $\sigma(q^*) > \sigma.\exp\left(\frac{1}{q^*}\right)$ such that $h(\sigma(q^*)) = 0$. (ii) Function $\sqrt{h}$ is: (a) C^∞ and positive over $\left[\bar{\sigma}, \sigma(q^*)\right[$; (b)

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[15] Mullainathan and Shleifer (2005) assume a quadratic Bernoulli utility, of the type $u_i(n_e^j) = \alpha n_e^j - \beta (n_e^j)^2$, with α and β positive. A simple calculation yields $U_i(\sigma_j, \mu_j, p) = \alpha\mu_j - \beta(\mu_j)^2 - \beta(\sigma_j)^2 - p$ for *any* continuous probability density, whether normal or not. U_i is strictly concave, and therefore strictly quasi-concave. It exhibits noise aversion over its whole domain. It is decreasing (resp. increasing) in μ_j if $\mu_j > \frac{\alpha}{2\beta}$ (resp. $\mu_j < \frac{\alpha}{2\beta}$). Therefore it exhibits bias aversion if and only if $\mu = \frac{\alpha}{2\beta}$. It is « prone-to-bias » otherwise.

[16] The frontier plays, within the present setup, a role analogous to the role of Minkowski's efficiency frontier in the capital asset pricing model, that is, it characterizes a locus in the mean-standard deviation plane where market equilibria must lie.

increasing over $\left[\bar{\sigma},\sigma.\exp\left(\frac{1}{q^*}\right)\right[$; (c) decreasing over $\left]\sigma.\exp\left(\frac{1}{q^*}\right),\sigma(q^*)\right[$; and (d) such that $\sqrt{h\left(\sigma\left(q^*\right)\right)}=\partial\sqrt{h\left(\sigma.\exp\left(\frac{1}{q^*}\right)\right)}=0$ and $\lim_{x\to\sigma(q^*)^-}\partial\sqrt{h(x)}=-\infty$. (iii) There exists $\varepsilon\in\mathbb{R}_{++}$ such that $\sqrt{h}$ is differentiably strictly concave over $\left]-\varepsilon+\sigma.\exp\left(\frac{1}{q^*}\right),\sigma(q^*)\right[$. (iv) For all $x\in\left[\bar{\sigma},\sigma(q^*)\right[$, $\partial\left[\mu+\sqrt{h(x)}\right]=-\frac{\partial_x\varphi\left(x,\mu+\sqrt{h(x)}\right)}{\partial_y\varphi\left(x,\mu+\sqrt{h(x)}\right)}$ and $\partial\left[\mu-\sqrt{h(x)}\right]=-\frac{\partial_x\varphi\left(x,\mu-\sqrt{h(x)}\right)}{\partial_y\varphi\left(x,\mu-\sqrt{h(x)}\right)}$. (v) There exists $\varepsilon\in\mathbb{R}_{++}$ and a function $g:\left]\mu-\varepsilon,\mu+\varepsilon\right[\to\mathbb{R}$ that is C^∞, and such that $g(\mu)=\sigma(q^*)$ and $(g(y),y)\in F(p^*)$ for all $y\in\left]\mu-\varepsilon,\mu+\varepsilon\right[$. Function g is: (a) increasing over $\left]\mu-\varepsilon,\mu\right[$; (b) decreasing over $\left[\mu,\mu+\varepsilon\right[$; (c) differentiably strictly concave over $\left]\mu-\varepsilon,\mu\right[\cup\left]\mu,\mu+\varepsilon\right[$; and (d) such that $\partial g(\mu)=\partial^2 g(\mu)=0$.

As a consequence of Proposition 5, $F(p^*)$ is a smooth (C^∞) one-dimensional manifold with boundary, compact, and symmetrical with respect to horizontal line $L=\left\{(\sigma_j,\mu_j)\in\mathbb{R}\times\mathbb{R}:\mu_j=\mu\right\}$. In the half-plane above L, it coincides with the graph of the restriction of $x\to\mu+\sqrt{h(x)}$ to $\left[\bar{\sigma},\sigma(q^*)\right]$. In the half-plane below L, it coincides with the graph of the restriction of $x\to\mu-\sqrt{h(x)}$ to $\left[\bar{\sigma},\sigma(q^*)\right]$. It has three non-degenerate critical points, namely, $\left(\sigma.\exp\left(\frac{1}{q^*}\right),\mu\pm\sqrt{h\left(\sigma.\exp\left(\frac{1}{q^*}\right)\right)}\right)$, and $\left(\sigma(q^*),\mu\right)$, the first two associated with a horizontal tangent and the third with a vertical tangent to $F(p^*)$. These properties hold true if the minimum level of idiosyncratic noise stemming from the media is small enough, that is, if $\bar{\sigma}<\sigma.\exp\left(\frac{1}{q^*}\right)$.

In terms of interpretation, the (potential) supply described by the frontier confronts the consumer with an arbitrage between noisy and biased information that we describe below. Note that it is more convenient, for graphical interpretation, to describe noise through the standard deviation gap $\sigma_j - \sigma$, rather than through the variance gap $\sigma_j^2 - \sigma^2$. Starting from minimal noise $\bar{\sigma} - \sigma$ and moving to the right along the upper or lower branches of the frontier, bias, as measured by $\left|\mu_j - \mu\right|$, and idiosyncratic noise, as measured by $\sigma_j - \sigma$, together increase up to maximum bias level $\sqrt{h\left(\sigma.\exp\left(\frac{1}{q^*}\right)\right)}$, reached at noise level $\sigma.\left(\exp\left(\frac{1}{q^*}\right) - 1\right)$.[17] Still moving to the right along the frontier, from critical points $\left(\sigma.\exp\left(\frac{1}{q^*}\right), \mu \pm \sqrt{h\left(\sigma.\exp\left(\frac{1}{q^*}\right)\right)}\right)$, bias decreases as noise increases, down to a minimum, null bias level, reached at maximum noise level $\sigma(q^*) - \sigma$.

Clearly, a distortion-averse consumer will purchase his news either at boundary points $\left(\bar{\sigma}, \mu \pm \sqrt{h(\bar{\sigma})}\right)$ or to the right of the vertical axis through critical points $\left(\sigma.\exp\left(\frac{1}{q^*}\right), \mu \pm \sqrt{h\left(\sigma.\exp\left(\frac{1}{q^*}\right)\right)}\right)$. The first type of choice means that the consumer's aversion to informational noise outweighs his aversion to informational bias. The second type of choice means, symmetrically, that his noise aversion is outweighed by his bias aversion. Note that the threshold that so divides the market into a conventional segment and a noisy segment is the news product that equates noise and squared bias (see footnote 17).

6-Informational non-convexities and demand-driven product differentiation

In this section, we provide a formal definition of the notion of *relative* bias or noise aversion outlined in the former sentence. We show that the non-convex structure of the set of alternatives implied by the convexity properties of the K-L divergence,[18] combined with consistent relative

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[17] As a consequence of Proposition 5, we get $\sigma_j^2 - \sigma^2 = \left(\mu_j - \mu\right)^2$ on the frontier, that is, noise is equal to squared bias at equilibrium supply, if and only if $\sigma_j = \sigma.\exp\left(\frac{1}{q^*}\right)$. This type of critical position corresponds to the singularities characterized by $-\frac{\partial_x \varphi\left(x, \mu + \sqrt{h(x)}\right)}{\partial_y \varphi\left(x, \mu + \sqrt{h(x)}\right)} = -\frac{\partial_x \varphi\left(x, \mu - \sqrt{h(x)}\right)}{\partial_y \varphi\left(x, \mu - \sqrt{h(x)}\right)} = 0$ (with $\partial_x \varphi(x, \mu + \sqrt{h(x)}) = \frac{x^2 - \sigma^2 - h(x)}{x^3} = 0$).

[18] More precisely, quasi-concave programming supposes the maximization of quasi-concave objective functions subject to non-negativity constraints defined from quasi-concave constraint functions. We show in the proofs of

aversion either to bias or to noise, induces the concentration of demand of distortion-averse consumers on the left and right ends of the frontier. We moreover establish that "prone-to-bias", noise-averse consumers choose their news outlets in the neighborhood of the left end of the market. Among the types of noise-averse consumers reviewed below, only one, namely, the relative bias-averse type, may choose its news provider at the "noisy" (i.e. right) end. All others choose it in the vicinity of the "conventional" (i.e. left) end.

Let $\psi_1 = \mu + \sqrt{h}$ and $\psi_2 = \mu - \sqrt{h}$. The two functions $\left[\bar{\sigma}, \sigma(q^*)\right] \to \mathbb{R}$ defined by $\sigma_j \to U_i\left(\sigma_j, \psi_1(\sigma_j), p^*\right)$ and $\sigma_j \to U_i\left(\sigma_j, \psi_2(\sigma_j), p^*\right)$ describe consumer i's utility along the upper and lower branches of frontier $F(p^*)$ respectively. Subject to the assumptions of Propositions 4 and 5, they are twice continuously differentiable over $\left]\bar{\sigma}, \sigma(q^*)\right[$, and their first derivatives read $\sigma_j \to \partial_\sigma U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right) + \partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right).\partial\psi_k(\sigma_j)$, $k \in \{1,2\}$.

We know from the implicit function theorem that $\partial\psi_k(\sigma_j) = -\dfrac{\partial_x\varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}{\partial_y\varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}$.

Supposing $\partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right) \neq 0$, we see therefore that the sign of $\partial_\sigma U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right) + \partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right).\partial\psi_k(\sigma_j)$ is equal to the sign of difference

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Theorems 1 and 2 below that an interior maximum of a quasi-concave U_i in $F(p^*)$ must be either a local maximum of U_i subject to $\varphi(\sigma_j, \mu_j) - \frac{1}{q^*} \geq 0$ or a local maximum of U_i subject to $-\varphi(\sigma_j, \mu_j) + \frac{1}{q^*} \geq 0$. The non-linear functions $\varphi - \frac{1}{q^*}$ and $-\varphi + \frac{1}{q^*}$ cannot be simultaneously quasi-concave. Moreover, function $\varphi - \frac{1}{q^*}$ is clearly *not* quasi-concave. And it is not clear whether $-\varphi + \frac{1}{q^*}$ is quasi-concave or not in general (see part (iii) of the proof of Proposition 5). Hence the non-convexity referred to in main text above. Note nevertheless that, according to this criterion, prone-to-bias consumers may confront a convex optimization problem if $-\varphi + \frac{1}{q^*}$ is quasi-concave (their interior maximum, if any, turns out to be a local maximum of U_i subject to $\varphi(\sigma_j, \mu_j) \leq \frac{1}{q^*}$). Note also that relative bias-averse consumers clearly confront a non-convex problem, their interior maximum being a local maximum of U_i subject to $\varphi(\sigma_j, \mu_j) \geq \frac{1}{q^*}$ (see footnotes 21 and 22 in the proofs of the theorems).

$-\frac{\partial_x \varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}{\partial_y \varphi\left(\sigma_j, \psi_k(\sigma_j)\right)} - \left(-\frac{\partial_\sigma U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right)}{\partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right)}\right)$ if $\partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right) > 0$, and to the sign of difference $-\frac{\partial_\sigma U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right)}{\partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right)} - \left(-\frac{\partial_x \varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}{\partial_y \varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}\right)$ if $\partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right) < 0$.

The ratio $-\frac{\partial_\sigma U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right)}{\partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right)}$ is the marginal rate of substitution of noise for bias at point $\left(\sigma_j, \psi_k(\sigma_j)\right)$ of the frontier. The ratio $-\frac{\partial_x \varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}{\partial_y \varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}$ may be viewed, accordingly, with some terminological leeway, as the marginal rate of transformation of noise for bias at $\left(\sigma_j, \psi_k(\sigma_j)\right)$. Thus the sense of variation of the utility of distortion-averse consumers at any point of the frontier of equilibrium supply depends, essentially, on two determinants: the sign of $\psi_k(\sigma_j) - \mu$, that is, whether the news sold by firms of type j overestimate or underestimate, on average, raw expectation μ; and the sign of $-\frac{\partial_\sigma U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right)}{\partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right)} - \left(-\frac{\partial_x \varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}{\partial_y \varphi\left(\sigma_j, \psi_k(\sigma_j)\right)}\right)$, that is, whether the consumer's marginal rate of substitution of noise for bias is larger or smaller than the corresponding marginal rate of transformation.

Clearly, the utility of a distortion-averse consumer is decreasing along the part of the upper and lower branches of the frontier corresponding to low "noise levels", ranging in $\left]\bar{\sigma} - \sigma, \sigma.\left(\exp\left(\frac{1}{q^*}\right) - 1\right)\right]$. This is a simple consequence of distortion aversion, combined with the fact that noise and bias both increase to the right along this part of the frontier.

There is no such simple consequence of distortion aversion when noise reaches the higher levels ranging in $\left]\sigma.\left(\exp\left(\frac{1}{q^*}\right) - 1\right), \sigma\left(q^*\right) - \sigma\right[$, because noise and bias move in opposite directions along this part of the frontier (i.e. bias decreases as noise increases). This source of complexity in individual decisions motivates the following (rough) distinction between two types of distortion-averse consumers, namely, consumers whose noise aversion outweighs their bias aversion, and consumers who have the symmetrical, opposite characteristic, when they confront a bias-noise arbitrage on the market. From here on, we distinguish between bias and noise aversion in an *absolute* sense, as defined in section 4, and in a *relative* sense, as defined below.

Relative noise (resp. bias) aversion: Let U_i be distortion-averse over $F(p^*)$. We say that U_i exhibits *relative noise* (resp. *relative bias*) *aversion* if $\left|-\frac{\partial_\sigma U_i\left(\sigma_j,\psi_k(\sigma_j),p^*\right)}{\partial_\mu U_i\left(\sigma_j,\psi_k(\sigma_j),p^*\right)}\right| > \left|-\frac{\partial_x \varphi\left(\sigma_j,\psi_k(\sigma_j)\right)}{\partial_y \varphi\left(\sigma_j,\psi_k(\sigma_j)\right)}\right|$ (resp. $\left|-\frac{\partial_\sigma U_i\left(\sigma_j,\psi_k(\sigma_j),p^*\right)}{\partial_\mu U_i\left(\sigma_j,\psi_k(\sigma_j),p^*\right)}\right| < \left|-\frac{\partial_x \varphi\left(\sigma_j,\psi_k(\sigma_j)\right)}{\partial_y \varphi\left(\sigma_j,\psi_k(\sigma_j)\right)}\right|$) for all $\sigma_j \in \left[\sigma.\exp\left(\frac{1}{q^*}\right),\sigma\left(q^*\right)\right[$ and all $k \in \{1,2\}$.

Unsurprisingly, consumers whose preferences exhibit noise aversion in both the absolute and the relative sense choose their news providers on the boundary of the frontier, that is, inside $\left\{\left(\bar{\sigma},\psi_1(\bar{\sigma})\right),\left(\bar{\sigma},\psi_2(\bar{\sigma})\right)\right\}$, corresponding to the set of news products that establish the level of informational noise to the accessible minimum. Consumers whose preferences exhibit relative bias aversion make their choice inside $\left\{\left(\bar{\sigma},\psi_1(\bar{\sigma})\right),\left(\bar{\sigma},\psi_2(\bar{\sigma})\right),(\sigma(q^*),\mu)\right\}$, which notably contains the news outlet that is free of informational bias (and maximizes informational noise). These facts are summarized in Theorem 1 below and illustrated in Figure 4. The detailed proof is developed in the appendix.

Theorem 1: Suppose that: $p^* > 0$ is a competitive equilibrium price of the media market ; the associate equilibrium quality-scale mix (q^*, z^*) is unique; $\bar{\sigma} < \sigma.\exp\left(\frac{1}{q^*}\right)$; and for all i, u_i is C^2 and strictly concave and U_i exhibits absolute bias aversion over $F(p^*)$. Then, for all i, (i) U_i has at least one maximum in $F(p^*)$, and (ii) the set of maxima of U_i in $F(p^*)$ is contained in $\left\{\left(\bar{\sigma},\psi_1(\bar{\sigma})\right),\left(\bar{\sigma},\psi_2(\bar{\sigma})\right)\right\} \cup \left\{(\sigma_j,\mu_j) \in F(p^*) : \sigma_j^2 - \sigma^2 > \left(\mu_j - \mu\right)^2\right\}$. Moreover, (iii) if U_i is quasi-concave and displays relative noise aversion, then the set of maxima of U_i in $F(p^*)$ is contained in $\left\{\left(\bar{\sigma},\psi_1(\bar{\sigma})\right),\left(\bar{\sigma},\psi_2(\bar{\sigma})\right)\right\}$, and (iv) if U_i is quasi-concave and displays relative bias aversion, then the set of maxima of U_i in $F(p^*)$ is contained in $\left\{(\bar{\sigma},\psi_1(\bar{\sigma})),(\bar{\sigma},\psi_2(\bar{\sigma})),(\sigma(q^*),\mu)\right\}$.

"Prone-to-bias" preferences, finally, are easily accommodated in this setup. Theorem 2 below shows that prone-to-bias, noise-averse consumers choose their news providers at the upward sloping part of the upper half of the frontier (i.e., the half-part located above the symmetry axis L) when they favor overestimating biases, that is, if their utility is increasing in μ_j whenever $\mu_j \geq \mu$. Likewise, prone-to-bias, noise-averse consumers who favor underestimating biases (i.e. whose utility is decreasing in μ_j whenever $\mu_j \leq \mu$) choose their news providers at the downward sloping part of the lower half of the frontier. These behaviors mitigate the property of maximal horizontal differentiation implied by Theorem 1 without altering its main structural features, namely, the propensity of noise-averse consumers to choose their news providers in the "conventional" part of the frontier, that is, inside $\left\{(\sigma_j,\mu_j)\in F(p^*):\sigma_j^2-\sigma^2<\left(\mu_j-\mu\right)^2\right\}$.

Theorem 2: Suppose that: $p^*>0$ is a competitive equilibrium price of the media market ; the associate equilibrium quality-scale mix (q^*,z^*) is unique; $\bar{\sigma}<\sigma.\exp\left(\frac{1}{q^*}\right)$; and u_i is C^2 and strictly concave. Suppose moreover that U_i is quasi-concave and is prone-to-bias in one of the following two senses: it is strictly increasing in its second argument over $\left\{(\sigma_j,\mu_j)\in F(p^*):\mu_j\geq\mu\right\}$; or it is strictly decreasing in its second argument over $\left\{(\sigma_j,\mu_j)\in F(p^*):\mu_j\leq\mu\right\}$. Then: (i) U_i has at least one maximum in $F(p^*)$; and (ii) the set of maxima of U_i in $F(p^*)$ is contained in $\left\{(\sigma_j,\mu_j)\in F(p^*):\sigma_j^2-\sigma^2<\left(\mu_j-\mu\right)^2\right\}$.

7-An interpretation in terms of journalistic practice

Central to the model is the description of journalism as accurate fact reporting. Accordingly, its properties may be interpreted as a (stylized) description of the good practices of this profession's elite. Journalism itself is construed here as a risky activity striking some balance between two conflicting aims: on the one hand, drawing attention to *novel* facts of supposedly general interest, preferably "breaking news" and ideally "scoops" (i.e., exclusive breaking news), and, on the other hand, guaranteeing the *accuracy* of the novel facts conveyed to public attention by spending sufficient time and other scarce resources checking and assessing them. Thus horizontal differentiation opposes two complementary styles of good journalistic practice. One style of journalism prioritizes novelty by privileging scoops, thereby increasing the chances of revealing "truth", that is, of performing accurate reporting in a *timely* manner, at the expense of severely deviating from the truth on many specific occasions. The other style of journalism shies away from novelty, thereby reducing the risk of missing the point by too large a margin on any specific occasion, at the expense of systematically "lagging behind" accurate reporting on average in the long run.

The model thus interpreted provides a rationale and partial explanation for the distinction, briefly referred to in our introduction, between "mainstream" and "alternative" news media. The type of product differentiation implied by this distinction is well established in political science.[19] Lyubareva et al. (2020) provide an empirical account of this fact for the French news media, which appears to be very close to the interpretation above. They evaluate the "originality" of news products by computing an index of linguistic distance between the news published by a media and the formulation of the same facts in the dispatches of the *Agence France Presse* (the basic common information source of the French press). The similarity graph they draw from these statistics (their Figure 1, p. 153) exhibits two characteristic features. First, the overall (linguistic) distance between the news published by the media of the studied sample appears small. And second, the graph concentrates a large fraction of the sample into two groups, one of them, by far the largest, concentrated in close proximity to the *Agence France Presse*, and the second, much smaller in size, in close proximity to *Mediapart*, at some distance from the *Agence France Presse*. The first group includes all the main titles of the French "mainstream" news media, whereas the second group is rich in innovative pure players.

Lyubareva et al. (2020) interpret originality in terms of information quality, and view it as a dimension of vertical differentiation of news outlets (they assume that, *ceteris paribus*, all consumers wish more originality). Our own interpretation emphasizes novelty and identifies the quality of information with its accuracy, construed as a bi-dimensional object involving an evaluation of both bias and noise. Thus, the type of differentiation at stake is viewed as horizontal. Note that the linguistic distance between the factual accounts published by the

[19] See for example the recent survey by the Pew Research Center (2021) relative to the news media of the USA: https://www.pewresearch.org/fact-tank/2021/05/07/broad-agreement-in-u-s-even-among-partisans-on-whichnews-outlets-are-part-of-the-mainstream-media/

various news media is commonly found to be small in the literature of political science (as this is actually the case in the study of Lyubareva et al.). This latter type of facts leans toward an interpretation of the distinction between mainstream and alternative news medias as a form of horizontal, rather than vertical, differentiation. That is, such facts support the view that the news provided by the major mainstream and alternative media, considered from the standpoint of the journalistic standards and skills they implement, actually are very close in quality.

8- Conclusion

We studied the fact reporting activity of non-manipulative news providers operating on competitive markets for news. In this setup, consumers' noise aversion results in the horizontal differentiation of news products. The general view sketched in the theorems suggests that most noise-averse consumers will actually choose their news provider in the vicinity of the "conventional" end of the market, and also that some, among the consistently "relative" bias-averse individuals, will choose it at the opposite, noisy end.

We noticed in the introduction that the notions of bias and noise mobilized in this paper match those of Kahneman et al. (2021). The latter do not include news media in their wide-ranging review of empirical findings relative to noise and human judgment. We concur with these authors in emphasizing the general relevance and importance of noise, both factually and normatively, as a basic determinant of the quality of information, and we claim that this general statement applies to the sub-field of news media as well.

Accurate fact reporting is a basic norm of good journalistic practice. It is debatable whether, and to what extent, the actual practices of journalism do conform with this professional norm. Nevertheless, it seems to us that intentional distortion, in the *literal* sense given to distortion in the present paper, is neither sustainable nor sustained in ordinary practice. Empirical evidence on this specific type of manipulative misreporting (i.e. the deliberate, systematic reporting of *falsified* facts) is mixed, to say the least, and the type of facts briefly mentioned in section 7 does not support the view that it is an ordinary or even a widespread practice of standard news media, whether mainstream or alternative. This remark does not apply to the more complex editorial practices involved in the notion of filtering bias of Gentzkow and al. (2015). They include the selection of editorially relevant topics, the interpretation of the corresponding facts, the way these facts are presented, commented, and included in an editorial narrative or framework. Bias, understood in the latter sense, is compatible with accurate fact reporting in the *practical* sense, that is, with the reporting of facts that are checked according to the ongoing journalistic standards for fact checking. It is extensively practiced, normatively sustainable, and sustained as a legitimate expression of the diversity of opinions, whether political or otherwise. We also mentioned in section 3 the specific issues relative to the assumption of perfect competition in the case of the market for news. We excluded firms' strategic motives for product differentiation from our setup for the sake of analytical clarity. Further research on this topic could reintroduce them by combining, for example, consumers' noise aversion with filtering bias and Hotelling price setting on behalf of firms.

The role that bias and noise play in the model, as determinants of the quality of information, and as characteristics (in Lancaster's sense) of news products, owes much to the assumption that raw facts are described through a univariate normal distribution. We conclude below with a brief outlook over possible extensions to more general hypotheses for future research.

A first, and actually straightforward extension to the case of multivariate normal distributions would presumably preserve the main properties of the univariate case, with some additional complexities due to the fact that the covariance matrix then substitutes for the standard deviation in the calculation of the KL-divergence. Other extensions of the same type can be considered, to cases of statistical distributions yielding tractable formulas for this calculation. Kullback (1978) provides a comprehensive list, which notably includes Poisson and multinomial distributions.

A second extension would substitute Blackwell's dominance criterion for KL-divergence, in the characterization of accurate fact reporting. We mentioned in the introduction that this was the main option retained by Gentzkow et al. (2015) in their review of media bias theory. We moreover emphasized, in section 4, the close connection between noise aversion and the concavity of Bernoulli utility function. A setup which would characterize, jointly, *accuracy* through Blackwell's criterion, and *noise aversion* through the concavity of Bernoulli utility functions, would have the great advantage of freeing the analysis from the specific characteristics of the statistical distributions describing raw data. A criterion of second-order stochastic dominance could be introduced, notably, in order to complement Blackwell's first-order dominance criterion in a way that would consistently integrate noise aversion in the evaluation of the quality of information. We suggest this type of approach as the most promising one, although presumably also the most demanding one, for future theoretical research on this topic.

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Appendix I: Kullback-Leibler divergence and the maximum likelihood criterion

We illustrate the well-known connection between K-L divergence minimization and likelihood maximization through the following basic textbook argument. Suppose that a media of type j collects, from the flow of raw data d, a sample of N pairwise distinct, identically independently distributed observations $S=\{d_1,...,d_N\}$. The empirical law of d built from sample S is $\hat{p}(s)=\frac{1}{N}\sum_{i=1}^{N}\delta(s-d_i)$, where $\delta:\mathbb{R}\to\mathbb{R}$ denotes the Dirac measure (i.e.

$\delta(s-d_i)$ is equal to 0 everywhere except at 0 where it is equal to 1). The firm believes that the true distribution is parametric, with unknown parameters θ. Let p_θ denote the corresponding distribution. The K-L divergence of p_θ relative to $\hat{p}$ reads: $D(\hat{p}\|p_\theta)=\sum_{s\in S}\hat{p}(s)\log\frac{\hat{p}(s)}{p_\theta(s)}$.

A simple calculation yields: $D(\hat{p}\|p_\theta)=\sum_{s\in S}\hat{p}(s)\log\hat{p}(s)-\frac{1}{N}\sum_{s\in S}\log p_\theta(s)$. Let $H(\hat{p})=-\sum_{s\in S}\hat{p}(s)\log\hat{p}(s)$ denote the Shannon entropy of the empirical distribution. The first term in the difference above is equal to $-H(\hat{p})$. The second term in the difference, $\frac{1}{N}\sum_{s\in S}\log p_\theta(s)$, is the log-likelihood of the parametric distribution (divided by the number of observations). For a firm of type j that wants to estimate the parameters θ from sample S, it is equivalent, in particular, to deriving θ from the maximization of the likelihood of p_θ or to computing it from the minimization of $D(\hat{p}\|p_\theta)$.

Appendix II: Examples of calculable supply equilibria

Example 2-Competitive supply equilibrium with convex costs

We suppose technically separable data-processing and broadcasting activities as in Example 1, and moreover suppose parabolic costs for each of them, that is, $c(q_j,z_j)=q_j^2+z_j^2$. The necessary first-order condition (f.o.c.) for profit maximization of Proposition 2-(i) readily implies that $p=2$, that is, the competitive equilibrium price is determined by technology in this case. The necessary second-order condition (s.o.c.) of Proposition 2-(ii) is then satisfied, as $p=2=\partial^2_{qz}c(q^*,z^*)+\sqrt{\partial^2_{qq}c(q^*,z^*).\partial^2_{zz}c(q^*,z^*)}$. The f.o.c. also implies that: $\frac{z^*}{q^*}=1$, that is, any profit-maximizing quality-scale mix (q^*,z^*) must be located on ray $\{(q_j,z_j)\in\mathbb{R}^2_+ : z_j=q_j\}$ (see Figure 2); and also $p.q^*.z^*=\partial_q c(q^*,z^*).q^*=\partial_z c(q^*,z^*).z^*$. Moreover, Euler's identity for homogeneous functions implies $2.c(q_j,z_j)=\partial_q c(q_j,z_j).q_j+\partial_z c(q_j,z_j).z_j$ for all (q_j,z_j), and therefore $p.q^*.z^*-c(q^*,z^*)=0$; that is, the equilibrium profit is null. One easily verifies that $2.q_j.z_j-c(q_j,z_j)=-(q_j-z_j)^2$ is null everywhere along ray $\{(q_j,z_j)\in\mathbb{R}^2_+ : z_j=q_j\}$ and negative otherwise. To sum up: the unique competitive equilibrium price of this example is

$p = 2$; any quality-scale mix (q^*, z^*) of $\{(q_j, z_j) \in \mathbb{R}^2_+ : z_j = q_j\}$ maximizes type j's profit at this price; the equilibrium profit is null.

Example 3-Symmetric log-linear supply function

We consider here a variant of Example 2, with an additively separable, symmetric, strictly convex cost function of the type $c(q_j, z_j) = q_j^\alpha + z_j^\alpha$, $\alpha > 2$. The f.o.c. for profit-maximization readily implies $\frac{z^*}{q^*} = 1$ and $q^* = z^* = \left(\frac{p}{\alpha}\right)^{\frac{1}{\alpha-2}}$. We obtain $\partial^2_{qz} c(q^*, z^*) + \sqrt{\partial^2_{qq} c(q^*, z^*).\partial^2_{zz} c(q^*, z^*)} = (\alpha - 1).p > p$ for all $p > 0$, which implies that $(q^*, z^*) = \left(\left(\frac{p}{\alpha}\right)^{\frac{1}{\alpha-2}}, \left(\frac{p}{\alpha}\right)^{\frac{1}{\alpha-2}}\right)$ is then the unique profit-maximizing quality-scale mix. If $p = 0$, then the unique profit-maximizing quality-scale mix is obviously the null supply $(0,0)$, generating a null profit. That is, the supply function of each firm of type j is well-defined over $\mathbb{R}_+$. It reads $(q_j(p), z_j(p)) = \left(\left(\frac{p}{\alpha}\right)^{\frac{1}{\alpha-2}}, \left(\frac{p}{\alpha}\right)^{\frac{1}{\alpha-2}}\right)$. Euler's identity for homogeneous functions and the f.o.c. together imply $\alpha.c(q^*, z^*) = \partial_q c(q^*, z^*).q^* + \partial_z c(q^*, z^*).z^* = 2.p.q^*.z^*$, and therefore $p.q^*.z^* - c(q^*, z^*) > p.q^*.z^* - \frac{\alpha}{2} c(q^*, z^*) = 0$ for all $p > 0$. That is, the maximized profit is positive if (and only if) the price is positive.[20]

[20] Note that $\frac{z_j}{q_j} = \frac{\partial_q c(q_j, z_j)}{\partial_z c(q_j, z_j)}$ at firm's equilibrium, which means that "proportion" $\frac{z_j}{q_j}$ is equal to the marginal rate of substitution $\frac{\partial_q c(q_j, z_j)}{\partial_z c(q_j, z_j)}$ at any profit-maximizing quality-scale mix (q_j, z_j). It seems reasonable to expect, under real conditions, a small marginal cost of broadcasting $\partial_z c(q_j, z_j)$, relative to the marginal cost of quality $\partial_q c(q_j, z_j)$, that is, a large marginal rate of substitution at equilibrium. Equilibrium conditions then imply a large ratio $\frac{z_j}{q_j}$, corresponding to the wide broadcasting of information sheets of equilibrium quality q_j. Naturally this feature of actual mass-media markets is not captured through the symmetric cost functions of the former two examples.

Appendix III: Proofs

Proof of Proposition 1: This follows, as a special case, from Kullback (1978), Theorem 3.1 of chapter 2. ■

Proof of Proposition 2: Propositions 2-(i) and 2-(ii) are clear enough. The proof of the remainder is a simple consequence of the computation of the following first and second partial derivatives of function φ. We get $\partial_x \varphi(x,y) = \dfrac{x^2 - \sigma^2 - (y-\mu)^2}{x^3}$, which readily implies Propositions 2-(iii) and 2-(iv). We have $\partial^2_{yy}\varphi(x,y) = \dfrac{1}{x^2} > 0$ for all $(x,y) \in \mathbb{R}_{++} \times \mathbb{R}$. This implies the strict convexity of partial functions $y \to \varphi(x,y)$ for all $x \in \mathbb{R}_{++}$.■

Proof of Proposition 3: Recall that we assume that the cost function is C^2 in $\mathbb{R}^2_{++}$ and strictly convex.

The differentiability assumption implies that the first-order necessary conditions and the second-order conditions for an interior solution to $\max\left\{p.q_j.z_j - c(q_j,z_j) : (q_j,z_j) \in [0,+\infty[\times [0,+\infty[\right\}$ hold true at (q^*,z^*) (e.g. Mas-Colell (1985), D1 and D2).

The first-order necessary conditions read $p.z^* - \partial_q c(q^*,z^*) = p.q^* - \partial_z c(q^*,z^*) = 0$. They entail Proposition 3-(i).

The Hessian matrix of $(q_j,z_j) \to p.q_j.z_j - c(q_j,z_j)$ at (q^*,z^*) reads $H(q^*,z^*) = \begin{pmatrix} -\partial^2_{qq}c(q^*,z^*) & p - \partial^2_{qz}c(q^*,z^*) \\ p - \partial^2_{zq}c(q^*,z^*) & -\partial^2_{zz}c(q^*,z^*) \end{pmatrix}$. Its diagonal elements are <0, by strict convexity of the cost function. Its determinant reads $\left|H(q^*,z^*)\right| = \partial^2_{qq}c(q^*,z^*).\partial^2_{zz}c(q^*,z^*) - \left(p - \partial^2_{qz}c(q^*,z^*)\right)^2$. Sylvester's criterion implies therefore that the matrix is negative semi-definite (resp. negative definite) if and only if $\left|H(q^*,z^*)\right| \geq 0$ (resp. $\left|H(q^*,z^*)\right| > 0$).

The second-order necessary condition for an interior solution to $\max\left\{p.q_j.z_j - c(q_j,z_j) : (q_j,z_j) \in [0,+\infty[\times [0,+\infty[\right\}$ (i.e. a negative semi-definite $H(q^*,z^*)$) entails Proposition 3-(ii).

Finally, suppose that $\left|p - \partial^2_{qz}c(q^*,z^*)\right| < \sqrt{\partial^2_{qq}c(q^*,z^*).\partial^2_{zz}c(q^*,z^*)}$. Then $H(q^*,z^*)$ is negative definite. Moreover, it remains so in some open neighborhood V of (q^*,z^*) in $\mathbb{R}^2_{++}$, by continuity of function $(q_j,z_j) \to \left|H(q_j,z_j)\right|$. This implies in turn that $(q_j,z_j) \to p.q_j.z_j - c(q_j,z_j)$ is strictly concave in V. Let us prove that (q^*,z^*) is the unique solution to $\max\left\{p.q_j.z_j - c(q_j,z_j) : (q_j,z_j) \in [0,+\infty[\times [0,+\infty[\right\}$. Suppose the contrary, that is, suppose that there exists $(\tilde{q}_j,\tilde{z}_j) \neq (q^*,z^*)$ that solves $\max\left\{p.q_j.z_j - c(q_j,z_j) : (q_j,z_j) \in [0,+\infty[\times [0,+\infty[\right\}$, and let us derive a contradiction. By assumption, we have $p.\tilde{q}_j.\tilde{z}_j - c(\tilde{q}_j,\tilde{z}_j) = p.q^*.z^* - c(q^*,z^*)$. For any real number $\lambda \in [0,1]$, let $(q^\lambda_j, z^\lambda_j)$ denote convex combination $\lambda.(\tilde{q}_j,\tilde{z}_j) + (1-\lambda).(q^*,z^*)$. We have $(q^\lambda_j, z^\lambda_j) \in V$ for any $\lambda > 0$ picked sufficiently close to 0. The strict concavity of $(q_j,z_j) \to p.q_j.z_j - c(q_j,z_j)$ in V then implies :

$$p.q^\lambda_j.z^\lambda_j - c(q^\lambda_j,z^\lambda_j) > \lambda.\left(p.\tilde{q}_j.\tilde{z}_j - c(\tilde{q}_j,\tilde{z}_j)\right) + (1-\lambda).\left(p.q^*.z^* - c(q^*,z^*)\right) = p.q^*.z^* - c(q^*,z^*),$$

the wished contradiction.■

Proof of Proposition 4: We have $V_i(\sigma_j,\mu_j) = \int_{-\infty}^{+\infty} u_i(n^j_e)\frac{1}{\sigma_j\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{(n^j_e-\mu_j)^2}{\sigma^2_j}\right)dn^j_e$. Let $v^j_e = \frac{n^j_e - \mu_j}{\sigma_j}$ and $g(v^j_e) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(v^j_e)^2\right)$, and apply the following change of variable: $V_i(\sigma_j,\mu_j) = \int_{-\infty}^{+\infty} u_i(v^j_e.\sigma_j + \mu_j).g(v^j_e)dv^j_e$.

Differentiating with respect to σ_j, we get: $\partial_{\sigma_j}V_i(\sigma_j,\mu_j) = \int_{-\infty}^{+\infty} \partial u_i(v^j_e.\sigma_j + \mu_j).v^j_e.g(v^j_e)dv^j_e$. Function g is positive and symmetrical, that is, $g(v^j_e) > 0$ and $g(v^j_e) = g(-v^j_e)$ for all $v^j_e \in \mathbb{R}$. Function ∂u_i is decreasing, by strict concavity of u_i. Therefore, for all $v^j_e > 0$, we have $v^j_e g(v^j_e) = v^j_e g(-v^j_e) > 0$ and $\partial u_i(-v^j_e\sigma^j_e + \mu_j) > \partial u_i(v^j_e\sigma^j_e + \mu_j)$. These inequalities together imply

$\partial u_i(-v_e^j\sigma_e^j+\mu_j).v_e^j.g(-v_e^j)>\partial u_i(v_e^j\sigma_e^j+\mu_j).v_e^j.g(v_e^j)$ for all $v_e^j>0$. Integrating both sides of the latter inequality, we get:

$$\int_0^{+\infty}\partial u_i(-v_e^j\sigma_e^j+\mu_j).v_e^j.g(-v_e^j)dv_e^j>\int_0^{+\infty}\partial u_i(v_e^j\sigma_e^j+\mu_j).v_e^j.g(v_e^j)dv_e^j$$

The left-hand side of this inequality is equal to $-\int_{-\infty}^{0}\partial u_i(v_e^j\sigma_e^j+\mu_j).v_e^j.g(v_e^j)dv_e^j$. Therefore we get $0>\partial_{\sigma_j}V_i(\sigma_j,\mu_j)=\partial_{\sigma_j}U_i(\sigma_j,\mu_j,p)$.

Differentiating with respect to μ_j, we get:

$$\partial_{\mu_j}U_i(\sigma_j,\mu_j,p)=\partial_{\mu_j}V_i(\sigma_j,\mu_j)=\int_{-\infty}^{+\infty}\partial u_i(v_e^j.\sigma_j+\mu_j).g(v_e^j)dv_e^j$$

Since the sign of $\partial u_i(v_e^j.\sigma_j+\mu_j)$ is indeterminate, the sign of $\partial_{\mu_j}U_i(\sigma_j,\mu_j,p)$ is indeterminate as well.■

Proof of Proposition 5: We suppose that $\sigma<\bar{\sigma}<\sigma.\exp\left(\frac{1}{q^*}\right)$.

(i) We have $\partial h(x)=4.\left(\frac{1}{q^*}-\log\left(\frac{x}{\sigma}\right)\right).x$. Therefore $\partial h(x)$ is >0 over $\left[\bar{\sigma},\sigma.\exp\left(\frac{1}{q^*}\right)\right[$ (and actually over $\left]0,\sigma.\exp\left(\frac{1}{q^*}\right)\right[$), =0 at $\sigma.\exp\left(\frac{1}{q^*}\right)$, and <0 over $\left]\sigma.\exp\left(\frac{1}{q^*}\right),+\infty\right[$. In particular, function h reaches a maximum at $x=\sigma.\exp\left(\frac{1}{q^*}\right)$ over $[\bar{\sigma},+\infty[$. Moreover, we have $h\left(\sigma.\exp\left(\frac{1}{q^*}\right)\right)=\left(\sigma.\exp\left(\frac{1}{q^*}\right)\right)^2-\sigma^2>0$ and $\lim_{x\to+\infty}h(x)=-\infty$. The continuity of h then implies the existence of at least one $x\in\left]\sigma.\exp\left(\frac{1}{q^*}\right),+\infty\right[$ such that $h(x)=0$. Since h is (strictly) decreasing over $\left]\sigma.\exp\left(\frac{1}{q^*}\right),+\infty\right[$, the latter is unique. It is denoted by $\sigma(q^*)$ below.

(ii) We have $h(\sigma)=\frac{2}{q^*}\sigma^2>0$ and $\partial h(x)>0$ for all $x\in\left[\sigma,\sigma.\exp\left(\frac{1}{q^*}\right)\right[$. Therefore, function h is positive over $\left[\sigma,\sigma.\exp\left(\frac{1}{q^*}\right)\right[$. The proof of part (i) moreover implies that h is positive also over $\left[\sigma.\exp\left(\frac{1}{q^*}\right),\sigma(q^*)\right[$, and that $h(\sigma(q^*))=0$. Therefore $\sqrt{h}$ is well-defined over $\left[\bar{\sigma},\sigma(q^*)\right]$, positive over $\left[\bar{\sigma},\sigma(q^*)\right[$, and such that $\sqrt{h(\sigma(q^*))}=0$. We have $\partial\sqrt{h(x)}=\frac{1}{2}.\frac{\partial h(x)}{\sqrt{h(x)}}$, which is therefore finite over $\left[\bar{\sigma},\sigma(q^*)\right[$, >0 over $\left[\bar{\sigma},\sigma.\exp\left(\frac{1}{q^*}\right)\right[$, =0 at $\sigma.\exp\left(\frac{1}{q^*}\right)$, and <0 over $\left]\sigma.\exp\left(\frac{1}{q^*}\right),\sigma(q^*)\right[$. In particular, $\sqrt{h}$ is increasing over $\left[\bar{\sigma},\sigma.\exp\left(\frac{1}{q^*}\right)\right[$, and decreasing over $\left]\sigma.\exp\left(\frac{1}{q^*}\right),\sigma(q^*)\right[$. It is clearly C^∞ over $\left[\bar{\sigma},\sigma(q^*)\right[$ and such that $\lim_{x\to\sigma(q^*)^-}\partial\sqrt{h(x)}=-\infty$.

(iii) We have $\partial^2 h(x)=-4.\left(1-\frac{1}{q^*}+\log\left(\frac{x}{\sigma}\right)\right)$. This implies that $\partial^2 h(x)<0$ for all $x\in\left]\sigma.\exp\left(\frac{1}{q^*}-1\right),+\infty\right[$. Since $\sigma.\exp\left(\frac{1}{q^*}-1\right)<\sigma.\exp\left(\frac{1}{q^*}\right)$, there exists a positive real number $\delta=\left(1-\frac{1}{e}\right).\sigma.\exp\left(\frac{1}{q^*}\right)$ such that $\partial^2 h(x)<0$ for all $x\in\left]-\delta+\sigma.\exp\left(\frac{1}{q^*}\right),+\infty\right[$.

We have $\partial^2\sqrt{h(x)}=\frac{1}{2\sqrt{h(x)}}.\left(\partial^2 h(x)-\frac{1}{2}.\frac{(\partial h(x))^2}{h(x)}\right)$ wherever it is well-defined. We established above that $\partial^2 h$ is negative over $\left]-\delta+\sigma.\exp\left(\frac{1}{q^*}\right),\sigma(q^*)\right]$, and that h is positive over $\left[\bar{\sigma},\sigma(q^*)\right[$. Therefore, there exists a positive real number $\varepsilon=\min\left\{\delta,\sigma.\exp\left(\frac{1}{q^*}\right)-\bar{\sigma}\right\}$ such that $\partial^2\sqrt{h(x)}<0$ for all $x\in\left]-\varepsilon+\sigma.\exp\left(\frac{1}{q^*}\right),\sigma(q^*)\right[$.

(iv) We have $\partial_y\varphi(x,y)=\frac{y-\mu}{x^2}\neq 0$ for all $x>0$ and all $y\neq\mu$. Part (iv) therefore is a simple consequence of part (i) and of the implicit function theorem applied to $\varphi(x,y)=\frac{1}{q^*}$ at any $(x,y)\in F(p^*)$ such that $x\neq\sigma(q^*)$.

(v) Recall that $F(p^*)=\left\{(\sigma_j,\mu_j)\in\left[\bar{\sigma},+\infty\right[\times\mathbb{R}:\varphi(\sigma_j,\mu_j)=\frac{1}{q^*}\right\}$. Function φ being C^∞ and such that $\partial_x\varphi(\sigma(q^*),\mu)=\frac{\sigma(q^*)^2-\sigma^2}{\sigma(q^*)^3}\neq 0$, the implicit function theorem implies the existence of $\varepsilon\in\mathbb{R}_{++}$ and of a function $g:\left]\mu-\varepsilon,\mu+\varepsilon\right[\to\mathbb{R}$ that is C^∞, and such that $g(\mu)=\sigma(q^*)$, $\varphi(g(y),y)=\frac{1}{q^*}$, and $\partial g(y)=-\frac{\partial_y\varphi(g(y),y)}{\partial_x\varphi(g(y),y)}$ for all $y\in\left]\mu-\varepsilon,\mu+\varepsilon\right[$. Let $\psi_1=\mu+\sqrt{h}$ and $\psi_2=\mu-\sqrt{h}$. Function g, restricted to $\left[\mu,\mu+\varepsilon\right[$ (resp. $\left]\mu-\varepsilon,\mu\right]$), is the local inverse of ψ_1 (resp. ψ_2).

$g\left(\left]\mu,\mu+\varepsilon\right[\right)$ and $g\left(\left]\mu-\varepsilon,\mu\right[\right)$ are open intervals diffeomorphic to $\left]\mu,\mu+\varepsilon\right[$ and $\left]\mu-\varepsilon,\mu\right[$ respectively. Let f (resp. k) denote the inverse of the restriction of $\sqrt{h}$ (resp. $-\sqrt{h}$) to $g\left(\left]\mu,\mu+\varepsilon\right[\right)$ (resp. $g\left(\left]\mu-\varepsilon,\mu\right[\right)$). By construction we have $g(y)=f(y-\mu)$ for all $y\in\left]\mu,\mu+\varepsilon\right[$ and $g(y)=k(y-\mu)$ for all $y\in\left]\mu-\varepsilon,\mu\right[$. Moreover, by symmetry, we have $g\left(\left]\mu,\mu+\varepsilon\right[\right)=g\left(\left]\mu-\varepsilon,\mu\right[\right)$ (this holds true because $g(y)=f(y-\mu)=k((2\mu-y)-\mu)=g(2\mu-y)$ for all $y\in\left]\mu,\mu+\varepsilon\right[$, and $2\mu-y$ runs over $\left]\mu-\varepsilon,\mu\right[$ when y runs over $\left]\mu,\mu+\varepsilon\right[$).

We know from parts (ii) and (iii) of the proof above that $\sqrt{h}$ is C^∞, decreasing and differentiably strictly concave over $g\left(\left]\mu,\mu+\varepsilon\right[\right)=g\left(\left]\mu-\varepsilon,\mu\right[\right)$ if ε is picked sufficiently close to 0. f is decreasing, as inverse of decreasing $\sqrt{h}$. Moreover, $\partial^2 f=-\frac{\partial^2\sqrt{h}\circ f}{\left(\partial\sqrt{h}\circ f\right)^3}$. Therefore $\partial g(y)=\partial f(y-\mu)<0$ and $\partial^2 g(y)=\partial^2 f(y-\mu)<0$ for all $y\in\left]\mu,\mu+\varepsilon\right[$. That is, g is decreasing and differentiably strictly concave over $\left]\mu,\mu+\varepsilon\right[$. We establish in the same way that g is increasing and differentiably strictly concave over $\left]\mu-\varepsilon,\mu\right[$ (since $\partial^2 k=-\frac{\partial^2\left(-\sqrt{h}\right)\circ k}{\left(\partial\left(-\sqrt{h}\right)\circ k\right)^3}$, $\partial^2\left(-\sqrt{h}\right)>0$ and $\partial\left(-\sqrt{h}\right)>0$).

We have $\partial_x \varphi(x,y) = \frac{x^2-\sigma^2-(y-\mu)^2}{x^3}$ and $\partial_y \varphi(x,y) = \frac{y-\mu}{x^2}$, and therefore $\partial g(y) = -\frac{g(y)\left(y-\mu\right)}{g(y)^2-\sigma^2-(y-\mu)^2}$ for all $y \in \left]\mu-\varepsilon, \mu+\varepsilon\right[$. Differentiating the latter identity, and observing that $\partial g(\mu)=0$, we get $\partial^2 g(\mu)=0$.■

Proof of Theorem 1: (i) We know from Proposition 5 that $F(p^*)$ is nonempty and compact under the assumptions of Theorem 1. The continuity of utility functions therefore implies the existence of at least one maximum of U_i in $F(p^*)$ for all i.

(ii) We know from Proposition 4 that, under the assumptions of Theorem 1, U_i exhibits distortion aversion in $F(p^*)$ for all i, that is, utility functions are decreasing both in $\left|\mu_j-\mu\right|$ and in σ_j over $F(p^*)$. Recall that $\psi_1 = \mu+\sqrt{h}$ and $\psi_2 = \mu-\sqrt{h}$. Noise aversion implies $\partial_\sigma U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right) < 0$ for all $x \in \left[\bar{\sigma}, \sigma.\exp\left(\frac{1}{q^*}\right)\right]$. Bias aversion implies $\partial_\mu U_i\left(\sigma_j, \mu_j, p^*\right) < 0$ if $\mu_j > \mu$ and $\partial_\mu U_i\left(\sigma_j, \mu_j, p^*\right) > 0$ if $\mu_j < \mu$. The latter and Proposition 5 then imply $\partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right).\partial\psi_k(\sigma_j) < 0$ for all $x \in \left[\bar{\sigma}, \sigma.\exp\left(\frac{1}{q^*}\right)\right]$, $k=1,\ 2$. Therefore functions $\sigma_j \to U_i\left(\sigma_j, \psi_1(\sigma_j), p^*\right)$ and $\sigma_j \to U_i\left(\sigma_j, \psi_2(\sigma_j), p^*\right)$ are decreasing over $\left[\bar{\sigma}, \sigma.\exp\left(\frac{1}{q^*}\right)\right]$, that is, $\partial_\sigma U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right) + \partial_\mu U_i\left(\sigma_j, \psi_k(\sigma_j), p^*\right).\partial\psi_k(\sigma_j) < 0$ for all $x \in \left[\bar{\sigma}, \sigma.\exp\left(\frac{1}{q^*}\right)\right]$, $k=1,\ 2$. This readily implies that the set of maxima of U_i in $F(p^*)$ is contained in $\left\{\left(\bar{\sigma}, \psi_1(\bar{\sigma})\right), \left(\bar{\sigma}, \psi_2(\bar{\sigma})\right)\right\} \cup \left\{(\sigma_j, \mu_j) \in F(p^*) : \sigma_j > \sigma.\exp\left(\frac{1}{q^*}\right)\right\}$. Finally, $\left\{(\sigma_j, \mu_j) \in F(p^*) : \sigma_j > \sigma.\exp\left(\frac{1}{q^*}\right)\right\} = \left\{(\sigma_j, \mu_j) \in F(p^*) : \sigma_j^2 - \sigma^2 > \left(\mu_j-\mu\right)^2\right\}$ as a consequence of Proposition 5.

(iii) In this third part of the proof, we assume that U_i is quasi-concave and displays relative noise aversion.

Let (σ^*,μ^*) denote a maximum of U_i in $F(p^*)$.

We first establish that (σ^*,μ^*) must be either a local maximum of U_i in $\left\{(\sigma_j,\mu_j)\in\mathbb{R}\times\mathbb{R}:\sigma_j\geq\bar{\sigma},\ \varphi(\sigma_j,\mu_j)\geq\frac{1}{q^*}\right\}$ or a local maximum of U_i in $\left\{(\sigma_j,\mu_j)\in\mathbb{R}\times\mathbb{R}:\sigma_j\geq\bar{\sigma},\ \varphi(\sigma_j,\mu_j)\leq\frac{1}{q^*}\right\}$. Suppose the contrary. Then there exist $(\hat{\sigma}_j,\hat{\mu}_j)\in\mathbb{R}\times\mathbb{R}$ and $(\tilde{\sigma}_j,\tilde{\mu}_j)\in\mathbb{R}\times\mathbb{R}$ such that : $\hat{\sigma}_j\geq\bar{\sigma}$, $\varphi(\hat{\sigma}_j,\hat{\mu}_j)>\frac{1}{q^*}$ and $U_i(\hat{\sigma}_j,\hat{\mu}_j)>U_i(\sigma^*,\mu^*)$; and $\tilde{\sigma}_j\geq\bar{\sigma}$, $\varphi(\tilde{\sigma}_j,\tilde{\mu}_j)<\frac{1}{q^*}$ and $U_i(\tilde{\sigma}_j,\tilde{\mu}_j)>U_i(\sigma^*,\mu^*)$. By continuity of φ, there exists some real number $\lambda\in\left]0,1\right[$ such that $\varphi(\lambda\hat{\sigma}_j+(1-\lambda)\tilde{\sigma}_j,\lambda\hat{\mu}_j+(1-\lambda)\tilde{\mu}_j)=\frac{1}{q^*}$. We have $\lambda\hat{\sigma}_j+(1-\lambda)\tilde{\sigma}_j\geq\bar{\sigma}$. And the quasi-concavity of U_i implies $U_i(\lambda\hat{\sigma}_j+(1-\lambda)\tilde{\sigma}_j,\lambda\hat{\mu}_j+(1-\lambda)\tilde{\mu}_j)>U_i(\sigma^*,\mu^*)$, a contradiction.

We suppose from now on that $\sigma^*>\bar{\sigma}$ and we derive a contradiction.

Suppose first that $\sigma^*>\bar{\sigma}$ and (σ^*,μ^*) is a local maximum of U_i in $\left\{(\sigma_j,\mu_j)\in\mathbb{R}\times\mathbb{R}:\varphi(\sigma_j,\mu_j)\geq\frac{1}{q^*}\right\}$. The first-order necessary conditions (f.o.c.) for a local maximum of U_i in $\left\{(\sigma_j,\mu_j)\in\mathbb{R}\times\mathbb{R}:\varphi(\sigma_j,\mu_j)\geq\frac{1}{q^*}\right\}$ read as follows (e.g. Mas-Colell (1985), D1): There are $(\lambda,\nu)\in\mathbb{R}_+\times\mathbb{R}_+$ such that

(a) $(\lambda,\nu)\neq 0$,

(b) $\nu.\left(\varphi(\sigma^*,\mu^*)-\frac{1}{q^*}\right)=0$,,

(c) and $\lambda.\partial_\sigma U_i(\sigma^*,\mu^*,p^*)+\nu.\partial_x\varphi(\sigma^*,\mu^*)=\lambda.\partial_\mu U_i(\sigma^*,\mu^*,p^*)+\nu.\partial_y\varphi(\sigma^*,\mu^*)=0$.

Since (σ^*,μ^*) is a maximum of U_i in $F(p^*)$ such that $\sigma^*>\bar{\sigma}$, we must have $\sigma^*>\sigma.\exp\left(\frac{1}{q^*}\right)$ by part (ii) of the proof above. Proposition 5 then implies $\partial_x\varphi(\sigma^*,\mu^*)\neq 0$. Moreover, absolute noise aversion implies $\partial_\sigma U_i(\sigma^*,\mu^*,p^*)<0$. The f.o.c. (a) and (c) above then imply that multipliers λ and ν are both >0.

The same conclusions apply, with obvious adaptations, if we suppose that $\sigma^* > \bar{\sigma}$ and (σ^*, μ^*) is a local maximum of U_i in $\left\{(\sigma_j, \mu_j) \in \mathbb{R}\times\mathbb{R} : \varphi(\sigma_j, \mu_j) \leq \frac{1}{q^*}\right\}$ (simply substitute f.o.c. (c') $\lambda.\partial_\sigma U_i(\sigma^*, \mu^*, p^*) - \nu.\partial_x\varphi(\sigma^*, \mu^*) = \lambda.\partial_\mu U_i(\sigma^*, \mu^*, p^*) - \nu.\partial_y\varphi(\sigma^*, \mu^*) = 0$ for f.o.c. (c) in the argument above).

Suppose now that $\sigma.\exp\left(\frac{1}{q^*}\right) < \sigma^* < \sigma\left(q^*\right)$. We established in Proposition 5 that, then, $-\frac{\partial_x\varphi\left(\sigma^*, \mu^*\right)}{\partial_y\varphi\left(\sigma^*, \mu^*\right)}$ is well-defined and $\neq 0$. Since λ, ν, $\partial_x\varphi\left(\sigma^*, \mu^*\right)$ and $\partial_y\varphi\left(\sigma^*, \mu^*\right)$ are all $\neq 0$, $\partial_\sigma U_i\left(\sigma^*, \mu^*, p^*\right)$ and $\partial_\mu U_i\left(\sigma^*, \mu^*, p^*\right)$ must be $\neq 0$ as well by f.o.c. (c) and also by f.o.c. (c'). F.o.c. (c) and (c') then both imply in turn that $\left|-\frac{\partial_\sigma U_i\left(\sigma^*, \mu^*, p^*\right)}{\partial_\mu U_i\left(\sigma^*, \mu^*, p^*\right)}\right| = \left|-\frac{\partial_x\varphi\left(\sigma^*, \mu^*\right)}{\partial_y\varphi\left(\sigma^*, \mu^*\right)}\right|$, which contradicts relative noise aversion. Therefore $\sigma^* = \sigma\left(q^*\right)$.

Up to this point, we have established that $\sigma^* > \bar{\sigma}$ implies $\sigma^* = \sigma\left(q^*\right)$. We now prove that $(\sigma(q^*), \mu)$ is a local (strict) *minimum* of U_i in $F(p^*)$.

From Proposition 5, there exists $\varepsilon \in \mathbb{R}_{++}$ and a C^∞ function $g :]\mu-\varepsilon, \mu+\varepsilon[\to \mathbb{R}$ such that $g(\mu) = \sigma(q^*)$ and $(g(y), y) \in F(p^*)$ for all $y \in]\mu-\varepsilon, \mu+\varepsilon[$. Consider function $W_i :]\mu-\varepsilon, \mu+\varepsilon[\to \mathbb{R}$ defined by $W_i(y) = U_i(g(y), y, p^*)$. It will suffice to prove that $(\sigma(q^*), \mu)$ is a (strict) minimum of the latter function.

For all $y \in]\mu-\varepsilon, \mu+\varepsilon[$, we have:

$$\partial W_i(y) = \partial_\sigma U_i(g(y), y, p^*).\partial g(y) + \partial_\mu U_i(g(y), y, p^*)$$

Absolute noise aversion implies $\partial_\sigma U_i(g(y), y, p^*) < 0$ for all $y \in]\mu-\varepsilon, \mu+\varepsilon[$. Absolute bias aversion implies $\partial_\mu U_i(g(y), y, p^*) < 0$ for all $y \in]\mu, \mu+\varepsilon[$ and $\partial_\mu U_i(g(y), y, p^*) > 0$ for all $y \in]\mu-\varepsilon, \mu[$ (and therefore $\partial_\mu U_i(g(\mu), \mu, p^*) = 0$ by continuity of the first derivative and of function g).

Function g is the local inverse of function $\psi_1 = \mu + \sqrt{h}$ (resp. $\psi_2 = \mu - \sqrt{h}$) over $]\mu, \mu+\varepsilon[$ (resp. $]\mu-\varepsilon, \mu[$), and we have in particular $\partial g(y) = -\frac{\partial_y\varphi\left(g(y), y\right)}{\partial_x\varphi\left(g(y), y\right)} = \frac{1}{\partial\psi_1(g^{-1}(y))}$

for all $y \in]\mu, \mu+\varepsilon[$ and $\partial g(y) = -\dfrac{\partial_y \varphi(g(y), y)}{\partial_x \varphi(g(y), y)} = \dfrac{1}{\partial \psi_2(g^{-1}(y))}$ for all $y \in]\mu-\varepsilon, \mu[$. We deduce from this fact and Proposition 5 that $-\dfrac{\partial_y \varphi(g(y), y)}{\partial_x \varphi(g(y), y)} < 0$ for all $y \in]\mu, \mu+\varepsilon[$ and $-\dfrac{\partial_y \varphi(g(y), y)}{\partial_x \varphi(g(y), y)} > 0$ for all $y \in]\mu-\varepsilon, \mu[$.

Relative noise aversion then implies that $\partial W_i(y) < 0$ for all $y \in]\mu-\varepsilon, \mu[$ and $\partial W_i(y) > 0$ for all $y \in]\mu, \mu+\varepsilon[$. This fact and the continuity of W_i imply in turn that $W_i(\mu) \geq W_i(y)$ for all $y \in]\mu-\varepsilon, \mu+\varepsilon[$, with a strict inequality for all $y \neq \mu$, the contradiction we were looking for. This concludes the proof of part (iii).

(iv) The reasoning developed in part (iii) identically applies to the proof of part (iv), with obvious adjustments implying that $(\sigma(q^*), \mu)$ is a local *maximum* of U_i in $F(p^*)$ (relative bias aversion implies that $\partial W_i(y) > 0$ for all $y \in]\mu-\varepsilon, \mu[$ and $\partial W_i(y) < 0$ for all $y \in]\mu, \mu+\varepsilon[$).[21]■

Proof of Theorem 2: Part (i) is established as part (i) of the proof of Theorem 1. Let us prove part (ii). We assume that U_i is quasi-concave and prone-to-bias. And we let (σ^*, μ^*) denote a maximum of U_i in $F(p^*)$. We have to prove that $\sigma^* \in \left[\bar{\sigma}, \sigma.\exp\left(\dfrac{1}{q^*}\right)\right[$.

If $\sigma^* = \bar{\sigma}$ there is nothing to prove. We therefore suppose from there on that $\sigma^* > \bar{\sigma}$.

We first establish that $\sigma^* \neq \sigma\left(q^*\right)$. This is done by means of a variant of the argument developed in the last part of the proof of part (iii) of Theorem 1. Let W_i be defined as in part (iii) of Theorem 1, and recall that $\partial W_i(y) = \partial_\sigma U_i(g(y), y, p^*).\partial g(y) + \partial_\mu U_i(g(y), y, p^*)$ for all

[21] Note that, as a by-product of the proof of part (iv) of Theorem 1, if $(\sigma^*, \mu^*) = (\sigma(q^*), \mu)$ is a maximum of U_i in $F(p^*)$, then it must be a local maximum of U_i subject to $\varphi(\sigma_j, \mu_j) \geq \dfrac{1}{q^*}$, in the case of relative noise-averse preferences. Precisely, in such a case we have $\partial_\sigma U_i(\sigma(q^*), \mu, p^*) < 0$, $\partial_x \varphi(\sigma^*, \mu^*) = \dfrac{\sigma(q^*)^2 - \sigma^2}{\sigma(q^*)^3} > 0$ and $\partial_\mu U_i(\sigma(q^*), \mu, p^*) = 0 = \partial_y \varphi(\sigma(q^*), \mu)$. The f.o.c. (a) and (c) or (a) and (c') then imply that the multipliers are both positive. The latter fact and inequalities $\partial_\sigma U_i(\sigma(q^*), \mu, p^*) < 0$ and $\partial_x \varphi(\sigma^*, \mu^*) = \dfrac{\sigma(q^*)^2 - \sigma^2}{\sigma(q^*)^3} > 0$ are then inconsistent with f.o.c. (c').

$y \in \left]\mu-\varepsilon, \mu+\varepsilon\right[$ for some $\varepsilon > 0$ and some C^{∞} function $g : \left]\mu-\varepsilon, \mu+\varepsilon\right[\to \mathbb{R}$ such that $g(\mu) = \sigma(q^*)$, $\partial g(y) < 0$ for all $y \in \left]\mu, \mu+\varepsilon\right[$ and $\partial g(y) > 0$ for all $y \in \left]\mu-\varepsilon, \mu\right[$. Absolute noise aversion implies $\partial_\sigma U_i(g(y), y, p^*) < 0$ for all $y \in \left]\mu-\varepsilon, \mu+\varepsilon\right[$. Prone to bias preferences imply that either $\partial_\mu U_i(g(y), y, p^*) > 0$ for all $y \in \left]\mu, \mu+\varepsilon\right[$ or $\partial_\mu U_i(g(y), y, p^*) < 0$ for all $y \in \left]\mu-\varepsilon, \mu\right[$. We have therefore either $\partial W_i(y) > 0$ for all $y \in \left]\mu, \mu+\varepsilon\right[$ or $\partial W_i(y) > 0$ for all $y \in \left]\mu-\varepsilon, \mu\right[$. This fact and the continuity of W_i imply in turn that either $U_i(g(y), y, p^*) > U_i(\sigma(q^*), \mu, p^*)$ for all $y \in \left]\mu, \mu+\varepsilon\right[$ or $U_i(g(y), y, p^*) > U_i(\sigma(q^*), \mu, p^*)$ for all $y \in \left]\mu-\varepsilon, \mu\right[$. Therefore $\sigma^* \neq \sigma\left(q^*\right)$.

At the beginning of the proof of part (iii) of Theorem 1, we established that, as a consequence of the quasi-concavity of U_i, any maximum of U_i in $F(p^*)$ must be either a local maximum of U_i in $\left\{(\sigma_j, \mu_j) \in \mathbb{R}\times\mathbb{R} : \sigma_j \geq \bar{\sigma},\ \varphi(\sigma_j, \mu_j) \geq \frac{1}{q^*}\right\}$ or a local maximum of U_i in $\left\{(\sigma_j, \mu_j) \in \mathbb{R}\times\mathbb{R} : \sigma_j \geq \bar{\sigma},\ \varphi(\sigma_j, \mu_j) \leq \frac{1}{q^*}\right\}$.

Suppose first that (σ^*, μ^*) is a local maximum of U_i in $\left\{(\sigma_j, \mu_j) \in \mathbb{R}\times\mathbb{R} : \sigma_j \geq \bar{\sigma},\ \varphi(\sigma_j, \mu_j) \geq \frac{1}{q^*}\right\}$. Since we assumed $\sigma^* > \bar{\sigma}$, we can suppose, equivalently, that (σ^*, μ^*) is a local maximum of U_i in $\left\{(\sigma_j, \mu_j) \in \mathbb{R}\times\mathbb{R} : \varphi(\sigma_j, \mu_j) \geq \frac{1}{q^*}\right\}$. Therefore (σ^*, μ^*) verifies the following set of first-order necessary conditions: There are $(\lambda, \nu) \in \mathbb{R}_+ \times \mathbb{R}_+$ such that

(a) $(\lambda, \nu) \neq 0$,

(b) $\nu.\left(\varphi(\sigma^*, \mu^*) - \frac{1}{q^*}\right) = 0$,,

(c) and $\lambda.\partial_\sigma U_i(\sigma^*, \mu^*, p^*) + \nu.\partial_x \varphi(\sigma^*, \mu^*) = \lambda.\partial_\mu U_i(\sigma^*, \mu^*, p^*) + \nu.\partial_y \varphi(\sigma^*, \mu^*) = 0$.

Since $\sigma^* \neq \sigma(q^*)$, we have $\partial_y \varphi(\sigma^*, \mu^*) \neq 0$ and $\mu^* \neq \mu$ by Proposition 5. Moreover: absolute noise aversion implies $\partial_\sigma U_i(\sigma^*, \mu^*, p^*) < 0$; and prone-to-bias preferences imply either $\partial_\mu U_i(\sigma^*, \mu^*, p^*) > 0$ if $\mu^* > \mu$ or $\partial_\mu U_i(\sigma^*, \mu^*, p^*) < 0$ if $\mu^* < \mu$. The f.o.c. (a) and (c) above then imply that multipliers λ and ν are both >0. The latter, $\partial_\sigma U_i(\sigma^*, \mu^*, p^*) < 0$ and f.o.c. (c) then imply in turn that $\partial_x \varphi(\sigma^*, \mu^*) \neq 0$.

The same conclusions apply, with obvious adaptations, if we suppose that $\sigma^* > \bar{\sigma}$ and (σ^*, μ^*) is a local maximum of U_i in $\left\{(\sigma_j, \mu_j) \in \mathbb{R} \times \mathbb{R} : \varphi(\sigma_j, \mu_j) \leq \frac{1}{q^*}\right\}$ (simply substitute f.o.c. (c') $\lambda.\partial_\sigma U_i(\sigma^*, \mu^*, p^*) - \nu.\partial_x \varphi(\sigma^*, \mu^*) = \lambda.\partial_\mu U_i(\sigma^*, \mu^*, p^*) - \nu.\partial_y \varphi(\sigma^*, \mu^*) = 0$ for f.o.c. (c) in the argument above).

From f.o.c. (c) and (c') we deduce then that $-\frac{\partial_\sigma U_i\left(\sigma^*, \mu^*, p^*\right)}{\partial_\mu U_i\left(\sigma^*, \mu^*, p^*\right)} = -\frac{\partial_x \varphi\left(\sigma^*, \mu^*\right)}{\partial_y \varphi\left(\sigma^*, \mu^*\right)}$. Absolute noise aversion and prone-to-bias preferences then imply that either $-\frac{\partial_x \varphi\left(\sigma^*, \mu^*\right)}{\partial_y \varphi\left(\sigma^*, \mu^*\right)} > 0$ if $\mu^* > \mu$ or $-\frac{\partial_x \varphi\left(\sigma^*, \mu^*\right)}{\partial_y \varphi\left(\sigma^*, \mu^*\right)} < 0$ if $\mu^* < \mu$. From this fact and Proposition 5 we deduce that $\sigma^* \in \left]\bar{\sigma}, \sigma.\exp\left(\frac{1}{q^*}\right)\right[$, which concludes the proof of Theorem 2.■[22]

[22] Note again that, as a by-product of the proof of Theorem 2, the sign of the multipliers λ and ν and of partial derivatives $\partial_\mu U_i\left(\sigma^*, \mu^*, p^*\right)$ and $\partial_y \varphi\left(\sigma^*, \mu^*\right) = \frac{\mu^* - \mu}{(\sigma^*)^2}$ are inconsistent with f.o.c. (c), that is, an interior maximum of U_i in $F(p^*)$ must be a local maximum of U_i subject to $\varphi(\sigma_j, \mu_j) \leq \frac{1}{q^*}$ in the case of noise-averse, prone-to-bias preferences.