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The Choice of Land Titling System and the Blockchain

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The Choice of Land Titling System and the Blockchain*

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Abstract

Should the advent of the blockchain lead to the reorganization or even the replacement of traditional land titling systems? In addressing this issue, we first generalize the model developed by Arruñada and Garoupa (2005) to study optimal land titling systems. Instead of considering only recording and registration alone, we examine an a priori infinite set of systems, each characterized by its quality (the probability that there is no forfeiture for a given plot of land) and its unit transaction cost. In this respect, the blockchain is viewed as a cost-efficient mechanism albeit not one providing the highest quality. We find that, despite the introduction of the blockchain, under some reasonable assumptions, it is still socially optimal to maintain traditional public land titling. In that case, the optimal quality of protection provided by traditional land titling must be either sufficiently high (and higher than that of the blockchain), or low enough (and lower than that of the blockchain). Yet under another set of assumptions, it is optimal to rely on the blockchain alone and to abandon traditional land titling.

Keywords: blockchain, land titling, forfeiture.

JEL Codes: O3, D23, L86, K25

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1 Introduction

Both the cost (broadly speaking) and the degree of protection of titled lands affect the functioning of the real-estate markets as well as the rest of the economy. For instance, costly and lengthy real-estate transactions slow workers' mobility and thus affect the labor market (Rupert and Wasmer 2012). Moreover, where property rights are not properly titled and are poorly protected, investments are deterred and growth becomes sluggish (D'Arcy *et al.*, 2021, Feder and Nishio, 1999). More adequate land titling may on the contrary facilitate access to credit markets and spur growth (Besley 1995; Alston, Libecap and Schneider 1996).[1]

The advent of blockchain land titling might challenge the design of contemporary titling systems. The blockchain, a particular type of distributed ledger technology (DLT), is a (theoretically) secure, transparent information storage and transmission technology. Due to its advantages in terms of cost, duration, and production of immutable information, the blockchain is a candidate to replace or supplement traditional land titling systems (Arruñada 2019, von Wangenheim 2020). The reduction of transaction costs through the diminishing role of trusted intermediaries is a key argument of blockchain proponents (Barbieri and Gassen 2017). Yet the blockchain encounters numerous legal and technical limits (Arruñada 2019, Barbieri and Gassen 2017, von Wangenheim 2020). For instance, running a blockchain like that associated with Bitcoin consumes vast amounts of electricity, which is likely to be detrimental to the environment. Furthermore, there are concerns about the quality of the data submitted to the blockchain and hijacking risks. These limits might explain why the number of real-estate-related applications of the blockchain remains limited in practice.

The aim of this paper is to study how an optimal land titling system should be set with the blockchain. To perform this study, we develop a general version of the model introduced by Arruñada and Garoupa (2005). We first examine the properties satisfied by an optimal titling system (absent any blockchain). By contrast with Arruñada and Garoupa (*ibid*), we do not restrict ourselves to considering two alternative titling systems (i.e., recording and registration). In our setting, a large set of protection levels is available. Any one of them can be associated with a given titling system and corresponds to a certain unit land titling cost. The higher the protection level, the higher the cost, and the lower the share of owners who can afford to resort to the titling system.[2] We then consider the blockchain, which we interpret as a system providing a high level of protection but at a strictly lower cost than the traditional system. We do not assume, however, that the blockchain is the most secure titling system (because, e.g., there is a certain risk of hijacking). Depending on the degree of protection and cost associated with traditional titling, we identify the conditions under which landowners prefer privacy (that is, no titling), traditional, or blockchain land titlings. We find that, while introducing the blockchain is always socially optimal, it is not always optimal to get rid of traditional titling. Where it is optimal to keep traditional land titling, its level of protection should either be sufficiently high (and higher than that of the blockchain), or low enough (and lower than that of the blockchain), but should never take intermediate values. While the location of the optimal quality of traditional titling is often indeterminate (the actual outcome depends on the distribution of land values), there are conditions under which it is possible to state whether it should be high or not.

The remainder of the paper is organized as follows. In the next section, we briefly review the land title registration literature. In section 3, we lay out a general model of land titling and study the properties satisfied by optimal systems. In section 4, we present the pros and cons of blockchain

[1] The idea that an efficient land titling system can facilitate access to credit through the use of property as collateral has been developed by De Soto (2000, 2001) and investigated by Besley, Burchardi, and Ghatak (2012).

[2] This hypothesis is in line with Arruñada and Garoupa (2005).

land titling, and we introduce the blockchain in our theoretical framework. Section 5 investigates the choice of a landowner between privacy, blockchain and traditional titlings. Section 6 presents our main results. First, we determine the conditions under which the blockchain makes traditional land titling unnecessary. Second, when using traditional titling remains optimal, we present some conditions under which its optimal quality is high or low. Section 7 concludes. Appendix A sets outs the symbols used in the paper. All the proofs are relegated to appendix B.

2 An Overview of the Law-and-Economics of Land Titling Systems

There are two main land title systems: the recording system and the registration system. For instance, England, Australia, and Germany (Grundbuch) have adopted a registration system, while France and the United States rely on the recording system. Under the recording system, the government keeps a public record of property transactions (the deeds). In the event of a dispute, the rightful claimant obtains the land, and the possessor may be financially compensated. On the contrary, in the registration system, there is a public register of land titles. Furthermore, the government certifies ownership rights at the time of the transaction. "The rightful claimant ordinarily cannot seek restoration of her title but only monetary compensation out of a public fund financed by registration fees" (Miceli *et al.* 2002, p. 566).[3]

Therefore, a first way to study land title systems is to focus on the way they address the following issue: who receives the land and who receives monetary compensation when a legitimate claim is made? From this perspective, Miceli and Sirmans (1995) study which system is better at promoting efficient exchange of land. They find that if transaction costs are low, both systems promote efficient exchange (a result in line with Coase's theorem). But if transaction costs are high, awarding the land to the true owner (rather than the possessor) may prevent it from ending up with the party who values it most. As the subjective value of land for its possessor is generally above its market price, and as transaction costs are generally high in a register system, this system is consistent in assigning title to the current property holder in the event of conflicting ownership claims.

Miceli *et al.*(2002) consider yet another issue. Because the registration and the recording systems apply different principles to resolve conflicting claims to land title, the nature of the system affects both the value of the insurance premium paid by property owners and the land value. They show that the price of land must be higher with the registration system. They also show that when the transaction costs associated with the registration system are higher than those of the recording system, the registration system is preferred by owners whose land commands a high value, whereas the recording system is preferred by owners whose land is of low value. In this regard, the distribution of equilibrium land prices matters for the choice of the title system.[4]

Arruñada and Garoupa (2005) present a model of land titling in order to study the different consequences of recording and registration. In contrast with the approach developed by Miceli and his co-authors, land prices are assumed to be fixed. On the other hand, Arruñada and Garoupa (2005) assume that people can buy and sell land without resorting to a titling system (this is the privacy

[3] See also Miceli and Sirmans 2005

[4] Miceli *et al.*(2011) study how title system characteristics affect real-estate assets and more generally different types of assets (see also, Miceli, Sirmans, and Turnbull 1998 and 2000). They show that when the risk of a claim is very high, the registration system is preferred over the recording system, despite higher transaction costs (and conversely).

option). They also assume that recording and registration differ in their costs and the degrees of protection offered against forfeiture. Registration is the costlier regime, but also the regime which provides the higher degree of protection. Recording comes at a lower cost, but is characterized by a higher probability of forfeiture. The third option, privacy, provides a low degree of protection, but implies no cost for the buyer and the seller. Which system is preferable depends on the distribution of land values.

Following Arruñada and Garoupa (2005), this paper assumes that a higher degree of protection goes with a higher cost. However, we consider a larger set of titling systems. Furthermore we introduce the blockchain. Therefore we look at three kinds of regimes: privacy, traditional land titling, and blockchain land titling. Because this paper is a generalization of Arruñada and Garoupa (2005)'s contribution, it shares its differences with other significant contributions to the Law-and-Economic analysis of land titling. In particular, we do not assume that the current owner values the land more than the claimant (as in Miceli and Sirmans 1995), nor do we consider the dynamic consequences of land titling systems, as in Miceli *et al.* (1998, 2000).

3 A General Model of a Land Titling System

Following Arruñada and Garoupa (2005), we assume that title values V are distributed according to a distribution function F over a set $\mathcal{V} = [0, \overline{V}]$, which admits a density function f that takes strictly positive values.

The quality of a land titling system θ is defined by the probability that a landowner will not face forfeiture. Here, the term "quality" refers to the degree of protection, or security, provided by a land titling system to the landowner. The set of possible qualities for the land titling system is the interval $[\underline{\theta}, \overline{\theta}]$, with $0 < \underline{\theta} < \overline{\theta} \leq 1$. Thus the expected wealth of the owner of a titled land of value V is θV.[5]

Let $c(\theta)$ be the function giving the (constant) unit cost per transaction associated with a land titling system whose quality is θ. This cost includes both a share of the fixed costs needed to establish the titling system and a variable cost (reflecting the expenditures required to run the system). Because in our setting titling is voluntary (as opposed to universal titling), it seems reasonable that the fixed cost be borne by all the users (that is, the owners who decide to titling their parcels).

Further, assume that $c : [\underline{\theta}, \overline{\theta}] \rightarrow \mathbb{R}_+$ is a twice-differentiable strictly convex increasing function and $c(\underline{\theta}) \geq 0$.[6] This assumption formalizes the idea that the higher the protection provided by a titling system, the higher the unit cost. For instance, registering property rights (in rem) is often considered to be more costly than mere recordation of deeds (if only because it takes time an resources to ascertain property rights). Yet, as Arruñada (2012) argues, because recordation is less secure, parcel owners must buy insurance. In particular in Europe, it seems that "registration systems are not only more effective but also less costly than recordation" (*ibid* p. 157) (but prices of titling services are probably not set optimally everywhere, and we do not know how the fixed costs are taken into account). However, Arruñada (2012) also states that for countries endeavoring to

[5] Arruñada and Garoupa (2005) consider two values of θ corresponding to a recording system and a registration system respectively, and assume that the latter provides a greater degree of protection than the former.

[6] The function $c(\theta) = \frac{\alpha\left(\theta-\underline{\theta}\right)^2}{2} + \beta$, with $\alpha > 0$ and $\beta \geq 0$, satisfies these properties.

develop a titling system, opting for registration might not be the best choice. That is because, for a registration to be effective, a country needs to have well-organized civil registries, a well-organized system for identifying land parcels, as well as an efficient court system. We interpret that advice as implying that it would be costlier to build a well-functioning registration system (high quality title is costly), rather than a recording system. All in all, we thus believe that from normative viewpoint, it is reasonable to assume that the unit cost increases with the quality of the titling system.[7]

We next consider whether an individual owning or endeavoring to own a piece of land of value V would choose to publicly titling it or rely on a pure private contract, an option we call privacy. We will suppose privacy entails limited transaction cost, but provides the lowest possible quality, i.e., $\underline{\theta}$.

3.1 Privacy vs public land titling

Assume that the quality θ of public land titling satisfies $\theta > \underline{\theta}$. Under titling, the net expected value of a title V is θV, minus the titling cost $c(\theta)$. Under privacy, the expected value is $\underline{\theta} V$, minus the transaction cost $c(\underline{\theta})$. Hence, a potential owner of a title with value equal to V would opt for titling if, and only if, the following condition holds:

$$V \geq \frac{c(\theta) - c(\underline{\theta})}{\theta - \underline{\theta}} \equiv V_{T/P}(\theta).$$

$V_{T/P}(\theta)$ is the threshold land value above which public titling is preferred to privacy, where T stands for titling and P for privacy. In words, for a landowner, it is worth resorting to public titling if the value added in terms of protection $(\theta - \underline{\theta})V$ is large enough to offset the increase in cost $c(\theta) - c(\underline{\theta})$ of such protection. This is only possible for sufficiently high value lands.

The next Lemma establishes that this threshold value increases with the quality of the land titling system.

Lemma 1. *$V_{T/P}(\theta)$ is increasing with θ, i.e. the land value for which one is indifferent between privacy and traditional registration increases with the degree of protection.*

In other words, everything else being equal, the higher the quality of the traditional system, the higher the proportion of landowners who renounce to traditional registration, and opt for privacy. Indeed, the cost of protection $c(\theta)$ rises with quality more than proportionally.

We shall make the following assumption:

Assumption 1. *The land value $V_{T/P}(\overline{\theta}) = \frac{c(\overline{\theta}) - c(\underline{\theta})}{\overline{\theta} - \underline{\theta}}$, for which one is indifferent between privacy and traditional registration with the highest quality $\bar{\theta}$, is less than the highest land value $\overline{V}$.*

This assumption means that some individuals choose to publicly titling their land when the public titling system has the highest quality but also the highest unit cost of transaction.

[7]Notice that if the unit cost of a transaction were decreasing with the quality of the titling system, all titling systems would use the highest possible quality at the lowest unit cost. This seems to be a highly implausible outcome. What remains disputable may be the fact that the unit cost is a convex function of the titling system quality. Convexity of the unit cost function, however, is no implausible, and simplifies the analysis considerably.

3.2 Optimal quality of the public titling system

We have seen above that the set of land values for which potential owners choose privacy is $[0, V_{T/P}(\theta)]$ and the set of land values for which resorting to public titling is worthwhile is $[V_{T/P}(\theta), \overline{V}]$. Using the distribution of land values, we can compute the aggregate (expected) land value given a land titling system with quality θ. We use this expected value as a *measure* of social welfare. This social welfare function W is written:

$$W(\theta) = \int_0^{V_{T/P}(\theta)} [\underline{\theta}V - c(\underline{\theta})]f(V)dV + \int_{V_{T/P}(\theta)}^{\overline{V}} \big[\theta V - c(\theta)\big] f(V)dV.$$

We define an optimal titling system as one associated with a value of θ that maximizes the social welfare function above.[8] The next proposition states a property satisfied by optimal titling systems:

Proposition 1. *When the quality of an optimal titling system θ^* is such that $\theta^* \in (\underline{\theta}, 1)$, then we have*

$$\int_{V_{T/P}(\theta^*)}^{\overline{V}} \big[V - c'(\theta^*)\big] f(V)dV = 0. \tag{1}$$

The condition above can be interpreted as follows. According to condition (1), the net private marginal benefit of the quality of the public titling system for a land value V (*i.e.*, $V - c'(\theta)$)) is negative for land values close to the threshold $V_{T/P}(\theta^*)$ and positive for land values close to $\overline{V}$. To understand this property, let us focus on the value of the land $V_{T/P}(\theta^*)$ such that its owner is indifferent between privacy and public titling. The effect of a marginal increase in the quality θ on this individual's payoff reads $V_{T/P}(\theta^*) - c'(\theta^*)$. Given that $V_{T/P}(\theta^*) = \frac{c(\theta^*) - c(\underline{\theta}^*)}{\theta^* - \underline{\theta}}$, one can see that the net marginal payoff $V_{T/P}(\theta^*) - c'(\theta^*)$ is negative.[9] To wit, what matters to the landowner indifferent between public titling and privacy is only the difference in the unit cost $c(\theta^*) - c(\underline{\theta})$ and not the marginal cost $c'(\theta^*)$. Because of this property, some of the publicly titled lands have values that are lower than the marginal cost of the land titling quality. The reverse conclusion is obtained for lands with high-values. The optimal value of land titling quality is such that the negative effects of a marginal increase in this quality on the expected net benefit of low land values are exactly compensated by the positive effects on the expected net benefit of high land values.

4 Blockchain Land Titling

In this section, we first briefly present the main characteristics of the blockchain technology. We also recall the conditions identified in the burgeoning literature on the law-and-economics of the blockchain, under which the blockchain might efficiently replace or supplement more traditional land titling systems. Second, building on our presentation of the blockchain, we introduce this technology in our model. We view it as a cost-efficient mechanism, albeit not one providing the highest quality.

[8]Note that we have ignored the fact that the individual benefit of public land titling for a would-be landowner (θV) is probably higher than its social value (since the owner's loss is at least in part gained by someone else in the economy). See Arruñada and Garoupa (2005, p. 716) for discussion.

[9]This property is linked to the convexity of $c(.)$, since we have $c(\underline{\theta}) - c(\theta) > c'(\theta)(\underline{\theta} - \theta)$.

4.1 Blockchain for land transactions

The blockchain technology is full of promises, notably in terms of disintermediation possibilities. This technology, however, has significant limitations, identified by recent literature.

4.1.1 What is the blockchain and how can it be used in land transactions?

The blockchain, a particular type of distributed ledger, is (*a priori*) a secure, transparent information storage and transmission technology. Each node of the blockchain (i.e., an individual or an organization) has a copy of the ledger and participates in its updating through a consensus process. Hence, each entry can be seen by all users, ensuring transparency. Once written, the added information is unmovable and time-stamped. The use of cryptographic procedures ensures that any attempt to change the recorded information will be detected. Therefore, in principle, the blockchain produces immutable information at a low cost.

Due to its advantages in terms of cost and security and its potential to disintermediate transactions, the blockchain appears to be a challenger to more traditional land title registration systems. More precisely one can anchor digitized land registers in the blockchain, digitize deed registers in a blockchain, or tokenize property rights in the blockchain. Von Wangenheim (2020) explores in detail the advantages and limitations of these different possibilities. In practice, several countries have introduced the blockchain technology in land titling. For instance, Sweden has adopted the blockchain technology to preregister deeds in order to reduce delays associated with real-estate transactions (see Proskurova and Dörry 2018) while Georgia has "anchored" (i.e. connected) its digital land registry to a blockchain.

4.1.2 Some limits to the use of the blockchain in land titling

The blockchain, however, encounters numerous limits, both legal and technical (Arruñada, 2019; Barbieri and Gassen, 2017; von Wangenheim, 2020). These limits might explain why the number of real-estate related applications of the blockchain remains limited.

Contracting property through the blockchain implies solving two challenges. Firstly, there must be an interface between the digital and the real world which makes it possible for claimants to get physical possession of the assets (holding an electronic code for entering the property is probably not enough). Secondly, the transfer of title, which amounts to holding a claim against specific individuals (*in personam* right) must also become property rights valid against the whole world (*in rem* right) (Arruñada, 2003). This second challenge is usually solved by a third party (be it a court or a registry and so on). This third party has a public function, in that it must be impartial to all and prevail over the parties to any given contract (Arruñada, 2017). It also defines the set of rights enforced *in rem* (often referred to as the *numerus clausus* of rights) and the mechanisms and evidentiary requirements for right holders to convey their consent with respect to intended transactions.[10] To solve this last challenge, notice that the blockchain must have a value as legal evidence.

In practice, there is a large spectrum of applications of the blockchain for land property, from blockchain-enabled peer-to -peer exchanges to blockchain-enabled intermediation. Blockchain-enabled peer-to -peer in property is a system where exchanges and titling of property rights happens on the blockchain without intermediary. In such utmost system, individuals would become custodians

[10] As we have already mentioned Arruñada (2019) p. 10 is skeptical that the blockchain can ever solve the two challenges.

of their rights (Arruñada, 2019). At the minimum, the legal requirements should be as follows (Arruñada, 2019, p. 243):

> Minimum necessary regulation would include: (1) defining the legal status of blockchain records to establish priority of claims and adjudicate property rights among conflicting claimants; (2) establishing a low and strict *numerus clausus*- exclusive of all unregisteblue rights-before coding a smart contract capable of handling property conveyancing and/or registration; (3) regulating the switch of title records or property rights to the blockchain register, a task which differs widely if mandatory or voluntary; if voluntary (as is likely inevitable in most cases), regulating any conflicts emerging from the resulting multiple sources of legal evidence, possible parallel sources of evidence and even overlapping registries; and (4) regulating the legal status of non-contractual property rights such as those derived from, e.g., judicial seizures, inheritance rights or even constraints rooted in land planning.

As to blockchain-enabled intermediation in property *per se*, there is a wide range of applications, from blockchain-enabled registration review to register of deeds on the blockchain. In each case, specific legal arrangements are also needed (Arruñada, 2019, p. 246):

> However, registering actual title records in a blockchain requires a permissioned blockchain to make it viable in terms of mining (validation) costs; a blockchain system of conveyancing or registration based on tokenized titles requires transforming property titles into negotiable instruments; and a blockchain register based on a smart contract would also require a strict, low numerus clausus of rights to make such a contract writable.

The required changes in the law needed to make blockchain titling working, however, are unlikely to be sufficient. Judges' whim matters because, as pointed out by Carol Rose (1988), simple and informative rules cannot always be applied without yielding unexpected consequences, and in those cases, judges tend to rely on "muddy" rules of decision based on equitable ideas (Rose, 1988) and may overcome statutory law (see, e.g., Arruñada, 2019, p. 249).

A significant obstacle, however, for giving legal value to blockchain land titling is the control of the data submitted to the blockchain, which is a key function of a land titling system. For example, title registration goes with an extensive *ex ante* examination of rights. No such examination is expected in a public blockchain. Thus, a (public) blockchain induces risk associated with the lack of independent verification, lack of disclosure of the participant's identity, and the risk of irregularities resulting therefrom. Some argue that the cost of property transactions may even increase rather than decrease due to the risk induced by the blockchain, as one would need specialized intermediaries, extended due diligence exercise, and title insurances.

In addition, Kaczorowska (2019) draws attention to a risk of double selling due to a technical characteristic of the blockchain system, as "there is no guarantee that the order in which transactions are received by the nodes is the same order in which new blocks are added."

Furthermore, both the decentralization of power and the democratic functioning of consensus are jeopardized by the existence of "mining farms" which concentrate mining power (a power necessary to ensure transaction security). Actually, a blockchain can be hijacked if one acquires enough computing power or exploit vulnerabilities in the code.[11] Thus the blockchain does not completely eliminate the risk of capture of a property.

[11] An well known example of an attack of the blockchain is that of the Decentralized Autonomous Organization D.A.O. on Etherum (New York Times, A Hacking of More Than $50 Million Dashes Hopes in the World of Virtual Currency, dated June 17, 2016).

Arruñada (2019) also highlights the limits of a peer-to-peer blockchain where individuals become custodians of their cryptographic keys as well as of the legal integrity of their rights. Individuals can hardly do without intermediaries in this hypothetically peer-to-peer system.

Moreover, in practice, land titling systems differ widely from one country to another. Because of legal diversity across jurisdictions, the benefit of using the blockchain in land titling is country-dependent (Arruñada 2019; Kaczorowska 2019; von Wangenheim 2020).

In this connection, recall that proof of ownership may be produced either by deed registrations (as in France, Italy, the Netherlands, and in most US counties), or by title registration (as in Australia, England and Wales, Germany, Poland, Spain, and Sweden). Thus, as the examination of the rights may be made before or after registration, some authors argue that the legal problems involved in the use of a public blockchain are less exacerbated in a deed-based system than in a title-based system (Arruñada 2019; Kaczorowska 2019; von Wangenheim 2020). Indeed, title registration requires an extensive check of the correctness of the basis for entry, while the role of registrars seems to be limited more to formalities under deed registration. Yet, even with deed registration, there are significant limits to the use of the blockchain. Arruñada (*ibid*) makes clear that this replacement is unlikely to be completel as contracts are necessarily incomplete and thus require a certain legal expertise.

As a matter of fact, however, in many civil law countries notaries advise, authenticate, and ascertain the legal capacities of the parties to a contract (Arruñada 1996). In other countries, lawyers represent the interests of their client during the land transaction. These activities cannot be fulfilled by the blockchain alone.

To wrap up, there seems to be a consensus with regard to the inadequacy of the blockchain in its pure form (meaning, public blockchain) to land titling registration. It does not seem plausible to design a public blockchain to replace a register. Potential challengers of traditional registration systems are private or consortium blockchains that allow governance to be set up and a leader to be identified (Legeais 2019). In the remaining part of the section, we rely on the above presentation to introduce the blockchain in our model of land titling.

4.2 Modeling blockchain land titling

We *interpret* the blockchain as being a new titling technology that produces a certain level of quality θ_B at a unit cost per transaction of $\tilde{a}$. We make the following assumptions in order to capture the main characteristics of the blockchain that we identified in the preceding subsection.

Assumption 2.

> *The quality of the blockchain is higher than the quality of privacy, but it is lower than the highest quality of traditional registration:* $\underline{\theta} < \theta_B < \overline{\theta}$*.The unit cost of the blockchain is lower than the unit cost of the traditional titling system that provides the same quality:* $\tilde{a} < c\big(\theta_B\big)$*.*

The first part of the assumption refers to the protection provided by the blockchain. This protection depends on two factors. First, there is a technological factor. The blockchain is free of the traditional threats to other titling technologies, like fire, human error (provided that the data are correctly entered in the system), and secure (due to its cryptographic properties). Second, there is a legal factor. The legal protection given by the blockchain depends on how it is regulated and on how judges consider the evidence it gives. As a consequence, it seems reasonable to assume that the expected degree of protection provided by the blockchain is higher than that of privacy. Yet as was seen above, the blockchain also entails its own risk, like hacking or wrongful entry.[12] Moreover, a blockchain system of registration will likely not provide the same protection of *in rem* rights as a full-fledged traditional title registration (if only because it is difficult to perform the registration review automatically, Arruñada (2019), p. 252). To account for those risks, we thus let $\theta_B < \overline{\theta}$.

The second part of assumption 1 encapsulates the idea that the blockchain is a potential cost-saving mechanism. That is, for a quality of land titling equal to θ_B, we assume that it is costlier to use the traditional public land titling system rather than the blockchain. If the inequality $\tilde{a} < c(\theta_B)$ does not hold, of course, the blockchain cannot develop. The cost-saving property of the blockchain, however, is affected by regulation. We thus assume that the legal protection provided by blockchain titling is sufficiently high so that landowners do not have to buy costly title insurance."

Under assumption 2, notice that the expected gain of using blockchain land titling for land of value V equals $\theta_B V - \tilde{a}$.

5 Individual Choices Between Blockchain, Traditional Land Titling, and Privacy

In this section, we study how a potential or an actual owner of a land title of value V chooses between blockchain land titling, traditional (and public) land titling, and privacy. We begin with the choice between blockchain and traditional land titling. We contrast two cases. In the first one, as opposed to the second one, traditional land titling offers more security than the blockchain. Then, we briefly consider the choice between blockchain land titling and privacy. We rely on the study of the titling choices to determine the set of land values associated with each of these choices. The determination of these choices will prove instrumental in the study of optimal titling systems when blockchain land titling becomes available.

5.1 Blockchain vs traditional land titling

Recall that the expected value of a land of value V under traditional land titling is $\theta V - c(\theta)$, versus $\theta_B V - \tilde{a}$ with blockchain land titling. Relying on traditional titling is the best choice whenever:

$$\theta V - c(\theta) > \theta_B V - \tilde{a},$$

or,

$$V\left(\theta - \theta_B\right) > c(\theta) - \tilde{a}.$$

In order to discuss the choice of titling system, we contrast two cases, namely the case where traditional land titling provides a quality of protection at least as high as blockchain land titling, and the case where traditional land titling provides a lower quality of protection than blockchain land titling.

[12] For instance, if h is the probability of hacking, we may have $\theta_B = (1 - h)\overline{\theta}$.

5.1.1 Traditional titling offers more security than blockchain titling

Let us first consider the case where traditional land titling offers a quality of protection greater than the blockchain, i.e., $\theta > \theta_B$. People prefer traditional land titling if the value of their land satisfies the following condition

$$V > \frac{c(\theta) - \tilde{a}}{\theta - \theta_B} \equiv V_{T/B}(\theta). \tag{2}$$

$V_{T/B}(\theta)$ is the land value threshold above which traditional land titling is preferred to the blockchain, where T stands for traditional titling and B for blockchain titling. Intuitively, it is worth using traditional land titling (which provides better protection at a higher cost) when the difference between the expected values of the title under both systems $(\theta - \theta_B)V$ is greater than the difference in costs $c(\theta) - \tilde{a}$. When V is high, it may be worth using traditional land titling to protect the property. On the contrary, the gain obtained by using traditional land titling (which is a fraction of V) is negligible compared to its cost when the value of land V is low.

Notice that some individuals prefer traditional land titling over blockchain land titling only if the following necessary condition holds:

$$\overline{V} > V_{T/B}(\theta). \tag{3}$$

In words, the highest possible land value must exceed the threshold value $V_{T/B}(\theta)$ in order to have some landowners who prefer traditional titling over the blockchain. This condition is less likely to be satisfied the closer θ is to the quality θ_B provided by the blockchain. That is because, relying on traditional land titling produces a difference in costs that exceeds the difference in expected values.

The next Proposition states the conditions under which some people would prefer traditional over blockchain land titling.

Proposition 2. *Suppose that the quality of traditional registration is higher than that of the blockchain ($\theta \in]\theta_B, \overline{\theta}]$)*

1. *If, for any of such quality, the land value $V_{T/B}(\theta)$ for which one is indifferent between blockchain and traditional registration exceeds the highest land value $\overline{V}$, then all landowners prefer blockchain to traditional land titling.*

2. *Assume instead that the land value $V_{T/B}(\overline{\theta})$ for which one is indifferent between the blockchain and traditional registration with the highest quality exceeds the highest value $\overline{V}$ of the land, but that there is a quality θ such that the land value $V_{T/B}(\theta)$ is lower than $\overline{V}$. Then, there is an interval $[\theta'_1, \theta'_2] \subset]\theta_B, \overline{\theta}[$ for the qualities, such that $V_{T/B}(\theta'_1) = V_{T/B}(\theta'_2) = \overline{V}$, and such that for any quality in this interval the land value $V_{T/B}(\theta)$ for which one is indifferent between the blockchain and traditional registration is lower than $\overline{V}$. Moreover, all owners whose land value lies within $[V_{T/B}(\theta), \overline{V}]$ prefer traditional to blockchain land titling.*

3. *Alternatively, assume that the land value $V_{T/B}(\overline{\theta})$ for which one is indifferent between the blockchain and traditional registration with the highest quality is lower than the highest land value. Then there exists a quality $\theta' \in]\theta_B, \overline{\theta}[$ such that the value $V_{T/B}(\theta')$ of the land of the owner indifferent between the two titling systems is equal to $\overline{V}$ and such that for any quality $\theta > \theta'$, the land value $V_{T/B}(\theta)$ of the owner indifferent between the two titling systems is lower than $\overline{V}$. For such a quality, all owners of land whose land value lies within $[V_{T/B}(\theta), \overline{V}]$ prefer traditional to blockchain land titling.*

The Proposition above states that traditional land titling can be discarded even when providing a higher quality than the blockchain (case 1. of Proposition 2). Indeed, choosing traditional land titling is advantageous only if the increase in expected value exceeds the increase in cost. This choice is possible only when the maximal value of land $\overline{V}$ exceeds the threshold value $V_{T/B}(\theta)$ (cases 2 and 3 of Proposition 2).

Figure 1 illustrates the different cases stated in Proposition 2.[13] Cases 1, 2 and 3 of Proposition 2 are represented respectively by $\overline{V} = \overline{V}_1$, $\overline{V} = \overline{V}_2$ and $\overline{V} = \overline{V}_3$. Notice that the threshold $V_{T/B}(\theta)$ decreases when the quality θ of the traditional land titling is slightly higher than θ_B. This is intuitive: as the quality of traditional land titling increases with respect to the quality of blockchain land titling, it is less necessary to own land of great value to choose the former. Indeed, while the unit cost is always higher with traditional titling, the (expected) benefit of this system increases with the value of the land. Only with land of great value can the increase in quality associated with traditional titling be beneficial for a landowner. But it is not always true that the threshold value $V_{T/B}(\theta)$ invariably decreases. Figure 1 depicts a case where $V_{T/B}(\theta)$ reaches a minimum in $\underline{\underline{\theta}}$ (with $\underline{\underline{\theta}} < \overline{\theta}$). If the minimum value of the threshold is greater than $\overline{V}$, as it is the case when $\overline{V} = \overline{V}_1$, no owner can actually benefit from the traditional system. To wit, the increase in quality remains too small for the traditional land titling system to be worth it (case 1. of Proposition 2). By contrast, when $\overline{V} = \overline{V}_2$, some owners can benefit from the traditional system if the degree of protection takes an intermediate value ($\theta \in [\theta_1', \theta_2']$), which represents case 2 of Proposition 2. When $\overline{V} = \overline{V}_3$, traditional titling might be favored if its degree of protection is sufficiently higher than that of the blockchain ($\theta > \theta'$), which represents case 3 of Proposition 2.

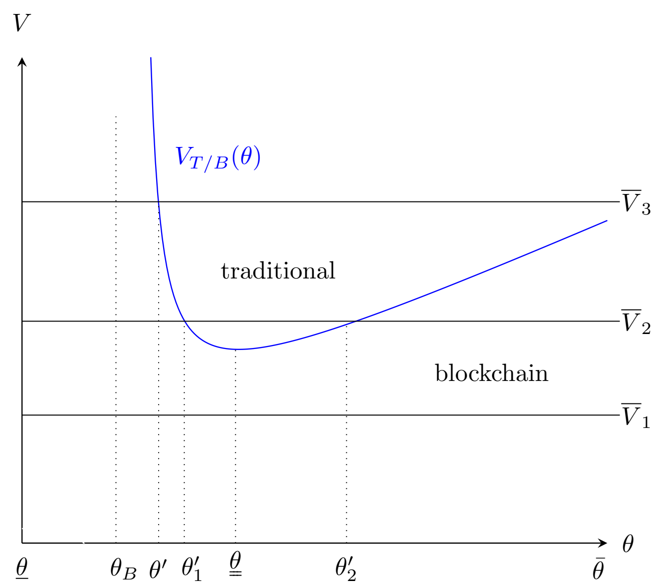

Figure 1: The threshold $V_{T/B}(\theta)$ when the quality of traditional registration is higher than that of the blockchain ($\theta \in]\theta_B, \bar{\theta}]$).

[13] In this figure, $\underline{\theta} = 1$, $\bar{\theta} = 2$, $\theta_B = 1.4$, $c(\theta) = .9(\theta - \underline{\theta})^2 + 0.005$, $\tilde{a} = .01$.

5.1.2 Traditional land titling provides less security than the blockchain.

Let us now consider the case where traditional land titling provides a lower quality of protection than blockchain land titling: $\underline{\theta} \leq \theta < \theta_B$. That this assumption is empirically founded is, of course, disputable. Yet there is no denying that it is relevant when the traditional land titling system is quite recent, *e.g.*, as in some Eastern European countries.

Furthermore, let us denote by $\hat{\theta}$ the quality level at which the cost of traditional titling equals the cost of blockchain titling, i.e., $c(\hat{\theta}) = \tilde{a}$.[14]

Then we can see that blockchain land titling is always preferred to traditional land titling when $\theta \in [\hat{\theta}, \theta_B]$. Indeed, by assumption, we know that: $\tilde{a} < c(\theta_B)$. Then when $\theta \in [\hat{\theta}, \theta_B[$, traditional land titling provides a lower quality than that of the blockchain titling while being costlier, and therefore *no one* prefers traditional to blockchain land titling.

The previous analysis suggests that traditional titling could be chosen if its quality is relatively low, so that its cost becomes significantly lower than that of blockchain titling. Indeed, whenever $\theta \in [\underline{\theta}, \hat{\theta}[$, so that $c(\theta) < \tilde{a}$, the individuals who prefer traditional land titling to blockchain land titling are those for whom the land value V is such that:

$$V < \frac{\tilde{a} - c(\theta)}{\theta_B - \theta} \equiv V_{T/B}(\theta). \tag{4}$$

The threshold above is always well-defined. To see this, recall that titles make take arbitrarily small values and that the expected loss in title values associated with a titling system is proportional these values. Thus there is always a landowner who prefers traditional to blockchain titling as the former is cheaper and the expected loss in the value of his title associated with traditional titling is negligible.

5.2 Blockchain land titling versus privacy

Let us now focus on the choice between blockchain land titling and privacy. A potential owner of a title with value equal to V would rely on blockchain titling rather than privacy if, and only if, the following condition holds:

$$V > \frac{\tilde{a} - c(\underline{\theta})}{\theta_B - \underline{\theta}} \equiv V_{B/P}.$$

$V_{B/P}$ is the threshold land value above which blockchain land titling is preferred to privacy, where B stands for blockchain and P for privacy. Again, blockchain titling is favored when the gain in the land titled expected value $(\theta_B - \underline{\theta})V$ exceeds the cost increase implied by the blockchain $\tilde{a} - c(\underline{\theta})$.

We shall suppose that some people would always choose blockchain titling rather than privacy. Thus, the following assumption will hold:

Assumption 3. $V_{B/P} = \frac{\tilde{a} - c(\underline{\theta})}{\theta_B - \underline{\theta}} < \overline{V}$.

The interpretation of this assumption is straightforward. The expected value added by the blockchain compared to privacy for the most valuable land $\overline{V}$ must exceed the increase in cost $\tilde{a} - c(\underline{\theta})$.

[14] As the unit cost function $c(.)$ is continuous and monotonic, there exists a unique value $\hat{\theta}$ such that $c(\hat{\theta}) = \tilde{a}$.

5.3 Sorting the threshold values $V_{T/B}(\theta), V_{T/P}(\theta)$, and $V_{B/P}$

Recall that an individual is indifferent between privacy and traditional land titling when the value of his property is equal to

$$V_{T/P}(\theta) = \frac{c(\theta) - c(\underline{\theta})}{\theta - \underline{\theta}}.$$

To sort the different thresholds we consider two cases in turn.

• **Traditional titling offers a quality of protection higher than blockchain titling ($\theta > \theta_B$).**

We obtain the following results.

Lemma 2. *Whenever traditional registration quality is higher than blockchain quality, the landowners who prefer traditional titling to the blockchain also prefer traditional titling to privacy.*

For all quality of traditional registration higher than that of the blockchain ($\theta_B < \theta$), the land value $V_{T/B}(\theta)$ for which one is indifferent between traditional registration and blockchain exceeds the land value for which one is indifferent between traditional registration and privacy, i.e. whenever people prefer traditional titling to the blockchain, they also prefer it to privacy.

This result greatly simplifies the analysis.[15] Intuitively, if people prefer traditional titling to the blockchain, it is because the benefit of traditional titling largely exceeds its cost (otherwise, the blockchain would be preferred as it provides a lower protection that is less expensive). So for these people, the cheapest protection, which is also the lowest one, would not be worthwhile.

Lemma 3. *Whenever traditional registration quality is higher that blockchain quality, the landowners who prefer traditional to blockchain titling also prefer blockchain titling to privacy.*

To grasp the gist of this Lemma[16], consider the owner of a land that is worth $V_{T/B}(\theta)$. By definition of this term, this is the land value such that its owner is indifferent between traditional and blockchain titling. But from Lemma 2, this owner prefers traditional titling to privacy (this is also true for all the owners whose land value is higher than $V_{T/B}(\theta)$). Therefore, the landowner indifferent between blockchain titling and privacy has a land whose value $V_{B/P}$ must be lower than $V_{T/B}(\theta)$.

To conclude the analysis of the case where $\theta_B < \theta$, observe that when the land value V is lower than $V_{T/B}(\theta)$ but higher than $V_{B/P}$, its owner chooses blockchain titling. On the other hand, if $V < V_{B/P}$, the landowner chooses privacy.

• **Traditional land titling offers a lower quality of protection than the blockchain ($\theta < \theta_B$).**

The next result illustrates the case where traditional land titling is *never* used when its quality is low.

Lemma 4. *Assume that the value $V_{T/B}(\theta)$ of the land of an owner indifferent between traditional and blockchain titling is decreasing with respect to the quality θ of traditional titling. Then privacy is always prefered to traditional land titling.*

[15]Formally, Lemma 2 implies that for all θ such that $\theta_B < \theta$, it holds that $V_{T/P}(\theta) < V_{T/B}(\theta)$.

[16]Formally, Lemma 3 asserts that for all θ such that $\theta_B < \theta$, $V_{B/P} < V_{T/B}(\theta)$.

When $V_{T/B}(\theta)$ is decreasing the higher the quality of the traditional system, the costlier it is and only owners of land with low values would prefer it to blockchain titling. Furthermore, we have seen in Lemma 1 that the value $V_{T/P}$ of the land of an owner indifferent between traditional titling and privacy is increasing with the quality of former. In addition, it turns out that $V_{T/B}(\underline{\theta}) \leq V_{T/P}(\underline{\theta})$.[17] Thus as the quality of traditional land titling increases, there is an increasing difference between $V_{T/B}(\theta)$ and $V_{T/P}(\theta)$. Therefore, when a landowner prefers traditional to blockchain titling, this is because she owns a land with relatively low value, but in that case, privacy is also preferred to traditional titling. Figure 2 illustrates Lemma 4.[18]

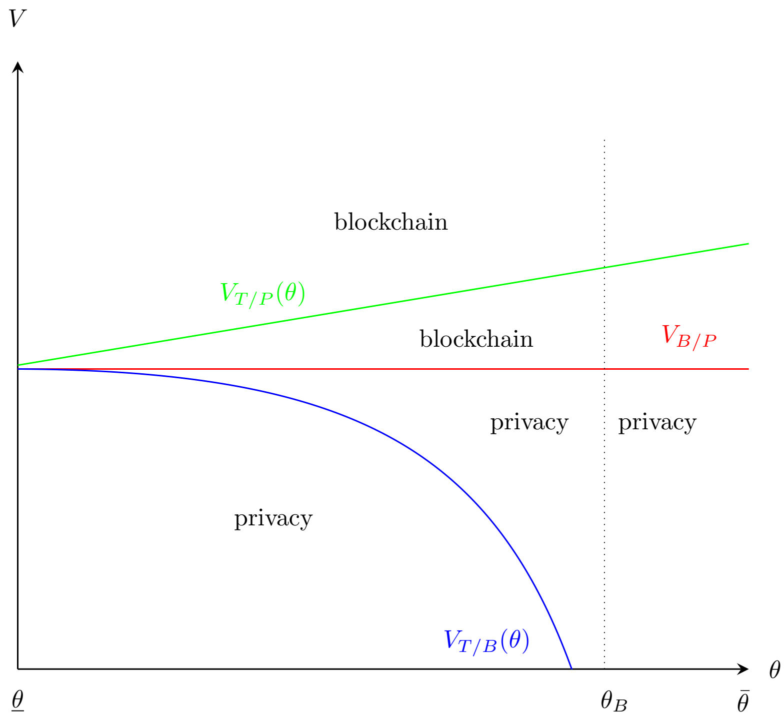

Figure 2: A case where $V'_{T/B}(\underline{\theta}) < 0$.

The following Lemma gives a condition under which the different thresholds are equal.

Lemma 5. *There is a quality $\check{\theta}$ of the traditional titling system which is lower than the blockchain quality ($\underline{\theta} < \check{\theta} < \theta_B$), such that both the land value $V_{T/P}(\check{\theta})$ for which one is indifferent between*

[17]The conditions $V'_{T/B}(\underline{\theta}) \leq 0$ is equivalent to $V_{T/P}(\underline{\theta}) = c'(\underline{\theta}) \geq \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}} = V_{T/B}(\underline{\theta}) = V_{B/P}$. The interpretation is that for $V_{T/B}(\theta) = \frac{\tilde{a}-c(\theta)}{\theta_B-\theta}$ to decrease at $\theta = \underline{\theta}$ it must be that $c(\theta)$ increases relatively faster at that quality level.

[18]In this case we have used the cost function $c(\theta) = \theta(\theta - \underline{\theta}) + .005$. The parameter values are the same as those used in the preceding figure, except that $\tilde{a} = 0.4$.

traditional titling and privacy and the land value $V_{T/B}(\check{\theta})$ for which one is indifferent between traditional and blockchain registration are equal to the (constant) land value for which one is indifferent between blockchain and privacy, if, and only if, $V_{T/B}(\theta)$ is increasing.

To understand this lemma, first observe that when the thresholds $V_{T/P}(\theta)$ and $V_{T/B}(\theta)$ are equal, then they are also equal to the threshold $V_{B/P}$. Indeed, whenever for a certain land value its owner is indifferent between traditional land titling and privacy, as well as between traditional and blockchain titlings, then this owner must be indifferent between blockchain titling and privacy.

Secondly, observe that under the condition stated in the lemma above, when the quality of traditional titling is close to its minimum value $\underline{\theta}$, one has $V_{T/B} > V_{T/P}$. That is because, as the quality provided by blockchain titling is greater than with traditional titling, and as the cost of the former is greater than the latter, to benefit from blockchain titling the value of land must be high enough. Now when the quality of traditional land titling improves, its cost rises more than proportionally and there are ever fewer land values for which traditional land titling is advantageous (that is because, the values of these lands must be lower and lower). This implies that $V_{T/B}(\theta)$ eventually goes to zero. But as $V_{T/P}(\theta)$ is increasing there must be a value of θ, $\check{\theta}$, such that the two thresholds $V_{T/P}(\theta)$ and $V_{T/B}(\theta)$ are equal. Figure 3 illustrates the preceding argument.[19]

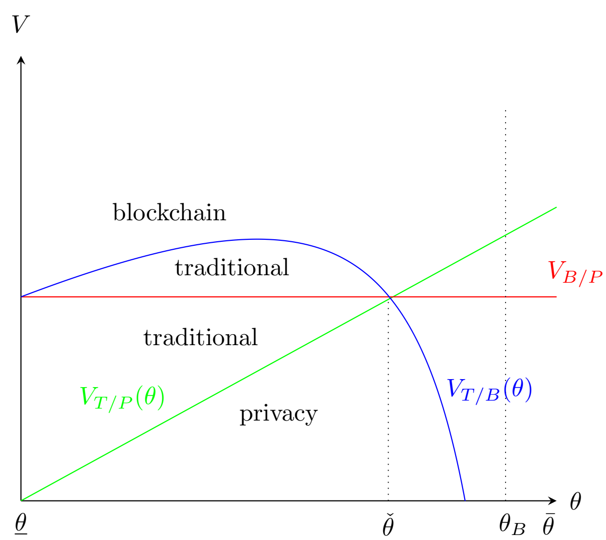

Figure 3: An example where $V'_{T/B}(\underline{\theta}) > 0$ and $V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta}) = V_{B/P}$.

5.4 Overview of individuals choices

Figure 4 summarizes individual choices between blockchain and traditional land titling and privacy.[20] Note that in order to save place, we concentrate on the case (depicted in Figure 3) where $V'_{T/B}(\underline{\theta}) >$

[19] In this figure we set $\underline{\theta} = 1$, $\bar{\theta} = 2$, $\theta_B = 1.4$, $c(\theta) = .6(\theta - \underline{\theta})^2 + 0.005$ and $\tilde{a} = .5$. Observe that $c'(\underline{\theta}) = 0 < \frac{\tilde{a} - 0.005}{\theta_B - \underline{\theta}} = 1.2375$.

[20] In this figure, $\underline{\theta} = 1$, $\bar{\theta} = 2$, $\theta_B = 1.4$, $c(\theta) = .5(\theta - \underline{\theta})^2 + 0.005$ and $\tilde{a} = .06$.

0.[21] The green line represents the threshold land value $V_{T/P}(\theta)$ above which traditional titling is preferred to privacy while the red line represents the constant threshold land value $V_{B/P}$ above which blockchain titling is preferred to privacy. The blue curve represents the threshold values $V_{T/B}(\theta)$. For traditional land titling quality above (below) that of blockchain titling (θ_B), this threshold is the land value above which traditional titling is (not) preferred to blockchain titling.

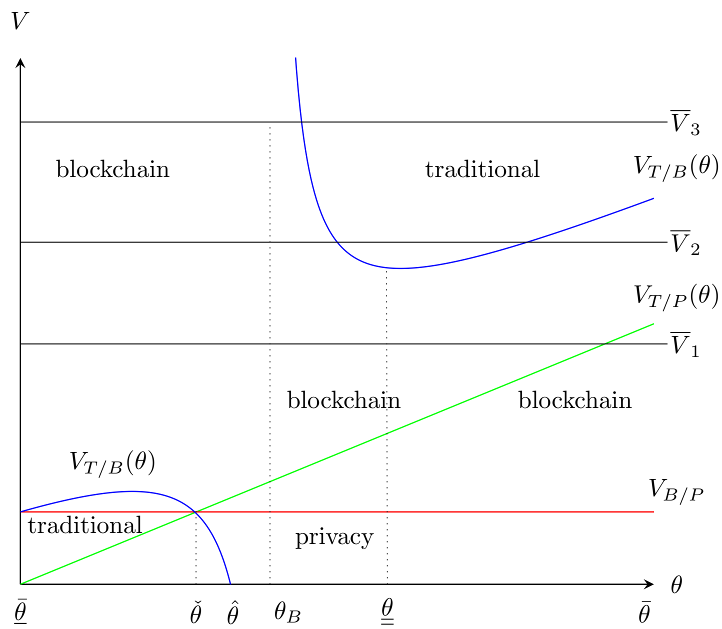

Figure 4: Combinations of V and θ that result in different choices of land titling, where $\overline{V}_1$, $\overline{V}_2$, $\overline{V}_3$ refer to three cases stated in Proposition 2.

The figure provides a map of the cases where traditional titling may be the best choice. Following Proposition 2, if $\overline{V} = \overline{V}_1$, this is impossible for high quality levels (that is, θ higher than θ_B). On the contrary, it is possible that traditional titling is preferred to blockain titling and to privacy when the maximal land value is relatively high, as it is the case for $\overline{V} = \overline{V}_2$ and $\overline{V} = \overline{V}_3$.

Regarding low quality levels, as $V'_{T/B}(\underline{\theta}) > 0$, according to Lemma 5, there is a quality $\check{\theta}$ of the traditional titling system such that $\underline{\theta} < \check{\theta} < \theta_B$ and $V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta}) = V_{B/P}$. The figure depicts the case where traditional titling is preferred to both blockchain titling and privacy, for low quality values and low land values.[22]

[21] We know that where $V'_{T/B}(\underline{\theta}) \leq 0$ no landowner chooses traditional titling when $\theta < \theta_B$. When $\theta_B < \theta$, individual choices are similar to those made when ${V'}_{T/B}(\underline{\theta}) > 0$.

[22] A more rigorous argument justifying these choices is provided in the proof of Proposition 4, Case 1.

6 Blockchain and Optimal Titling Systems

In this section, we study optimal titling systems when blockchain titling becomes available. We first focus on the case where blockchain ousts traditional land titling (subsection 6.1), meaning that whatever the quality of provided by this system, none will use it. Secondly, when it is optimal to keep traditional land titling, we concentrate on determining whether its quality should be higher than that of blockchain titling, or lower (subsection 6.2). Thirdly, we investigate the effect of the blockchain on the optimal quality of the traditional titling system (subsection 6.3).

6.1 Blockchain titling ousts traditional titling

The next proposition sums up the conditions under which it is optimal to replace traditional land titling with blockchain titling.

Proposition 3. *Assume that the land value for which one is indifferent between the blockchain and traditional titling decreases with quality θ when this quality is close to its lowest value $\underline{\theta}$ ($V'_{T/B}(\underline{\theta}) \leq 0$). Assume also that when the quality is above that of the blockchain, the land value for which one is indifferent between blockchain and traditional titling exceeds the highest land value $\overline{V}$. Then, whatever its quality θ, people will never use the traditional land titling system. They will either use blockchain titling or resort to privacy.*

To understand this proposition, let us consider the following three sets of quality for a traditional land titling system. This quality may be higher than that of blockchain titling ($\theta \in]\theta_B, \overline{\theta}]$). It may be intermediate ($\theta \in [\hat{\theta}, \theta_B]$), which is lower than that of the blockchain, and its unit cost greater than the blockchain's (in that case, traditional land titling is of no use). Finally, the quality and the unit costs of traditional land titling may both be lower than their counterparts with the blockchain ($\theta \in [\underline{\theta}, \hat{\theta}]$).

Assume that the quality of traditional land titling is higher than that of the blockchain. Under the conditions set in Proposition 3, landowners do not use traditional land titling despite the high degree of protection provided. Indeed, using traditional land titling would imply bearing a cost increase that can never be compensated by a sufficiently large expected gain in holding land, as the minimum land value at which the increase in quality is worthwhile is above the maximum value of land in the economy considered. Hence, the landowners resort to either privacy or blockchain titling. Choosing the latter is triggered by a land value being above the threshold $V_{B/P}$.

Consider now the case where the quality of traditional land titling is lower than the quality at which traditional and blockchain land titlings have the same cost, $c(\hat{\theta}) = \tilde{a}$. Under the assumptions of the Proposition above, whenever a landowner prefers traditional to blockchain titling (namely when $V \leq V_{T/B}(\theta)$) he also prefers privacy to the former. Thus, landowners actually choose between blockchain titling and privacy.

To sum up, when the quality cost of the traditional titling system grows more quickly than its benefit, whether the quality level is low or high, blockchain titling is a better alternative. Therefore, while blockchain titling suffers from defects (that is, it is not free, and it is less secure than one might hope), it may still be the best alternative titling system (for all those who do not choose privacy).

The form taken by social welfare is then as follows

$$\mathcal{W}_B(\theta) = \int_0^{V_{B/P}} \underline{\theta} V f(V) dV + \int_{V_{B/P}}^{\overline{V}} [\theta_B - \tilde{a}] f(V) dV. \tag{5}$$

In the expression above, the first integral corresponding to the expected value of untitled lands, and the second one is the expected value of blockchain titled lands. Notice that social welfare no longer depend on the quality of traditional land titling.

6.2 Traditional land titling remains

When do we need traditional land titling to maximize social welfare? If the conditions of Proposition 3 are met, we know that no one ever relies on traditional titling. But when these conditions are not satisfied, we show that the optimal quality is always such that some people rely on traditional titling and others on blockchain titling.

Proposition 4 states the different cases where traditional land titling remains, and provides some insights on the location of its optimal quality.

Proposition 4.

1. *Low optimal quality. Assume that the land value for which one is indifferent between the blockchain and traditional titling increases with quality θ when this quality is close to its lowest value $\underline{\theta}$ ($V'_{T/B}(\underline{\theta}) > 0$). Moreover, assume that when the quality is above that of the blockchain, the land value for which one is indifferent between blockchain and traditional titling exceeds the highest land value $\overline{V}$. Then, it is optimal to use traditional land titling and its optimal quality is lower than $\check{\theta}$.*

2. *High optimal quality. Assume that the land value for which one is indifferent between the blockchain and traditional titling decreases with quality θ when this quality is close to its lowest value $\underline{\theta}$ ($V'_{T/B}(\underline{\theta}) \leq 0$). Assume also that when the quality is above that of the blockchain, the land value for which one is indifferent between blockchain and traditional titling is below the highest land value $\overline{V}$. Then, it is optimal to use traditional land titling and its optimal quality is lower than $\check{\theta}$.*

3. *Indeterminate optimal quality. Finally, assume that the land value for which one is indifferent between the blockchain and traditional titling increases with quality θ when this quality is close to its lowest value $\underline{\theta}$ ($V'_{T/B}(\underline{\theta}) \leq 0$). In addition, suppose that when when the quality is above that of the blockchain, the land value for which one is indifferent between blockchain and traditional titling is below the highest land value $\overline{V}$. Then, it is optimal to use traditional land titling and its optimal quality is either lower than $\check{\theta}$ or higher than θ' or in $[\theta'_1, \theta'_2]$.*

In all three cases, there is an optimal quality of traditional land titling. This optimal quality balances the marginal increase in the average expected quality of land traditionally titled (net of the marginal (unit) cost), and the reduction in the size of the set of these plots (this trade-off is akin to that seen in section 3).

The analysis of the optimal quality of the traditional land titling system leads us to identify two different situations. In the first situation, it is possible to locate the optimal quality of the system, which can be either low (Proposition 4, case 1), or high (Proposition 4, case 2).

Under the assumptions of case 1 (low quality), for low quality values ($\theta < \check{\theta}$), some landowners find it worthwhile to use traditional land titling instead of the blockchain (the land values of these owners lie in between $V_{T/P}$ and $V_{T/B}(\theta)$). To understand why social welfare is greater when the traditional land titling quality is lower than $\check{\theta}$, note that all landowners who rely on traditional titling could have chosen blockchain titling or privacy. But if a landowner resorts to an alternative titling system, his decision must improve his payoff. As all the other owners stick to their choices (whatever the quality of the traditional system, they never resort to it), their expected gains are unchanged. Therefore, social welfare must increase. It is thus it optimal to use traditional titling, and the degree of protection provided by traditional titling should be low (and cheap).

On the contrary, under the assumptions of case 2, while it is optimal to keep traditional titling, the degree of optimal protection provided by this titling must be high (but costly). Traditional rather than blockchain titling increases the expected payoff of the individuals with the land of the highest value. The reasoning is similar to that of case 1 of Proposition 4, that is, if one owner changes his titling choice, his new decision must improve his payoff. Notice that in all cases, some landowners will rely on the blockchain.

By contrast with the first two cases, in case 3 of Proposition 4, the location of the optimal quality of traditional titling is indeterminate. Whether traditional land titling quality if low or high, there will always be some landowners who use traditional titling. That is why one cannot precisely locate the optimal quality of traditional land titling.

Let us consider more precisely the indeterminate optimal quality case. To understand what matters for the choice of the optimal quality it is useful to review in turn the expression of social welfare when the quality of the traditional titling system is either low or high.

When the best quality θ_{**} of traditional land titling is among the lowest, that is $\theta_{**} \in [\underline{\theta}, \check{\theta}]$, then the social welfare function is written as follows:

$$W_B(\theta_{**}) = \int_0^{V_{T/P}(\theta_{**})} \underline{\theta} V f(V) dV + \int_{V_{T/P}(\theta_{**})}^{V_{T/B}(\theta_{**})} (\theta_{**} V - c(\theta_{**}))\, f(V) dV + \int_{V_{T/B}(\theta_{**})}^{\overline{V}} (\theta_B V - \tilde{a})\, f(V) dV. \quad (6)$$

When one uses traditional land titling with the best high quality (higher than θ', or in $[\theta_1'; \theta_2']$), say θ^{**}, the social welfare function reads:

$$\mathcal{W}_B(\theta^{**}) = \int_0^{V_{B/P}} \underline{\theta} V f(V) dV + \int_{V_{B/P}}^{V_{T/B}(\theta^{**})} [\theta_B V - \tilde{a}] f(V) dV + \int_{V_{T/B}(\theta^{**})}^{\overline{V}} [\theta^{**} V - c(\theta^{**})] f(V) dV. \quad (7)$$

To determine the best quality of traditional land titling one needs to compare the social welfare as expressed in (6) and (7), when θ maximizes either (6) or (7). Unfortunately, in general nothing can be said about the comparison.

To better understand what determines the outcome of the comparison, it is useful to rely on the following Lemma.

Lemma 6. *The following inequalities hold*

$$V_{T/P}(\theta_{**}) < V_{B/P} < V_{T/B}(\theta_{**}) < V_{T/B}(\theta^{**}) \leq \overline{V}. \tag{8}$$

We build on this lemma to make the following remarks.

1. All the owners for whom the land value V is higher than $V_{T/B}(\theta^{**})$ are better off with a high-quality traditional land titling system.
2. All the owners whose land values V are in $]V_{B/P}, V_{T/B}(\theta_{**})[$ are worse off with a high-quality land titling system. They prefer low-quality traditional titling to the blockchain, but they prefer the latter to high-quality traditional titling.
3. All the owners whose land value V are in $]V_{T/P}(\theta_{**}), V_{T/B}(\theta_{**})[$ are worse off with a high-quality land titling system. They prefer the low-quality land titling system to privacy (or the blockchain), but they prefer privacy to the blockchain.
4. The other owners are indifferent between the two alternatives.

It follows from the remarks above that in this indeterminate case the optimal quality of traditional land titling depends on the distribution of land values.

Consider for instance an economy where land values are often large. Introducing the blockchain benefits owners of land whose values are relatively low because it decreases the probability of forfeiture at a cost lower than that of the traditional land titling. On the other hand, for the owners of highly valued land it is better to rely on traditional titling rather than blockchain titling. That is because, the increase in the expected value of land with traditional titling is higher than that obtained with the blockchain, and still more than compensates the rise in the unit cost. Since the owners of lowly valued land are few and the values of the latter are low, using the blockchain results at best in a slight increase in social welfare. It then follows that to maximize social welfare, one needs to choose a high quality for traditional titling.

6.3 The effect of the blockchain on the traditional titling system

The aim of this last section is to give some insights about the impact of the introduction of blockchain titling on the quality of the registration system. Should the quality be degraded or upgraded in response to the introduction of the blockchain?

To answer this question, let us focus on the cases identified in Proposition 4 where traditional land titling remains.[23] In the first two cases presented in Proposition 4, we know where lies the optimal quality of the titling system when blockchain titling becomes available, that is, below or above that provided by blockchain titling. In the former case we have Crummy Titling, whereas in the second case we have Cadillac Titling. In the last case of Proposition 4, the optimal quality is indeterminate. Let us consider these three cases in turn.

[23] We set aside the case where blockchain titling ousts traditional titling. The conditions under which this case exists are given in Proposition 3.

Crummy Titling

In the first case of Proposition 4, the optimal quality (in presence of blockchain) θ_{**} is lower than that of the blockchain (θ_B). For this quality θ_{**} of traditional land titling, we have $\theta_{**} \in [\underline{\theta}, \check{\theta}]$, and the social welfare function is written as follows:

$$W_B(\theta_{**}) = \int_0^{V_{T/P}(\theta_{**})} \underline{\theta} V f(V) dV + \int_{V_{T/P}(\theta_{**})}^{V_{T/B}(\theta_{**})} (\theta_{**} V - c(\theta_{**})) f(V) dV + \int_{V_{T/B}(\theta_{**})}^{\overline{V}} (\theta_B V - \tilde{a}) f(V) dV. \quad (9)$$

Let us first assume that the initial optimal degree θ^* of quality of the traditional titling system before the introduction of the blockchain is higher than $\check{\theta}$ ($\theta^* \in [\check{\theta}, \bar{\theta}]$). Then, it is immediate that $\theta_{**} < \theta^*$. That is because, the new optimal quality of traditional titling cannot lie in $[\check{\theta}, \bar{\theta}]$. In such a case, there is a *decrease* in the optimal quality provided by the traditional titling system.

Now assume that $\theta^* < \check{\theta}$. In order to understand why it may be optimal to decrease the quality of traditional titling, suppose we stick with the quality θ^* once the blockchain is available. Since the value of social welfare is given by equation (9) (where θ_{**} is replaced by θ^*), the marginal value of social welfare reads

$$\int_{V_{T/P}(\theta^*)}^{V_{T/B}(\theta^*)} [V - c'(\theta^*)] f(V) dV. \quad (10)$$

But by definition of θ^* and equation (1) we know that

$$\int_{V_{T/P}(\theta^*)}^{\overline{V}} \left[V - c'(\theta^*)\right] f(V) dV = 0. \quad (11)$$

Since $V - c'(\theta^*) > 0$ for values of V close to $\overline{V}$, it follows that:

$$\int_{V_{T/P}(\theta^*)}^{V_{T/B}(\theta^*)} [V - c'(\theta^*)] f(V) dV < 0. \quad (12)$$

Thus, one can increase social welfare by decreasing the quality of traditional titling. Since we do not know if the social welfare is concave or single-peaked, however, it could be that the maximum value of social welfare is achieved with $\theta_{**} > \theta^*$.

Cadillac Titling

In the second case of Proposition 4, the optimal quality θ^{**} (in presence of blockchain) of traditional titling is higher than that of the blockchain (θ_B).

Let us first assume that the optimal quality of the traditional titling system without the blockchain θ^* is below θ' (or θ'_1). If $\theta^* \in [\underline{\theta}, \theta']$, then it is immediate that $\theta^{**} > \theta^*$. Indeed, the new optimal quality of traditional titling cannot lie into $[\underline{\theta}, \theta']$. In such a case, introducing the blockchain results in a *increase* in the optimal quality provided by the traditional titling system.

Suppose now that $\theta_B < \theta^*$ (otherwise, we obtain the same conclusion as above). Recall that because θ^{**} higher than θ' (or else in $[\theta'_1; \theta'_2]$), the social welfare function reads:

$$\mathcal{W}_B(\theta^{**}) = \int_0^{V_{B/P}} \underline{\theta} V f(V)dV + \int_{V_{B/P}}^{V_{T/B}(\theta^{**})} [\theta_B V - \tilde{a}] f(V)dV$$

$$+ \int_{V_{T/B}(\theta^{**})}^{\overline{V}} [\theta^{**} V - c(\theta^{**})] f(V)dV. \quad (13)$$

In order to understand why it may be optimal to increase traditional titling quality when blockchain titling is introduced, suppose that we stick with the previous optimal quality θ^* of traditional titling. Since the value of social welfare is given by equation (13) (where θ^{**} is replaced by θ^*), the marginal value of social welfare reads

$$\int_{V_{T/B}(\theta^*)}^{\overline{V}} \left[V - c'(\theta^*)\right] f(V)dV. \quad (14)$$

From Lemma 2, we have $V_{T/B}(\theta) > V_{T/P}(\theta)$ when $\theta_B < \theta$. But by definition of θ^* we know that:

$$\int_{V_{T/P}(\theta^*)}^{\overline{V}} \left[V - c'(\theta^*)\right] f(V)dV = 0. \quad (15)$$

This implies that

$$\int_{V_{T/B}(\theta^*)}^{\overline{V}} \left[V - c'(\theta^*)\right] f(V)dV > 0. \quad (16)$$

That is because, for values of V close to $V_{T/P}(\theta^*)$, $V - c'(\theta^*) < 0$. Therefore, one can increase social welfare by increasing its quality. However, as we do not know if the welfare function is concave or single-peaked, it could be that its global maximum is reached for values of θ lower than θ^*.

In both cases (Cadillac and Crummy titling), it is worth noticing that the optimal quality of traditional deviates, upwards or downwards, from the quality provided by the blockchain, after the introducing of the latter. The economies in cost, or else, the value added by the traditional titling system must be high enough to worth it, for some landowners.

Indeterminate optimal titling quality

Finally, consider the last case of Proposition 4, where the location of the new optimal titling quality is indeterminate. We have two candidates (either Crummy or Cadillac Titlings). Then the location of the new optimal quality will depend on the distribution of land values.

To see this, suppose that a distribution of land values first-order stochastically dominates another one. This implies that the first distribution gives more weights to the land of high values. In this case, it is likely that Cadillac titling is the optimal solution. That is because, in the case of Crummy titling, owners of lands of high values rely on blockchain titling. But since they rely on traditional titling when Cadillac titling is used, this implies that the landowners who choose traditional titling are better off with this solution. Again, as the proportion of lands with high values is relatively large, social welfare is likely to be greater with Cadillac rather than with Crummy titling.

7 Discussion

The previous results were obtained under a set of assumptions that were made to simplify the analysis. In this section, we consider three issues that pertain to the relevance of these results, namely insurance titling, different blockchain qualities and switching costs.

• **Title insurance**

Following Arruñada and Garoupa (2005) basic model, we have not considered title insurance costs, notably the cost of hiring conveyancers, in our analysis.[24] Title insurance would depend on the characteristics of the titling system: the lower the protection the higher the demand, and the insurance expenses. A necessary condition for our results to hold is that title insurance cannot make landowners indifferent between two titling systems. Otherwise, one could be indifferent between, e.g., resorting to privacy with insurance and traditional registration (without insurance). Yet, our results would certainly be modified if landowners were characterized by their heterogeneous ability to insure at low cost. Notice, however, that in some countries insurance titling is not a relevant issue. For instance, in France only the notaries insure themselves again titling mistakes (of course, the insurance costs are passed on to landowners through titling fees, albeit in a uniform way). Hence, what matters is how title insurance is organized.

• **Different types of blockchain**

While we have assumed that there is continuum of quality for traditional titling, we have considered only one type of blockchain. A more complete analysis would thus have to take into account a set of blockchains (see subsection 4.1 for a quick overview of the different kinds of blockchain). But in relation to social welfare maximization, we would thus have to look for a pair of qualities (one for the traditional system, one for the blockchain). This would be quite intricate. That is because, for any pair of qualities, we would have to analyze the titling choices made by the landowners. Yet, we suspect that we would still have crummy titling. Under crummy titling, the only way traditional titling is used is because it is cheap. That could still be the case with different qualities of blockchain titling. First of all, for any quality of the traditional titling system, we can imagine that there would be only one blockchain titling choice (that choice would balance the benefit and the cost of titling). Then, it could still be the case that traditional titling with low quality but with low cost could be used by some landowners. The case for Cadillac titling can be justified in a similar way.

• **Switching costs**

When multiple titling systems are being simultaneously in place, owners whose lands are titled and who prefer an alternative titling system may face switching costs. While our analysis may seem to imply that all registrations are first registrations, we believe that our conclusions are not limited to this case, provided that switching costs are negligible. Of course, switching for another titling system is likely to raise the risk of conflicting claims.[25] Yet, since this difficulty will affect every potential switcher all the switching costs could be socialized and pass on to the landowners in a uniform way and we would be back to a setting quite similar to the one used in this paper (that is, the cost of a titling system would be the same for all owners, whether they already own a land or not).

[24] As a referee notices, title insurance is mot insurance per see as it does not rely on pooling risks. It is actually purging conflicting claims. See Arruñada (2002) and (2012) (pp. 188).

[25] We are grateful for a referee for drawing our attention to this fact.

8 Conclusion

In this paper, we have first generalized the model developed by Arruñada and Garoupa (2005) to study optimal land titling systems. In our setup, there is a priori an infinite set of systems, each characterized by its quality (namely, the probability that a given plot of land does not face forfeiture) and the unit transaction cost. We have used our model to study optimal titling systems when blockchain titling becomes available. In particular, we have addressed the following question: Can it be optimal to get rid of the traditional land titling?

We have found that, despite the introduction of the blockchain, and under reasonable assumptions, it is still socially optimal to keep the traditional land titling system. However, it is possible that the traditional land titling system must be adapted. We indeed show that the optimal quality of protection provided by traditional titling must be either sufficiently high (and higher than that of the blockchain), or low enough (and lower than that of the blockchain).

Yet, the advent of the blockchain in land titling may also render traditional titling unnecessary. This is the case when the expected additional value of traditional titling can never compensate for the additional cost, by comparison with the blockchain; and when the marginal cost of increasing the degree of protection beyond that of privacy is too high.

Interestingly, the conditions determining the optimal titling systems are in principle observable (*e.g.*, the distribution of land prices, the risk of hacking, etc.). Consequently, the theoretical approach developed in this paper can be useful in shaping the reform of titling systems.

Beyond the study of the three issues raised in the discussion section, a natural topic for further research is to study how notaries, for instance, are likely to adapt to the introduction of blockchain titling, and what public policies a country should follow to induce them to choose the optimal quality of the land titling system. Another topic for further research is to consider that changing a land titling system may be costly and this should be taken into account in devising optimal public titling systems when blockchain registration is available. Lastly, it would be interesting, albeit challenging, to examine the design of optimal titling system when the cost of quality is not everywhere convex increasing.

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A Notations

- V: land title value.
- $\overline{V}$: maximal land title value.
- θ: probability that a landowner will not face forfeiture. This term reflects the degree of protection provided by a land-titling system to the landowner, or equivalently the quality of the land titling system.
- $\underline{\theta}$: the lowest degree of protection. The lowest degree of protection is provided by privacy.
- $\overline{\theta}$: the highest degree of protection.
- θ_B: the degree of protection provided by the blockchain.
- $c(\theta)$ unit cost per transaction associated with a traditional land title system whose quality is θ.
- $V_{T/P}(\theta)$: threshold value of the title above which traditional land titling if preferred to privacy, with $V_{T/P}(\theta) = \frac{c(\theta)-c(\underline{\theta})}{\theta-\underline{\theta}}$.
- $V_{T/B}(\theta)$: threshold value of the title above (or below) which traditional land titling if preferred to blockchain titling, with $V_{T/B}(\theta) = \frac{c(\theta)-\tilde{a}}{\theta-\theta_B}$.
- $V_{B/P}$: threshold value of the title above which blockchain land titling if preferred to privacy, with $V_{B/P} = \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}}$.
- $W(\theta)$: social welfare function when blockchain technology is not available.
- $W_B(\theta)$: social welfare function when blockchain technology is available.
- $\tilde{a}$: unit cost per transaction associated with the blockchain.
- θ'_1 and θ'_2: degrees of protection such that $V'_{T/B}(\theta'_1) = V'_{T/B}(\theta'_2) = \overline{V}$ in case 2 of Proposition 2, with $\theta'_1 < \theta'_2$.
- θ': degree of protection such that $V_{T/B}(\theta') = \overline{V}$ in case 3 of Proposition 2.
- $\underline{\underline{\theta}}$ is the value of θ that minimizes $V_{T/B}(\theta)$.
- $\hat{\theta}$: value of θ such that $c(\hat{\theta}) = \tilde{a}$, the degree of quality of traditional land titling such that the unit cost of transaction under traditional land titling equals the unit cost of transaction under blockchain land titling.
- $\check{\theta}$: value of θ such that $V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta}) = V_{B/P}$, which exists under an assumption specified in Lemma 5.

B Proofs

Proof of Lemma 1

We have

$$\frac{d}{d\theta}V_{T/P}(\theta) = \frac{d}{d\theta}\left(\frac{c(\theta)-c(\underline{\theta})}{\theta-\underline{\theta}}\right) = \frac{c'(\theta)(\theta-\underline{\theta})-\big(c(\theta)-c(\underline{\theta})\big)}{\theta-\underline{\theta}}.$$

Since $c(.)$ is strictly convex, we get

$$0 \leq c'(\theta)(\theta-\underline{\theta})-\big(c(\theta)-c(\underline{\theta})\big).$$

The result follows.

Proof of Proposition 1

Assume that the optimal registration system is such that $\theta \in (\underline{\theta}, 1)$. Then using Leibnitz's rule, this optimal system satisfies the following necessary condition

$$\underline{\theta}\frac{d}{d\theta}\left(V_{T/P}(\theta)\right)V_{T/P}(\theta)f\left(V_{T/P}(\theta)\right) - \frac{d}{d\theta}V_{T/P}(\theta)\left(\theta V_{T/P}(\theta)-c(\theta)\right)f\left(V_{T/P}(\theta)\right) + \int_{V_{T/P}(\theta)}^{\overline{V}}\big(V-c'(\theta)\big)f(V)dV = 0, \quad (17)$$

which reduces to

$$\int_{V_{T/P}(\theta)}^{\overline{V}}\big(V-c'(\theta)\big)f(V)dV = 0. \quad (18)$$

Proof of Proposition 2

1. Case $\overline{V} \leq \min_\theta V_{T/B}(\theta)$. The conclusion is straightforward by definition of $V_{T/B}(\theta)$.
2. Case $\min_\theta V_{T/B}(\theta) < \overline{V} < V_{T/B}(\overline{\theta})$. In that case, it must be that $V'_{T/B}(\overline{\theta}) > 0$. Otherwise, $V'_{T/B}(\overline{\theta}) \leq 0$, and as $\min_\theta V_{T/B}(\theta) < \overline{V} < V_{T/B}(\overline{\theta})$, it must be that $V_{T/B}(\theta)$ reaches a maximum in $]\theta_B, \overline{\theta}[$. Observe, however, that, $V''_{T/B}(\theta) = \frac{c''(\theta)}{\theta-\theta_B}$, when $V'_{T/B}(\theta) = 0$. Therefore, there cannot be any maximum for $V_{T/B}$. So it must be that $V'_{T/B}(\overline{\theta}) > 0$. As $V_{T/B}(\theta) > \overline{V}$ when θ is close to θ_B, $V_{T/B}$ reaches a global minimum on $]\theta_B, \overline{\theta}[$. The existence of θ'_1 and θ'_2 follows easily.
3. Case $V_{T/B}(\overline{\theta}) < \overline{V}$. Suppose that $V'_{T/B}(\overline{\theta}) > 0$. Then, the existence of θ' is immediate. Moreover, there is only one value of θ such that $V_{T/B}(\theta) = \overline{V}$. Otherwise, $V_{T/B}$ would have a local maximum which is impossible (see the proof on the preceding case). Now suppose that $V'_{T/B}(\overline{\theta}) \leq 0$. Then, we have $V'_{T/B}(\theta) \leq 0$ for all θ in $]\theta_B, \overline{\theta}]$. Otherwise, there would be a local maximum, and as was already seen, this is impossible.

Proof of Lemma 2

We have

$$V_{T/B}(\theta) - V_{T/P}(\theta) = \frac{c(\theta) - \tilde{a}}{\theta - \theta_B} - \frac{c(\theta) - c(\underline{\theta})}{\theta - \underline{\theta}} \tag{19}$$

$$\geq \frac{c(\theta) - c(\theta_B)}{\theta - \theta_B} - \frac{c(\theta) - c(\underline{\theta})}{\theta - \underline{\theta}} \tag{20}$$

$$= \frac{-c(\underline{\theta}) - (-c(\theta))}{\underline{\theta} - \theta} - \frac{-c(\underline{\theta}) - (-c(\theta))}{\theta_B - \theta} \tag{21}$$

$$\geq 0 \tag{22}$$

where the last inequality follows from the fact that $-c(.)$ is concave and $\underline{\theta} < \theta_B < \theta$.

Proof of Lemma 3

For all $\theta > \theta_B$, we have

$$V_{T/B}(\theta) - V_{B/P} = \frac{c(\theta) - \tilde{a}}{\theta - \theta_B} - \frac{\tilde{a} - c(\underline{\theta})}{\theta_B - \underline{\theta}} \tag{23}$$

$$= \frac{\big(c(\theta) - \tilde{a}\big)(\theta_B - \underline{\theta}) - \big(\tilde{a} - c(\underline{\theta})\big)(\theta - \theta_B)}{(\theta - \theta_B)(\theta_B - \underline{\theta})}$$

$$= \frac{c(\theta)(\theta_B - \underline{\theta}) - \tilde{a}\big(\theta_B - \underline{\theta} + \theta - \theta_B\big) + c(\underline{\theta})(\theta - \theta_B)}{(\theta - \theta_B)(\theta_B - \underline{\theta})}$$

$$= \frac{c(\theta)(\theta_B - \underline{\theta}) - \tilde{a}\big(\theta - \underline{\theta}\big) + c(\underline{\theta})(\theta - \underline{\theta} + \underline{\theta} - \theta_B)}{(\theta - \theta_B)(\theta_B - \underline{\theta})}$$

$$= \frac{\big(c(\theta) - c(\underline{\theta}\big)(\theta_B - \underline{\theta}) - \big(\tilde{a} - c(\underline{\theta}\big)(\theta - \underline{\theta})}{(\theta - \theta_B)(\theta_B - \underline{\theta})}$$

$$= \frac{\theta - \underline{\theta}}{\theta - \theta_B}\left[\frac{c(\theta) - c(\underline{\theta})}{\theta - \underline{\theta}} - \frac{\tilde{a} - c(\underline{\theta})}{\theta_B - \underline{\theta}}\right] \tag{24}$$

$$> \frac{\theta - \underline{\theta}}{\theta - \theta_B}\left[\frac{c(\theta) - c(\underline{\theta})}{\theta - \underline{\theta}} - \frac{c(\theta_B) - c(\underline{\theta})}{\theta_B - \underline{\theta}}\right] \tag{25}$$

where the last inequality follows from Lemma 1 and the fact that $\tilde{a} < c(\theta_B)$ and $\theta_B < \theta$.

Proof of Lemma 4

Let us first show that under the assumptions of the Lemma we have $V'_{T/B}(\theta) \leq 0$. To see this observe that

$$V'_{T/B}(\theta) = \frac{-c'(\theta)\big(\theta_B - \theta\big) + \tilde{a} - c(\theta)}{\big(\theta_B - \theta\big)^2}. \tag{26}$$

We have

$$V'_{T/B}(\theta) = \frac{\phi(\theta)}{\big(\theta - \theta_B\big)^2} \tag{27}$$

where

$$\phi(\theta) = -c'(\theta)\big(\theta_B - \theta\big) + \tilde{a} - c(\theta), \tag{28}$$

and $\phi'(\theta) = -c''(\theta)\big(\theta_B - \theta\big) \leq 0$. Then $V'_{T/B}(\underline{\theta}) \leq 0$ implies that $\phi(\underline{\theta}) \leq 0$, hence that $V_{T/B}(\theta)$ is non-increasing. As $V_{T/B}(\underline{\theta}) = \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}}$, then $V_{T/B}(\theta) \leq \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}}$ for all $\theta \leq \hat{\theta}$. Moreover, one can check that

$$V'_{T/B}(\underline{\theta}) = \frac{1}{\big(\theta_B - \underline{\theta}\big)}\big[V_{B/P} - c'(\underline{\theta})\big]. \tag{29}$$

As by assumption, $V'_{T/B}(\underline{\theta}) \leq 0$ then $V_{B/P} \leq c'(\underline{\theta})$. Now we have

$$\lim_{\theta \to \underline{\theta}} V_{T/P}(\theta) = \lim_{\theta \to \underline{\theta}} \frac{c(\theta) - c(\underline{\theta})}{\theta - \underline{\theta}} = c'(\underline{\theta}) \geq \frac{\tilde{a} - c(\underline{\theta})}{\theta_B - \underline{\theta}} \equiv V_{B/P}, \tag{30}$$

and as the function $\big(c(\theta) - c(\underline{\theta})\big)/(\theta - \underline{\theta})$ is increasing, it follows that $V_{T/B}(\theta) \leq V_{B/P} \leq V_{T/P}(\theta)$ for all $\theta \leq \hat{\theta}$. We deduce from these inequalities that no landowner ever chooses traditional land titling.

Proof of Lemma 5

(Necessity). Suppose that there exists a quality $\check{\theta} > \underline{\theta}$ such that $V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta}) = V_{B/P}$. Then

$$V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta}) \iff \frac{c(\check{\theta}) - c(\underline{\theta})}{\check{\theta} - \underline{\theta}} = \frac{\tilde{a} - c(\underline{\theta})}{\theta_B - \underline{\theta}} = V_{B/P}. \tag{31}$$

Since $c\big(\theta_B\big) > \tilde{a}$, this implies that the value $\check{\theta}$ such that (31) holds is such that $\check{\theta} < \theta_B$. It is even lower than $\hat{\theta}$ since $V_{T/B}(\theta)$ takes negative values when θ is in $[\hat{\theta}, \theta_B[$. Now, since $V_{T/P}(\theta)$ is increasing, this implies that $\lim_{\theta \to \underline{\theta}} V_{T/P}(\theta) = c'(\underline{\theta}) < \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}}$.
(Sufficiency). Assume $\lim_{\theta \to \underline{\theta}} V_{T/P}(\theta) = c'(\underline{\theta}) < \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}}$. Since $V_{T/B}(\underline{\theta}) = \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}}$, $V_{T/B}(\hat{\theta}) = 0$, $V'_{T/B}(\underline{\theta}) > 0$ and $V_{T/P}(\theta)$ is increasing, there is a value $\check{\theta}$ such $V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta})$. It also follows from equation (24) in the proof of Lemma 3 that $V_{T/P}(\check{\theta}) = \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}}$.

Proof of Proposition 3

In order to prove the proposition, we need to consider the three following cases:

- $\theta \in]\theta_B, \bar{\theta}]$:

  By assumption $\overline{V} \leq V_{T/B}(\theta)$, $\forall \theta \in]\theta_B; \overline{\theta}]$). Thus no one uses traditional land titling. The conclusion follows from Proposition 2, case 1. Landowners resort to privacy when the value of their land is below $\frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}} \equiv V_{B/P}$.

- $\theta \in [\hat{\theta}, \theta_B[$:

  It can immediately be seen that whenever θ is in $[\hat{\theta}, \theta_B]$, $V_{T/B}(\theta) \leq 0$. Therefore, blockchain land titling will always be preferred to traditional titling. Moreover, people whose land value V is greater than $V_{B/P}$ will prefer the blockchain titling to privacy.

- $\theta \leq \hat{\theta}$. The conclusion follows from Lemma 4.

Proof of Proposition 4

• Firstly, assume that $V'_{T/B}(\underline{\theta}) > 0$ and $\overline{V} \leq \min_{\theta \in]\theta_B, \overline{\theta}]} V_{T/B}(\theta)$. As was observed in the Proof of Proposition 3, when θ is either in $[\hat{\theta}, \theta_B[$ or in $]\theta_B, \overline{\theta}]$, no one uses traditional land titling. Suppose then that θ is lower than $\hat{\theta}$. From Lemma 5 we know that when $c'(\underline{\theta}) < \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}} = V_{B/P}$, there exists $\check{\theta}$ such that $V_{T/B}(\check{\theta}) = V_{T/P}(\check{\theta}) = V_{B/P}$. Moreover, from the proof of Lemma 4, equation (29) ensures that $c'(\underline{\theta}) < \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}} = V_{B/P}$ whenever $V'_{B/T}(\underline{\theta}) > c'(\underline{\theta})$. Now, when $\theta \in]\check{\theta}, \hat{\theta}]$, we have $V_{T/B}(\theta) < V_{T/P}(\theta)$ (this follows from the fact that when $\theta = \check{\theta}$, $V_{T/B}(\check{\theta}) = V_{T/P}(\check{\theta})$, $V_{T/P}(\theta) = \frac{c(\theta)-c(\underline{\theta})}{\theta-\underline{\theta}}$ is increasing (Lemma 1), and as was seen in the proof of Lemma 2, $V_{T/B}(\theta) - V_{T/P}(\theta) = \frac{\theta_B-\theta}{\theta-\underline{\theta}}\left(\frac{c(\theta)-c(\underline{\theta})}{\theta-\underline{\theta}} - \frac{\tilde{a}-c(\underline{\theta})}{\theta_B-\underline{\theta}}\right)$. Thus traditional land titling is never used when $\theta \in]\check{\theta}, \hat{\theta}]$. Then consider the case where $\theta \in]\underline{\theta}, \check{\theta}[$. We have $V_{T/P}(\theta) < V_{B/P}$, and thus all the landowners whose values are in $[V_{T/P}(\theta), V_{B/P}]$ prefers traditional to blockchain land titling. Notice that all the other landowners stick to their choices. Since the set of landowners who choose traditional land titling are not negligible, it follows that the social welfare function takes a greater value than when $\theta \geq \check{\theta}$. Thus the optimal quality of traditional land titling is lower than $\check{\theta}$.

• Next assume that $V'_{T/B}(\underline{\theta}) \leq 0$ and $\min_{\theta \in]\theta_B, \overline{\theta}]} V_{T/B}(\theta) < \overline{V}$. We can use the proof of Proposition 3 (case where $\theta \leq \hat{\theta}$) to show that when θ is lower than $\hat{\theta}$, no one uses traditional land titling. According to Proposition 2, case 2, we know that if $\min_\theta V_{T/B}(\theta) < \overline{V} < V_{T/B}(\overline{\theta})$, then, there is an interval $[\theta'_1, \theta'_2]$ within $]\theta_B, \overline{\theta}]$ such that $V_{T/B}(\theta'_1) = V_{T/B}(\theta'_2) = \overline{V}$, and such that for all θ within this interval, $\overline{V} > V_{T/B}(\theta)$. All owners of land whose value lies within $[V_{T/B}(\theta), \overline{V}]$ prefer traditional to blockchain land titling when $\theta \in [\theta'_1, \theta'_2]$. Because these owners are better off than when $\theta < \theta_B$, and because all the other individuals stick to their titling choices, the social welfare function takes a greater value than if $\theta \leq \theta_B$. The conclusion follows. The same reasoning applies when $V_{T/B}(\overline{\theta}) < \overline{V}$.

• Finally assume that $V'_{T/B}(\underline{\theta}) > 0$ and $\min_{\theta \in]\theta_B, \overline{\theta}]} V_{T/B}(\theta) < \overline{V}$. By the same arguments used in the two preceding cases, we can show that when $\underline{\theta} < \theta < \check{\theta}$ and $\theta \in [\theta'_1, \theta'_2]$, some landowners will choose traditional titling. It follows that the social welfare function depends on θ in these two cases. But we cannot decipher whether it reaches it maximum with low or with high quality.

Proof of Lemma 6

Given the definitions of $V_{B/P}$, $V_{T/P}(\theta_{**})$, $V_{T/B}(\theta_{**})$ and $V_{T/B}(\theta^{**})$ it is sufficient to prove that $V_{T/B}(\theta_{**}) < V_{T/B}(\theta^{**})$. But we have seen in the proof of Lemma 4 that $V'_{T/B}(\theta) = 0$ implies $c(\theta) = \tilde{a} + c'(\theta)[\theta - \theta_B]$. Using this expression, observe that when $\theta < \theta_B$, $V_{T/B}(\theta)$ reaches its maximum value at a quality θ_1 such that $V_{T/B}(\theta_1) = c'(\theta_1)$ (this follows from the definition of $V_{T/B}$ and the equality $c(\theta) = \tilde{a} + c'(\theta)[\theta - \theta_B]$). Likewise, when $\theta > \theta_B$, $V_{T/B}(\theta)$ reaches its minimum value at a quality θ_2 such that $V_{T/B}(\theta_2) = c'(\theta_2)$. Since the function $c(.)$ is strictly convex, it follows that $V_{T/B}(\theta_1) < V_{T/B}(\theta_2)$ and therefore that $V_{T/B}(\theta_{**}) < V_{T/B}(\theta^{**})$.