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Corporate Cartels as a Criminal Network

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Abstract

Firms may belong to many cartels, resulting in interconnected networks of corporate cartels that share practices, strategies, and information. Consequently, each firm can be viewed as a node, and each cartel as the links between those nodes within a larger network of cartels. Using Graph Theory, we develop a model of collusive behavior within a network of cartels to examine how its organization affects their stability. Firms face a "*centrality* trade-off": the more cartels they belong to, the higher their profits, but the less stable each cartel is. We characterize one general network shape that maximizes cartel stability: the *denser* the network, the more stable the cartels. In other words, cartels are more resilient when firms participate in multiple *overlapping* cartels.

Keywords: cartels, network, antitrust, recidivism
JEL Codes: K21, L14, L41

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1 Introduction

The harmfulness of corporate cartels is a consensual issue among economists. This assessment has led nearly 40 countries to criminalize corporate cartels[1] (OECD, 2020), sometimes imposing severe penalties, including hefty prison sentences and substantial fines, of up to 14 years of imprisonment, and US$20 million upon culpable corporate executives. Additionally, corporate cartels can be seen as a white-collar criminal network, where firms act as nodes and cartels as the links connecting them.[2] The incentive to collude depends not only on the behavior of firms within a cartel but also on the choices their co-conspirators make outside this cartel, resulting from this interconnected network of collusions. Considering such networks of cartels may be crucial when looking at cartel dynamics.

This paper investigates whether cartel stability is facilitated by the existence and shape of a network of corporate cartels. Building on the framework developed by Ballester, Calvó-Armengol, and Zenou (2006), we apply it to the context of corporate cartels by assuming an exogenous network in which the firms' participation constraint (PC) is fulfilled ex-ante, meaning that firms have already decided to join the network of cartels. Our focus is therefore on firms' incentives to either remain in or deviate from the cartels they belong to, captured by the incentive compatibility constraint (ICC), which we interpret as *cartel stability*. We depart from the preexisting antitrust literature by considering that the firms' ICC is affected by network-related variables, and analyze how their *centrality*, known as their number of connections (or co-conspirator), affects the cartel stability. We first characterize a "*centrality trade-off*" faced by firms: the higher their centrality within the network, the greater the profits they can earn, but the less stable each of the cartels they belong to becomes. Last, we identify the *network shapes*, defined as the way cartels are connected, that maximize cartel stability, which we refer to as *optimal network shapes*. Both notions of the centrality trade-off and the optimal network shapes are closely related to the concept of *cartel recidivism* and *multiple offending*, which we broadly define as the simultaneous or sequential participation, after being sanctioned or not, of firms in multiple cartels.

Our main findings are organized into two main topics: *(i)* the identification of optimal firms' centrality, cartel size, and collusive effort, to maximize the stability of the network of cartels, *(ii)* the characterization of optimal shapes of networks of cartels.

(i) By analyzing the critical discount factor, we conclude that increasing the degree centrality, which is the number of connections a firm has, is always associated with greater cartel instability. However, this result does not mean that firms have no interest to be not connected at all. Indeed, the more cartels they belong to, the higher their profits, but each cartel to which they belong is individually more unstable, resulting in a "*centrality trade-off*". There is also an optimal cartel size and collusive effort, such that cartel stability is maximized.

(ii) We characterize two general network shapes that maximize the cartel stability. When the network is *connected*, implying that for each cartel there is at least one multiple-offending firm, the cartel stability increases with the network *density*. The network density is a measure of graph theory that compares the actual number of connections of a given network with the maximum number of connections that network could have. In other words, it provides an idea of the overall connectedness of the nodes within a given

[1] As of 2020, 36 countries and territories had criminalized bid-rigging and corporate cartels (see Appendix A.1).
[2] See a data-visualization of the "*Private International Cartels*" database in Appendix A.2.

network. The link between the cartel stability and network density means that the more connected the firms are with the same co-conspirators, the more stable those cartels are. Thus, under those conditions, cartel recidivism can be doubly beneficial, as it not only brings higher profits to the multi-offender firms but also strengthens the stability of the cartels to which these firms belong.
As an extension, we also study the case where the network is not restricted to be connected, in which case the *clustered network* is the most stable. A clustered or isolated network is defined in our setup as a network in which each cartel is not connected to any other cartel. This shape of network implies that there is no multi-offending firm in any cartel of that network. This finding means that the most stable networks are the ones within which cartel recidivism is totally eliminated, but the cartels composing such networks are hard to destabilize.

The theoretical literature on networks of corporate cartels is very scarce. Belleflamme and Bloch (2004) study bilateral collusive agreements, assuming that each firm is initially present in a national market and can enter foreign ones. Market-sharing agreements are modeled as reciprocal agreements not to compete, and the set of bilateral agreements gives rise to a collusive network among firms. The model analyzes the conditions under which collusive networks are stable and efficient, and how market characteristics affect the formation and durability of these networks. Belleflamme and Bloch (2004) apply their model to Cournot oligopoly with homogeneous products and auctions. However, even if part of our research question is related, their way of modeling the network of cartels is different. On one hand, they model cartels as bilateral noncompete agreements, whereas we consider that a cartel can include multiple firms forming what graph theory defines as cliques. On the other hand, they assume that firms can join and leave cartels, thus studying both PC and ICC, whereas we focus exclusively on the ICC. Also, we consider the presence of a competition agency that has a crucial role in cartel stability and thus the network shape, which they do not. Thus, although our methods are similar, our results are complementary and address different issues.

Grau-Carles and Cuerdo-Mir (2016) is one of the first empirical article to use graph theory to study corporate cartels. Their analysis focuses on the Vitamins Cartel (1989-1999)[3], which was active in 12 different markets and products, involving 13 firms. Grau-Carles and Cuerdo-Mir (2016) uses several graph-theoretic measures (e.g., degree, closeness, and betweenness centrality), and measures of network cohesion (e.g., subgraphs, connectivity, and clustering) to identify the dominant firm within this "super-cartel", defined as the one most likely to have played a central role in organizing this small cartel network. Beyond this study, a broader empirical literature has examined the use of network analysis in antitrust and cartel research (e.g., Gupta et al., 2010; Gabardo and Lopes, 2014; Wachs and Kertész, 2019; Harrington Jr and Imhof, 2022; Huber and Imhof, 2023). Those articles highlight that applying graph theory to corporate cartels can be a useful analytical tool. We strengthen the main idea of their analysis by showing that there is a theoretical link between firms' centrality and cartel stability, and that this network dimension of cartels should therefore be taken into account.

In contrast, there is a broad literature on corporate cartels, either theoretical (e.g., Motta and Polo, 2003; Harrington, 2018a), empirical (e.g., Levenstein and Suslow, 2006; Forsbacka, Le Coq, and Marvão, 2023), or experimental (e.g., Bigoni et al., 2012). Researchers looked at many aspects of corporate cartels, from the determinants of their formation, such as the estimated detection probability (Bryant and Eckard,

[3]European Commission's Decision, Case COMP/E-1/37.512 - Vitamins, November 21, 2001.

1991) or their duration (Broos et al., 2016), to those of their recidivism (e.g., Connor, 2010; Marvão, 2023). They also studied the optimal sanctions, including fines (e.g., Connor and Lande, 2008; Combe and Monnier, 2011), prison sentences (e.g., Werden, Hammond, and Barnett, 2011), leniency programs (e.g., Spagnolo, 2004; Bigoni et al., 2012; Marvão and Spagnolo, 2018), and alternative sanctions such as asset divestitures (Harrington, 2017; Harrington, 2018b). The main difference between these articles and the literature on networks of cartels is that the interactions between different cartels are either neglected or underestimated. Within this antitrust literature, our paper mainly contributes to the strand focusing on cartel stability and recidivism (e.g., Levenstein, Marvão, and Suslow, 2015; Marvão, 2016; Marvão, 2023). We show that accounting for the interconnection between cartels can have either a negative or positive impact, depending on the structure of these connections. We find that recidivism may be even more problematic than previously suggested in the literature, as it can allow firms not only to increase their profits but also to further enhance the stability of the cartels they belong to.

Last, our paper is also related to the literature on multimarket contact collusion (e.g., Bernheim and Whinston, 1990; Ciliberto and Williams, 2014; Ryu, Reuer, and Brush, 2020; Wilson, 2025). In our framework, firms may participate in several cartels operating in different markets, which creates a form of interconnection between cartels. However, unlike most papers in this literature (e.g. Bernheim and Whinston, 1990; Poppius, 2020; Ryan-Charleton and Galavan, 2024), we do not consider the possibility for firms to punish their partners in one market following a deviation in another. In our model, the interconnection between cartels arises not from cross-market punishment mechanisms but from the detection probability, which depends on the firms' centrality within the network. Nonetheless, our results are consistent with the main insights of the multimarket contact literature. Networks of corporate cartels composed of firms that participate in several cartels with identical members are more stable than those composed of distinct firms. In other words, having multimarket contacts enhances cartel stability, even though the underlying mechanism is not cross-market punishment through shared cartels, but rather the spreading of detection costs across all common cartels. Thus, the use of graph theory reinforces the findings of the multi-market contact literature, showing that, under certain conditions, such interconnections can enhance cartel stability.

The remainder of this paper is organized as follows. The next section (section 2) presents the network of cartels (section 2.1), the Competition policy (section 2.2), and the firms' profit functions and optimal deviation (section 2.3). Section 3 studies how the critical discount factor varies with the variables of interest. Last, section 4 analyzes the shapes of the networks of cartels that ensure the greatest stability.

2 The model

Consider n firms that belong to at least one cartel, such that their participation constraint (PC) is fulfilled. For each cartel in which it is involved, the firm i faces a binary choice: either sticking to or deviating from the collusion. We consider an infinitely repeated game, with a symmetric and static firms' discount factor denoted $\delta \in [0\,,1]$.

We restrict our attention to stationary collusive agreements supported by grim trigger strategies. Hence, any deviation by a firm at the period t is observed at $t+1$ by the other firms of this cartel, and definitively puts an end to the collusion, returning to the competitive game. Thus, once a firm has decided to deviate from a cartel, this firm has no other choice than to compete with the firms from that cartel in the subsequent periods[4].

2.1 The network of corporate cartels

Any network of corporate cartels $\boldsymbol{g}$ is a pair $\boldsymbol{g} = (N, M)$, where $N = \{1, 2, ..., n\}$ is a finite set of n colluding firms and $M = \{M_1, M_2, ..., M_m\}$ a finite set of m corporate cartels. Hence, each element of the set M is itself a subset composed of colluding firms, such that any cartel $M_a \subseteq N$. The size of a cartel M_a, defined as the number of firms within this cartel, is denoted $|M_a| = card(M_a)$, with $2 \leq |M_a| \leq n$, meaning that a cartel, to exist, is composed of at least 2 firms, and at most of all the colluding firms of the network.

Following the definition of Newman (2018), "a network is, in its simplest form, a collection of points joined together in pairs by lines. In the jargon of the field the points are referred to as *vertices* or *nodes* and the lines are referred to as *edges*.". Hence, nodes represent individual entities, while edges denote relationships or interactions between these entities.

Definition 1 – Network of corporate cartels. The pair $(N,\ M)$ forms the network of corporate cartels $\boldsymbol{g}$, where:

(i) the set N constitutes the nodes of the network, which are the colluding firms.

(ii) the set M constitutes the edges connecting these nodes, which are the corporate cartels.

For any network of corporate cartels $\boldsymbol{g}$, let $\boldsymbol{G}$ be the adjacency matrix of dimension $n \times n$ of entry $g_{i,j}$, that contains all the collusive interactions between firms. For any two firms $i, j \in N$ with $i \neq j$, the pair-wise relationship between these firms is captured by a binary variable, $g_{i,j} \in \{0, 1\}$, for which $g_{i,j} = 1$ if firm i is *connected*[5] to firm j, and $g_{i,j} = 0$ if that they are not. We assume reciprocal and non-oriented relationships, i.e., $g_{i,j} = g_{j,i}$, and set $g_{i,i} = 0$, such that firms are not connected to themselves.

For any cartel M_a, the firms within this cartel are all connected, so that each cartel is a *complete subgraph* or a *clique*. A *subgraph* is defined as a subset of nodes and links taken from a larger graph (here the network of corporate cartels $\boldsymbol{g}$), and is called *complete* (or a clique) if all nodes in this subgraph are connected. This assumption relies on the fact that, in most cases, the exact internal organization of cartels is unknown — that is, the existence of the internal links between their members. Hence, it is assumed that whenever firms belong to the same cartel, they are all in direct contact with one another, without any hierarchical structure or specific organization in the diffusion of information.

Last, the *degree centrality* provides a simple and intuitive measure of a node's relative importance within a network, defined as the number of connections of that node. In our framework, the degree centrality of a firm corresponds to the total number of its co-conspirators across all cartels. We use this measure as a proxy to identify the most "active" agents[6] in a given network of cartels $\boldsymbol{g}$.

[4]See a stage game illustration in Appendix B.1.

[5]The terms "connected", "linked", and "belong to the same cartel" are used interchangeably because if a firm i is connected or linked to a firm j, it means they belong to the same cartel.

[6]As defined by Zenou (2016).

Definition 2 – Degree centrality (1). The degree centrality of any firm i within the network $\boldsymbol{g}$ is the number of links it has with other firms and is denoted $DC(i)$, and is formally set as:

$$DC(i) = \sum_{j \in N} g_{i,j}$$

2.2 Competition policy

A competition agency may detect firms' collusive agreement M_a with a probability $p_a \in [0\,,1]$. If a firm is detected, it incurs a fine F, for which $F > 0$.

We consider the detection probability to be endogenous to two variables. The first variable is the collusive effort (y_a)[7], defined as the overcharge paid by consumers because of the cartel M_a, such that $y_a \in [\varepsilon\,; p^M]$, where ε is the smallest (considered as infinitesimal) collusive price above the competition price and p^M is the monopoly price.[8] We assume that the collusive effort is set at the cartel level and that the firms within a cartel put the same effort into the collusion. The other variable impacting the detection probability is the degree centrality, denoted $DC(i)$, defined in section 2.1 as the number of connections of firm i. The detection probability is increasing in both the collusive effort and the degree centrality.

We assume that the detection probability of a firm does not necessarily depend on its own centrality but may also rely on the centrality of its neighboring firms. This assumption implies that if a firm i has a low degree centrality but is connected to a central firm j, then i faces the same risk of being detected as firm j. This *domino effect* is, however, limited to direct connections and does not spread to the entire network. The intuition behind this assumption is that when a firm that has participated in a cartel is detected, the competition agency launches an investigation within that firm, and obtains full knowledge of the other cartels in which it is involved. Hence, if that firm is initially detected because of a cartel M_a, but is also involved in a cartel M_b, then the firms of that cartel M_b are also detected. Nevertheless, the domino effect of detection stops at this stage, and only firms directly linked to this firm are detected.[9]

Definition 3 – Detection probability. For any firms i and j that belong to the cartel M_a within the network of corporate cartels $\boldsymbol{g}$, the detection probability of i is denoted:

$$p_a(y_a, \boldsymbol{g}) = \phi\,[y_a, max_{i,j \in M_a}\,\{DC(i), DC(j)\}]$$

ϕ is a normalizing constant such that, for any cartel M_a in the network of cartels $\boldsymbol{g}$, the detection probability $p_a(y_a, \boldsymbol{g}) \in [0\,,1]$ is fulfilled.

Note that even if a firm is detected when all the firms from its cartel have decided to stick to the collusion, then this cartel is re-formed in the next period with the same firms[10]. Thus, cartel detection is treated as a potential cost associated with engaging in prohibited behavior. Also, evidence of the existence of a cartel is destroyed at the end of each period, which means that the probability of detection is independent of the number of periods, and firms that have left a cartel in period t, no longer run the risk of being caught and punished in the subsequent periods.

[7] Similarly to Liu et al. (2012) which uses criminal effort.

[8] Close to Harrington and Chen (2006).

[9] This assumption is close to the case of domino detections that occurred in the lysine and citric acid cartels (1991-1995), where a manager from ADM & Co., involved in the lysine cartel and collaborating with the FBI, enabled federal agents to uncover the citric acid cartel, in which ADM & Co. was also engaged.

[10] As in Motta and Polo (2003).

2.3 Profit functions and optimal deviation

For any firm i involved in the cartel M_a, we let the deviation profit be $\pi^D_{i,a}(y_a, |M_a|)$, the cartel profit $\pi^C_{i,a}(y_a, |M_a|)$, and the competition profit $\pi^P_{i,a}$, with $\pi^D_{i,a}(y_a, |M_a|) > \pi^C_{i,a}(y_a, |M_a|) > \pi^P_{i,a}$.

The collusive effort denoted y_a has been defined as the overcharge paid by consumers because of the cartel M_a, whereas the cartel size denoted $|M_a|$ is the number of firms within the cartel M_a. We consider that the cartel profit increases concavely with the collusive effort and the cartel size, while the deviation profit increases convexly with those two variables.[11]

Note that for any firm i, the decision to stick to the collusion or deviate must be made for each cartel in which the firm i is involved. For example, a firm that is involved in two cartels, M_a and M_b has 4 options. It can decide to remain in both cartels, deviate from only one of them (either M_a or M_b), or deviate from both. Last, when a firm deviates from a cartel, that cartel is considered definitively and completely broken, and all links connecting firms under that cartel disappear.

The graph theory literature[12] has identified two conditions for defining network stability.[13] For simplicity, we consider in our setup a given network of corporate cartels where firms cannot create new links.[14] Hence, the cartel stability (or instability) only comes from the firms' ICC, and any modification of the network is due to a cartel break-up.

Definition 4 – ICC and network stability. Any given corporate network $\boldsymbol{g}$ is considered to be stable if any colluding firm that is linked to another has a strict incentive to maintain this link.

The fact that firms want to create a new link is tantamount to looking at the participation constraint (PC). For simplicity, we assume that the PC is always fulfilled, meaning that we focus on the firms' decision whether to stay in or leave the cartels to which they belong. Focusing on the firms' ICC and assuming that the firms' discount factor is exogenous, we can state:

Lemma 1. In any given network of corporate cartels $\boldsymbol{g}$, under the grim trigger strategy, deviation is the most profitable when it occurs in the first period.

Proof. As discussed in section 14.2 of Belleflamme and Peitz (2015), if one firm decides to deviate in period t, the best response of the other firms is to deviate at period $t-1$. Thus, by iteration, if a firm has to *optimally* deviate, it will do so from the first period. □

Despite considering a network of corporate cartels, the optimal firms' deviation occurs in the first period because of the grim trigger strategy, in line with the antitrust literature (e.g. Belleflamme and Peitz, 2015). The existence of a corporate cartel network leaves this result unchanged, because the network affects only the probability of detection, which matters for firms' profits and hence for cartel stability, but not for the structure of the supergame.The consequence of Lemma 1 is interesting from a network stability perspective since it means that any network of cartels $\boldsymbol{g}$ is permanently stable from the second period onward.

[11] Close to Ivaldi et al. (2003), Harrington and Chen (2006), and Broos et al. (2016).

[12] e.g., Jackson and Wolinsky (2003); Goyal and Joshi (2003).

[13] The first condition states that any agent linked to another in the network has a strict incentive to maintain the link, while the second assumes that any agent not linked has no strict incentive to form a link with each other.

[14] As in the section 3.1 of Liu et al. (2012).

3 Critical discount factor and centrality trade-off

For any firm i that belongs to the network of corporate cartels $\boldsymbol{g}$, its present value (PV) of the overall profits obtained from sticking to the cartel M_a is denoted $PV_{i,a}^C$ and is computed as follows:

$$PV_{i,a}^C = \frac{1}{1-\delta_a} \cdot [\, \pi_{i,a}^C(\, y_a, |M_a|) \, - p_a(y_a, \boldsymbol{g})F] \tag{1}$$

Equation (1) shows the present value of any colluding firm i that chooses to indefinitely stick to the cartel M_a ($PV_{i,a}^C$). Recall that δ_a is the firm's discount factor for a cartel M_a, i.e., the parameter that captures the firm's preference for the future. For each period, the colluding firm i earns a cartel profits ($\pi_{i,a}^C(\, y_a, |M_a|)\,$) but faces an expected sanction ($p_a(y_a, \boldsymbol{g})F$) imposed by the Competition Agency.

As shown in Lemma 1, the optimal deviation occurs in the first period. Hence, for any firm i that belongs to a cartel M_a in a given network $\boldsymbol{g}$, its present value (PV) of the overall profits obtained from the optimal deviation of its cartel M_a is denoted:

$$PV_{i,a}^D = \pi_{i,a}^D(\, y_a, |M_a|) \, - p_a(y_a, \boldsymbol{g})F + \frac{\delta_a}{1-\delta_a} \cdot \pi_{i,a}^P \tag{2}$$

Equation (2) shows the present value of any colluding firm i that chooses to deviate from the cartel M_a ($PV_{i,a}^D$). In the first period, the firm breaks the collusion and perceives a deviation profit ($\pi_{i,a}^D(\, y_a, |M_a|)\,$) but still runs the risk of being caught by the Competition Agency and paying a fine ($p_a(y_a, \boldsymbol{g})F$). The other firms observe the cartel break-up in the second period and punish the deviating firm by imposing competitive profits ($\pi_{i,a}^P$) for all the subsequent periods.

Thus, any corporate cartel M_a is stable if, for any firm i that belongs to the network of cartels $\boldsymbol{g}$, its present value of sticking to the collusion is greater than its present value obtained by the *optimal* deviation such that its incentive compatibility constraint (ICC) is fulfilled. Considering a homogeneous and static discount factor, if the ICC is satisfied, all the cartels are stable, so is the network $\boldsymbol{g}$. We thus obtain the critical discount factor ($\underline{\delta_a}$) of the firms in the cartel M_a as follows

$$PV_{i,a}^C > PV_{i,a}^D$$

$$\Leftrightarrow \delta > \frac{\pi_{i,a}^D(\, y_a, |M_a|) \, - \pi_{i,a}^C(\, y_a, |M_a|)}{\pi_{i,a}^D(\, y_a, |M_a|) \, - \pi_{i,a}^P - p_a(y_a, \boldsymbol{g})F} \equiv \underline{\delta_a} \tag{3}$$

Proposition 1. The network of corporate cartels $\boldsymbol{g}$ is stable if, for any cartel M_a, the discount factor δ is greater than $\underline{\delta_a}$.

Proof. Immediate from equation (3), see Appendix C.1 for the proof of equation (3) and Appendix C.2 for its existence conditions. □

Proposition 1 sets out the sufficient condition for a collusive equilibrium for any firm i that belongs to a cartel M_a. The minimum discount factor is decreasing with the cartel profits ($\pi_{i,a}^C$), which tends to make it easier to collude. The effect of the deviation profits ($\pi_{i,a}^D$), the competition profits ($\pi_{i,a}^P$), and the expected sanction ($p_a(y_a, \boldsymbol{g})F$) on the minimum discount factor is the opposite. We can therefore derive the following proposition, which constitutes one of the central results of our analysis.

Proposition 2. For any firm i in a given network of corporate cartels $\boldsymbol{g}$, the higher the degree centrality, the lower the cartel stability.

Proof. Recall that the degree centrality of a firm i is its number of connections. The only function that depends on the degree centrality is the probability of detection (p_a), which is convexly increasing. Hence, all the other things being equal, when the degree centrality increases, so does the probability of detection, which increases the critical discount factor, and thus makes the collusions harder to sustain. $\square$

Proposition 2 states that the more connected the firms are, the less stable the cartels. Isolating the effect of the degree centrality on the critical discount factor without modifying the cartel size is tantamount to studying the impact of multiple offending on cartel stability. The intuition is that the more a firm is involved in cartels or is connected to central firms, the greater the probability of detection and so the cartel instability.

Corollary 1 – The centrality trade-off. The higher the firms' degree centrality, the greater their profits, but the lower the stability of each cartel to which they belong.

Proof. Immediate from equation (2) and proposition 2. The greater the number of cartels a firm belongs to, the higher its profits ($\pi^C_{i,a} > \pi^P_{i,a}$). However, as the detection probability increases with the number of connections, the stability of each cartel is more difficult to achieve. $\square$

Although the detection probability increases with the degree centrality, meaning that the more connected firms are, the less stable their cartels become, firms face a "*centrality trade-off*". On the one hand, the more cartels firms belong to, the higher their profits. On the other hand, firms are unlikely to have an incentive to remain completely unconnected, as this would mean staying in competition, which is not beneficial for them. However, being connected to the entire network may also not be desirable, as this would result in a high detection probability and, consequently, greater cartel instability.

Note that in our model, firms cannot create new links. This centrality trade-off merely explains why firms might prefer to participate in multiple cartels simultaneously, even though deviating from one would increase the stability of the other cartels they are part of.

The cartel size ($|M_a|$) and the collusive effort (y_a) affect the critical discount factor differently from degree centrality. There exists an optimal cartel size and collusive effort (from the firms' perspective) that minimizes the critical discount factor, thereby maximizing cartel stability. Both variables exhibit *pro-collusive* and *pro-competitive* effects. Increasing the cartel size promotes stability by increasing cartel profits (pro-collusive effect) but undermines it by amplifying deviation profits and the detection probability (pro-competitive effect). Note that the detection probability is not directly linked to the cartel size, but increasing the cartel size always make the degree centrality increases too, which in turn to increase the detection probability. The mechanism is the same with the collusive effort, which, when it increases, is associated with an increase in cartel profit (pro-collusive effect) but also with an increase in deviation profit and the probability of detection (pro-competitive effect).

These results are useful as they provide information on how the critical discount factor varies with effort in collusion, cartel size, and, most importantly, degree centrality. We can thus deepen our analysis of the network of corporate cartels by studying how those cartels must be connected to enhance their stability.

We call *network shape* the way cartels are connected (or organized), and investigate which one guarantees the highest stability in the following section.

4 Network shape and cartels' stability

We have previously identified that there exists an optimal cartel size that maximizes cartel stability, while firms' centrality always affects negatively the cartel stability. Combining these two insights allows us to examine the *optimal network shape*, defined as the configuration of connections among cartels that maximizes their stability. By network shape of cartels, we refer to the way in which cartels are interconnected, which can be characterized by four key variables: cartel size, firms' centrality, the total number of cartels, and the total number of firms. Analyzing the optimal shape of cartel networks is crucial from a policy standpoint. Understanding which network structures maximize cartel stability may help competition authorities target the most problematic configurations and identify firms that play a stabilizing role within the network.

In what follows, we define the criteria under which different networks of cartels can be considered comparable, which is a crucial step in our analysis. Then, we use these criteria, together with the results obtained in section 3, to identify the optimal shape of cartel networks.

4.1 Comparing networks of cartels

Comparing networks of cartels can be challenging, as the networks under study must be made sufficiently similar to allow for a meaningful comparison. The two extreme network shapes are the empty network, in which nodes are not connected at all, and the complete network, in which nodes are all connected. However, there are many other possible alternative shapes of networks (e.g., Goyal and Joshi, 2003; Borgatti, 2006; Liu et al., 2012). In this section, we restrict our attention to *connected networks* of cartels denoted $\boldsymbol{g^c}$, defined by Newman (2018) as networks in which there is a path[15] between every node of that network. See Appendix D.1 for an extension in which we relax the assumption of an initial connected network of cartels.

Figure 1 illustrates how the shape of a given connected network is modified when changing the cartel size (Figure 1b and 1c), the total number of cartels (Figure 1d and 1e), and the firms' degree centrality (Figure 1f) by one, compared to a benchmark example (Figure 1a).

In the benchmark example of Figure 5, the network of cartels $\boldsymbol{g}$ is composed of 2 cartels of 3 firms ($M = \{M_1, M_2\}$, $M_1 = \{1, 2, 3\}$, $M_2 = \{3, 4, 5\}$, $m = 2$), such that there are 5 firms ($N = \{1, 2, ..., 5\}$, $n = 5$). The Figure 1b revisits the benchmark example, modifying the size of cartel 2 by decreasing it from 3 to 2 firms ($M_2 = \{3, 4, 5\}$ becomes $M_2 = \{3, 4\}$), while, in the Figure 1c, the cartel 2 is larger of 1 additional firm than the benchmark example ($M_2 = \{3, 4, 5\}$ becomes $M_2 = \{3, 4, 5, 6\}$). The next two panels modify the total number of cartels in the network, reducing it to a single cartel in the Figure 1d ($m = 2$ becomes $m = 1$) and increasing it to three cartels in the Figure 1e ($m = 2$ becomes $m = 3$). Finally, the Figure 1f reduces the centrality of firm 3 by one connection, which is considered the most

[15] A path is defined (Newman, 2018) as any sequence of nodes such that every consecutive pair of nodes in the sequence is connected by an edge in the network.

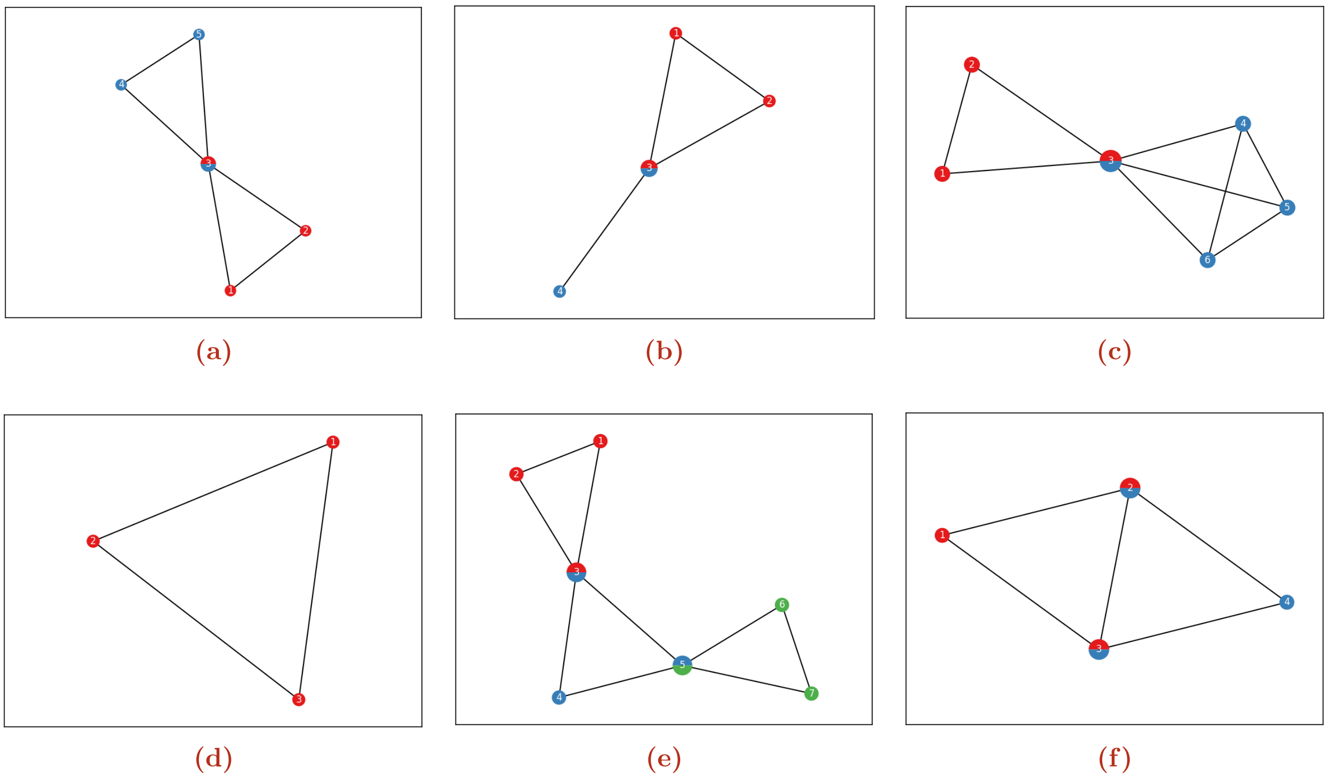


Figure 1: Example of 6 connected networks with different network shapes

central firm in the benchmark network ($DC(3) = 4$ becomes $DC(3) = 3$). As explained above, the comparability of these networks is not straightforward: can the first network of cartels be compared to the others, and if so, what would be the scope of such a comparison?

An essential condition for comparing networks of cartels in order to identify an *optimal shape* is that when varying the key variable, as few third-party variables as possible should be altered; and if some of these variables must be affected, it is important that their impact on cartel stability remains minimal.

Decreasing the cartel size of a collusion means artificially removing firms from that given cartel (as in Figure 1b), while increasing it means adding new firms to that same cartel (as in Figure 1c). In other words, the cartel size affects the network shape by modifying the size of the cliques within the network.

However, the comparison between the benchmark example (Figure 1a) and both Figure 1b and 1c is problematic. When the size of a cartel is reduced, the centrality of all the firms in that cartel is also reduced, as is the total number of firms in the network.[16] While the variation in the total number of firms in the network does not affect the discount factor and can therefore be neglected, the change in the centrality of all the firms in the modified cartel is highly relevant for the study of network stability. Moreover, if the cartel whose size is modified contains the most central firm of the network, as in Figure 1b and 1c, the detection probability of several cartels may be affected by the indirect variation in the centrality of that firm. Thus, in Figure 1b, where the size of cartel 2 is reduced, firm 3 sees its centrality decrease ($DC(3) = 4$ becomes $DC(3) = 3$). Since it is the most central firm of the network, it directly

[16] The mechanism is the same (but opposite) when increasing the cartel size, which increases firms' centrality and the total number of firms within the network.

affects the detection probability of the firms in both cartel 1 and cartel 2, and the change in its centrality induces a change in the detection probability. As shown in Figures 1b and 1c, modifying the cartel size therefore has indirect effects that make it difficult to isolate the impact of this change on network stability.

Reducing or increasing the total number of cartels in a given network, by contrast, means eliminating or adding an entire clique to it. Thus, the total number of cartels directly determines the number of cliques in the network. However, comparing the benchmark example with both Figure 1d and 1e also raises issues. Indeed, modifying the number of cartels in a connected network alters the centrality of certain firms ($DC(3) = 4$ becomes $DC(3) = 2$ in Figure 1d, and $DC(5) = 2$ becomes $DC(5) = 4$ in Figure 1e), and also changes the total number of firms in the network ($n = 5$ becomes $n = 3$ in Figure 1d and $n = 7$ in Figure 1e). Thus, when the total number of cartels is modified, it is no longer possible to isolate its impact from the resulting changes in firms' centrality.

Last, regarding firms' degree centrality, increasing it is not consistent with the study of an optimally stable network, as Proposition 1 demonstrated that cartel stability is negatively related to degree centrality. The only relevant variation in firms' degree centrality is therefore to diminish it, which requires decreasing the number of connections of the most central firm in a given cartel. In this sense, comparing Figure 1a with Figure 1f does not raise any major issue and allows for the fairest possible comparison of networks of corporate cartels. Indeed, when connections are removed from the most central firm in the network, no other variable is altered, apart from the total number of firms in the network ($n = 5$ becomes $n = 4$ in Figure 1f), which we previously explained is only a minor limitation. For all other variables of the model, removing a connection from the central firm has no impact. Indeed, the size of the cartels in the network remains the same ($|M_1| = |M_2| = 3$ in both Figure 1a and 1f), as does the total number of cartels ($m = 2$).

Thus, whether varying the cartel size, the total number of cartels, or the firms' centrality, only the latter allows for a clean isolation of effects and a meaningful comparison of network stability across configurations. We therefore must proceed by reasoning in *quasi* all other things equal, keeping networks with the same cartel size and the same number of cartels, but varying only the firms' degree centrality, and the total number of firms within the networks.

4.2 Optimal network shape and density

As we have explained the reasons why our comparison focuses on networks with a given number of cartels and a fixed cartel size, while varying firms' centrality, we now proceed to identify the network shapes that ensure the highest level of stability.

We can rewrite the degree centrality by expressing it as a function of the number and size of the cartels to which a firm i belongs.

Definition 5 – Degree centrality (2). The degree centrality of any firm i in a network of cartels $\boldsymbol{g}$ denoted $DC(i)$ is:

$$DC(i) = \sum_{a \in M} (|M_a| - 1) - \sum_{i \neq j} \max(0, |C_{i,j}| - 1)$$

Where $|M_a| - 1$ is the number of connections that the firm i has in the cartel M_a, and the set $C_{i,j}$ is the cartels in which the firms i and j belong together.

The first part of the equation counts the total number of links that firm i can have, by adding up the size of all the cartels to which it belongs, but subtracting 1 for each of those cartels, since a firm cannot be connected to itself. The second part of the equation removes all the links that this firm i has in common with the other firms to which it is connected, since two firms belonging to several cartels are only connected by one link ($g_{i,j} \in \{0, 1\}$) as explained in section 2.1.

Definition 6 – Network density. We call a *dense* network a network in which the actual number of links is close to the maximum number of links in this network, that is formally set by Newman (2018) as:

$$d(\boldsymbol{g}) = \frac{l}{n(n-1)}$$

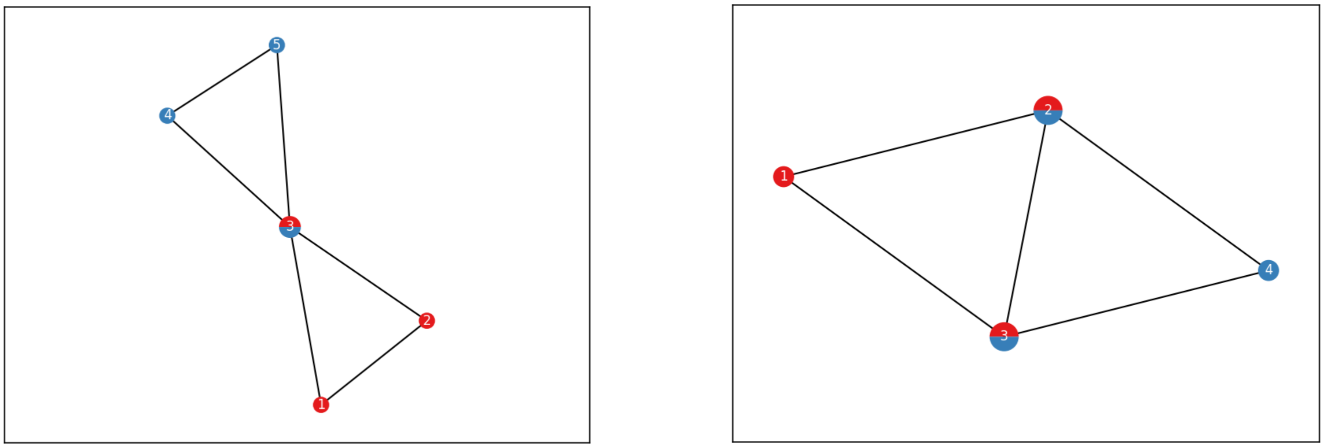
Where $d(\boldsymbol{g})$ is the network $\boldsymbol{g}$'s density, l the number of links, and n the number of nodes (in our setup, the number of firms), where $n(n-1)$ is the maximum number of links in an undirected network. Definitions 5 and 6 thus allow us to state the following central proposition regarding the optimal shape of a connected network.

Proposition 3. For any given *connected network* of corporate cartels $\boldsymbol{g^c}$ with the same number of cartels and cartel size, the *denser* the network, the greater its stability.

Proof. For a given connected network of cartels $\boldsymbol{g^c}$, with the same number and size of cartels, decreasing the maximum centrality of a firm can only be achieved by increasing its number of common links, denoted $C_{i,j} - 1$ in Definition 5. By increasing firm i's number of common links, one mechanically removes a link and a firm from the cartel network (see Figures 1a and 1f). However, given the definition of density, in a connected network of cartels, removing a link and a firm mechanically increases the density of the modified network, since the loss of the link in the numerator does not offset the loss of the firm in the denominator. Hence, we obtain an inverse relationship between firms' centrality and density, where decreasing the centrality of firm i increases the density of the network $\boldsymbol{g^c}$. See Appendix D.2 for the complete proof. □

To illustrate Proposition 3, let us consider a network of 2 corporate cartels ($M = \{M_1, M_2\}$) composed of 3 firms ($|M_1| = |M_2| = 3$). If we restrict our attention to connected networks, the 2 possible network shapes are the ones depicted in Figure 2.[17] In the Figure 2a, firm 3 is a recidivist, as it belongs to both cartels, while the other ones are single offenders. In Figure 2b, firms 2 and 3 are recidivists, as they belong to both cartels, while firms 1 and 4 are single offenders. In the first case, firm 3 has the highest degree centrality, such that $DC(3) = 4$, and the network density denoted $d(\boldsymbol{g}^{\text{left}})$ is equal to 0.6, while in the second case, firms 3 and 4 are the most central with $DC(3) = DC(4) = 3$, and the network density denoted $d(\boldsymbol{g}^{\text{right}})$ is equal to 0.83. Proposition 2 shows that cartel stability decreases with firms' degree centrality. In our illustration, the Figure 2b depicts the most stable network, which also has the highest density. Holding the number and size of cartels constant, we find an inverse relationship: denser networks exhibit lower maximum degree centrality, and thus, higher cartel stability.

[17] Excluding the network in which all the firms belong to the same two cartels.



(a) **(b)**

Figure 2: Example of two connected networks with two cartels of three firms

This result is due to the fact that when a firm already has a connection with another one because of a cartel, if this firm is involved in a second cartel with that same firm, their connection is not counted a second time ($g_{i,j} \in \{0, 1\}$). Thus, firms may have an interest in forming cartels with firms with which they are already connected to minimize their probability of detection. One possible interpretation is that firms involved together in multiple cartels face lower costs of maintaining their links. Indeed, if two firms i and j participate in both cartels M_a and M_b, the cost of maintaining the link, denoted $g_{i,j}$, is spread across cartels a and b. By contrast, if different firms were involved in each cartel, firm i would have to bear not only the cost of maintaining its link with j, but also an additional cost for its link with firm k.

The result of Proposition 3 follows directly from Definitions 5 and 6: given that we compare networks with the same number and size of cartels, reducing the maximum centrality of a firm mechanically increases the density of the new network. The denser a connected network is, the higher the number of common links, which contributes to lowering firms' centrality. In other words, the denser the network, the lower the degree of centrality of the most central firm, which in turn improves the stability of cartels in the network. Hence, firms have an incentive to participate in as many common cartels as possible with other firms, since the detection probability associated with such links is shared across several cartels. This result is therefore concerning, as it implies that multiple offending, provided it occurs with common partner firms, is doubly profitable: it increases the profits of cartelized firms while simultaneously enhancing the stability of those cartels. One low-cost solution for competition agencies is to impose much harsher sanctions on these firms (such as firms 2 and 3 in Figure 2b), given their role in enhancing cartel stability.

5 Conclusion

We have shown that taking into account networks of corporate cartels substantially changes the analysis hitherto carried out in the antitrust literature.

First, we show that increasing the firms' centrality is always associated with greater instability of the cartels to which those firms belong. However, it does not mean that firms have no incentive to be connected at all. Indeed, firms face a "*centrality trade-off*" between maximizing their centrality in the network, enabling them to extract high profits (pro-collusive effect), but at the risk of being detected more easily by the Competition Agency (pro-competitive effect).

Second, we characterize two main optimal shapes of networks of cartels that provide the highest stability. The first optimal network shape is when we restrict our attention to *connected* networks, for which we find that the cartels' stability is increasing with the network density. In connected networks of cartels, firms benefit from joining cartels with common partners, which can be interpreted as a reduction of coordination costs due to shared membership, thus enhancing cartel stability. This finding offers a new rationale for cartel recidivism: beyond the pursuit of higher profits, it may strengthen collusive agreements. Consequently, recidivism can pose an even greater concern for competition agencies, as it not only signals deterrence failure but also fosters more resilient cartels. One possible solution is to impose harsher penalties on these firms, not because they are multi-offenders, but because they contribute to reinforcing the stability of the cartels to which they belong.

The other optimal shape of networks of cartels emerges when we extend our analysis to include all types of network shapes, whether connected or not. In this case, the most stable structure is the clustered network, where each firm belongs to only one cartel at a time. Under such a configuration, cartel recidivism is entirely eliminated, but the remaining cartels are significantly more stable and harder to deter, which presents another kind of challenge for competition agencies. At first glance, the absence of recidivism appears to be good news for competition agencies, as one of their main objectives is to deter the recurrence of cartels. However, since the networks being compared contain a fixed total number of cartels, this implies that even without recidivism, the same number of cartels remains active, and, in addition, these cartels are more stable. This result is, however, more restrictive, as it relies on the assumption that no firm, in the whole network, participates in multiple cartels.

This paper thus bridges the antitrust literature and network economics by highlighting the importance of incorporating networks into the analysis of corporate cartels. Future research could extend this setup by endogenizing the formation of networks of corporate cartels, or by studying the optimal design of sanctions and leniency programs in the presence of such networks.

References

Ballester, Coralio, Antoni Calvó-Armengol, and Yves Zenou. 2006. “Who’s who in networks. Wanted: The key player.” *Econometrica* 74 (5):1403–1417.

Belleflamme, Paul and Francis Bloch. 2004. “Market sharing agreements and collusive networks.” *International Economic Review* 45 (2):387–411.

Belleflamme, Paul and Martin Peitz. 2015. *Industrial organization: markets and strategies*. Cambridge University Press.

Bernheim, B Douglas and Michael D Whinston. 1990. “Multimarket contact and collusive behavior.” *The RAND Journal of Economics* :1–26.

Bigoni, Maria, Sven-Olof Fridolfsson, Chloé Le Coq, and Giancarlo Spagnolo. 2012. “Fines, leniency, and rewards in antitrust.” *The RAND Journal of Economics* 43 (2):368–390.

Borgatti, Stephen P. 2006. “Identifying sets of key players in a social network.” *Computational & Mathematical Organization Theory* 12:21–34.

Broos, Sébastien, Axel Gautier, Jorge Marcos Ramos, and Nicolas Petit. 2016. “Analyse statistique des affaires d’ententes dans l’UE (2004-2014).” *Revue économique* 67 (HS1):79–94.

Bryant, Peter G and E Woodrow Eckard. 1991. “Price fixing: the probability of getting caught.” *The Review of Economics and Statistics* :531–536.

Ciliberto, Federico and Jonathan W Williams. 2014. “Does multimarket contact facilitate tacit collusion? Inference on conduct parameters in the airline industry.” *The RAND journal of economics* 45 (4):764–791.

Combe, Emmanuel and Constance Monnier. 2011. “Fines against hard core cartels in Europe: the myth of overenforcement.” *The Antitrust Bulletin* 56 (2):235–275.

Connor, John M. 2010. “Recidivism revealed: private international cartels 1991-2009.” *Available at SSRN 1688508* .

Connor, John M and Robert H Lande. 2008. “Cartel overcharges and optimal cartel fines.” .

Forsbacka, Tove, Chloé Le Coq, and Catarina Marvão. 2023. “Cartel birth and death dynamics: Empirical evidence.” *International Journal of Industrial Organization* 89:102932.

Gabardo, Ademir Cristiano and Heitor Silvério Lopes. 2014. “Using social network analysis to unveil cartels in public bids.” In *2014 European Network Intelligence Conference*. IEEE, 17–21.

Goyal, Sanjeev and Sumit Joshi. 2003. “Networks of collaboration in oligopoly.” *Games and Economic behavior* 43 (1):57–85.

Grau-Carles, Pilar and Miguel Cuerdo-Mir. 2016. “Using Network Theory to Detect Dominant Cartel Firms.” *Journal of Competition Law & Economics* 12 (3):541–556.

Gupta, Samir, Michael Polonsky, Arch Woodside, and Cynthia M Webster. 2010. "The impact of external forces on cartel network dynamics: Direct research in the diamond industry." *Industrial Marketing Management* 39 (2):202–210.

Harrington, Joseph E. 2017. "The deterrence of collusion by a structural remedy." *Economics letters* 160:78–81.

———. 2018a. "Developing competition law for collusion by autonomous artificial agents." *Journal of Competition Law & Economics* 14 (3):331–363.

———. 2018b. "A proposal for a structural remedy for illegal collusion." *Antitrust LJ* 82:335.

Harrington, Joseph E and Joe Chen. 2006. "Cartel pricing dynamics with cost variability and endogenous buyer detection." *International Journal of Industrial Organization* 24 (6):1185–1212.

Harrington Jr, Joseph E and David Imhof. 2022. "Cartel screening and machine learning." *Stan. Computational Antitrust* 2:133.

Huber, Martin and David Imhof. 2023. "Flagging cartel participants with deep learning based on convolutional neural networks." *International Journal of Industrial Organization* 89:102946.

Ivaldi, Marc, Bruno Jullien, Patrick Rey, Paul Seabright, and Jean Tirole. 2003. "The economics of tacit collusion." .

Jackson, Matthew O and Asher Wolinsky. 2003. *A strategic model of social and economic networks*. Springer.

Levenstein, Margaret C, Catarina Marvão, and Valerie Y Suslow. 2015. "Preventing cartel recidivism." *Antitrust* 30:81.

Levenstein, Margaret C and Valerie Y Suslow. 2006. "What determines cartel success?" *Journal of economic literature* 44 (1):43–95.

Liu, Xiaodong, Eleonora Patacchini, Yves Zenou, and Lung-Fei Lee. 2012. "Criminal networks: Who is the key player?" .

Marvão, Catarina. 2016. "The EU leniency programme and recidivism." *Review of Industrial Organization* 48 (1):1–27.

———. 2023. "Cartel activity and recidivism." In *Research handbook on cartels*. Edward Elgar Publishing, 332–350.

Marvão, Catarina and Giancarlo Spagnolo. 2018. "Cartels and leniency: Taking stock of what we learnt." In *Handbook of Game Theory and Industrial Organization, Volume II*. Edward Elgar Publishing, 57–90.

Motta, Massimo and Michele Polo. 2003. "Leniency programs and cartel prosecution." *International journal of industrial organization* 21 (3):347–379.

Newman, Mark. 2018. *Networks*. Oxford University Press.

OECD. 2020. "OECD Competition Trends." .

Poppius, Hampus. 2020. "Multimarket Contact and Collusion in Online Retail." *Working papers* (2020: 5).

Ryan-Charleton, Tadhg and Robert J Galavan. 2024. "Multimarket contact between partners and strategic alliance survival." *Strategic Management Journal* 45 (10):1988–2017.

Ryu, Wonsang, Jeffrey J Reuer, and Thomas H Brush. 2020. "The effects of multimarket contact on partner selection for technology cooperation." *Strategic Management Journal* 41 (2):267–289.

Spagnolo, Giancarlo. 2004. "Divide et impera: Optimal leniency programs." *Available at SSRN 716143* .

Wachs, Johannes and János Kertész. 2019. "A network approach to cartel detection in public auction markets." *Scientific reports* 9 (1):10818.

Werden, Gregory J, Scott D Hammond, and Belinda A Barnett. 2011. "Deterrence and detection of cartels: Using all the tools and sanctions." *The Antitrust Bulletin* 56 (2):207–234.

Wilson, Kyle. 2025. "Local Competition, Multimarket Contact, and Product Quality: Evidence From Internet Service Provision: K. Wilson." *Review of Industrial Organization* 66 (4):439–467.

Zenou, Yves. 2016. "Key players." *Oxford Handbook on the Economics of Networks* :244–274.

Appendix

A Introductory illustrations

A.1 Appendix A.1 - The criminalization of corporate cartels

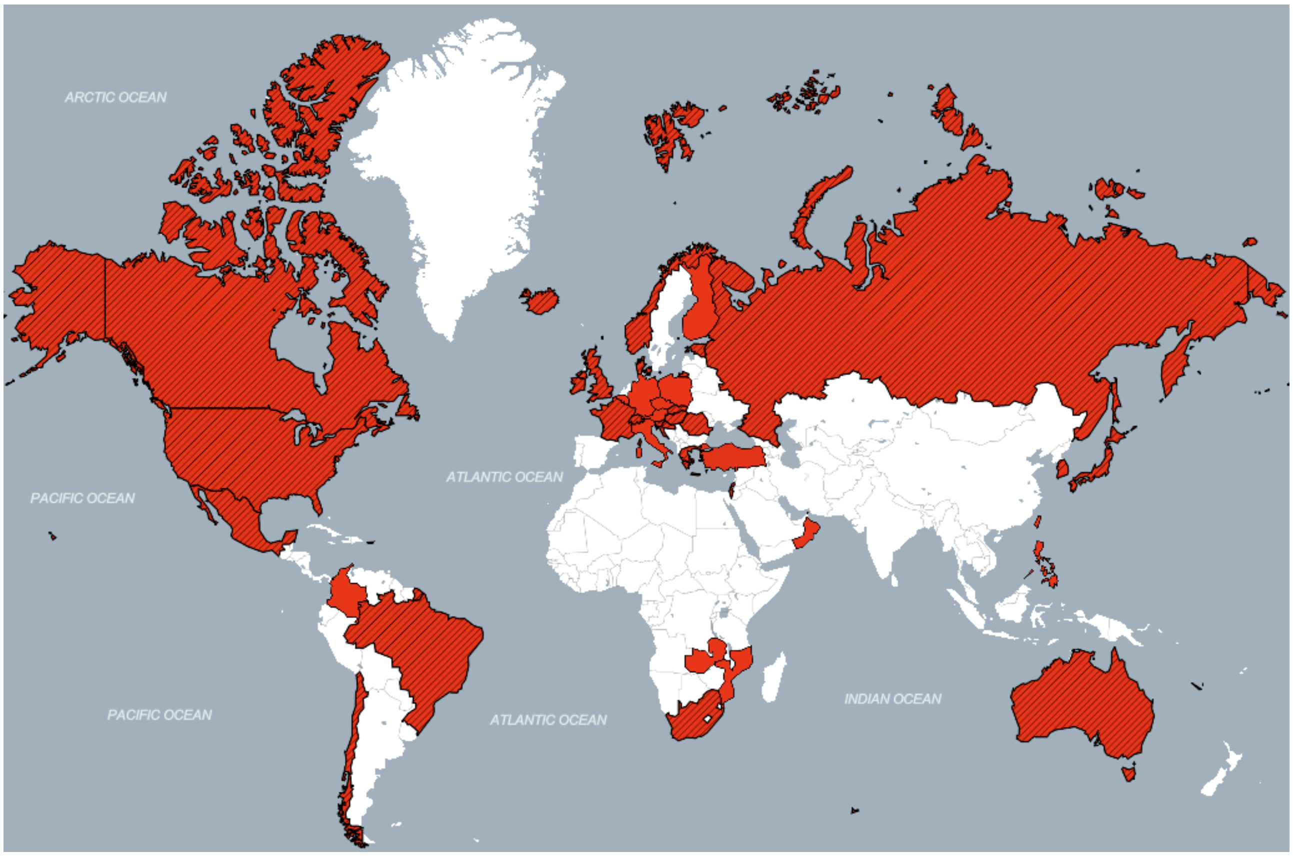


Figure 3: World map of the countries that criminalize corporate cartels (<u>data:</u> OECD, 2020)
<u>In red:</u> countries that criminalize only bid-rigging;
<u>In hatched red:</u> countries that criminalize both bid-rigging and corporate cartels

A.2 Appendix A.2 - The network organization of corporate cartels

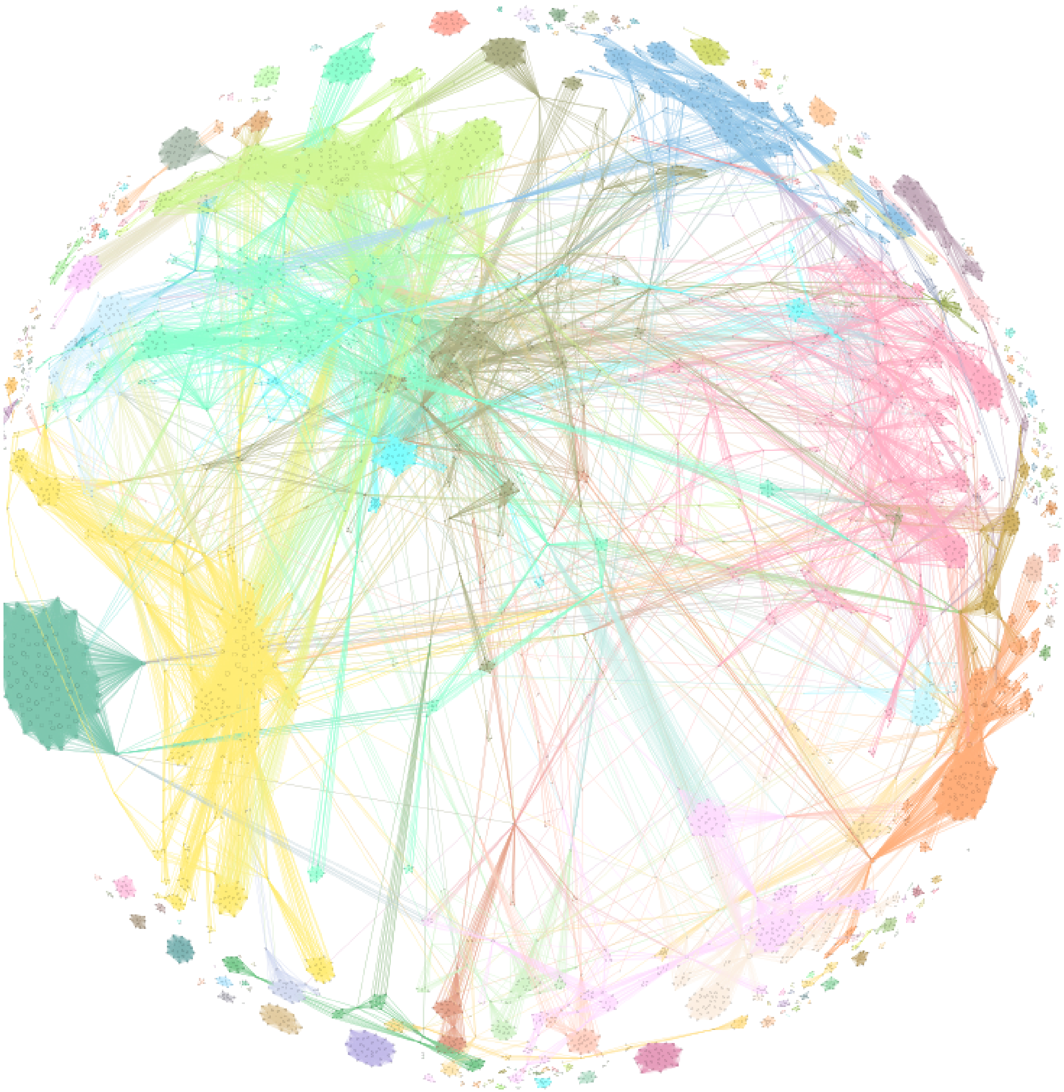


Figure 4: Data-visualization of a corporate cartels' network (data: "*PIC*" database, 1990-2019)

B Modeling illustration

B.1 Appendix B.1 - Stage game

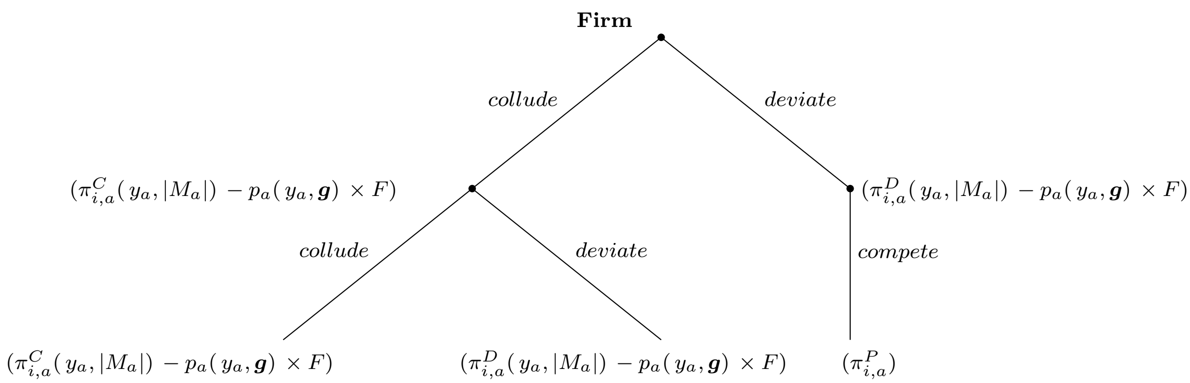


Figure 5: Stage game of any firm i's trade-off to stay or leave a cartel M_a in a corporate cartels' network $\boldsymbol{g}$

At $t = 1$, any firm i that initially belongs to a cartel M_a faces a trade-off, for which it has to decide whether to stick to the collusion (left part of the tree) or to deviate from it (right part of the tree).

- If they respect the collusion, firm i earn cartel profits $(\pi^C_{i,a}(y_a, |M_a|))$, while supporting the expected sanction $(p_a(y_a, \boldsymbol{g})F)$.
- If they decide to deviate from the collusion, firms earn deviation profits $(\pi^D_{i,a}(y_a, |M_a|))$ and still run the risk of being caught by the Competition Agency $(p_a(y_a, \boldsymbol{g})F)$.

At $t = 2$ and for all subsequent periods, firm i faces a different trade-off depending on their decision in the previous period $(t = 1)$.

- If firm i has sustained collusion at $t = 1$, it faces the same trade-off.
- If firm i has deviated at $t = 1$, it earns competition profits $(\pi^P_{i,a})$ for all next periods, and cannot form the cartel M_a again.

C Discount factor-related proofs

C.1 Appendix C.1 - Proof of equation (3)

$$\frac{1}{1-\delta_i} \cdot [\,\pi^C_i(y_a, |M_a|) - p_a(y_a, \boldsymbol{g}) \times F] > \pi^D_i(y_a, |M_a|) - p_a(y_a, \boldsymbol{g}) \times F + \frac{\delta_i}{1-\delta_i} \cdot \pi^P_i$$

$$\Leftrightarrow \frac{1}{1-\delta_i} \cdot [\,\pi^C_i(y_a, |M_a|) - p_a(y_a, \boldsymbol{g}) \times F] - \pi^D_i(y_a, |M_a|) + p_a(y_a, \boldsymbol{g}) \times F - \frac{\delta_i}{1-\delta_i} \cdot \pi^P_i > 0$$

$$\Leftrightarrow \pi^C_i(y_a, |M_a|) - p_a(y_a, \boldsymbol{g}) \times F - (1-\delta_i) \cdot \pi^D_i(y_a, |M_a|) + (p_a(y_a, \boldsymbol{g}) \times F)(1-\delta_i) - \delta_i \cdot \pi^P_i > 0$$

$$\Leftrightarrow \pi^C_i(y_a, |M_a|) - \pi^D_i(y_a, |M_a|) + \delta_i \cdot \pi^D_i(y_a, |M_a|) - \delta_i \cdot p_a(y_a, \boldsymbol{g}) \times F - \delta_i \cdot \pi^P_i > 0$$

$$\Leftrightarrow \delta_i \cdot \pi^D_i(y_a, |M_a|) - \delta_i \cdot p_a(y_a, \boldsymbol{g}) \times F - \delta_i \cdot \pi^P_i > -\pi^C_i(y_a, |M_a|) + \pi^D_i(y_a, |M_a|)$$

$$\Leftrightarrow \delta_i > \frac{\pi^D_i(y_a, |M_a|) - \pi^C_i(y_a, |M_a|)}{\pi^D_i(y_a, |M_a|) - \pi^P_i - p_a(y_a, \boldsymbol{g}) \times F} \equiv \underline{\delta_i}$$

C.2 Appendix C.2 - Existence conditions of equation (3)

$$\delta \in [0;1]$$

We then have to check if, and under which conditions, the discount factor fulfilled these two equations:

1. $\frac{\pi_i^D(y_a,|M_a|)-\pi_i^C(y_a,|M_a|)}{\pi_i^D(y_a,|M_a|)-\pi_i^P-p_a(y_a,\boldsymbol{g})\times F} \geq 0$

2. $\frac{\pi_i^D(y_a,|M_a|)-\pi_i^C(y_a,|M_a|)}{\pi_i^D(y_a,|M_a|)-\pi_i^P-p_a(y_a,\boldsymbol{g})\times F} \leq 1$

Existence condition 1:

$$\frac{\pi_i^D(y_a,|M_a|)-\pi_i^C(y_a,|M_a|)}{\pi_i^D(y_a,|M_a|)-\pi_i^P-p_a(y_a,\boldsymbol{g})\times F} \geq 0$$

As we set that $\pi^D > \pi^C > \pi^P$, it is straightforward that:

$$\pi_i^D(y_a,|M_a|) - \pi_i^C(y_a,|M_a|) > 0$$

The existence condition 1. is then fulfilled if and only if:

$$\pi_i^D(y_a,|M_a|) - \pi_i^P - p_a(y_a,\boldsymbol{g}) \times F > 0$$

As the competition profit (π^P) tends to be small, the crucial point of this subsequent condition is that the deviation profits (π_i^D) have to be higher than the expected sanction ($p_i(y_a,\boldsymbol{g})F$).

Existence condition 2:

$$\frac{\pi^D(y_a,|M_a|)-\pi^C(y_a,|M_a|)}{\pi^D(y_a,|M_a|)-\pi^P-p_a(y_a,\boldsymbol{g})\times F} \leq 1$$

This condition is fulfilled if and only if:

$$\pi_i^D(y_a,|M_a|) - \pi_i^C(y_a,|M_a|) \leq \pi_i^D(y_a,|M_a|) - \pi_i^P - p_a(y_a,\boldsymbol{g}) \times F$$
$$\pi_i^C(y_a,|M_a|) \geq \pi_i^P + p_a(y_a,\boldsymbol{g}) \times F$$

D Optimal network shapes

D.1 Appendix D.1 - Extension: a general optimal network shape

In this extension, we allow the given network not being *connected*, on the opposite of Section 4, meaning that there may not be a path between every node of that network. Recall that according to Newman (2018), a path is defined as any sequence of nodes such that every consecutive pair of nodes in the sequence is connected by an edge in the network. As for Section 4, the goal of this extension is to study which network shape guarantees the highest cartel stability, but in a more general context: the network of cartels can take any shape, as long as respecting all the assumptions set in Section 2.1.

Definition 7 A *cluster network* is a network formed from the disjoint union of complete graphs.

In our setup, a *cluster network* of cartels is a network in which there is at least a cartel in which the whole firms from this cartel belong to only that cartel. In such a case, there is no path connecting this cartel to any other cartel in the network.

Proposition 4. For any given networks of cartels $\boldsymbol{g}$ with the same number of cartels and cartels size, the *cluster network* is the more stable.

Proof. Following Defitinion 6, the firm i's degree centrality is:

$$DC(i) = \sum_{a \in M} (|M_a| - 1) - \sum_{i \neq j} \max(0, |C_{i,j}| - 1)$$

- If the firm i belongs to single cartel:

$$DC(i) = (|M_a| - 1)$$

- If the firm i belongs to several cartels:

$$DC(i) = \sum_{a \in M} (|M_a| - 1) - \sum_{i \neq j} \max(0, |C_{i,j}| - 1)$$

However, for a given cartel size M_a:

$$(|M_a| - 1) < \sum_{a \in M} (|M_a| - 1) - \sum_{i \neq j} \max(0, |C_{i,j}| - 1)$$

Because the critical discount factor is increasing with the degree centrality (Proposition 1), cartels are more stable when the degree centrality is the lowest, i.e. when the cartels' network is *clustered*, such that no path connects any two cartels. □

D.2 Appendix D.2 - Proof of Proposition 3

For any given *connected networks* of cartels $\boldsymbol{g^c}$ with the same number of cartels and cartels size, the *denser* the network, the greater its stability.

Proof. Following Defitinion 5, the firm i's degree centrality is:

$$DC(i) = \sum_{a \in M} (|M_a| - 1) - \sum_{i \neq j} \max(0, |C_{i,j}| - 1)$$

- If the firm i belongs to several cartels with only different firms:

$$DC(i) = \sum_{a \in M} (|M_a| - 1)$$

as no node $j \neq i$ is present in more than one cartel with i, $|C_{i,j}| < 1$

- If the firm i belongs to several cartels with common links:

$$DC(i) = \sum_{a \in M} (|M_a| - 1) - \sum_{i \neq j} \max(0, |C_{i,j}| - 1)$$

However, for a given cartel size M_a:

$$\sum_{a\in M}(|M_a|-1)-\sum_{i\neq j}\max(0,|C_{i,j}|-1)<\sum_{a\in M}(|M_a|-1)$$
$$\exists j\neq i:|C_{i,j}|>1\implies\sum max(0,|C_{i,j}|-1)>0$$

Because the critical discount factor is increasing with the degree centrality (Proposition 1), cartels are more stable when the degree centrality is the lowest, i.e. when firms share the maximum number of common cartels, leading to the *densest* network. □