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Central Bank Digital Currency and Bank Risk: Welfare and Policy Implications

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CYRIL MONNET* ASGERDUR PETURSDOTTIR† MARIANA ROJAS-BREU‡

*University of Bern, Study Center Gerzensee, Switzerland
†University of Bath, United Kingdom
‡Université Paris-Panthéon-Assas, CRED, France.

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Cyril Monnet*
University of Bern
Study Center Gerzensee

Asgerdur Petursdottir†
University of Bath

Mariana Rojas-Breu‡
Université Paris-Panthéon-Assas

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Abstract

We study the effect introducing interest-bearing central bank digital currency (CBDC) has on bank intermediation, risk-taking and welfare. We model a CBDC that competes with bank deposits as a medium of exchange. Monopolistic banks issue deposits to lend to productive investment projects. CBDC does not lead to disintermediation, but it can distort bankers' investment decisions. To retain risk-averse depositors, banks need to compete with a risk-free asset (CBDC), which leads them to adjust their risk exposure and hold a safer loan portfolio. This can lead to overinvestment in risk-free (less productive) loans which is sub-optimal from a social point of view. If depositors are highly risk averse and risk-free projects are scarce in the economy, a CBDC that bears interest may lead to an overall welfare loss. Interest rate on reserves then becomes an important policy tool to crowd-out sub-optimal investment and mitigate banking sector risk.

Keywords: CBDC, interest on reserves, banking, disintermediation

*cyril.monnet@unibe.ch
†aop21@bath.ac.uk
‡mariana.rojas-breu@u-paris2.fr

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1 Introduction

Central banks around the world are contemplating issuing their own digital money which would be available to everyone, a so-called retail central bank digital currency (CBDC). The motivation for issuing a CBDC differs between countries. However, as depicted in the latest BIS survey on CBDC (Di Iorio et al. (2024)), around 80% of central banks state that preserving the role of central bank money is an important driver for introducing a CBDC. This comes on the back of a decline in the use of physical currency and the increased adoption of privately issued cryptoassets.[1]

As a liability of the central bank, CBDC would be a risk-free digital means of payment, denominated in the national unit of account. It can therefore be considered a risk-free substitute to bank deposits. Thus, many have raised concerns that CBDC may have an adverse effect on the economy. Particularly, banks may lose depositors who prefer holding CBDC. The resulting contraction of bank deposits may reduce bank lending and impact real economic activity, and even lead to bank runs and financial instability.

The above concerns have contributed to the delay in the issuance of CBDC and lead most central banks to indicate that they do not intend to pay interest on CBDC[2] since they fear remuneration will exacerbate the negative effects of CBDC. However, the digital nature of CBDC allows for the possibility of paying interest rates on public money. Therefore, studying whether opportunities are missed by excluding remuneration and considering it in terms of the monetary policy toolkit is important.[3]

Many studies (cf. Andolfatto (2021), Chiu et al. (2023) and Williamson (2022a)) have already studied the question of disintermediation – the extent to which banks may lose deposits and reduce lending due to CBDC. Those studies find that for low to moderate

[1] According to the latest survey of central bankers published by the Bank for International Settlements (Di Iorio et al (2024)), 94% of central banks are engaged in some sort of work on CBDC, ranging from research phase to having already implemented CBDC.

[2] According to Di Iorio et al (2024).

[3] It is worth noting that historically, central bank reserves were unremunerated in the U.S., until the Fed was granted permission in 2006 and began paying interest on reserves in 2008 to have an extra tool for the conduct of monetary policy. (See for instance Walter and Haltom (2009).

levels of CBDC interest rates bank intermediation increases. That is because bankers respond to an increase in the interest rate on CBDC by increasing the remuneration on their deposits. Consequently, the demand for deposits increases, and banks can increase lending. This effect is due to a well-known increase in payment efficiency when the payment device pays an interest (Williamson 2022a).

However, the existing literature, by abstracting from bank risk, has not looked at how CBDC will affect the business model of banks. Will banks take more or less risk on their asset side when they must compete with CBDC for funding? Beyond disintermediation, answering this question is important for the stability of the financial system and for proposing the best policy response to complement the introduction of a CBDC. We conduct our analysis for different levels of remuneration of CBDC and for different types of economies in terms of productivity of investment projects and scarcity of safe assets.

We construct a model using Lagos and Wrigxht (2005) with the following ingredients. A monopolist bank has the technology to lend to entrepreneurs who are endowed with productive projects. The bank can monitor entrepreneurs to make their project risk-free, while the project of unmonitored entrepreneurs remain risky. Even if risky, projects still have a positive expected return, which implies that risk taking by the bank through unmonitored investment can be socially valuable. Besides lending to entrepreneurs, the bank can also invest in interest-bearing central bank reserves, a safe asset. The bank however does not have equity and needs to issue uninsured deposits to finance loans to entrepreneurs. Deposits can serve as a means to pay, as does an interest bearing CBDC issued by the central bank. Given it competes with CBDC, the bank chooses a portfolio consisting of safe and risky assets and the deposit contract that ensures the participation of depositors to maximize its profits. Doing so, the bank takes into consideration the remuneration on CBDC and that risk averse depositors prefer using a safe rather than a potentially risky means of payment. We will show that, in such an environment, the introduction of an interest-bearing CBDC is non-neutral for the allocation of resources and thus for social welfare.

First, we establish conditions for which risky deposits arise in equilibrium. When the marginal productivity of investment projects is high, the bank is able to profitably monitor a sizeable volume of entrepreneurs. Under those conditions, when CBDC bears no or low interest rate, banks can compete with CBDC while holding a safe asset portfolio. As a result, deposits bear no risk, because the share of safe assets composing the bank's portfolio is sufficient to service all the deposits. However, when risk-free assets are scarce and CBDC is convenient enough, the bank must rely on risky projects to offer a deposit remuneration that competes with CBDC. Under those conditions, deposits are endogenously risky.

Then we consider how changes to the interest rate paid on CBDC, decided by the central bank, affects the business model of banks. Specifically, the bank can act through two channels: increase the interest rate paid on deposits and reduce their portfolio risk.

Studying the deposit rate channel, we show that the bank responds to an increase in the interest rate paid on CBDC by increasing the remuneration on deposits. This is a well-established result in the existing literature. However, given our focus on risk, we demonstrate that when deposits are safe, the bank only needs to react through this channel to attract and retain deposits. Thus, under safe deposits, the bank pays the same interest rate on deposits as is paid on CBDC. When deposits are risky, however, a spread arises between the interest rate paid on bank deposits and CBDC. Hence, the bank responds to an increase in the interest rate paid on CBDC by adjusting the risk premium on deposits.

Furthermore, the portfolio channel becomes active under risky deposits in which the bank responds to an increase in the interest rate paid on CBDC by holding a safer asset portfolio. This channel becomes more important with more risk averse depositors. The bank uses the portfolio channel to reduce the level of deposit risk by investing more in monitored loans. However, because monitored loans become prohibitively costly due to the convex monitoring cost, the bank over-invests in monitored loans relative to the social optimum.

This shortage of safe profitable assets has policy implication for the central bank. The monetary authority can alleviate the investment inefficiency and risk to depositors by increasing the interest rate paid on reserves beyond the rate paid on CBDC, allowing the bank access to an attractive safe asset. Then, the bank can make its asset portfolio safer by investing more in reserves rather than in monitored loans. Investing in reserves also allows the bank to increase its payment to depositors when risky loans fail, thus reducing the risk they bear. Then setting the appropriate positive interest rate spread between reserves and CBDC improves welfare. The key to this result is to recognise that CBDC and reserves are different. Whereas CBDC competes with the bank's liabilities, reserves are only used by the bank as an asset to hoard liquidity. Hence, the interest on CBDC affects the return on loans via the bank's funding cost, while the interest on reserves does not have such an effect.

For an economy with an abundance of highly productive and safe investment projects, overall welfare is improved the higher the interest rate paid on CBDC. This result is due to the payment efficiency of CBDC. Higher interest rate on money makes liquidity cheaper, increasing payment efficiency, and for a given inflation rate a higher level of profitable investment can be supported through bank lending. In such economies, the cost for banks of retaining deposits is low, as both deposits and CBDC are risk-free.

On the other hand, in economies with a scarcity of highly productive safe assets and very risk averse agents, increasing the remuneration on CBDC can worsen overall welfare. Due to risk aversion, the bank is obliged to excessively reduce the risk of their balance sheet to attract and retain depositors. In that case, the benefit of a higher remuneration on CBDC, consisting of higher welfare for depositors and higher payment efficiency in investment spending, is offset by the foregone payout of risky projects.

The above result provides a rationale for a low to moderate interest rate on CBDC, accompanied by a positive spread between the interest paid on reserves and CBDC. This configuration clearly comprises an unremunerated CBDC. Our result that retail CBDC should have no to moderate remuneration relies on investment efficiency and thus differs from the one based on the need for financial independence of the central bank vis-à-vis the Treasury, as suggested by Williamson (2022b). Furthermore, while our analysis is intended to capture the convenience of using CBDC and its effects on the banking sector in the most stylized way by focusing on remuneration, other features of CBDC as a means of payment could be explicitly incorporated.

Related literature As in Calomiris and Kahn (1991) who show that the threat of withdrawal by depositors will discipline banks, we demonstrate that the introduction of CBDC disciplines bankers since it affects the value of depositors outside option. While they focus on the effect it has on the cost of bank capital, our focus is on the investment mix and level of disintermediation of the banking sector.

We contribute to the growing literature on CBDC by studying the effect its introduction has on the investment decision of banks and thus bank risk, since the existing literature has mainly considered a single type of bank asset while focusing on disintermediation of the banking sector (cf. Andolfatto (2021), Chiu et al. (2023), Brunnermeier and Niepelt (2019)). Fernandez-Villaverde et al. (2021) build a Diamond-Dybvig model to assess how CBDC would affect banks' maturity transformation, while Kahn et al. (2018) explore the incentives of private banks to distribute CBDC to their customers. Skeie (2021) studies the banking sector response to the issuance of a CBDC in a high inflation country and Garratt et al. (2023) study the effects of CBDC on the banking sector when banks differ in their market share.

The importance of CBDC when there is a shortage of safe assets is at the heart of Keister and Sanches (2023). We contribute to this strand of research by including a monitoring decision by banks which endogenizes the bank's balance sheet risk. Therefore, we take another approach to studying financial system risk, relative to e.g. Cecchetti and Schoenholtz (2017), Williamson (2022a), Keister and Monnet (2022), Bidder et al. (2024), who all take the view that CBDC endangers stability because it facilitates bank runs (à la Diamond and Dybvig (1983)). Rather we show that banks adapt to CBDC competition as a depositors' risk-free outside option by changing the structure of their balance sheet.[4]

[4]By considering another aspect of bank activities, the provision of credit lines, Piazzesi and Schneider (2020) argue that CBDC can have negative effects.

In an extension of their benchmark model, Ahnert et al. (2024) include monitoring decision of banks with the purpose of studying the impact of CBDC on financial stability due to risk-taking decisions on the banks' asset side. They find that monitoring effort increases with remuneration of CBDC leading to more stable bank asset side, while higher CBDC remuneration also increases the risk of a bank run, thus having an ambiguous effect on financial stability. Our paper focuses instead on the effects of CBDC on the efficiency of banks' financing of the real economy in the presence of risk. We also highlight the role of monetary policy and the optimal spread between the interest rate paid on reserves and CBDC in the face of scarcity of safe assets and bank risk. This complements papers such as Jiang and Zhu (2021) and Garratt et al. (2023) who study how the introduction of a CBDC affects the pass-through of monetary policy depending on the degree of competition in the banking market and consider a policy mix given by the interest rate on reserves and the interest rate on CBDC.

The paper is organized as follows. Section 2 presents the model and discusses the main assumptions. Section 3 presents the equilibrium equations that hold with safe and risky deposits. In Section 4 we analyze the effects of an interest-bearing CBDC on the equilibrium allocations for the different equilibrium regions. Section 5 analyzes the impact on welfare of the interest rate on CBDC as well as the spread between the interest rate on reserves and the one on CBDC. Section 6 illustrates by means of an example how varying the inflation rate compares to changing the interest rate on CBDC. Section 7 concludes.

2 Environment

The model builds on the Lagos and Wright (2005) framework. Time is discrete and continues forever. The economy is populated by a continuum of four types of infinitely-lived agents: buyers, sellers, suppliers, entrepreneurs, with discount factor $\beta \in (0, 1)$, as well as bankers that live for one period. There also exists a consolidated government which we refer to as the central bank.

Each period is divided into three subperiods. The first is a competitive capital goods market (KM), the second a decentralized goods market (GM) and the third a frictonless centralized market where settlement takes place (SM). A capital good, k, is produced in the KM. A perishable consumption good, c, is produced and consumed in the GM. Finally, the numeraire x is produced and consumed in the SM.

2.1 Agents, preferences, technology and nominal assets

Buyers and sellers participate in the GM and the SM every period. In the SM, buyers and sellers can produce the numeraire using a linear production technology. The agents receive linear utility from its consumption. In the GM, buyers cannot produce, but receive utility $u(c)$ from consuming c units of the consumption good, with $u'(c) > 0$ and $u''(c) < 0$. Sellers produce the consumption good at linear cost c, while they do not want to consume in the GM.

Buyers and sellers preferences can be represented by their within period utility functions:

$$\begin{aligned} U_d(c, x_d) &= u(c) + x_d \\ U_s(c, x_s) &= -c + x_s \end{aligned}$$

where x_d and x_s are buyers' and sellers' net consumption of the numeraire, respectively.

Buyers and sellers are randomly matched in the GM with probability 1 and buyers make a take-it-or-leave-it offer to the seller. We assume buyers need a medium of exchange to acquire GM consumption due to lack of commitment and sellers inability to enforce debt repayments.

Suppliers and entrepreneurs participate in the KM and the SM. In the KM, suppliers work to produce capital good, k, at linear cost. Entrepreneurs are endowed with investment projects, with each project requiring a unit of the capital good as input. We assume there are $\overline{h}$ available projects, where $\overline{h}$ can be made arbitrarily large. Entrepreneurs buy capital from suppliers in the competitive KM at nominal price $\widetilde{\rho}$. Suppliers are hand-to-mouth agents and thus use the proceeds from selling capital to consume the numeraire in the subsequent SM, receiving linear utility x_p.

As in the GM, we assume lack of commitment and limited enforcement in the KM. Therefore, entrepreneurs require a medium of exchange to buy k, which they borrow from the banker.

Bankers have access to a costly monitoring technology and are thus willing to lend to entrepreneurs. A monitored investment project is risk-free since it delivers output R with certainty. Bankers choose a measure q of projects to monitor, incurring cost $\kappa(q) > 0$, with $\kappa'(q) > 0$ and $\kappa''(q) > 0$. Bankers also choose a measure n of non-monitored investment projects. These projects are risky as they deliver output R with probability p and 0 otherwise.[5] The shock is correlated meaning the banker cannot diversify across risky projects. Non-monitored projects have a positive net present value, $pR > 1$.

Bankers are monopolistic. Therefore, entrepreneurs receive no profits from their investment projects since the banker has all the bargaining power. Bankers use their profits to consume the SM numeraire, receiving utility x_b.

Bankers fund their asset side by issuing bank deposits, d, which is tradable debt. With ϕ the price of CBDC in terms of the numeraire, the real value of the bank's debt is $\delta \equiv \phi d$. Bankers promise to pay interest rate i_d with $(1 + i_d)\,\delta$ the gross real value of deposits at redemption. Since bankers are monopolistic the depositor has no bargaining power. Deposits are safe when the banker can always honor the contract $(1 + i_d)\,\delta$. Otherwise, the banker defaults, which can occur when non-monitored projects do not pay a return, and deposit holders share the safe assets of the banker.

In addition to bank deposits there are two other types of nominal assets, central bank digital currency (CBDC) and reserves, both issued by the central bank.[6] CBDC is a digital

[5] In a more general specification the no/low monitoring activity would incur a lower (positive) cost. The cost of monitoring q and n projects would be $\kappa\,(q + \nu n)$ with $\nu < 1$. In this setup, $\nu = 0$.

[6] The central bank could also issue physical currency (cash), but we focus on the effect of CBDC instead. Unremunerated CBDC is equivalent to cash. When CBDC is remunerated, it is preferred to cash unless we assume additional trading frictions.

currency available to all agents, that pays gross interest $(1+i_m)$. Buyers acquire z real units of CBDC in the SM. On the other hand, reserves is a digital currency that is only available to bankers. It pays gross interest $(1+i_r)$. There is no reserve requirement and bankers are free to choose their desired nominal amount of reserves $\widetilde{r} \geq 0$, with real value $r = \phi\widetilde{r}$. The central bank stands ready to exchange its liabilities at par.

We focus on stationary monetary policies where the total liabilities of the central bank grow at a constant gross rate $\gamma > \beta$. The gross inflation rate is $\gamma \equiv \frac{\phi}{\phi_{+1}}$, where ϕ is the price of CBDC in terms of the numeraire at time t and ϕ_{+1} at time $t+1$. We focus on policies where $i_m < \frac{\gamma}{\beta} - 1$. The central bank runs the Friedman rule when $i_m = \frac{\gamma}{\beta} - 1$.

The government has taxation powers, which can be used to pay interest rates on central bank liabilities, and rebate any excesses to buyers using lump-sum transfers, T. We abstract from government asset purchases.

2.2 Exchange and model timeline

In the GM, sellers accept both bank deposits and CBDC as a medium of exchange. Whereas in the KM, the sellers, which we call suppliers, only accept CBDC. Our assumption is that suppliers are unbanked, e.g. due to spatial separation. Since entrepreneurs need CBDC as medium of exchange in the KM, bankers fund them by issuing deposits in exchange for CBDC from buyers. Bankers can also invest their funds in reserves. Given portfolio (q, n, r), bankers gross expected return is $Rq + pRn + (1+i_r)\,r$. The model is represented in Figure 1.

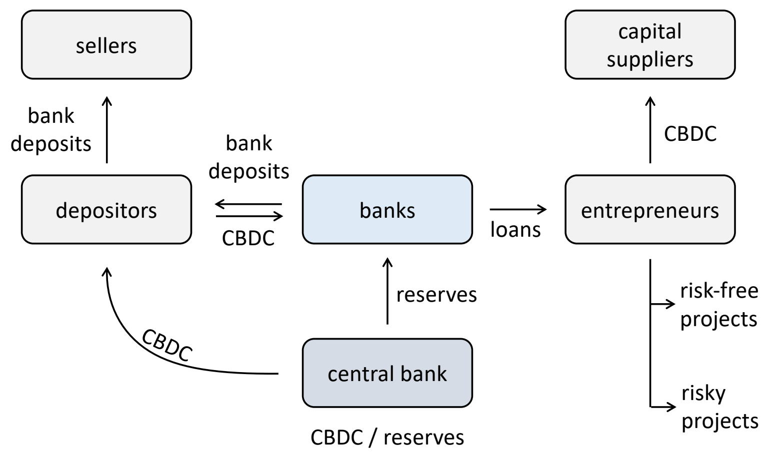

Figure 1. The model

The timing of the model is as follows. Buyers work for CBDC in the SM at $t-1$. At the beginning of time t, buyers meet with a banker and choose how much CBDC to exchange for bank deposits. Bankers that obtain CBDC from buyers can lend to entrepreneurs and decide on the measure of projects to monitor. Entrepreneurs use the borrowed CBDC to buy capital goods from suppliers in the KM. At the beginning of the GM, agents learn whether the non-monitored projects pay a return and whether the banker can honor the deposit contract. Hence, buyers and sellers know the value of deposits when they trade, and buyers bear the deposit risk. In the SM at time t, the payoff of projects materializes, entrepreneurs repay their loans and holders of CBDC, reserves and bank deposits receive interest payment. Figure 2 summarizes the timing of the model.

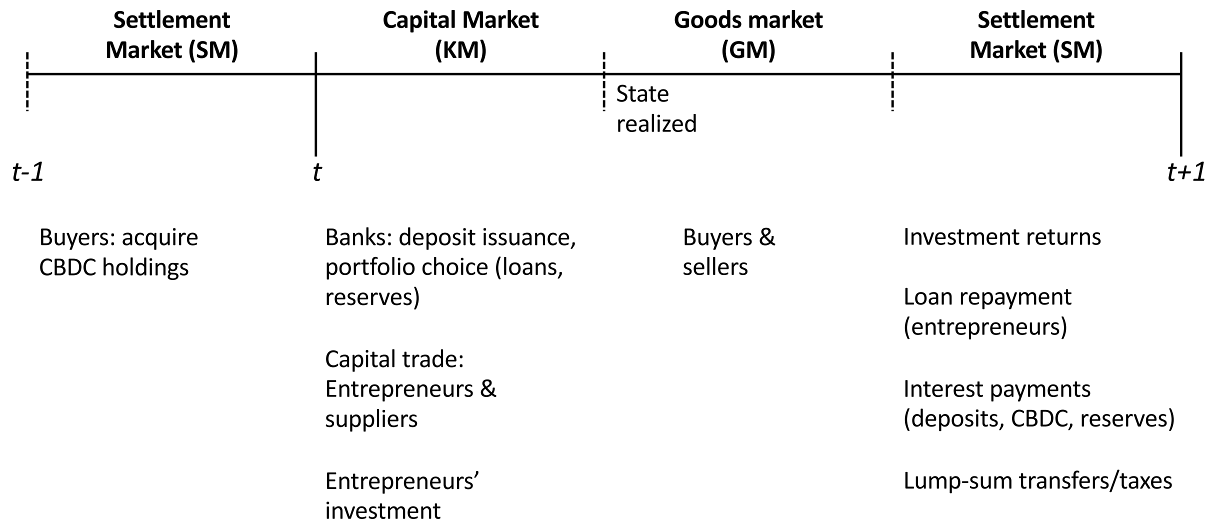

Figure 2. Timeline

2.3 Motive for central bank money

We build the model to focus on the potential contraction of the demand for deposits and its resulting effects on bank intermediation and financing of the real economy, due to the introduction of a risk-free, potentially interest-bearing, form of money issued by the central bank. It is widely accepted that deposits are a convenient source of funds for banks, as long as depositors do not all withdraw at once. In our model, the bank acquires CBDC with deposits and later lends CBDC to entrepreneurs (because suppliers are unbanked) while maybe retaining only a fraction in reserves. Therefore, our bank intermediates funds, and within a period, it operates as a fractional reserve bank. In this sense, the bank creates money.

There are two components to the bank's funding cost. Firstly, deposits must pay a high enough interest rate to compensate buyers for holding deposits rather than CBDC. Secondly, the bank must acquire enough CBDC to cover the needs of entrepreneurs, irrespective of the relative returns of money and deposits. This cost is intrinsically related to the banks' need for central bank money, as determined by exogenous factors. These could be related to policy (e.g. reserve requirements) or technological constraints as in our model. The assumption that suppliers are unbanked is thus useful to provide a motive for the use of central bank money and to capture part of the cost of bank loans.

It is worth emphasizing that remunerating CBDC has a negative effect on banks' profit because it increases its funding cost. However, we will see that is also has a positive effect on banks' profit because a higher remuneration rate increases payment efficiency, in the sense that a smaller amount of CBDC is needed to fund the same number of projects.

2.4 Comparing inflation and interest rate on CBDC

Our analysis keeps the level of inflation γ fixed while focusing on the effects of the interest rates on CBDC and reserves instead. Therefore we consider a long run target for inflation, while the central bank can adjust interest rates on a shorter horizon to control the vagaries of the economy. While we do not consider it, our setting may be extended to incorporate aggregate temporary shocks, say to the probability of success of risky projects p, that the central bank may take into account when setting interest rates without changing its long-run inflation target.

2.5 Optimal allocation benchmark

The optimal allocation is the solution to the problem of a planner who seeks to maximize welfare, while being subject to the same technological constraints on the return of investment projects as the agents. The planner solves

$$\max_{q,n,r} u(c) - c + Rq + pRn - q - n - \kappa(q)$$

subject to

$$q + n \leq \overline{h} \tag{1}$$

In words, the planner maximizes the trade surplus of buyers and sellers and the utility from consuming the return on investment projects, net of capital and monitoring costs.

The level of investment is bounded by the number of available projects, captured by the capacity constraint (1).

Lemma 1 *The planner sets buyers' consumption to* c^* *with* $u'(c^*) = 1$*, invests* q^* *in risk-free projects, with*

$$\kappa'(q^*) = R(1 - p) \tag{2}$$

and n^* *in risky projects, with* $n^* = \overline{h} - q^*$*.*

Planner sets consumption for buyers such that sellers produce the socially efficient level c^*. Furthermore, the planner invests q^*, defined by (2), into risk-free projects and the rest of the $\overline{h}$ available projects into risky projects. Thus, the planner utilises all the available investment projects. For a low $\overline{h}$, the planner only invests in risk-free projects, whereas for higher levels the planner invests in both risk-free and risky projects. Hence, investing in risky projects can be efficient.[7]

3 Equilibrium Analysis

3.1 Suppliers' problem

Suppliers choose how much capital k to produce in exchange for CBDC by equating the marginal cost of production with the marginal benefit. Each unit of capital is sold for $\rho = \phi\widetilde{\rho}$ units of real CBDC that pay $1 + i_m$ in the SM. With linear production technology it costs -1 to produce a unit of capital. Thus, a supplier is indifferent whether to produce capital whenever

$$\rho(1 + i_m) = 1. \tag{3}$$

From (3) we see that the real price of capital is inversely related to the interest rate paid on CBDC. This captures the payment efficiency aspect of CBDC: for a higher i_m, a given amount of CBDC allows for higher level of capital investment.

[7] The planner distributes the proceeds from investment to agents in the SM. How they are distributed is irrelevant for the solution, since all agents have linear utility in the SM.

3.2 Buyer's problem

In any period t, buyers wish to consume the GM good produced by sellers. Needing a means to pay, buyers acquire z_t real units of CBDC in the SM at $t-1$, which they can choose to exchange for bank deposits before the GM opens at time t. However, since the banker has all the bargaining power, buyers do not gain anything from acquiring bank deposits and obtain the same payoff as when they use only CBDC with sellers. Thus, the solution of the buyer's problem is the same as when the buyer only uses CBDC when trading with sellers.

In the GM, the buyer makes a take-it-or-leave-it offer to the seller to produce c_t in exchange for s_t real units of CBDC. At the beginning of the SM at $t-1$, and holding $\widetilde{z}_{t-1} = z_{t-1} - s_{t-1}$ real units of CBDC, the buyer solves

$$W_{t-1}\left(\widetilde{z}_{t-1}\right) = \max_{z_t, x_{d,t-1}, c_t, s_t} \left\{x_{d,t-1} + \beta\left[u\left(c_t\right) + W_t\left(z_t - s_t\right)\right]\right\}$$

subject to the budget constraint, $x_{d,t-1} + \gamma z_t = T_t + \widetilde{z}_{t-1}$ and the seller's participation constraint, $c_t = s_t(1+i_m)$. Except at the Friedman rule, the buyer's optimal choice is to hold the exact amount of CBDC that corresponds to the payment made to the seller. Therefore, $s_t = z_t$.

In a stationary equilibrium real variables are time invariant, with $z_t = z$ and $c_t = c$. The buyer's choice of real balances is then given by $c = z(1+i_m)$ and

$$u'\left[z\left(1+i_m\right)\right] = \frac{\gamma}{\beta\left(1+i_m\right)} \tag{4}$$

The buyer's real CBDC holding, z, can increase or decrease with i_m depending on the curvature of the utility function and thus whether the income or substitution effect dominates. However, the purchasing value of the buyer's money holdings, $z\left(1+i_m\right)$, always increases along with an increase in i_m.

3.3 Bankers problem

Bankers maximize profits by selecting items on their liability and asset sides of their balance sheet. On their liability side, they select a deposit contract (i_d, δ) consisting of an interest rate i_d and an amount of deposits δ. On their asset side, they select their investment portfolio (q, n, r). Given its investment plan, the banker needs to design the deposit contract such that it attracts at least $\rho(q+n)$ real units of CBDC, while taking into account depositors' outside option of CBDC. Buyers who accept the deposit contract become depositors. They can buy at most $c_h = (1+i_d)\,\delta + (1+i_m)(z-\delta)$ from sellers in the GM, unless the banker is unable to honor the deposit contract due to failure of non-monitored projects. In that case depositors can buy at most $c_\ell = \ell + (1+i_m)(z-\delta)$, where ℓ is the liquidation value of the banker.

The banker could offer a return i_d that allows depositors to consume more than c^* in the GM. In that case, the depositor would choose to consume c^* in the GM and keep $c_h - c^*$ units of real balances to consume in the following SM. With this in mind, we define the function $\upsilon(c_h)$ with $\upsilon(c_h) = u(c_h)$ if $c_h \leq c^*$ and $\upsilon(c_h) = u(c^*) + c_h - c^*$ if $c_h > c^*$.[8]

The problem of the banker is:

$$\begin{aligned} \max_{q,n,r,\delta,i_d} \quad & p\left[R\,(q+n) + (1+i_r)\,r - (1+i_d)\,\delta\right] \\ & + (1-p)\max\left[Rq + (1+i_r)\,r - (1+i_d)\,\delta, 0\right] - \kappa(q) \end{aligned} \tag{5}$$

subject to

$$\rho(q+n) + r \leq \delta \tag{6}$$

$$\delta \leq z \tag{7}$$

$$q, n, r \geq 0 \tag{8}$$

$$p\upsilon(c_h) + (1-p)\,u(c_\ell) \geq u\left[(1+i_m)\,z\right] \tag{9}$$

[8]More precisely, $\upsilon(c_h)$ is defined as $\upsilon(c_h) = \max_{c \leq c_h} u(c) + c_h - c$. This function is well defined, always positive, increasing and (weakly) concave with

$$\upsilon'(c_h) = \begin{cases} u'(c_h) & \text{if } c_h \leq c^* \\ 1 & \text{if } c_h > c^* \end{cases}$$

with

$$c_h = (1+i_d)\,\delta + (1+i_m)\,(z-\delta) \tag{10}$$

$$c_\ell = \min\left[c_h, \ell + (1+i_m)\,(z-\delta)\right] \tag{11}$$

Expression (5) captures the banker's profit function. Deposits are safe if the banker always has enough resources to honour the deposit contract, with $Rq + (1+i_r)\,r \geq (1+i_d)\,\delta$. Otherwise, deposits are risky and the banker liquidates its safe assets to pay depositors when non-monitored projects fail, with $\ell = Rq + (1+i_r)\,r$. Depositors can then acquire c_ℓ of the consumption good from sellers, where c_ℓ is defined in (11).

The resource constraint is captured by (6) and states that the banker cannot invest more than the amount of deposits that it issued. Under the resource constraint the price of capital ρ is taken as given from (3) – with $\rho = (1+i_m)^{-1}$ there is an inverse relationship between the price of capital and interest rate paid on CBDC. The deposit constraint (7) states that the banker cannot issue deposits for more CBDC than what the buyer brings. Lastly, (9) is the buyer's participation constraint.

The banker will invest all issued deposits, either into projects or reserves. Thus the resource constraint (6) always binds. The participation constraint (9) also binds, because the banker sets i_d such that the buyer is equally well off between the option of using CBDC and the option of depositing CBDC and using instead bank deposits as a medium of exchange in the GM. Hence, when deposits are safe, the banker sets $i_d = i_m$. Otherwise, the banker sets $i_d > i_m$ to compensate the depositor for the risk they bear. Furthermore, we can show that the deposit constraint (7) weakly binds. When it does not bind, the banker and depositors are indifferent between holding more deposits or CBDC. Hence, we assume without loss that $\delta = z$. The interest rate on deposits i_d is determined by the participation constraint (9).

Let λ denote the Lagrange multiplier on the banker's resource constraint. The firt-order condition on monitored loans q yields

$$R\left[p+(1-p)\frac{u'(c_\ell)}{\upsilon'(c_h)}\right]\leq\kappa'(q)+\frac{\lambda}{1+i_m} \tag{12}$$

with equality if $q>0$. On the right of (12), the marginal cost of monitored loans is given by the cost of monitoring an additional project and the opportunity cost of using an additional unit of CBDC for investment in q rather than r or n, as expressed by λ. On the left of (12), the benefit of an additional monitored loan is simply its return R under a safe deposit contract with $c_\ell = c_h$. If the deposit contract is risky and $c_\ell < c_h$, the marginal benefit of monitored loans is higher than R because these loans are safe and can be used to relax the depositor's participation constraint.

The first-order condition on n gives

$$pR\leq\frac{\lambda}{1+i_m} \tag{13}$$

and it states that the marginal benefit of non-monitored loans given by their expected return pR cannot exceed their marginal (opportunity) cost.

The first-order condition on the real amount of reserves r yields

$$(1+i_r)\left[p+(1-p)\frac{u'(c_\ell)}{\upsilon'(c_h)}\right]\leq\lambda \tag{14}$$

The marginal benefit of reserves is their return, $(1+i_r)$, if the deposit contract has no risk, but higher with risky deposits since in that case reserves help mitigate the risk for depositors and ensure their acceptance of the contract offered by the bank.

In Section 4, we characterize the banker's investment plan (q,n,r) for different levels of i_m.

4 Interest-bearing central-bank money

In order to analyze the equilibrium effects of remunerating central-bank digital currency, we distinguish the case where banks never fail and deposits are safe, and the case where banks fail with probability $(1-p)$ and deposits are risky. We make the following two assumptions.

Assumption 1 $c^* > Rq^*$.

Assumption 1 implies that we focus on the interesting case where the efficient investment in risk-free projects does not yield a high enough return to provide the efficient consumption level. Under this assumption, deposits may turn out to be risky in equilibrium.

Assumption 2 *i_r and i_m are such that $1 + i_r < (1 + i_m)pR$.*

For ease of exposition, we assume policies such that the expected return of investing in a risky project pR is higher than its shadow cost $(1+i_r)/(1+i_m)$. Indeed, undertaking an additional risky project only requires $1/(1 + i_m)$ units of reserves, thanks to the payment efficiency of CBDC. This assumption however does not imply that the bank will always find it profitable to invest in risky projects, because depositors are risk averse. Also, since $pR > 1$, Assumption 2 does not rule out $i_r > i_m$, but bounds the spread $i_r - i_m$ from above.

For given i_m and i_r, government transfers T (taxes if negative value) satisfy the central bank budget constraint

$$(\gamma - 1)\, z = T + i_m\, (z - r) + i_r r \tag{15}$$

where z denotes real CBDC balances and r denotes the measure of reserves. On the left, the central bank's revenue comes from money issuance. On the right, the central bank expenditures are the transfers and the interest payments. Notice that if interest rates i_m and i_r differ, interest payments depend on whether banks use part of their CBDC holdings to acquire reserves.

We fully characterize the equilibrium in the following proposition.

Proposition 2 *There is a unique equilibrium which is characterized as follows,*

> *(Region 1): If $i_m \leq i_m^1$, the bank only invests in safe projects ($q > n = r = 0$), deposit contracts are safe $c_h = c_\ell = (1 + i_m)z(i_m)$.*

(Region 2): If $i_m^1 < i_m \le i_m^2$, the bank invests in safe and risky projects ($q, n > r = 0$), deposit contracts are safe $c_h = c_\ell = (1 + i_m)z(i_m)$.

(Region 3): If $i_m^2 < i_m \le i_m^3$, the bank invests in safe and risky projects ($q, n > r = 0$), deposit contracts are risky $c_h > (1 + i_m)z(i_m) > c_\ell$.

(Region 4): If $i_m > i_m^3$, the bank invests in safe and risky projects and reserves ($q, n, r > 0$), deposit contracts are risky $c_h > (1 + i_m)z(i_m) > c_\ell$.

Region 1 exists if $R \ge R^1$, Region 2 exists if $R \ge R^2$ and Region 3 exists if $R \ge R^3$, with $R^3 < R^2 < R^1$.

In the rest of this section we further characterize the existence of each equilibrium region and discuss some comparative statics.

Region 1 : Safe deposit contracts backed by safe projects only When R is sufficiently high and i_m relatively low, the banker uses all its CBDC holdings to invest in monitored loans as their marginal benefit exceeds their marginal cost, with $R(1-p) \ge \kappa'(q)$. Note from Lemma 1 that in this region $q \le q^*$. The cost of monitoring remains low due to the small size of q, and the overall return from investing in monitored loans is higher than the expected return on risky loans and reserves, so that the bank does not invest in those assets and $n = r = 0$. As we explained above, with safe deposits, the monopolistic banker sets $i_d = i_m$. The banker obtains $z(i_m)$ deposits, which are worth $(1 + i_m)z(i_m)$ units of the investment good.

The comparative statics in Region 1 is straightforward. An increase in the payment efficiency of CBDC – through a higher i_m – means that a higher number of projects can be funded with the CBDC holdings that buyers deposit with bankers, so q increases. However, as the monitoring costs increase as a result, the wedge between the returns of the monitored and risky loans is reduced.

For $i_m > i_m^1$ – with i_m^1 being the solution to $q^* = (1 + i_m^1)z(i_m^1)$ – the value of CBDC holdings (in units of the investment good) that buyers deposit with the bank is that high that monitoring all ensuing loans would be too costly for the banker. Then the equilibrium switches to Region 2.

Region 2 : Safe deposit contracts backed by safe and risky projects When i_m lies between i_m^1 and i_m^2, the banker chooses to hold a portfolio of both monitored and non-monitored projects, with $q = q^*$ and $n = (1 + i_m)z(i_m) - q^*$. The banker still offers a safe deposit contract because, even if the risky projects fail, the banker can make the depositors whole and pay them $(1+i_m)z(i_m)$ using the return on monitored projects. Thus i_m^2 solves $Rq^*(R) = (1 + i_m^2)z(i_m^2)$ which implies that $i_m^1 < i_m^2$.

In Region 2, increasing i_m leads to increased intermediation but only in risky projects, since q is constant at q^* in this region. As in Region 1, in order to retain depositors the banker is compelled to increase the deposit rate i_d to keep up with the increase in i_m. For $i_m > i_m^2$, however, it is too costly for the banker to increase remuneration on deposits while keeping them safe. Then the equilibrium switches to Region 3.

The existence of Region 2 requires a lower value of R than the existence of Region 1 because in Region 2 the bank no longer invests only in safe projects.

The following Lemma summarizes the effect of i_m on the deposit rate and intermediation when deposits are safe.

Lemma 3 *Let $0 \leq i_m \leq i_m^2$. An increase in i_m has a positive effect on the deposit interest rate i_d and on total bank lending $q + n$.*

In Regions 1 and 2, while the increase in i_m entails an increase in the bank's funding costs, it does not result in a reduction in bank lending. The reason is clear; the payment efficiency of CBDC increases with i_m, which relaxes the banker's resource constraint (6) and fosters loan profitability. Overall, the level of lending is increasing in i_m for all $i_m \leq i_m^2$.

Region 3 : Risky deposit contracts with investments in monitored and risky projects, but no reserves For $i_m > i_m^2$, the banker can no longer offer full insurance to depositors as the return from monitored projects is insufficient to cover the value of the deposited CBDC in all states. The deposit contract becomes risky and the consumption of depositors differs across states, with $c_h > c_\ell$ and $c_\ell = Rq$.[9]

As long as $R > R^3$ and $i_m^2 < i_m \leq i_m^3$, the bank invests only into monitored and non-monitored loans, since it prefers monitored loans rather than reserves as a safe asset. Combining the first-order conditions on q and n, (12) and (13), we obtain

$$R\,(1-p)\,\frac{u'(Rq)}{u'(c_h)} = \kappa'\,(q) \tag{16}$$

The bank chooses q to equate the expected marginal benefit of monitored projects with their marginal cost. Relative to risky projects, the benefit of monitored projects accrue when risky projects fail (with probability $1-p$) in which case the return on monitored projects R serves to close the wedge between consumption in the good and the bad state. The choice of q and c_h must satisfy the depositors' participation constraint so that

$$pu(c_h) + (1-p)u(Rq) = u((1+i_m)z(i_m)) \tag{17}$$

Equations (16) and (17) can be solved for q and c_h jointly.

It is easy to show that $q(i_m)$ and $c_h(i_m)$ are both increasing in i_m. Intuitively, as the interest rate on CBDC increases the banker needs to make its deposit contract more valuable in order to attract risk-averse depositors. Hence, to increase the payout in the bad state, the banker increases its investment in monitored projects in line with the increase in i_m. As a result, the banker holds a higher level of monitored projects than the planner solution suggests, as a comparison between (2) and (16) shows.

At the same time, with the increase in i_m, the investment in safe projects becomes too costly and insufficient to guarantee a payment in the bad state commensurate to i_m. Hence, to attract depositors, the banker has to increase the remuneration in the good state even further. This increases the wedge between payments in the good and bad states leading

[9] We can show that $c_h < c^*$ so that $v\,(c_h) = u\,(c_h)$ in Region 3. In order to rule out the case where $c_h \geq c^*$ in this region, we assume that $p \leq \bar{p}$ where $\bar{p}$ solves $(1-\bar{p})\,u'\left[\kappa'^{-1}\left(\bar{p}R\,(R\varepsilon-1)\right)R\right] = \bar{p}\,(R\varepsilon-1)$ with $\varepsilon = \frac{1+i_m}{1+i_r}$. Intuitively, if p is relatively low depositors greatly care about deposits' liquidation value and hence the banker needs to ensure a high liquidation value rather than just promising a redemption value higher than c^* that is only fulfilled in the good state.

to increased risk to depositors. Therefore, since in this region the banker lacks access to a risk-free asset that offers a suitable return, the interest rate level on CBDC leads to bankers making inefficient investment decisions and results in risk to depositors. As $q(i_m)$ increases with i_m, investment in monitored projects becomes prohibitively expensive for $i_m > i_m^3$, and the banker starts investing in reserves. The equilibrium then switches to Region 4.

Region 4 : Risky deposit contracts with investments in monitored and risky projects, and reserves When $i_m > i_m^3$ and q is relatively high, the net return on monitored projects becomes too low and the banker starts investing in reserves. The equilibrium is then defined by the following system of equations,

$$R(1-p)\frac{u'(c_\ell)}{v'(c_h)} = \kappa'(q) \tag{18}$$

$$pv(c_h) + (1-p)u(c_\ell) = u((1+i_m)z(i_m)) \tag{19}$$

$$\frac{\kappa'(q)}{R} = p\left(R\frac{1+i_m}{1+i_r} - 1\right) \tag{20}$$

with $c_\ell = Rq + (1+i_r)r$. Notice that we have to keep the utility function $v(\cdot)$, as it could be that $c_h > c^*$ in the good state.

When $i_m = i_r$, (20) implies that q is constant in Region 4 and as a consequence the degree of insurance, as captured in (18) by the ratio of marginal utilities across states, is also constant. Still, consumption is increasing in i_m in both states, owing to the interest rate paid on reserves, and to the increased investment in risky projects.

Within this region, we can show that for levels of $i_m > i_m^4$ for some $i_m^4 < \gamma/\beta - 1$, CBDC represents such a profitable outside option that the bank needs to offer depositors a payment higher than c^* in the good state. In this case, any further increases in i_m, while keeping $i_m = i_r$, induce the bank to increase its investment into risky projects. Once the buyer is satiated in the good state with $v'(c_h) = u'(c^*) = 1$, increases in i_m do not exacerbate the dispersion of marginal utilities across states even if the bank no longer increases investment in q or r. This favours the bank investing in risky projects.

The following Lemma summarizes the effect of an increase in i_m on equilibrium variables under a risky deposit contract in Regions 3 and 4 when $i_m = i_r$.

Lemma 4 *Let $i_m^2 < i_m \leq \frac{\gamma}{\beta} - 1$ and $i_m = i_r$.*

i) An increase in i_m has a positive effect on the deposit interest rate i_d. The banks' liquidation value is increasing in i_m for $i_m < i_m^4$ and is constant for $i_m \geq i_m^4$.[10]

ii) Risk-free loans q increase with i_m for $i_m < i_m^3$ and stay constant for $i_m \geq i_m^3$, while risky loans may increase or decrease with an increase in i_m for $i_m \leq i_m^4$ and unambiguously increase for $i_m > i_m^4$.

Policy mix i_r and i_m We next analyze how the spread $i_r - i_m$ impacts the optimal investment choice of the bank.

If the interest rate on reserves is larger than the interest rate on CBDC, $i_r > i_m$, issuing deposits is cheap because the generated funds can be invested in reserves. As a result, the deposit constraint binds for all $i_m < \gamma/\beta - 1$. However, in that case, the change in the spread $i_r - i_m$ is crucial to determine the level of risk faced by depositors. From (18) and (20) we can state the following Lemma.

Lemma 5 *Let $i_m > i_m^3$. An increase in the spread $i_r - i_m$ entails a reduction in q and in $u'(c_\ell)/v'(c_h)$.*

If the central bank increases the spread between i_r and i_m, the return on reserves increases by more than the required expected return on deposits. Hence, the banker invests more in reserves and reduces the over-investment in monitored projects. The reason for this effect of a positive spread is that, while i_m favours investment in monitored loans through the use of CBDC as a means to acquire capital, i_r does not have such an effect because reserves are only used to hoard liquidity. The increase in the spread narrows the wedge between depositor's marginal utility in the good and bad states and increases the overall level of insurance. Therefore, when depositors are faced with risk in the banking

[10] If $i_m \geq i_m^4$, $c_h \geq c^*$ and hence $u'(c_h) = 1$ in (18). Since q is fixed on the right in (18), on the left $Rq + (1 + i_r)r$ must be constant so $(1 + i_r)r$ is not moving with i_m (r decreases when i_r increases).

sector, reserves and their remuneration become an important tool to mitigate the level of risk and investment inefficiency.

5 Welfare

In this section, we analyze the welfare effect of an interest-bearing CBDC. We define welfare in a given period as the sum of buyers' surplus and bankers' profits (the other agents in the economy always obtain zero net utility):

$$\mathcal{W} = u(c) - \gamma z + T + Rq + pRn + r(1 + i_r) - \kappa(q) - pc_h - (1 - p) c_\ell$$

Since buyers obtain utility $u(c)$ in the GM and work $\gamma z - T$ in the SM, from (4) and (15) an increase in i_m with $i_m = i_r$ unambiguously improves buyers' surplus. However, it has two opposing effects on bankers' profit. On the one hand, the increase in i_m improves the depositor's outside option and forces bankers to increase the remuneration it offers to depositors – this has a negative effect on bankers' profit. On the other, the increase in i_m makes paying with CBDC cheaper, effectively reducing the price of capital and rendering the investment in projects more profitable – this increases banks' profits. Which effect dominates depends on the equilibrium region.[11] Proposition 6 summarizes the results of a joint increase in i_m and i_r on welfare, with $i_m = i_r$. For this proposition, we let $u(x) = \frac{x^{(1-\sigma)}}{1-\sigma}$, with σ the coefficient of relative risk aversion.

Proposition 6 *Let $i_m = i_r$. i) For both low and high levels of i_m ($i_m < i_m^3$ and $i_m > i_m^4$), welfare strictly improves when i_m increases. For intermediate levels of i_m ($i_m^3 \leq i_m \leq i_m^4$), an increase in i_m is welfare-improving if σ is relatively low and welfare-worsening if σ is relatively high. ii) The Friedman rule, with $i_m = i_r = \gamma/\beta - 1$, does not ensure achieving the first-best.*

[11] The increase in i_m may also foster buyers' money demand, depending on the curvature of the utility function, and provide the bank with more funds to invest. Note that in previous papers, like Chiu et al. (2023) and Andolfatto (2021), there is no investment effect through the payment efficiency of CBDC because CBDC is not used as a means of payment in the capital market as it is in our model.

The welfare effect of an increase in i_m is unambiguous for either low or high levels of i_m. When the interest rate on CBDC is small (Regions 1 to 3), the bank is particularly eager to increase its investment and meeting the depositors' outside option is relatively affordable. In that case, the reduction in the price of capital due to the increase in i_m leads to an increase in profits for banks even if they must provide a higher remuneration on deposits. Therefore, increasing the interest rate offered on CBDC improves overall welfare in the region where $i_m \leq i_m^3$, which equires a relatively high R. With R high, investment projects are so profitable that the effect of a lower price of capital from an increase in i_m is particularly powerful to ensure a welfare improvement.

On the other extreme, when i_m, and hence the value of the depositor's outside option, is high ($i_m > i_m^4$), the depositor attains c^* in the GM unless the bank fails. Further increases in i_m only lead the bank to invest more in risky projects and n increases. Then whether the return on investment is captured by the bank or the depositor is neutral for overall welfare, since both obtain linear utility. The only relevant effect from higher i_m is the ability to increase the volume of risky loans, which is clearly beneficial for welfare because of their positive expected net return.

For intermediate levels of i_m ($i_m^3 \leq i_m < i_m^4$), an increase in i_m may however considerably reduce the profit of banks.[12] The increase in i_m still implies a greater ability to invest, but to compete with a risk-free remunerated CBDC the bank cannot allocate funds solely based on assets' returns. So in this range the CBDC safety disciplines the bank. As a result it does not invest "too much" in risky assets.[13] To assess the effect of an increase in i_m on banks' profits, the degree of depositors' risk aversion is key. If depositors are not that risk averse, the bank is relatively free to take advantage of the higher availability of

[12] In Region 3 and Region 4 when $c_h \leq c^*$, an increase in i_m increases profits if $Rpu'(c_h) > u'(c)$. The expected return on risky projects must be high, as well as the marginal utility obtained by the depositor, in which case the bank needs not greatly increase the remuneration it offers to retain the depositor when the interest rate on CBDC increases. On the contrary, if $u'(c)$ is high, the depositor's outside option is highly valued and this is harmful for bank's profits.

[13] This situation may arise even for i_m rather low if productive safe assets are scarce, with R lower than R^3 (in which case Regions 1 to 3 cannot exist and the economy is necessarily in Region 4).

resources when choosing its portfolio. Its profits may increase, or slightly decrease, but in any case overall welfare improves. If, however, depositors are highly risk averse, it is too costly for the bank to keep up with the depositor's outside option when taking risk. This lead to a reduction in the bank's profit and in overall welfare.

Effect of the spread $i_r - i_m$ on welfare Next we consider the effect on welfare of the spread between i_r and i_m, as considered in Lemma 5. We show that a positive spread may be associated with higher welfare than a no-spread policy. Notice that in Proposition 6 the (likely) positive welfare effect does not come solely from the increase in i_m, but also from the joint increase in i_r. The overall cost of holding money is reduced when the central bank increases both policy rates. Consider two cases in the region where banks hold reserves, one with $i_m = i_r$ and the other with $i_m < i_r$. As Figure 3 shows, welfare may turn out to be higher under a positive spread. This is more likely in an economy with a higher p,[14] because it implies a higher return on non-monitored loans which reduces the socially optimal number of monitored loans q^* given by (2). The bank is inclined to invest more in non-monitored loans n with higher p but, to mitigate the additional risk, it also has to increase its investment in monitored loans q. Hence, while a higher p is associated with a lower optimal q^*, it implies a higher equilibrium value of q. To limit this investment inefficiency, it is welfare improving to set a positive spread $i_r - i_m$ that reduces q for a larger range of values of i_m if p is relatively high.

[14] Figure 3 plots welfare for $i_m \in [0, i_r]$ with $i_r = 0.034$ (blue curve) and for $i_m = i_r \in [0, 0.034]$ (yellow curve) in Region 4 and $c_h \leq c^*$. The assumed functional forms and parameters are $u(x) = u_0 \frac{x^{(1-\sigma)}}{1-\sigma}$ with $u_0 = 1.4$ and $\sigma = 1.5$, $\kappa(q) = \kappa_0 q^\eta$ with $\kappa_0 = 3.5$ and $\eta = 3$, $R = 1.3$, $\beta = 0.93$ and $\gamma = 1.02$.

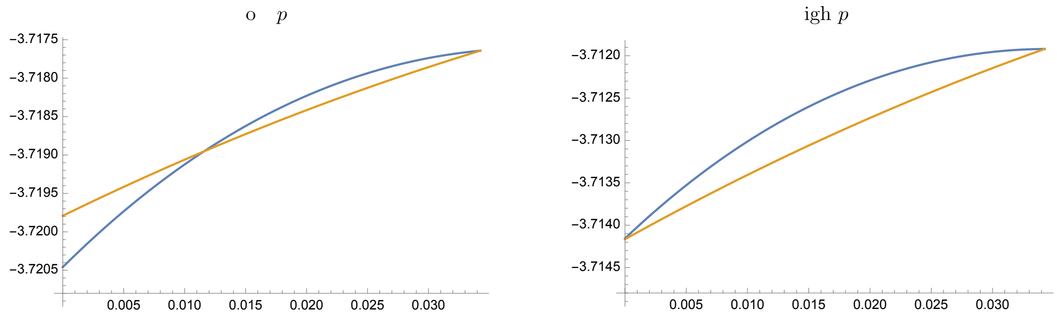

Figure 3. Welfare as a function of i_m: $i_r - i_m > 0$ (blue), $i_r - i_m = 0$ (yellow), for $p = 0.8$ (left) and $p = 0.82$ (right).

Finally, notice that although the Friedman rule may provide the maximum attainable level of welfare, it does not necessarily implements the first-best. The reason is that the Friedman rule does not solve the main friction present in this model: While banks possess the technology to increase output in the economy, risk-averse depositors are the ones that hold the resources to carry out productive projects. Even if the cost of holding money is zero, as is the case at the Friedman rule, the optimal level of risky investment may not be attained if depositors do not bring enough money balances to banks.[15]

6 Comparing policies

Since the money growth rate γ and the interest rate on CBDC i_m are policy tools at the disposal of the central bank that both affect the payment efficiency of CBDC, it is legitimate to ask whether a reduction in γ is equivalent to an increase in i_m. Notice that in the regions where banks do not hold reserves, the level of i_r is irrelevant, and the effects of a decrease in γ or an increase in i_m on the equilibrium allocation are analogous. Both policies affect $c(i_m, \gamma) = (1 + i_m) z(i_m, \gamma)$ and thereby the depositor's participation constraint and the banker's resource constraint in exactly the same manner.

[15]In particular, depositors must bring enough money balances that the banker exhausts all available risky projects once the optimal investment in monitored projects is attained; i.e., $\bar{h} - q^*$.

A reduction in γ and an increase in i_m do not have the same effects, though, if the bank holds reserves and $i_m < i_r$.[16] In that case, these policies affect similarly the participation constraint of depositors, but their effects on the resource constraint of the banker differ: While γ and i_m have the same effect on the maximum amount of capital that can be acquired with the deposited CBDC, equal to $c(i_m, \gamma) = (1 + i_m) z(i_m, \gamma)$, only i_m alters the price of capital and thereby the set of combinations of q and n, on the one hand, and r, on the other, that satisfy the constraint. As discussed in Section 5, this effect comes from the fact that CBDC is used as a medium of exchange in the capital market while reserves are not. In turn, by inducing different variations in reserve holdings, the two policies have different fiscal cost implications.

Table 1: Effect of a reduction in γ and increase i_m (fixing effect on consumption)

	$\downarrow \gamma$	$\uparrow i_m$
consumption c	+1.7%	+1.7%
money holdings z, deposits δ	+2.01%	+1%
monitored lending q	+0%	+1.87%
non-monitored lending n	+1.78%	+11.15%
reserves r	+2.68%	−6%
transfers T	−199.5%	−84.5%
welfare	+0.2%	+0.25%

Note: Starting from $\gamma = 1.02$ and $i_m = 0$, the reduction in γ consists in setting $\gamma = 1.0099$ and the increase in i_m consists in setting $i_m = 0.01$, holding the same change in c.

The following example illustrates the differential effect of these policies. Consider an economy with a positive spread $i_r - i_m$ where p is relatively small so that the spread

[16] If $i_m = i_r$, a reduction in γ and an increase in i_m that equally affect c have identical effects on welfare. With $i_m = i_r$ in this region, both q and the ratio of marginal utilities of c_h and c_ℓ become independent not only of γ but also of i_m. For a same value of c, the values of c_h and c_ℓ that result from (18) and (19) are the same. Given that q is determined by (20) and fixed, the value of c_ℓ uniquely determines the value of $r(1 + i_m) = r(1 + i_r)$. The value of r is therefore different whether γ has been reduced or i_m increased, as for z, but the value of $r(1 + i_m)$ is the same. Then the value of n and, in the end, the level of welfare, are also identical.

between i_r and i_m is not efficient. We let $p = 0.78$, $R = 1.4$, $\beta = 0.92$. We further assume $u(x) = u_0 \frac{x^{(1-\sigma)}}{1-\sigma}$, with $u_0 = 1.4$ and $\sigma = 0.5$, and $\kappa(q) = \kappa_0 q^{\eta}$, with $\kappa_0 = 3.5$ and $\eta = 3$. We consider an initial monetary policy characterized by $\gamma = 1.02$, $i_m = 0$ and $i_r = 0.03$, that entails CBDC holdings $z = 1.595$ and positive reserve holdings by banks, with $r = 0.789$. The resulting lump-sum monetary transfers are 0.008. The level of investment in monitored and non-monitored loans is, respectively, $q = 0.193$ and $n = 0.613$. With the assumed preference parameters, a reduction of γ to $\gamma = 1.0099$ or, alternatively, an increase in i_m to $i_m = 0.01$ have an identical (positive) effect on c. Money holdings increase in both cases, although the increase is stronger in the case of a reduction in γ ($z = 1.627$ vs. $z = 1.610$). In terms of banks' investments, the reduction in γ implies an increase in r to $r = 0.810$ and an increase in n to $n = 0.623$, with no effect on q. By contrast, the increase in i_m implies a decrease in r to $r = 0.741$, an increase in q to $q = 0.197$ and an increase in n to $n = 0.681$. The fiscal cost of these policies also differ. While setting $\gamma = 1.0099$ and $i_m = 0$ entails lump-sum negative transfers, the policy that combines $\gamma = 1.02$ with $i_m = 0.01$ entails lump-sump positive transfers (-0.008 vs. 0.001). Therefore, the former policy requires that the central bank has taxation power.

Comparing the two policies in terms of investment allocation, reducing γ and keeping a low value of i_m is beneficial because it mitigates the overinvestment in q. However, increasing i_m and keeping a higher value of γ allows a higher investment in non-monitored loans, therefore closer to its socially optimal level. Overall, given the effects on investment and lump-sum transfers, an increase in i_m delivers a higher level of social welfare than a reduction in γ in this example. Table 1 summarizes the percent changes in equilibrium variables for the two alternative policies. Given the unequal effects of varying γ and i_m, this example shows that considering an interest-bearing CBDC is not superfluous even if the central bank has the same ability to change both policy variables.

7 Conclusion

We analyzed a simple model of risk taking by banks, where the bank issues debt (or deposits) to fund its assets and depositors can choose to hold remunerated central bank digital currency. We showed that the level of CBDC remuneration is important for the choices made by the bank in equilibrium. A low remuneration makes funding scarce, and banks invest funds in safe (costly to monitor) projects. Because funding is relatively scarce, the overall monitoring cost remains low which makes safe projects viable. As the CBDC remuneration increases so does the availability of funding for banks: The bank offers more favorable conditions to depositors, thus generating more funds. As a consequence, while their debt is still safe, banks now start taking more risk because the relatively high monitoring cost reduces the viability of safe projects. With a high remuneration, liquidity is so abundant compared to the availability of safe projects that there is a shortage of safe assets for banks – this is endogenous in our model as these assets become prohibitively costly to monitor. This high remuneration of CBDC has similar effects on banks' behavior than a lower policy rate and induces banks to "search for yields" to compete with CBDC. The debt of banks becomes risky with investment in safe assets being low relative to investment in risky assets. At some point of the liquidity surplus, the bank is forced to invest in reserves to preserve some degree of safety of its deposits. In this context, we show that while depositors' welfare improves with CBDC remuneration, the equilibrium can be inefficient with an overall welfare worsening. The central bank can use the spread between the interest rate on CBDC and the one on reserves to correct the inefficiencies.

The issuance of a CBDC may raise questions regarding the usefulness and appropriate design of banking regulations, on different dimensions such as the level of competition in the banking sector or the requirements imposed on banks to reduce their vulnerability to runs. Our setting makes abstraction of those policies, since we see it as a benchmark to study the effects of CBDC on the banking sector if those policies are absent. A more comprehensive study of CBDC should consider the introduction of CBDC and the design of bank policies jointly.

Our model is simple but still capture some interesting investment behavior by banks. It would of course be also interesting to study the effect of the banks being able to issue deposits directly to firms when lending. The liquidity/funding effect of remunerating CBDC, which is important in our setting, may then be somewhat tamed. Also, analyzing how the presence of cash plays in the equilibrium with CBDC could lead interesting insights. Finally, while banks can default in equilibrium, we do not study the damaging effects of financial stability. We leave these extensions for future work.

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Appendix

Proof of Lemma 1

The planner's program is to solve

$$\max_{q,n,c} u(c) - c + Rq + pRn - \kappa(q) - q - n$$

subject to

$$q + n \leq \overline{h} \tag{21}$$

with ε Lagrangian multiplier for the capacity constraint. The first order condition on c is $u'(c^*) = 1$. The first-order conditions on q and n yield, respectively,

$$R - \kappa'(q^*) - 1 = \varepsilon$$

$$pR - 1 = \varepsilon$$

Thus we obtain

$$\kappa'(q^*) = (1 - p) R$$

and with $\varepsilon > 0$, using (21), we obtain $n^* = \overline{h} - q^*$.

Proof of Proposition 2

Region 1. Under a safe deposit contract $Rq + (1 + i_r)r - (1 + i_d)\delta \geq 0$ and the banker problem reduces to

$$\max_{q,n,r,i_d,\delta} Rq + pRn + (1 + i_r)r - (1 + i_d)\delta - \kappa(q)$$

subject to $q, n, r, \delta \geq 0$ and

$$\frac{q+n}{1+i_m} + r \leq \delta \tag{22}$$

$$\delta \leq z \tag{23}$$

$$u\left[(1 + i_d)\delta + (1 + i_m)(z - \delta)\right] \geq u\left[(1 + i_m)z\right] \tag{24}$$

We denote as λ, ω and μ the Lagrangian multiplier for (22), (23) and (24), respectively. The first order conditions on q, n, r and i_d yield

$$(1+i_m)\left[R-\kappa'(q)\right] \leq \lambda \tag{25}$$

$$(1+i_m)pR \leq \lambda \tag{26}$$

$$1+i_r \leq \lambda \tag{27}$$

$$\mu = 1/u'(c_h) \tag{28}$$

From (25) and (26), the bank chooses $q>0$ and $n=r=0$ if

$$\kappa'(q) < R(1-p) \tag{29}$$

and

$$\kappa'(q) < R - \frac{1+i_r}{1+i_m} \tag{30}$$

Condition (29) holds if $q<q^*$. If the banker uses the deposited z to invest in q only and it is sufficiently small then $R-\kappa'(q)>pR$, since $\kappa'(0)=0$ and $\kappa'(q)$ is increasing in q. Therefore, (25) must hold as an equality and (26) as an inequality in that case. Hence $n=0$. From Assumption 2, we consider the case $(1+i_m)pR>1+i_r$. Then if (26) holds as an inequality (27) must also hold as an inequality, so that condition (30) is implied by condition (29). Therefore, if $n=0$, then $r=0$ as well.

Using (28) and $\lambda=(1+i_m)[R-\kappa'(q)]$, the first-order condition on δ gives

$$(1+i_m)\left[R-\kappa'(q)-1\right] \leq \omega \tag{31}$$

As demonstrated, $q>0$ and $n=0$ and thus $R-\kappa'(q)>pR>1$. Hence $\omega>0$ and thus (23) implies $\delta=z$. Therefore, $q=(1+i_m)z=c$. With $\mu=1/u'(c_h)>0$, (24) yields $i_d=i_m$. We verify that $Rq>(1+i_d)\delta=c=q$ in this region and thus the deposit contract is safe.

In Region 1, $c=(1+i_m)z=q<q^*$ and hence from (4) for $\gamma/(\beta(1+i_m))>u'(q^*)$. Let $i_m^1=\gamma/(\beta u'(q^*))-1$. Since c is increasing in i_m, it follows that Region 1 exists for

$i_m \leq i_m^1$, provided that $i_m^1 \geq 0$. Since q^* is increasing in R from (2), there is value of R that we denote as R^1, that solves $i_m^1 = \gamma/\left(\beta u'\left(q^*\left(R^1\right)\right)\right) - 1 = 0$, such that $i_m^1 \geq 0$ for $R \geq R^1$. The existence of $i_m^1 \leq \gamma/\beta - 1$ requires that $u'(q^*) \geq 1$ which always holds under the assumption that $c^* > Rq^*$.

Since $c = (1 + i_m)z$ is increasing in i_m and $q = c$, $dq/di_m > 0$ for $i_m \leq i_m^1$.

Region 2. Conjecture that the banker offers a safe deposit contract for $i_m^1 < i_m \leq i_m^2$ with i_m^2 defined by $Rq^*(R) = u'^{-1}\left[\gamma/\left(\beta\left(1 + i_m^2\right)\right)\right]$. Thus the bank's first-order conditions are as in (25)-(28) and (31). Equation (27) must hold as an inequality since $1 + i_r < (1 + i_m)pR$ from Assumption 2, so $r = 0$. For $i_m > i_m^1$, $q^* < u'^{-1}\left[\gamma/\left(\beta\left(1 + i_m\right)\right)\right]$ and thus if the bank chose $q > 0$ and $n = 0$ as in Region 1 condition (26) would be violated. Therefore $n > 0$ and both (25) and (26) hold with equality. This implies $R - \kappa'(q) = pR > 1$ and hence $\omega > 0$ in (31) implying $\delta = z$. Therefore $q + n = (1 + i_m)\, z = c$. With $\mu = 1/u'(c_h) > 0$, (24) yields $i_d = i_m$. From the definition of i_m^2 we verify that $Rq > (1 + i_d)\,\delta = c$ in this region and thus the deposit is safe. From the definitions of R^1 and R^2, we have $R^1 > R^2$ since $R > 1$ and q^* is increasing in R. Since i_m^1 solves $c\left(i_m^1\right) = q^*$ and thus satisfies $u'(q^*) = \gamma/\left(\beta\left(1 + i_m^1\right)\right)$, while i_m^2 solves $c\left(i_m^2\right) = Rq^*$ and satisfies $u'(Rq^*) = \gamma/\left(\beta\left(1 + i_m^2\right)\right)$, it follows that $i_m^1 < i_m^2$ considering that $R > 1$. The existence of $i_m^2 \leq \gamma/\beta - 1$ requires that $u'(Rq^*) \geq 1$ and always holds under the assumption that $c^* > Rq^*$.

Since $c = (1 + i_m)z$ is increasing in i_m and $c = n + q^*$, $dq/di_m = 0$ and $dn/di_m > 0$ for $i_m^1 < i_m \leq i_m^2$.

Region 3. Suppose that $i_m^2 < i_m \leq i_m^3$ where i_m^3 is defined below. For $i_m > i_m^2$ the equilibrium described for Region 2 no longer exists, since $c > Rq^*$ and hence the safe deposit contract would be violated.

Conjecture that in this region $q, n > 0$, $r = 0$ and $c_\ell = \ell + (1 + i_m)(z - \delta)$, with $c_\ell < c_h$, where $\ell = Rq$ is the liquidation value of the bank. The bank's first-order conditions (12)-

(14) become

$$(1+i_m)\left[R\left(p+(1-p)\frac{u'(c_\ell)}{u'(c_h)}\right)-\kappa'(q)\right]=\lambda$$
$$(1+i_m)\,pR=\lambda$$
$$(1+i_r)\left[p+(1-p)\frac{u'(c_\ell)}{u'(c_h)}\right]\leq\lambda$$

These conditions yield

$$R(1-p)\frac{u'(c_\ell)}{u'(c_h)}=\kappa'(q) \tag{32}$$

and

$$\kappa'(q)\leq pR\left(R\frac{1+i_m}{1+i_r}-1\right) \tag{33}$$

Since $c_h > c_\ell$, $u'(c_h) < u'(c_\ell)$ which from (2) and (32) implies that in this region $q > q^*$. In this conjectured equilibrium, we have $Rq^* < c_\ell = Rq < c_h$. Suppose instead that $c_\ell = c_h$. That would entail $c_\ell = c_h = c$ from the depositor's participation constraint and $\kappa'(q) = R(1-p)$ from (32), implying $q = q^*$ and hence $c_\ell = c_h = c \leq Rq^*$. However, for $i_m > i_m^2$, $c > Rq^*$, which leads to a contradiction.

The system of equations for c_h and q in this equilibrium is then

$$R(1-p)\frac{u'(Rq)}{\kappa'(q)} = u'(c_h) \tag{34}$$
$$u(c) = pu(c_h)+(1-p)\,u(Rq) \tag{35}$$

From (35), if i_m increases, q and/or c_h must increase, and (34) requires that q and c_h both increase or decrease. It follows that $dq/di_m > 0$ and $dc_h/di_m > 0$. Furthermore, from (34), since $dq/di_m > 0$ and hence $\kappa'(q)$ increases with i_m, it follows that $u'(c_\ell)/u'(c_h)$ increases with i_m. To verify that for $i_m^2 < i_m \leq i_m^3$ it is optimal to set $r = 0$, note that condition (33) holds with inequality at $i_m = i_m^2$, since $\kappa'(q^*) = R(1-p) < pR(R(1+i_m)/(1+i_r)-1)$ given Assumption 2. When i_m increases, the left side of (33) increases since $dq/di_m > 0$ in this region. The right side may stay constant if $i_m = i_r$ or increase if $i_m < i_r$ but in that case only up to an upper bound given Assumption 2. It follows that there is a threshold value of i_m, that we define as i_m^3, above which it is optimal for the bank to hold reserves on top of safe and risky projects, and thus (33) holds with equality. Consider first the case $i_m = i_r$ (we will consider the generic case $i_m \leq i_r$ below). Then i_m^3 is uniquely determined by

$$\begin{aligned} u\left(c\left(i_m^3\right)\right) &= pu\left[u'^{-1}\left[\frac{(1-p)\,u'\left[\kappa'^{-1}\left(pR\left(R-1\right)\right)R\right]}{p\left(R-1\right)}\right]\right] \\ &+ (1-p)\,u\left[\kappa'^{-1}\left(pR\left(R-1\right)\right)R\right] \end{aligned} \tag{36}$$

with $q\left(R\right) = \kappa'^{-1}\left[pR\left(R-1\right)\right]$ given (33). Denote as R^3 the value of R that solves (36) when $i_m^3 = 0$ and notice that the right side of (36) is increasing in R. It follows that $i_m^3 \geq 0$ holds if $R > R^3$.

To verify that $R^3 < R^2$, notice that $R^2 q^*\left(R^2\right) = c\left(R^3\right) = u'^{-1}\left(\gamma/\beta\right)$. Thus, we need to compare $R^2 q^*\left(R^2\right)$ and $c\left(R^3\right)$ to assess whether $R^3 < R^2$ holds. Consider some value of R that we denote R^*. If $c\left(R^*\right) > R^* q^*\left(R^*\right)$, that would imply that the function $c\left(R\right)$ requires a lower R than the function $Rq^*\left(R\right)$ to become equal to $u'^{-1}\left(\gamma/\beta\right)$, and hence $R^3 < R^2$. Consider $c\left(R\right)$ as the value of c defined in (36) for $i_m = 0$, which lies between the values of c_h and c_ℓ in (36). First, it is easy to verify that $c_\ell < c_h$ since $pR > 1$. Second, for any value R^*, the value of $c_\ell\left(R^*\right)$ in (36) satisfies $c_\ell\left(R^*\right) > R^* q^*\left(R^*\right)$. Since from (36) $c_\ell\left(R^*\right) < c\left(R^*\right) < c_h\left(R^*\right)$, and $c_\ell\left(R^*\right) > R^* q^*\left(R^*\right)$, it follows that $c\left(R^*\right) > R^* q^*\left(R^*\right)$. We can then conclude that $R^3 < R^2$.

For the generic case $i_m \leq i_r$, i_m^3 is defined as the value of i_m that solves

$$u\left(c\left(i_m^3\right)\right) = pu\left(u'^{-1}\left[R\left(1-p\right)\frac{u'(q\left(i_m^3\right)R)}{\kappa'(q\left(i_m^3\right))}\right]\right) + (1-p)\,u\left(q\left(i_m^3\right)R\right) \tag{37}$$

where the expression for c_h comes from (34) and $q\left(i_m^3\right)$ is determined by

$$\kappa'(q\left(i_m^3\right)) = pR\left(R\frac{1+i_m^3}{1+i_r} - 1\right) \tag{38}$$

Denoting now R^3 as the value of R that solves (37) when $i_m^3 = 0$, given that the right side of (37) is increasing in R, it follows that $i_m^3 \geq 0$ holds if $R > R^3$. The same reasoning used before can be applied to the generic case $i_m \leq i_r$ to get to the same conclusion that $R^3 < R^2$.

Until now we have conjectured that the equilibrium described for Region 3 exists for $i_m^2 < i_m \leq i_m^3$ where i_m^3 denotes the value of i_m above which $r > 0$, with $c_\ell < c_h$. However, we must consider an alternative possible upper threshold value of i_m for Region 3, that we define as $\tilde{\imath}_m^3$, above which $c_h \geq c^*$ while the bank does not hold reserves. To obtain $\tilde{\imath}_m^3$, consider the equilibrium equations that prevail for $c_h \geq c^*$ that are

$$\begin{aligned} R\left(1-p\right)\frac{u'(Rq)}{\kappa'(q)} &= 1 \\ u\left(c\right) &= p\left[u\left(c^*\right)+c_h-c^*\right]+\left(1-p\right)u\left(Rq\right) \end{aligned} \tag{39}$$

Therefore $\tilde{\imath}_m^3$ is defined by

$$u\left(c\left(\tilde{\imath}_m^3\right)\right) = pu\left(c^*\right)+\left(1-p\right)u\left(Rq\right) \tag{40}$$

where q solves (39). Given that $dq/di_m > 0$ in Region 3, the threshold $\tilde{\imath}_m^3$ would be smaller than the threshold i_m^3 if the value of q that solves (39) is smaller than the value of q that solves (38). We can explicitly solve for $q\left(\tilde{\imath}_m^3\right)$ by using (40)

$$q\left(\tilde{\imath}_m^3\right) = \frac{1}{R}u^{-1}\left[\frac{u\left(c\left(\tilde{\imath}_m^3\right)\right)-pu\left(c^*\right)}{1-p}\right] \tag{41}$$

and therefore we can rewrite (39) as follows

$$\kappa'\left(\frac{1}{R}u^{-1}\left[\frac{u\left(c\left(\tilde{\imath}_m^3\right)\right)-pu\left(c^*\right)}{1-p}\right]\right) = R\left(1-p\right)u'\left(u^{-1}\left[\frac{u\left(c\left(\tilde{\imath}_m^3\right)\right)-pu\left(c^*\right)}{1-p}\right]\right) \tag{42}$$

Since the left side is increasing in $\tilde{\imath}_m^3$ and the right side is decreasing in $\tilde{\imath}_m^3$, this equation yields a unique solution for $\tilde{\imath}_m^3$. Note that the left side in (42) is decreasing in R, while the right side is increasing in R. Conversely, the left side in (42) is increasing in $\tilde{\imath}_m^3$, while the right side is decreasing in $\tilde{\imath}_m^3$. Therefore (42) defines a positive relationship between R and $\tilde{\imath}_m^3$: if R is sufficiently high, then $\tilde{\imath}_m^3 \geq 0$.

Recall that we have assumed that $p \leq \bar{p}$, where $\bar{p}$ solves $\left(1-\bar{p}\right)u'\left[\kappa'^{-1}\left(\bar{p}R\left(R\varepsilon-1\right)\right)R\right] = \bar{p}\left(R\varepsilon-1\right)$ with $\varepsilon = \frac{1+i_m}{1+i_r}$ ($\bar{p}$ solves $\left(1-\bar{p}\right)u'\left(R\kappa'^{-1}\left(\bar{p}R\left(R-1\right)\right)\right) = \bar{p}\left(R-1\right)$ if $i_m = i_r$).

We now show that if $p \leq \bar{p}$ then $i_m^3 \leq \tilde{\imath}_m^3$ (so that if $p < \bar{p}$ then $i_m^3 < \tilde{\imath}_m^3$). If $i_m^3 \leq \tilde{\imath}_m^3$ this means that the value of q that solves (38) is smaller than the value of q that solves (39). This in turn implies that $\kappa'\left(q\left(i_m^3\right)\right) \leq R(1-p)\,u'\left(q\left(i_m^3\right)\right)$, which using (38) may be rewritten as

$$p\left(R\frac{1+i_m^3}{1+i_r}-1\right) \leq (1-p)\,u'\left[R\kappa'^{-1}\left(pR\left(R\frac{1+i_m^3}{1+i_r}-1\right)\right)\right] \tag{43}$$

Notice that the left side in (43) is increasing in p, while the right side is decreasing in p. Hence for a given i_m there is a unique $p = \bar{p}$ such that (43) holds with equality. We then confirm that the inequality in (43) holds if $p \leq \bar{p}$. This ensures that $c_h < c^*$ for $i_m \leq i_m^3$. Since a smaller p implies a greater q in (39) and a smaller q in (38), it follows that for $p \leq \bar{p}$ the value of q that solves (33) with equality is smaller than the value of q that solves (39) and hence $\tilde{\imath}_m^3 > i_m^3$. Since c_h in (37) is smaller than c^*, it follows from (37) and (40) that $c\left(\tilde{\imath}_m^3\right) > c\left(i_m^3\right)$ with $\tilde{\imath}_m^3 > i_m^3$.

Region 4 In Region 4 q solves

$$\kappa'(q) = pR\left(R\frac{1+i_m}{1+i_r}-1\right) \tag{44}$$

The system of equations that solve for c_h and r for $i_m > i_m^3$ is

$$p + (1-p)\frac{u'\left(Rq + r\left(1+i_r\right)\right)}{v'(c_h)} = \frac{1+i_m}{1+i_r}pR \tag{45}$$

$$pv\left(c_h\right) + (1-p)\,u\left(Rq + r\left(1+i_r\right)\right) = u\left(c\right) \tag{46}$$

Therefore if i_m increases while keeping $\frac{1+i_m}{1+i_r}$ fixed then q and the ratio of marginal utilities across the good and the bad states are unaffected while both c_h and c_ℓ increase (which implies that $r\left(1+i_r\right)$ increases and hence that $r > 0$).

Since $dc_h/di_m > 0$ in this region and $c_h > c^*$ for $1 + i_m = \gamma/\beta$ there must exist i_m^4 with $i_m^3 \leq i_m^4 < \gamma/\beta$ such that for $i_m = i_m^4$ we have $c_h = c^*$ and for $i_m > i_m^4$ we have $c_h > c^*$. From (45), it follows that $dc_\ell/di_m = 0$ for $i_m > i_m^4$.

After studying the conditions on R for the existence of the different regions, we can conclude that there are are values of $i_m \geq 0$ for which Region 1 exists provided that

$R \geq R^1$, for which Region 2 exists provided that $R \geq R^2$ and for which Region 3 exists provided that $R \geq R^3$, with $R^3 < R^2 < R^1$. Regarding Region 4, notice that it must be that $r > 0$ independently of the value of R. Otherwise c_ℓ would equal Rq, with q given by (44), and (46) would imply a higher value of c_h than the one that obtains with $r > 0$. This would entail a higher value of $u'(c_\ell)$ and a lower value of $v'(c_h)$ than the ones that obtain with $r > 0$ and the first-order condition (45) would not hold.

Proof of Lemma 3

As shown in the proof of Proposition 2, $dq/di_m > 0$ and $dn/di_m = 0$ for $0 \leq i_m \leq i_m^1$, while $dq/di_m = 0$ and $dn/di_m > 0$ for $i_m^1 < i_m \leq i_m^2$. Hence $d(q+n)/di_m > 0$ for $0 \leq i_m \leq i_m^2$. Since $i_d = i_m$ for $i_m^1 < i_m \leq i_m^2$, it follows that $di_d/di_m = 1 > 0$.

Proof of Lemma 4

From the proof of Proposition 2, $dq/di_m > 0$ and $dc_h/di_m > 0$ for $i_m^2 < i_m \leq i_m^3$, while $dq/di_m = 0$ and $dc_h/di_m > 0$ for $i_m^3 < i_m \leq \gamma/\beta - 1$ and $dc_\ell/di_m = 0$ for $i_m > i_m^4$. Therefore i_d is increasing in i_m for $i_m^2 < i_m \leq \gamma/\beta - 1$. The bank's liquidation value is $c_\ell = Rq$, thus increasing in i_m for $i_m^2 < i_m \leq i_m^3$, and it is $c_\ell = Rq + r(1+i_r)$, increasing in i_m for $i_m^3 < i_m \leq i_m^4$ and constant for $i_m > i_m^4$. Since c_ℓ and q are constant for $i_m > i_m^4$, then $r(1+i_r)$ here equal to $r(1+i_m)$ is constant for $i_m > i_m^4$. Taking into account that the bank's resource constraint (6) gives $c = q + n + r(1+i_m)$ for $i_m > i_m^4$, we deduce that $dn/di_m = dc/di_m > 0$.

Proof of Lemma 5

An equilibrium with $r > 0$ exists for $i_m > i_m^3$. In this case q is fixed for a given $(1+i_m)/(1+i_r)$ and solves (44). It is immediate from (44) and (45) that an increase in the spread $i_r - i_m$ entails a reduction in q and in the ratio $u'(c_\ell)/v'(c_h)$. Since the right side in (45) decreases when the spread increases, the left side must decrease as well. This rules out a positive effect on c_h accompanied by a negative (or nil) effect on $r(1+i_r)$. It follows that $r(1+i_r)$ must increase for (45)-(46) to hold.

Proof of Proposition 6 i) To compute welfare, we only consider buyers' surplus and banks' profits since suppliers and sellers obtain zero per-period utility in equilibrium. We show the effect of an increase in i_m for each region.

Region 1

In Region 1 when $i_m < i_m^1$ and bankers only invest in risk-free projects, overall welfare is

$$\mathcal{W}_1 = u(c) - \gamma z + T + Rq - \kappa(q) - c$$

that consists of the sum of buyers' utility from consuming in the GM, $u(c)$, net of the disutility of working in the SM, $-\gamma z + T$, and banks' profits that consist of the return on risk-free projects Rq, net of the monitoring cost $\kappa(q)$ and the payment to depositors c. In this region $c = (1 + i_m) z = q$ and $T = (\gamma - 1) z - i_m z$ from (15). Thus welfare simplifies to

$$\mathcal{W}_1 = u(c) - c + Rq - \kappa(q) - q \tag{47}$$

Using that $c(i_m) = q(i_m) = (1 + i_m) z$ and taking the total derivative yields

$$\frac{d\mathcal{W}_1}{di_m} = \left[u'(c) - 1\right] \frac{dc}{di_m} + \left[R - 1 - \kappa'(q)\right] \frac{dq}{di_m}$$

With $u'(c) - 1 > 0$ and $dc/di_m > 0$ for all $i_m \leq \gamma/\beta - 1$, the first term on the right hand side is positive. With $R - \kappa'(q) > pR > 1$ in this region and $dq/di_m > 0$ (see proof of Proposition 2), the second term on the right side is also positive. Therefore, $d\mathcal{W}_1/di_m > 0$ in Region 1.

Region 2

Considering Region 2, where the banker invests in both risk-free and risky projects and $i_m^1 \leq i_m \leq i_m^2$, buyer's surplus is as in Region 1 and the payment to depositors is $c = (1 + i_m) z = q + n$. Hence overall welfare is

$$\mathcal{W}_2 = u(c) - c + Rq + pRn - \kappa(q) - q - n \tag{48}$$

Taking the total derivative yields

$$\frac{d\mathcal{W}_2}{di_m} = \left[u'(c) - 1\right]\frac{dc}{di_m} + \left[R - 1 - \kappa'(q)\right]\frac{dq}{di_m} + (pR - 1)\frac{dn}{di_m} \tag{49}$$

Again with $u'(c) - 1 > 0$ and $dc/di_m > 0$ for all $i_m \leq \gamma/\beta - 1$, the first term on the right side is positive. Since $dq/di_m = 0$ and $dn/di_m > 0$ (see proof of Proposition 2) and $pR > 1$, bank profits are increasing in i_m, with the second term on the right side equal to zero and the third term being positive. Therefore, $d\mathcal{W}_2/di_m > 0$ in Region 2.

Region 3

Consider Region 3, for $i_m^2 \leq i_m \leq i_m^3$, with $n, q > 0$ and $r = 0$. Buyers' surplus is as in Regions 1 and 2 and thus increasing in i_m. Before computing the overall effect of i_m on welfare, we analyze the effect of i_m on banks' profits. In this region profits are

$$Rq + pRn - \kappa(q) - pc_h - (1-p)\,c_\ell$$

which can be rewritten as

$$pRc - \kappa(q) - pc_h$$

since $c_\ell = Rq$ and $c = q + n$ for $i_m^2 < i_m \leq i_m^3$.

The change in profits owing to an increase in i_m is

$$pR\frac{dc}{di_m} - \kappa'(q)\frac{dq}{di_m} - p\frac{dc_h}{di_m} \tag{50}$$

where dq/di_m and dc_h/di_m may be computed from (16) and (17) as follows

$$\frac{dc_h}{di_m} = \frac{R^2(1-p)\,u''(qR) - u'(c_h)\,\kappa''(q)}{u''(c_h)\,\kappa'(q)}\frac{dq}{di_m}$$

and

$$\left[pu'(c_h)\,\frac{R^2(1-p)\,u''(qR) - u'(c_h)\,\kappa''(q)}{u''(c_h)\,\kappa'(q)} + (1-p)\,u'(qR)\,R\right]\frac{1}{u'(c)}\frac{dq}{di_m} = \frac{dc}{di_m} \tag{51}$$

and thus are both positive.

Using (16), (50) can be rewritten as

$$\left[R(1-p)\,p\frac{Ru''(qR)-u'(qR)\,\kappa''(q)\,/\kappa'(q)}{u''(c_h)\,\kappa'(q)}+\kappa'(q)\right]\left[\frac{pRu'(c_h)}{u'(c)}-1\right]\frac{dq}{di_m} \tag{52}$$

Since the first term in brackets in (52) is positive, for profits to be increasing in i_m it must be that $pRu'(c_h) > u'(c)$. Since $u'(c_h) > 1$ in this region, a sufficient condition for profits to increase with i_m is $pR \geq u'(c)$. For $i_m^2 < i_m < i_m^3$, we have $u'(c) < u'(Rq^*)$ and hence if $pR \geq u'(Rq^*)$ then the condition is verified for $i_m = i_m^2$ and the condition $pR \geq u'(c)$ also holds for $i_m^2 < i_m < i_m^3$.

Using (16) and (51), from (52) the effect of i_m on overall welfare is

$$\left[\frac{(pR-1)\,u'(c_h)}{u'(c)}+u'(c_h)-1\right]\left[R(1-p)\,p\frac{Ru''(qR)-u'(qR)\,\kappa''(q)\,/\kappa'(q)}{u''(c_h)\,\kappa'(q)}+\kappa'(q)\right]\frac{dq}{di_m}$$

Since $pR > 1$ and $u'(c_h) > 1$, an increase in i_m improves welfare in Region 3.

Region 4

Subregion 4.1 Consider the subregion $i_m^3 < i_m < i_m^4$ and let $i_m = i_r$. Profits are

$$Rq + pRn + r(1+i_r) - \kappa(q) - pc_h - (1-p)\,c_\ell$$

which can be rewritten as

$$pRc - p(R-1)(c_\ell - Rq) - \kappa(q) - pc_h$$

since $c = q + n + r(1+i_m)$ and $c_\ell = Rq + r(1+i_r)$ in this region, and we have assumed that $i_m = i_r$.

Differentiating with respect to i_m, and taking into account from (20) that q is fixed for $i_m = i_r$, yields

$$pR\frac{dc}{di_m} - p(R-1)\frac{dc_\ell}{di_m} - p\frac{dc_h}{di_m} \tag{53}$$

From (18) and (19), we obtain

$$\frac{dc_h}{di_m} = \frac{R(1-p)\,u''(c_\ell)}{\kappa'(q)\,u''(c_h)}\frac{dc_\ell}{di_m}$$

and

$$\frac{dc_\ell}{di_m} = \frac{u'(c)}{pu'(c_h)\frac{u'(c_h)}{u'(c_\ell)}\frac{u''(c_\ell)}{u''(c_h)} + (1-p)\,u'(c_\ell)}\frac{dc}{di_m}$$

Therefore (53) becomes

$$\frac{\left[Rpu'(c_h) - u'(c)\right]\frac{u'(c_h)}{u'(c_\ell)}\frac{u''(c_\ell)}{u''(c_h)} + R(1-p)\,u'(c_\ell) - u'(c)(R-1)}{pu'(c_h)\frac{u'(c_h)}{u'(c_\ell)}\frac{u''(c_\ell)}{u''(c_h)} + (1-p)\,u'(c_\ell)} p\frac{dc}{di_m}$$

Combining (18) and (20) for $i_m = i_r$, this expression simplifies to

$$\left[Rp - \frac{u'(c)}{u'(c_h)}\right]\frac{dc}{di_m}$$

Similarly to Region 3, profits go up with i_m if $Rpu'(c_h) > u'(c)$ and go down if $Rpu'(c_h) < u'(c)$. Therefore, the effect of an increase in i_m on overall welfare is determined by

$$\left[Rp - \frac{u'(c)}{u'(c_h)} + u'(c) - 1\right]\frac{dc}{di_m}$$

Hence the effect of i_m on welfare is negative if

$$pR - 1 + u'(c)\left(1 - \frac{1}{u'(c_h)}\right) < 0$$

This inequality may be rewritten as

$$u'(c_h) < \frac{1}{1 + \frac{(pR-1)\beta(1+i_m)}{\gamma}} \tag{54}$$

Since in this region $u'(c_h) > 1$, a necessary condition is $pR - 1 < \frac{\gamma}{\beta(1+i_m)} = u'(c)$. Let $u(x) = \frac{x^{(1-\sigma)}}{1-\sigma}$ and thus $u'(x) = x^{-\sigma}$. Then the buyer's participation constraint (19) becomes

$$p\frac{c_h^{1-\sigma}}{1-\sigma} + (1-p)\frac{c_\ell^{1-\sigma}}{1-\sigma} = \frac{c^{1-\sigma}}{1-\sigma}$$

and gives c_ℓ as a function of c_h

$$c_\ell = \left(\frac{c^{1-\sigma} - pc_h^{1-\sigma}}{1-p}\right)^{\frac{1}{1-\sigma}} \tag{55}$$

Combining (18) and (20), we obtain

$$p + (1-p)\frac{u'(c_\ell)}{u'(c_h)} = pR \tag{56}$$

With the assumed utility function, (56) becomes

$$\frac{c_h}{c_\ell} = \left(\frac{p(R-1)}{1-p}\right)^{\frac{1}{\sigma}}$$

Inserting (55) gives

$$c_h = \left[\frac{\left(\frac{p(R-1)}{1-p}\right)^{\frac{1-\sigma}{\sigma}} c^{1-\sigma}}{1-p+p\left(\frac{p(R-1)}{1-p}\right)^{\frac{1-\sigma}{\sigma}}}\right]^{\frac{1}{1-\sigma}}$$

Therefore (54) becomes

$$c_h^{-\sigma} = \left[\frac{\left(\frac{p(R-1)}{1-p}\right)^{\frac{1-\sigma}{\sigma}} c^{1-\sigma}}{1-p+p\left(\frac{p(R-1)}{1-p}\right)^{\frac{1-\sigma}{\sigma}}}\right]^{\frac{-\sigma}{1-\sigma}} < \frac{1}{1+\frac{(pR-1)\beta(1+i_m)}{\gamma}}$$

Rearranging, this condition can be rewritten as

$$\frac{p(R-1)}{1-p} > \left[\frac{\gamma}{\beta(1+i_m)} + pR - 1\right]\left[1-p+p\left(\frac{p(R-1)}{1-p}\right)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \tag{57}$$

Notice that $p(R-1)/(1-p) > 1$ and $\gamma/(\beta(1+i_m)) + pR - 1 > 1$. The value of the term $1-p+p\left(\frac{p(R-1)}{1-p}\right)^{\frac{1-\sigma}{\sigma}}$ depends on the value of σ: If $\sigma < 1$ it is higher than 1 and if $\sigma > 1$ it is lower than 1. Suppose $\sigma > 1$. Then (57) becomes

$$\left[\frac{p(R-1)}{1-p}\right]^{\frac{\sigma-1}{\sigma}} > \left[\frac{\gamma}{\beta(1+i_m)} + pR - 1\right]^{\frac{\sigma-1}{\sigma}}\left[1-p+p\left(\frac{p(R-1)}{1-p}\right)^{\frac{1-\sigma}{\sigma}}\right]^{-1}$$

and can be rearranged as

$$\left(\frac{p(R-1)}{1-p}\right)^{\frac{1-\sigma}{\sigma}}\left[\left(\frac{1}{\frac{\gamma}{\beta(1+i_m)} + pR - 1}\right)^{\frac{1-\sigma}{\sigma}} - p\right] < 1-p$$

Therefore, a sufficient condition for an increase in i_m to be welfare worsening is

$$\left(\frac{\frac{p(R-1)}{1-p}}{\frac{\gamma}{\beta(1+i_m)}+pR-1}\right)^{\frac{1-\sigma}{\sigma}} < 1-p$$

Assuming that the term in brackets is higher than 1, the condition requires that σ is sufficiently high.

Suppose now that $\sigma < 1$. The inequality (57) becomes

$$\left(\frac{p\left(R-1\right)}{1-p}\right)^{\frac{1-\sigma}{\sigma}}\left[\frac{1}{\left[\frac{\gamma}{\beta(1+i_m)}+pR-1\right]^{\frac{1-\sigma}{\sigma}}}-p\right] > 1-p$$

Notice that if σ is sufficiently small the term $\left[\frac{\gamma}{\beta(1+i_m)}+pR-1\right]^{\frac{1-\sigma}{\sigma}}$ is that big that the expression between brackets on the left is negative. This implies that the above inequality cannot hold and thus the increase in i_m is welfare improving in that case.

Subregion 4.2

Finally, for $i_m^4 < i_m \leq \frac{\gamma}{\beta} - 1$, $q, n, r > 0$, $c_h \geq c^*$, the expression for profits is

$$Rq + pRn + r\left(1+i_r\right) - \kappa\left(q\right) - pc_h - \left(1-p\right)c_\ell$$

In the case $i_m = i_r$, it be can be rewritten as

$$p\left[\left(R-1\right)\left(q+n\right)+c-c_h\right]-\kappa\left(q\right)$$

Totally differentiating with respect to i_m yields

$$p\left[\left(R-1\right)\left(\frac{dq}{di_m}+\frac{dn}{di_m}\right)+\frac{dc}{di_m}-\frac{dc_h}{di_m}\right]-\kappa'\left(q\right)\frac{dq}{di_m}$$

From (44), q is fixed and independent from i_m if and $i_m = i_r$. Hence the above expression simplifies to

$$p\left[\left(R-1\right)\frac{dn}{di_m}+\frac{dc}{di_m}-\frac{dc_h}{di_m}\right] \tag{58}$$

With $i_m = i_r$, the equations that solve for r, n and c_h are

$$p\left[u\left(c\right)+c_h-c\right]+\left(1-p\right)u\left(Rq+r\left(1+i_m\right)\right) = u\left(c\right) \tag{59}$$

$$q+n+r\left(1+i_m\right) = c \tag{60}$$

$$p+\left(1-p\right)u'\left(Rq+r\left(1+i_m\right)\right) = pR \tag{61}$$

From (61), if i_m increases r must decrease, since c_ℓ and q are unaffected by i_m. Since $r\left(1+i_m\right)$ stays constant, we deduce from (60) that $dc/di_m = dn/di_m > 0$. In addition from (59) we obtain

$$\frac{dc_h}{di_m} = \left[\frac{\left(1-p\right)u'\left(c\right)}{p}+1\right]\frac{dc}{di_m}$$

Therefore (58) becomes

$$\left[p\left(R-1\right)-\left(1-p\right)u'\left(c\right)\right]\frac{dc}{di_m}$$

Hence profits increase with i_m if $p\left(R-1\right)-\left(1-p\right)u'\left(c\right) > 0$ which implies $pR > p+\left(1-p\right)u'\left(c\right)$. Since the left side of this inequality is increasing in p and the right side is decreasing in p, it follows that profits are increasing in i_m if p is sufficiently high and are decreasing in i_m otherwise.

The effect of an increase in i_m on overall welfare is given by

$$\left[u'\left(c\right)-1\right]\frac{dc}{di_m}+\left[p\left(R-1\right)-\left(1-p\right)u'\left(c\right)\right]\frac{dc}{di_m}$$

that can be simplified to

$$\left[pR-1+p\left(u'\left(c\right)-1\right)\right]\frac{dc}{di_m} > 0$$

Therefore an increase in i_m is unambiguously welfare improving in subregion 4.2 where $c_h > c^*$.

Proof of Proposition 6 ii) To assess whether the Friedman rule allows achieving the first-best allocation, recall the assumption that the central bank has taxation power. Then it is possible to set $i_m = i_r = \gamma/\beta - 1$. In that case, transfers become

$$\phi T = z(\gamma - 1) - (\gamma/\beta - 1) z = z\gamma (1 - 1/\beta) < 0$$

From Lemma 1 the optimal solution satisfies $u'(c^*) = 1$, $\kappa'(q^*) = R(1-p)$ and $n^* = \bar{h} - q^*$, with $\bar{h}$ arbitrarily high.

The Friedman rule implies that the buyers' money holdings satisfy $z(1+i_m) \geq c^*$ where c^* satisfies $u'(c^*) = 1$. If the buyer does not contract (deposit) with the banker, then the buyer consumes c^* in the GM. The banker has no CBDC to lend to entrepreneurs, and hence no investment is made: $q = n = 0$.

If the buyer does contract (deposits) with the banker, and as assumed $c^* > Rq^*$, when the bad state occurs, the (optimal) investment in the safe asset is not enough to cover for the optimal consumption of the buyer: Sellers are willing to work an amount c_ℓ where $c_\ell = Rq + r(1+i_m)$. If $r = 0$ and $q = q^*$, then $c_\ell = Rq^* < c^*$. The solution is also given by $c_h > c^*$, $q = q^*$ and $n = n^*$. This is not optimal because $c_\ell < c^*$. Alternatively, $c_\ell = c^*$, but this requires that $q > q^*$ and/or $r > r^* = 0$. If $q > q^*$, the solution is given by $c_\ell = c_h = c^*$, $c_\ell = Rq + r(1+i_m)$, and hence $n \leq \bar{h} - q < \bar{h} - q^*$ which implies $n < n^*$. If $q = q^*$ and $r > 0$, the resource constraint of the banker gives $z(1+i_m) = q + n + r(1+i_m)$. If $q = q^*$ and r satisfies $c^* = Rq^* + r(1+i_m)$, from the resource constraint we obtain $n = z(1+i_m) - q^* - r(1+i_m)$ that can be rewritten as $n = z(1+i_m) - q^* - c^* + Rq^*$. Let $\tilde{z}$ denote the amount of money holdings that the buyer brings into the SM, with $\tilde{z} = z(1+i_m) - c^*$ (notice that the buyer is indifferent between bringing money holdings $c^*/(1+i_m)$ and any higher amount). The resource constraint becomes $n = \tilde{z} - q^* + Rq^*$. Therefore if $\tilde{z} \geq \bar{h} - Rq^*$, then the first-best can be attained, and if $\tilde{z} < \bar{h} - Rq^*$ the first-best cannot be attained since $n < n^* = \bar{h} - q^*$.