Competition and welfare effects of bailout policies

June 2019

Noé Ciet∗, Marianne Verdier†
Abstract

In this paper, we analyze the welfare effects of bailout policies when banks compete with switching costs. We compare no-bailout policies to systematic bailouts. We argue that no-bailout policies increase the interest rates paid by borrowers ex ante (i.e., before a shock), whereas they may reduce the interest rates paid by consumers who are not credit constrained ex post. Such policies increase social welfare ex post if borrowers can easily switch banks and if the credit constraints are not too severe.

Keywords: Bailout, Bank Failure, Switching Costs, Resolution Policies.
JEL Codes: L1, L5, G2.

1 Introduction

During the 2008 crisis, several governments resorted to bailout policies either through recapitalizations, asset relief programs or public guarantees on bank debt. Such interventions raised concerns on their impact on banks’ incentives to take risks. Since then, the regulatory agenda has focused on designing new policy instruments to avoid bank bailouts.¹ The main objective of regulators is to protect financial stability, mostly through the prevention of systemic contagion and bank runs. Yet, bailout (or no-bailout) policies also impact banks’ pricing strategies in the credit market and therefore, borrowers’ access to credit.

Credit markets are characterized by frictions due to imperfect competition, information asymmetries, transaction and switching costs. In this paper, we compare the welfare effects of no-bailout policies and systematic bailouts when banks compete with switching costs. If the credit constraints are not too severe, we show that no-bailout policies may lower interest rates and benefit consumers who are not credit constrained if switching costs are high enough. We also identify the conditions on the level of switching costs and the severity of funding constraints faced by banks such that a no-bailout policy increases social welfare.

Understanding the interplay between resolution policies, competition and welfare in credit markets is necessary, as rescue plans must often be jointly cleared by governments, legislators and competition authorities.² The importance of borrower mobility in designing banking regulation echoes a report by the UK Independent Commission on Banking (ICB). Chaired by Lord John Vickers, the commission concluded in 2011 that the presence of switching costs in banking markets

¹The introduction of new safety-net tools, from higher capital requirements to more adequate resolution policies, is expected to sign a "farewell to bailout" (Benczur and al., 2017). For example, the Dodd-Frank Act in the United-States has introduced strict limits on the government ability to conduct bailouts (see the title XI of the Dodd-Frank Act). Bailouts may only be conducted during a systemic crisis and cannot be targeted to particular institutions. In Europe, the Banking Recovery and Resolution Directive limits public recapitalization to solvent institutions. Moreover, it requires sufficient burden-sharing, restructuring plans and minimum competitive distortions.

²Such issues are also often addressed by financial supervisors and competition authorities independently from each other. Several researchers ask whether it is optimal to conduct financial stability policies separately from competition policies (See Beck, 2008, Vickers, 2010). Following the financial crisis in the UK, in December 2013, the Financial Services (Banking Reform) Act gave the Prudential Regulatory Authority a secondary competition objective. Even if competition policy issues may sometimes appear as secondary, banks’ market power before and after a shock ultimately impacts the welfare effects of bailout policies and borrowers’ access to credit.
is central to financial regulation. Unlike other initiatives in OECD countries, the Vickers report articulates the competition policy in banking markets with the regulatory agenda. Moreover, it reviews a large set of regulatory instruments to decrease switching costs such as the regulation of account closure, account transfer fees, data portability and prohibition of early repayments. In our paper, we analyze whether a reduction of switching costs may create positive welfare effects if the state decides to renounce to systematic bailouts.

We model two banks which compete à la Hotelling for two periods to offer credit to consumers. Consumers are homogenous in terms of risks but incur switching costs at the second period if they do not remain with their initial bank. Banks are able to price discriminate at the second period between their insider and outsider consumers. Between the two periods, two different types of shocks (i.e., systemic and bank-specific) may occur and change the competitive conditions on the credit market. If a systemic shock occurs, all banks face a reduction of their funding sources. If they remain active at the second period, they are constrained on the amount of credit they can supply to their consumers. If a bank simultaneously faces a specific shock, it fails, unless the state intervenes to support its activity.

We compare two different games. In the first game, the state is able to commit not to bailout banks. In the second game, the state intervenes systematically to bailout banks between the two periods when it is necessary. Banks make different expectations on the continuation value of their activity according to the state’s bailout (or no-bailout) policy. This impacts their pricing strategies at the first period of competition.

Our framework enables us to analyze the impact of bailout and no-bailout policies on the interest rates paid by insider and outsider borrowers at the first and the second period according to the severity of the funding constraints and the level of switching costs. We show that no-bailout policies increase the interest rates paid by borrowers at the first period compared to systematic bailouts. This result is caused by banks’ ability to price discriminate at the second period between their insider and outsider consumers who incur switching costs. Under a systematic bailout policy, banks expect to compete without funding constraints at the second period. Because borrowers face switching costs, they have incentives to lower their interest rate at the first period compared to a static game of competition to extract rents from their insiders. Those rents increase with the profitability of the credit market and the level of switching costs. At the second period, the interest rate charged to insider (resp., outsider) consumers increases (resp., decreases) with the level of switching costs. The choice of the first period interest rate reflects banks’ trade-off between extracting rents from their insider and outsider consumers at the second period.

Under no-bailout policies, banks expect to operate either under monopoly or duopoly at the second period, either with or without funding constraints. The uncertainty of the market structure impacts banks’ trade-off between first period and second period profits. As banks expect to lose their relationships with their insider consumers when they attract them at the first period, they have fewer incentives to decrease their first period interest rates. Consequently, the interest rate at the first period is lower under a systematic bailout policy than under a no-bailout policy. At the second period, interest rates charged to insider and outsider consumers are no longer strategic complements.

---

3On p.17, the report states that "One of the reasons for long-standing problems of competition and consumer choice in banking and financial services more generally has been that competition has not been central to financial regulation".

4Following Vickers report, the program "Banking for the 21st Century" implements both information disclosure between banks and an online service to facilitate and guarantee switching. It also enables consumers to access their aggregate information to take advantage of comparison websites. However, it was simply designed to facilitate entry of new participants following a period of huge bank mergers. In our paper, we support the view that these instruments can also be used to make a no-bailout policy welfare-enhancing. For a survey of policies in OECD countries dealing with switching costs in banking, see OECD (2009).
when a bank becomes a monopoly or when it faces funding constraints. This is because consumers’ outside option is to renounce to credit if they do not switch. If switching costs and banks’ funding sources are high enough with respect to the profitability of the credit market, a bank-specific shock may lower interest rates for insider consumers, even if one of the two banks fails. Therefore, in our model, no-bailout policies may generate a fall in the interest rates paid by borrowers who have already built a banking relationship. This effect arises for a given level of risk of the bank’s investment.

Finally, we make two main claims regarding the effects of a systematic bailout on consumer and social welfare. First, we acknowledge that a systematic bailout provides some benefits to consumers compared to a no-bailout policy, through a better market coverage and a larger set of switching options. The effect of a systematic bailout may be negative only for consumers who do not wish to switch banks if possible. Second, we argue that a reduction of switching costs and credit profitability may increase the relative welfare-efficiency of no-bailout policies compared to bailouts, if the funding constraints faced by banks are soft enough. If this condition holds, policies aiming at reducing switching costs may complement the decision to renounce to systematic bailouts.

The reminder of the paper is as follows. In Section 2, we present the literature that is related to our study. In Section 3, we present our model. In Section 4, we determine the interest rates chosen by banks when the market structure is uncertain at the second period. In Section 5, we compare the equilibrium with a systematic bailout policy and when there is no-bailout. Finally, we conclude.

2 Related literature

Our article is related to two different strands of research on the banking industry, namely a literature analyzing the impact of a bailout on competition and a literature studying the role of switching costs.

An abundant literature stemming from Keeley (1990) indirectly analyzes the competitive effects of deposit and liquidity insurances through moral hazard and implicit subsidies. This literature underlines that risky activities are sensitive to a trade-off between higher revenues and the preservation of banks’ charter value (Perotti and Suarez, 2002). Bailout policies have ambiguous effects on social welfare. On the one hand, they increase banks’ risk-taking because of limited liability in case of failure. On the other hand, they may increase banks’ monitoring efforts because they reduce the externalities associated with contagion (Dell’Ariccia and Ratnovski, 2013).

Another strand of the literature analyzes the effects of a bailout policy on credit prices. First, bailout policies may benefit aided banks and increase their market power. Ex ante, too-big-to-fail banks benefit from implicit subsidies and moral hazard, and therefore compete on an unequal level-playing field with smaller banks. Second, bailout policies may exert positive externalities on overall banking competition. One can expect public support to be beneficial to borrowers ex ante if implicitly subsidized banks pass through lower funding costs into lower credit interest rates. Also, as argued by a failing-firm defense (Vives, 2016), a bailout enables the number of competitors to remain high and it decreases interest margins. This failing-firm argument is empirically tested

---

5Based on these concerns, state aid was sometimes conditional on activity restrictions. For instance, restrictions and divestments in their retail activities were imposed respectively on Northern Rock and on RBS following public support (Beck and al. 2010).

6In the banking industry, the relationship between an increase in the number of competitors and lower interest rates is not straightforward (Degryse, Kim and Ongena, 2009). Furthermore, this relationship may depend on the business cycle. There is indeed empirical evidence that bank loan markups tend to move countercyclically (Mandelman, 2011, Olivero, 2010, Aliaga-Diaz and Olivero, 2010). Those movements arise even independently from the variations of borrowers’ riskiness during the business cycle because of the presence of switching costs.
by Calderon and Schaeck (2014), who show that public aid decreases margins and Lerner index in countries experiencing public support relative to countries which did not. In our paper, we enrich the failing-firm argument by making a distinction between insider and outsider borrowers. Our theoretical results show that borrowers who are already related to a solvent bank may not benefit from the preservation of a competing bank. Indeed, under duopoly, banks are able to extract rents from their installed base of consumers because of switching costs.

Only a few studies derive more precisely the competitive effects of a bailout policy on the credit market. Acharya and Yorulmazer (2007) show that the resolution policy may impact banks’ herding behavior when the latter choose the correlation of their portfolio. If each bank expects to buy its failing competitor, both banks choose ex ante an uncorrelated portfolio in order to maximize their ex post rents. However, under a systematic bailout policy, both banks choose to perfectly correlate their portfolio if the bailout guarantee is more valuable than this expected rent. Bertsch, Calcagno and Le Quement (2015) note that a systematic bailout increases the lifespan of banks, such that it makes tacit coordination more easily sustainable.

Hakenes and Schnabel (2010) analyze whether market discipline by investors limit or exacerbate the competitive distortions induced by a bailout. Focusing on the effect of lower refinancing costs on risk-taking, they show that a bailout unambiguously leads to higher risks for a protected bank only if the banking system is transparent (i.e. investors can observe the risk level of their bank). In either case, non-protected banks react by taking on more risks if they expect a higher probability of bailout for their competitor. This is empirically confirmed by Gropp (2011), who finds that moral hazard is limited for banks benefiting from implicit public guarantees, whereas implicit guarantees increase the risk-taking behavior of non-protected banks.

Our article differs from this literature in two directions. First, if focuses on the impact ex ante and ex post of bailouts on price competition, while Hakenes and Schnabel (2010) only account for ex ante effects. Most importantly, it is, to our knowledge, the first theoretical attempt to provide insights on the effect of a systematic bailout on the interest rates and borrowers’ welfare according to the level of switching costs. In a paper that is closely related to ours, Stenbakka and Takalo (2016) study the relationship between switching costs and financial stability on the market for deposits. They find that lower switching costs for inherited consumer relationships increase the probability of bank failure at the second period. By contrast, the intensified competition for deposits at the first period decreases the probability of a bank failure. Our work offers another perspective, as we model competition for loans and funding constraints on the credit market.

Second, this article is motivated by the literature on the effect of relationship lending on credit competition, especially the existence of switching costs and poaching prices.7 There is strong evidence that switching costs play an important role in shaping competition in the banking industry (Shy, 2002, Kim et al., 2003, and Degryse and Ongena., 2009, for a survey of the empirical literature) but also that consumers incur specific costs of switching after a branch exit (Bonfim, Nogueira and Ongena, 2017). The existence of poaching in the banking sector is also empirically confirmed by several studies, including Degryse and Bouckaert (2001), Hauswald and Marquez (2008) or Ioannidou and Ongena (2010). For instance, Carbo-Valverde et al. (2009) provide evidence that banks price deposit more aggressively to win consumers in Spanish regions characterized by higher migration levels, showing evidence of lock-in effects in banking retail markets. Based on the models of Fudenberg and Tirole (2000) and Chen (1997), theoretical papers by Gehrig and Stenbacka (2007) and Ahn and Breton (2014) model poaching in the banking sector to study respectively the effects of information disclosure and securitization. We contribute to this literature by adding the

7The literature on price discrimination defines poaching prices as discounted prices aimed at attracting customers from a competitor (Fudenberg and Tirole, 2000).
possibility that a bank can fail or face funding constraints in a market with switching costs.

Our results are also related to the literature revisiting the effect of switching costs on average prices when markets feature product differentiation. Following Dubé et al. (2009), Shin and Sudhir (2009) and Cabral (2016) argue that the relationship between switching costs and average prices is U-shaped. Their theoretical analysis may explain why, depending on the market considered, prices may either increase or decrease with switching costs. We highlight that on markets experiencing failures or capacity constraints, the rent-extraction effect of switching costs may vanish and therefore prices may decrease with switching costs for all consumers.

Our work is also related to a large empirical literature on the bank lending channel. This literature shows that liquidity shocks and bank failures reduce consumer access to credit during financial crisis. In particular, several papers show that the effect of a shock on borrowers depends on the strength of the lending relationship. However, this literature does not explain the competitive mechanisms underlying banks’ intertemporal trade-off between increasing or lowering their credit supply.\(^8\)

3 The model

We build a two-period model in which banks compete for borrowers who incur switching costs. At the end of the first period, each bank may experience a shock that triggers a failure. Our model enables us to analyze how a systematic bailout policy impacts the interest rates charged by banks before and after the bailout according to the level of switching costs and the severity of the funding constraints.

**Banks** Two banks \(A\) and \(B\) compete à la Hotelling to offer credit to retail consumers in a two-period game. The profit of bank \(k \in \{A, B\}\) at period \(l \in \{1, 2\}\) on the credit market is \(\pi^l_k\). Both banks are exogenously located at the two extremes of a linear city of length one, bank \(A\) being at point 0 and bank \(B\) at point 1, respectively. Banks set different interest rates for their borrowers at each period. Moreover, at the second period, they are able to “poach” the consumers of their rival by attracting them with a lower interest rate.\(^9\) At the first period, bank \(k\) sets the interest rate \(r^1_k\) to retail borrowers that maximizes the expected discounted value of its profit over the two periods. The common discount factor is denoted by \(\delta\) and we assume that \(\delta < 1\). At the second period, the interest rate charged by bank \(k\) to its consumers (the "insiders") is \(r^2_k\), whereas the interest rate charged to the consumers of its competitor (the "outsiders") is \(r^o_k\).

**Retail Borrowers** On the Hotelling line, there is a continuum of borrowers, whose preferences are uniformly distributed on \([0, 1]\) and invariant over time.\(^10\) At each period, a borrower needs one dollar to invest in a homogenous project which returns \(\rho\) with probability \(p\) and 0 with probability \(1 - p\).\(^11\) The expected return of the project is \(R = pp\). At the end of each period, the loan is

---

\(^8\)Because of a "flight to quality", it is problematic to compare borrowers before and after a shock in order to estimate the effect of the shock on interest rates and competition (see Bernanke 1993), unless micro-level data is available.

\(^9\)Therefore, in our setting, we allow for two types of price discrimination, that is, price discrimination across periods and consumer poaching. In the Online Appendix C, we show that a price-discrimination strategy is a Nash equilibrium of the game in which banks choose whether or not to price discriminate.

\(^10\)One interpretation of this assumption is that the differentiation in the services provided by each bank remains unchanged between the two periods.

\(^11\)Since we model a constant demand for credit, our model does not take into account a strategic behavior of banks to over lend in favorable periods of the credit cycle.
reimbursed if the investment is successful and the borrower defaults otherwise. The borrower is protected by limited liability.\textsuperscript{12} The return of the project is perfectly observable, so there is no moral hazard.

At each period, a borrower chooses between borrowing from bank $A$, bank $B$ and not borrowing, which brings him a reservation utility of zero.\textsuperscript{13} Borrowers have a transportation cost of $t > 0$ per unit of length. The transportation cost can be either interpreted as the degree of differentiation between both banks or the cost of reaching a bank branch. The information on the expected return of the project is known by banks at no cost. In contrast to banks, borrowers are assumed to be myopic, i.e. they maximize their utility in the first period without considering the consequences of their first period choice on the utility that they obtain in the second period.\textsuperscript{14} At the second period, consumers can remain with the same bank, decide not to borrow or switch to the competing bank and incur a switching cost $s > 0$.

**Shocks** At each period, each bank issues loans by raising one unit of short term debt (e.g., deposits, commercial papers, wholesale funding), in exchange for an exogenous return $c > 0$ at each period. Banks rely exclusively on short term debt, which exposes them to liquidity risk and failure.\textsuperscript{15}

At $t = 1$, with probability $\phi > 0$, there is no systemic shock and all short-term creditors roll over their debt. With probability $1 - \phi > 0$, a systemic shock hits both banks and all creditors may not roll over their debt. Each bank $k$ loses an amount of funding sources that depends on the realization of banks’ specific shocks. Banks are symmetric with respect to their individual probability of experiencing a specific shock. If bank $k$ faces a specific shock, it loses all its funding sources. Therefore, it exits the market unless the government intervenes.\textsuperscript{16} If bank $k$ does not face any specific shock, it keeps some (but not all) of its funding sources. The amount of remaining funds is conditional on the other bank being solvent. If its competitor fails, bank $k$ becomes a monopoly and its funding sources amount to $\lambda_m < 1$. If both banks remain active and constrained, their funding sources amount to $\lambda_d < \lambda_m$.\textsuperscript{17}

As banks’ funding sources are reduced after a systemic shock, their lending capacity is constrained at the second period. We assume that if a bank is constrained, the efficient credit rationing rule apply, that is, the bank serves first the consumers who have the highest willingness-to-pay for credit.\textsuperscript{18}

\textsuperscript{12}The borrowers’ risk is constant across periods. Therefore, the effects that we highlight in our model are not caused by variations in the borrowers’ riskiness during recession periods.

\textsuperscript{13}Hence we assume that borrowers do not hold multiple credit relationship at a same period. Empirical evidence on multi-relationship lending suggests both large variations between countries and firm sizes (Neuberger and Räthke, 2009). In our model, we allow for a different relationship at a refinancing stage, so that the two credit lines do not partly overlap only to simplify exposure. Also, we focus on long-term credit, which is on average more likely to be singular than liquidity services (Ongena and Smith, 2000).

\textsuperscript{14}Our model does not apply to sophisticated borrowers who could choose to overborrow at the first period to compensate for the probability to face credit constraints at the second period. Hence, we do not take into account the effects of an intertemporal allocation of the borrowers’ wealth on prices (Jeanne and Korinek, 2011).

\textsuperscript{15}Unlike Bolton and Oehmke (2018), we do not model bank’s trade-off between short-term and long-term debt. In our setting, the bank’s debt is runnable and therefore, orderly resolution is impossible.

\textsuperscript{16}We assume that in that case, the bank is unable to borrow from alternative private investors. Such an assumption can be justified by asymmetric information on banks’ assets in crisis times. Indeed, as argued by Flannery (1996), secondary markets for banks’ assets are plagued by a winner’s curse. Hence, the fire sales premium increases in crisis times.

\textsuperscript{17}If its competitor fails, the bank benefits from a transfer of funds. Hence, we assume that the reduction in funding sources per bank is stronger under duopoly than under monopoly. Note that we do not make any assumption on the difference in aggregate funding sources depending on the market structure.

\textsuperscript{18}In our Hotelling model, the willingness-to-pay for credit depends on the consumer’s location. Therefore, banks serve the closest consumers on the Hotelling line when they are constrained.
Given that a systemic shock occurs, the market structure at the second period depends on the realization of banks' specific shocks. If there is no systemic shock, both banks are active and unconstrained. Table 1 below summarizes the expected market structure after a systemic shock if there is no government intervention. The term into parenthesis refers to the conditional probability that a scenario occurs given the realization of a systemic shock.

<table>
<thead>
<tr>
<th>Bank B/Bank A</th>
<th>Specific shock</th>
<th>No specific shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific shock</td>
<td>Both banks exit ($q_0$)</td>
<td>Bank A monopoly ($q_{mc}$)</td>
</tr>
<tr>
<td>No specific shock</td>
<td>Bank B monopoly ($q_{mc}$)</td>
<td>Constrained duopoly ($q_{dc}$)</td>
</tr>
</tbody>
</table>

The bailout mechanism In case of a systemic shock, the government can issue guarantees on banks' liabilities to restore the confidence in the banking system.\textsuperscript{19} Such guarantee packages are offered on all banks' bonds.\textsuperscript{20} We assume that guaranteed bonds are bought by investors against a null interest rate. In exchange for the guarantee, the government charges a fee $c$ per dollar of insured liability, which corresponds to the market return demanded by investors. This fee can be either interpreted as a direct funding cost, an opportunity cost of funds or a measure of the guarantee premium. Banks use newly-issued bonds to compete on the credit market at the second period and to reimburse previous investors if needed. If the government does not offer public support to banks, the latter face funding constraints at the second period that impact their lending capacities.

Assumptions: We make the following assumptions:

(A1) $t \geq s$.

Assumption (A1) ensures that both banks poach some of their competitor’s borrowers at the second period at the equilibrium of the game.

(A2) $\lambda_d \in (0, 1/2)$.

Assumption (A2) ensures that if banks are constrained by their lending capacity, there is an equilibrium in pure strategies to the subgame in which banks choose their prices at the second period.

(A3) $\lambda_m \in (1/2, 1)$.

Assumptions (A3) implies if a bank operates as a monopoly at the second period, it has enough funds to lend to some consumers of the failed bank.\textsuperscript{21}

(A4) $\delta \phi \geq 1/2$.

(A5) $R \geq R \equiv \max\{c + 3t/2 - s/3, c + t + s/2\}$.

Assumptions (A4) and (A5) ensure that the market is covered at both periods when banks compete without funding constraints.

\textsuperscript{19}Therefore, we assume that a systemic shock is realized before the idiosyncratic shock. The government supports the banking sector before the idiosyncratic shock could be realized or it may be forbidden to target bailout. For a discussion of state-supported schemes for financial institutions, see Beck, Coyle, Dewatripont, Freixas and Seabright (2010).

\textsuperscript{20}This was the case in practice in 2008 in developed countries. From October 2008 to May 2010, around 1 060 billion euros of bank bonds were insured by government guarantee in developed countries (Levy and Schich, 2010).

\textsuperscript{21}There is empirical evidence that borrowers are less credit constrained in markets where banks enjoy more market power (See Bergstresser, 2010), which could also be a motivation for our assumption.
Timing of the game: The timing of the game is as follows:

- At $t = 0$, each bank $k$ sets an interest rate $r^1_k$. Borrowers choose from which bank to borrow and whether or not to borrow. They invest in their project, the outcome of the investment is realized and they repay their loans if the project is successful.
- At $t = 1$, shocks may occur. If a systemic shock occurs, the government can bail out the banking sector by issuing government guarantees.
- At $t = 2$, each active bank $k$ chooses the interest rates $r^i_k$ for its insiders and $r^o_k$ for its outsiders, respectively. Borrowers choose from which bank to borrow and whether or not to borrow. They invest in their project, the outcome of the investment is realized and they repay their loans if the project is successful.

4 Competition between banks under uncertainty

In this section, we analyze competition between banks when the market structure is uncertain at the second period. We denote by $\tilde{x}_D$ the location of the consumer who is indifferent between borrowing from A or B at the first period. Given that both banks are unconstrained at the first period, we have

$$\tilde{x}_D = \frac{t + p(r^1_B - r^1_A)}{2t}.$$  \hfill (1)

4.1 Competition at the second period

We analyze the interest rates charged by banks according to the market structure that prevails at the second period.

4.1.1 Duopolistic market structure with no constraints

If both banks remain in the market at the second period, consumers can choose between staying in their initial bank and switching to its competitor. We denote by $(\tilde{x}_k)^{du}$ the indifferent consumer between staying with its first-period bank $k$ and switching to bank $k'$, where the superscript $du$ stands for "duopoly unconstrained". The indifferent consumers $(\tilde{x}_A)^{du}$ and $(\tilde{x}_B)^{du}$ are given by

$$(\tilde{x}_A)^{du} = \frac{1}{2} + \frac{p(r^i_A)^{du} - (r^i_A)^{du}}{2t} + s,$$

and

$$(\tilde{x}_B)^{du} = \frac{1}{2} + \frac{p[(r^i_B)^{du} - (r^i_A)^{du}]}{2t} - s.$$  \hfill (2)

At the second period, each bank $k$ chooses the interest rates $(r^i_k)^{du}$ and $(r^o_k)^{du}$ that maximize its profit at the second period given by

$$(\pi_k^2)^{du} = (x^i_k)^{du} (p(r^i_k)^{du} - c) + (x^o_k)^{du} (p(r^o_k)^{du} - c),$$

where if $k = A$, we have $(x^i_A)^{du} = (\tilde{x}_A)^{du}$ and $(x^o_A)^{du} = (\tilde{x}_B)^{du} - \tilde{x}_D$. If $k = B$, we have $(x^i_B)^{du} = (1 - (\tilde{x}_B)^{du})$ and $(x^o_B)^{du} = \tilde{x}_D - (\tilde{x}_A)^{du}$.

\footnote{\textsuperscript{22}In Appendix A1, we show that under (A1)-(A5) the market is covered at the second period in a symmetric equilibrium and that both banks have an incentive to poach their outsider consumers. Moreover, in Appendix A2, we show that under Assumptions (A1)-(A5), the market is covered at the first period.}
Solving for the first-order conditions of profit-maximization, we find that the interest rate charged to bank $k$’s insider consumers at the equilibrium of stage 2 is given by

$$ (r^i_k)^{du} = \frac{3c + 2t + s + p(r^1_k - r^1_k)}{3p}, $$

whereas the interest rate charged to bank $k$’s outsider consumers at the equilibrium of stage 2 is given by

$$ (r^o_k)^{du} = \frac{3c + t - s - p(r^1_k - r^1_k)}{3p}. $$

Bank $k$ offers a discount to its outsider consumers. From (3) and (4), this discount is equal to $(t + 2s)/(3p) - (r^1_k - r^1_k)$ and it is decreasing with the size of bank $k$’s initial market share, the proportion of successful projects and increasing with the level of switching costs. It is interesting to note that the price charged to insider consumers is twice as elastic to the degree of product differentiation as the price charged to outsider consumers, reflecting the fact that a bank exerts a stronger market power on its insiders.

Replacing for $(r^i_k)^{du}$ and $(r^o_k)^{du}$ into Eq.(2), we find that the profit of bank $k$ at the equilibrium of stage 2 is given by

$$ (\pi^2_k)^{du} = \left[ \frac{2t + s + p(r^1_k - r^1_k)}{18t} \right]^2 + \left[ \frac{t - s - 2p(r^1_k - r^1_k)}{18t} \right], $$

where the first part of $(\pi^2_k)^{du}$ corresponds to the profit of bank $k$ on its insider borrowers, and the second part, the profit of bank $k$ on its outsider borrowers.

### 4.1.2 Duopoly under financial constraints

From Assumption (A2) and efficient credit rationing, we know that if both firms are active and constrained, each of them serves at most a measure $\lambda_d$ of consumers. Their respective markets do not overlap and they enjoy local monopoly power over their borrowers. We denote by $(\pi^2_A)^{dc}$ bank $A$’s profit if both firms are active and constrained at the second-period. In Appendix A3.a, we show that depending on the severity of the funding constraints, bank $A$ may either serve only its insiders or both its insiders and its outsiders (with financial constraints). If if $\lambda_d \leq \tilde{x}_D$, bank $A$ only serves its insiders and makes a profit given by $(\pi^2_A)^{dc} = (\pi^2_A)^{dc}$. If $\lambda_d > \tilde{x}_D$, bank $A$ serves both its insiders and its outsiders and makes a profit given by $(\pi^2_A)^{dc} = (\pi^2_A)^{dc}$.

### 4.1.3 Monopoly market under financial constraints

We denote by $(\pi^2_A)^{mc}$ bank $A$’s profit if it is a constrained monopoly at the second period. In Appendix A3.b, we show that depending on the severity of the funding constraints, bank $A$ may either serve only its insiders, both its insiders and its outsiders (with financial constraints) or both its insiders and outsiders without financial constraints. If $\lambda_m \leq \tilde{x}_D$, bank $A$ serves only its insiders and makes a profit given by $(\pi^2_A)^{mc} = (\pi^2_A)^{mc}$. If $\lambda_m \in (\tilde{x}_D, \tilde{\lambda}_D)$, bank $A$ serves its entire insider market. However, it is constrained on its outsider market and makes a profit given by $(\pi^2_A)^{mc} = (\pi^2_A)^{mc}$. If $\lambda_m \geq \tilde{\lambda}_A$, where $\tilde{\lambda}_A \equiv (R - c + t\tilde{x}_D - s)/(2t)$, bank $A$ is unconstrained and makes a profit given by $(\pi^2_A)^{mc} = (\pi^2_A)^{mc}$. 

10
4.2 Competition at the first period

4.2.1 Under systematic bailout policy

Under a systematic bailout policy, the state intervenes by issuing guarantees if there is a systemic shock. Therefore, the market structure is certain at the second period as banks expect to operate as an unconstrained duopoly. The expected discounted value of bank $k$’s profit at the first period is given by

$$\pi_k = \pi_1^k + \delta (\pi_2^k)_{du},$$  \hspace{1cm} (6)

where

$$\pi_1^k = \tilde{x}_D(pr_k^1 - c),$$  \hspace{1cm} (7)

and $(\pi_2^k)_{du}$ is given by (5).

Each bank $k$ maximizes the discounted value of its profit at the first period. Taking the derivative of Eq.(6) with respect to $r_1^k$, we have that

$$\frac{\partial \pi_k}{\partial r_1^k} = \frac{\partial \pi_1^k}{\partial r_1^k} + \delta \frac{\partial (\pi_2^k)_{du}}{\partial r_1^k}.$$

From (5), we have $\frac{\partial (\pi_2^k)_{du}}{\partial r_1^k}|_{r_1^A=r_1^B} = -p(s)/3t$ if both banks remain unconstrained at the second period. Therefore, at a symmetric equilibrium, banks charge a lower interest rate at the equilibrium of stage 1 than they would in absence of competition at the second stage. Indeed, if bank $k$ charges a higher interest rate at the first period, this decreases its profit at the second stage of competition. It becomes valuable to build a large insider market share at the first stage by offering a discount to consumers at the first period before extracting rents on this market at the second period.

Proposition 1 gives the interest rates charged by banks at the symmetric equilibrium if there is a systematic bailout.

**Proposition 1.** i) Under a systematic bailout, at the first period each bank $k \in \{A, B\}$ charges an interest rate given by

$$r_1^k = \frac{c + t}{p} - \delta \sigma_{du},$$  \hspace{1cm} (8)

where $\sigma_{du} = 2s/(3p)$.

ii) At the second period, under a systematic bailout, each bank $k$ charges an interest rate given by $(r_i^1)_{du} = (3c + 2t + s)/3p$ to its insiders and $(r_o^1)_{du} = (3c + t - s)/3p$ to its outsiders, respectively.

**Proof.** See Appendix B1. ■

At the first period, each bank sets an interest below its optimum under static competition (i.e., $(c + t)/p$) by offering a discount $\delta \sigma_d$ to its consumers. Indeed, banks trade off between extracting rents from their consumers at the first and the second period. By choosing to lower the first period interest rate, banks soften competition at the first period to attract more insiders from which they will be able to extract rents at the second period. At $r_1^k$ given by (8), the marginal loss on the first period profit is exactly equal to the marginal gain on the second period profit. The discount offered at the first period increases with switching costs, reflecting banks’ market power on their insiders. At the second period, the interest rate charged to their insiders increases with switching costs. By constrast, banks lower the interest rate charged to their outsiders at the second period to compensate for switching costs.

Replacing for the interest rates of Proposition 1 in Eq. (5), (6) and (7), we find banks’ profits at the equilibrium of the game, which can be found in Appendix B1.
4.2.2 Under a no-bailout policy

If the state does not intervene to bailout banks, the market structure and banks’ lending capacities are uncertain at the second period. With probability \( \phi \), there is no systemic shock and banks do not face funding constraints. Each bank \( k \) makes a profit given by \( (\pi_k^2)^{du} \) in Eq. (5). With probability \( (1 - \phi)q_{dc} \), both banks remain active with funding constraints and make a profit \( (\pi_k^2)^{dc} \). With probability \( (1 - \phi)q_{mc} \), only bank \( k \) remains active as a monopoly and makes a profit given by \( (\pi_k^2)^{mc} \). With probability \( (1 - \phi)q_o \), both banks exit the market and make zero profits.

If there is no bailout, each bank \( k \) maximizes the expected discounted value of its profit given by

\[
\pi_k = \pi_k^1 + \delta E(\pi_k^2),
\]

where

\[
E(\pi_k^2) = \phi(\pi_k^2)^{du} + (1 - \phi)(q_{dc}(\pi_k^2)^{dc} + q_{mc}(\pi_k^2)^{mc}).
\]

To ensure the existence a symmetric equilibrium, we make the following assumption:

\[ (A6) \ s > t(1 - \lambda_d). \]

**Lemma 1.** If both banks are able to poach some consumers if they compete as an unconstrained duopoly competition at the second period, the game admits a unique symmetric Nash equilibrium at the first period.

**Proof.** Appendix B2. ■

Assumption (A6) ensures that banks have incentives to reduce their interest rates at the first period, even if they expect to operate as a constrained (local) monopoly at \( t = 2 \). Such a condition is satisfied if the marginal gains of attracting an insider consumer exceed the marginal losses of renouncing to an outsider at the first period.

On the contrary, if \( s < t(1 - \lambda_d) \), symmetric pricing at the first period may no longer be a Nash equilibrium. In this case, there will exist Nash equilibria in which firms set asymmetric interest rates at the first period.\(^{23}\) If (A6) does not hold, it is profitable for one bank to increase its first period interest rate to lose some market share, because the marginal benefit from price-discrimination between insiders and outsiders is higher than the gain of serving only insiders.

At the symmetric equilibrium of the first period, \( (\pi_k^2)^{dc} \) in Eq.(9) is equal to \( (\pi_k^2)^{dc} \) given in Eq. (14). Also, let \( \lambda_m = (R - c + t/2 - s)/2t \) denote the threshold on the funding constraints such that a monopolistic bank is forced to renounce to serve some outsiders in a symmetric equilibrium. If \( \lambda_m \leq \lambda_{m}^{\phi} \), \( (\pi_A^2)^{mc} \) in Eq.(9) is equal to \( (\pi_A^2)^{mc} \) given in Eq. (15), and it is equal to \( (\pi_A^2)^{mu} \) given in Eq. (16) otherwise.

Proposition 2 gives the interest rates charged by banks at the symmetric equilibrium if there is no bailout.

**Proposition 2.** i) Under a no-bailout policy, at the first period each bank \( k \in \{A, B\} \) charges an interest rate given by

\[
r_k^1 = \frac{c + t}{p} - \delta[\phi \sigma_{du} + (1 - \phi)q_{mc} \sigma_{mc}],
\]

where \( \sigma_{mc} = (s - t(1 - \lambda_m))/p \) if there are severe funding constraints under monopoly (i.e., \( \lambda_m \leq \lambda_m^{\phi} \)). Otherwise, \( \sigma_{mc} = (R - c - 3t/2 + s)/2p \) if funding constraints are softer (i.e., \( \lambda_m > \lambda_m^{\phi} \)).

ii) At the second period, if there is no systemic shock, each bank \( k \) charges the interest rate \( (r_k^o)^{du} \) to its insiders (resp., \( (r_k^o)^{du} \) to its outsiders) given in Proposition 1.

\(^{23}\)See Appendix B2 for details.
If there is a systemic shock, the interest rates depend on the market structure that follows the realization of the idiosyncratic shocks. If only bank $k$ remains active with severe funding constraints (i.e., $\lambda_m \leq \lambda_m^c$), it charges the interest rates $(r_k^i)^{mc} = (R - t/2)/p$ to its insiders and $(r_k^o)^{mc} = (R - t\lambda_m - s)/p$ to its outsiders, respectively. Otherwise, if $\lambda_m > \lambda_m^c$, it charges $(r_k^o)^{mu} = (R + c - t/2 - s)/2p$ to its outsiders. If both banks remain active, they face severe funding constraints and serve only their insiders by charging them an interest rate given by $(r_A^i)^{dc} = (R - t\lambda_d)/p$.

**Proof.** See Appendix B2. ■

If there is no bailout, the same logic as for Proposition 1 applies, that is, banks set a lower interest rate than the optimum under static competition. However, there are two differences. First, the total discount charged at the first period is a weighted average of the expected market structure at the second period. Second, it depends on the severity of the funding constraints.

The total discount can be analyzed as follows. With probability $\phi$, there is no systemic shock in the economy and banks compete without constraints, such that the optimal discount $\delta\sigma_d$ is the same as the one banks set under a systematic bailout policy. With probability $(1 - \phi)q_{mc}$, bank $k$ remains active in the market as its competitor fails without any support from the state and the discount offered is $\delta\sigma_{mc}$. The discount depends on the severity of the funding constraints faced by banks under monopoly at the second period. If banks expect to be very constrained under monopoly such that they can no longer serve all their outsiders, the discount increases with the severity of the funding constraints. This reflects a lower profitability of the outsider market at the second period. Otherwise, if there is a sufficient share of stable funding sources, the severity of the funding constraints does not impact banks’ trade-off between serving insiders and outsiders.

With probability $(1 - \phi)(q_{mc} + q_0)$, bank $k$ does not discount its first period interest rate, because it expects to exit the market at the second period. Finally, with probability $(1 - \phi)q_{dc}$, bank $k$ expects to operate as a constrained duopoly. Since it cannot serve its entire insider market at the symmetric equilibrium, the trade-off between extracting rents from consumers at the first and the second period disappears.

Replacing for the interest rates of Proposition 2 into Eq. (5), (6), (14), (15) and (16), we find banks’ profits at the equilibrium of the game, which are given in Appendix B2.

## 5 The impact of a systematic bailout policy

In this section, we analyze the impact of a systematic bailout policy on the interest rates charged by banks, on consumer surplus and social welfare.\textsuperscript{24}

### 5.1 The impact of a systematic bailout policy on interest rates

In Proposition 3, we compare the impact of a systematic bailout policy on the interest rates charged by banks ex ante and ex post.

We define the threshold on the switching costs that will be useful for our comparison of the interest rates charged to consumers as $s_r \equiv 3(R - c - 7t/6)$. Moreover, we introduce different thresholds on funding constraints for insiders and outsiders, respectively given by $\lambda_i \equiv (R - c - (2t + s)/3)/t$ and $\lambda_o = (R - c - (t + 2s)/3)/t \geq \lambda_i$.

**Proposition 3.** i) A systematic bailout policy always reduces the interest rate at the first period. ii) If both banks remain active after a systemic shock, banks charge a higher interest rate to their insider consumers under a systematic bailout policy if $s > s_r$ and $\lambda_d > \lambda_i$ and a lower interest rate

\textsuperscript{24}In Online Appendix D, we analyse the impact of a systematic bailout on banks’ profit.
otherwise. If there is a systemic shock and one bank faces an idiosyncratic shock, banks charge a higher interest rate to insider consumers under a systematic bailout policy if \( s > s_r \) and a lower interest rate if \( s \leq s_r \).

If only one bank remains active after a systemic shock, it charges a higher interest rate to its outsider consumers under a systematic bailout policy if \( s > s_r \) and \( \lambda_m > \lambda_o \) and a lower interest rate otherwise.

Proof. Appendix C1 and C2

A systematic bailout policy reduces the first period interest rate compared to a no-bailout policy. If the regulator can credibly commit to bail out the banks, it impacts their expectations at the first period when the latter choose their interest rates. As shown in Proposition 1 and 2, banks reduce their first period interest rates compared to a static game of competition because they internalize the impact of their first period choices on the second stage of competition. The discount offered in expectation of a monopolistic structure \( \sigma_m \) is higher than the discount offered in expectation of a duopoly \( \sigma_d \) if the financial constraint is soft enough and the expected return on investment is large.\(^{25}\) Indeed, each bank has incentives to build a large insider market from which it is able to extract high rents if investments are profitable. However, even when banks expect a high return on investment, each bank has a relatively low probability of becoming a monopoly at the second period ((1 - \( \phi \))q_{mc} \leq 1/2 by symmetry between both banks) and has limited incentives to discount its first period interest rate. Therefore, a systematic bailout always implies a reduction of the interest rate charged to borrowers at the first period (i.e., \( \delta[\phi\sigma_d + (1 - \phi)q_{mc}\sigma_{mc}] \leq \delta\sigma_d \)).

By contrast, a systematic bailout may or may not increase the interest rates charged to borrowers at the second period. It increases the second period interest rates if the proportion of stable funding sources and the level of switching costs are high relative to the net profitability of the credit market (i.e., \( R - c \)).

This phenomenon is explained by the differences of elasticities of consumer demand under monopoly and duopoly, which generate different pricing strategies according to the level of switching costs. The demand for credit from outsiders is more sensitive to the level of switching costs under monopoly than under duopoly, as their outside option is to renounce to credit instead of remaining with their home bank. Therefore, the interest rates charged to outsiders are more sensitive to the level of switching costs under monopoly than duopoly. The interest rate charged to outsiders may become lower under monopoly when switching costs are high. This effect is reinforced by the strategic complementarity between the interest rate charged to the insiders and to the outsiders under duopoly. Indeed, it becomes less costly to attract outsiders under duopoly as switching costs become higher.

For insider consumers, the interest rate charged under monopoly is not related to the level of switching costs, as their outside option (i.e., no loan) does not depend on switching costs. The negative difference in sensitivity to switching costs between the two rates is now only due to insider consumers being locked-in under duopoly competition. If switching costs are high and if the profitability of the credit market is low, a no-bailout policy decreases the interest rates charged to insider consumers.

In our model, a bank may charge lower interest rates under monopoly than under competition because perfect price discrimination is impossible. A (local) monopoly’s optimal pricing is therefore to cover the market, until it faces a financial constraint if this is the case. On the contrary, under duopoly, banks are able to segment their markets between their closest insiders and outsiders, and therefore to extract higher rents. When switching and transportation costs are high enough, markets

\(^{25}\)The discount offered in expectation of a monopoly is higher than the discount offered in expectation of a duopoly if \( \lambda_m \geq 1 - s/3t \), and if \( R \geq c + 3t/2 + s/3 \) when \( \lambda_m \geq \lambda_o \).
of different consumers are highly segmented under duopoly, such that interest rates may be higher than under monopoly.

**Comparative statics** In Corollary 1, we compare the impact of a systematic bailout on the interest rates paid by the insiders and the outsiders if one bank exits the market.

**Corollary 1.** If a monopoly is financially constrained (i.e, \( \lambda_m < \lambda_m^c \)), a systematic bailout policy impacts more the interest rates paid ex post by outsider consumers than the interest rates paid by insiders if \( \lambda_m < \min \{5/6 - s/(3t), 2(R - c - (3t + s)/4)/t\} \), and a systematic bailout impacts more the interest rate paid by insiders otherwise.

If a monopoly is not financially constrained (i.e, \( \lambda_m \geq \lambda_m^c \)), a systematic bailout policy impacts twice as more the interest rates paid ex post by insider consumers as the interest rates paid by outsiders.

**Proof.** Appendix D. ■

Corollary 1 shows that a systematic bailout policy has a stronger impact on the interest rates paid by outsider consumers if there are severe financial constraints. This reflects the fact that financial constraints limit banks’ ability to exert their market power on their insider consumers.

From Proposition 3, if switching costs are low (\( s < s_r \)), a systematic bailout is profitable for both types of consumers. It is more profitable for outsiders than insiders, if a monopoly faces strong financial constraints under a no-bailout (i.e., \( \lambda_m < 5/6 - s/(3t) \)). Otherwise, a bailout policy is more profitable for insiders.

If switching costs are higher (\( s > s_r \)), a systematic bailout is not profitable for insiders. However, it can still be profitable for outsiders if a monopoly faces strong financial constraints under a no-bailout (i.e \( \lambda_m < \lambda_o \)). If financial constraints are very severe (i.e., \( \lambda_m < 2(R - c - (3t + s)/4)/t \)), the positive effect of a systematic bailout on the price paid by outsiders is stronger than the negative effect on the price paid by the insiders. Hence, a systematic bailout impacts more the interest rate paid by outsiders. Otherwise, a bailout policy impacts more insider consumers.

### 5.2 The impact of a systematic bailout policy on consumer surplus

In Proposition 4, we provide results on comparisons of consumer surplus under a systematic bailout and a no-bailout policy.\(^{26}\) For this purpose, we denote by \( R_{cs} \equiv c + t(\lambda_d^2 + (31t^2 + 16st - 2s^2)/36t) \) the threshold on the market profitability such that a systematic bailout policy may increase consumer surplus ex post and by \( \lambda_{cs} \) the maximum of the two possible roots of \( R_{cs} = R \).

**Proposition 4.** A bailout policy always increases the average consumer surplus ex ante.

A bailout policy always increases the average consumer surplus ex post if one bank fails. If both banks remain active with financial constraints, if \( s \geq t(7 - \sqrt{42}) \), \( R \leq R_{cs} \) and \( \lambda_d \geq \lambda_{cs} \), a bailout policy decreases consumer surplus and it increases consumer surplus otherwise.

**Proof.** Appendix E. ■

A systematic bailout policy has various effects on consumer surplus. As seen in Proposition 3, it impacts the prices paid by consumers both at the first and the second period. This first effect may either increase or decrease consumer surplus. Furthermore, a systematic bailout policy may improve market coverage at the second period, as fewer consumers may be left out of the credit market. This second effect increases consumer surplus. Finally, a bailout impacts consumer switching behavior

---

\(^{26}\)In our model, consumer surplus is different from borrower surplus as some outsider consumers do not borrow under a no-bailout policy at the second period. We focus on consumer surplus to take this effect into account in our comparison.
at the second period. Indeed, it offers consumers the option to switch, instead of staying with their home bank. By contrast, if there is no bailout, some consumers are forced to switch at the second period otherwise they do not obtain any credit (if one bank fails) or they do not have the option to switch because of credit rationing (if both banks remain active with financial constraints).²⁷

At the first period, consumer surplus is always higher if banks expect a systematic bailout, because the latter charge a lower interest rate (see Proposition 3).²⁸ At the second period, the bailout may either increase or decrease interest rates, increases market coverage, and may either increase or decrease switching costs. In our setting, the positive impact of the bailout on market coverage dominates the other forces in case the impact on interest rates or switching costs is negative, if one bank fails. However, if both banks remain active with financial constraints, the efficient rationing rule minimizes transportation costs, and no consumer incurs switching costs. Credit rationing can be beneficial to borrowers if switching cost are high enough for two reasons. First, it reduces the rents that banks can extract from their insiders, compared to a bailout situation where insiders would have been locked-in anyway. Second, it decreases the benefits of switching for outsiders in case there is a bailout. To sum-up, if enough consumers can still get access to credit and banks’ ability to extract rents is low, a no-bailout policy increases consumer surplus if both banks remain active.

Whatever the shock, a bailout policy has ex post different effects on consumers according to their switching behavior. We distinguish between four types of borrowers. First, consumers who are left out of the credit market because of credit rationing are better-off if there is a bailout. Second, borrowers who never switch and those who always switch whatever the resolution policy are worse-off under a systematic bailout if they pay higher prices after the bailout. Indeed, for those consumers, the bailout policy does not impact their switching nor their transportation costs. Third, outsider borrowers who are forced to switch when their home bank fails obtain lower interest rates because of poaching, but this does not fully compensate for higher switching and transportation costs. Therefore, they are better-off under a bailout.²⁹ Fourth, the effect of a bailout for insider borrowers who would be forced to stay with their home bank because of a bank failure or funding constraints is ambiguous.³⁰

To understand how variations in the interest rates impact this latter group of borrowers, we focus on the first borrower (among them) who stays with his home bank under a bailout. Indeed, he is indifferent between staying with his bank and switching. If the interest rate for insiders is lower under a bailout, his utility is higher under a bailout. Since all other borrowers pay higher transportation costs if they are forced to stay, this implies that their utility is higher under a bailout. From Proposition 3, this holds if the financial constraint on insiders is severe or if switching costs are low \((\lambda_d < \lambda_i \text{ or } s < s_r \text{ given in Proposition 3})\). On the contrary, if both the financial constraint on insiders is soft and switching costs are high \((\lambda_d \geq \lambda_i \text{ and } s \geq s_r\)), the indifferent consumer is better-off under a no-bailout. Some consumers close to him might be better-off as well. However, because the financial constraint is higher than \(\lambda_i\), a majority of insiders in this group are located further away from their home bank. Those borrowers are better-off if they switch banks, as the additional transportation costs implied by switching are low. Therefore, a bailout policy always

²⁷The effect of a systematic bailout on transportation costs is ambiguous, because it depends on the difference in the switching interval and on the fact that on average outsiders incur more transportation costs under a no-bailout policy. Therefore, when switching costs are low and return on investment large, transportation costs are higher under a no-bailout policy.

²⁸A systematic bailout has no impact on market coverage at the first period, nor on switching costs.

²⁹We can show that the poaching rate under monopoly is lower than the rate for insiders under duopoly if a monopoly is unconstrained, or if it is constrained and \(R < c + t\lambda_m + 2t/3 + 4s/3\).

³⁰At equilibrium, this case is feasible if the financial constraint of a bank is higher than \((2t + s)/6t\), such that insiders who never switch are already served.
increases the average consumer surplus of this group.

It is also interesting to examine how switching costs impact the magnitude of the loss in average consumer surplus if the state does not intervene to bailout a bank. When switching costs increase, the difference in consumer surplus under a no-bailout and a systematic bailout policy is reduced. This is because higher switching costs soften competition under duopoly. Furthermore, the surplus of insider consumers under a monopoly does not depend on switching costs, while the surplus of outsider consumers increases following the reduction of interest rates.

5.3 The impact of a systematic bailout policy on welfare

We now analyze whether a systematic bailout policy increases social welfare, defined as the sum of banks’ joint profit and consumer surplus. At the first period, since the market is covered under both regimes, total welfare is equal to the average benefit of credit for consumers (i.e., \( R - c - t/4 \)). Indeed, the interest rate is transferred from consumers to banks. It follows that total welfare is constant at the first period and does not depend on the resolution regime.

In Proposition 5, we analyze whether a systematic bailout policy increases social welfare if there are neither administrative nor funding costs. For this purpose, we denote by \( R_w = c - (36t^2\lambda_d^2 - 11t^2 - 8st + 10s^2)/(36t - 72t\lambda_d) \) the threshold on the profitability of the credit market such that a systematic bailout policy increases social welfare and by \( \lambda_w \) the maximum of the two possible roots of \( R_w = R \).

**Proposition 5.** If there are no costs associated to a bailout, a bailout policy decreases social welfare ex post if both banks remain active and

i) \( R < R_w \) if \( s < t(\sqrt{519}/2 - 10)/11 \)

ii) \( R < R_w \) and \( \lambda_d > \lambda_w \) if \( s \geq t(\sqrt{519}/2 - 10)/11 \)

and it increases welfare otherwise.

**Proof.** Appendix F.

To understand Proposition 5, we denote by \( \Delta W_k \) the difference in welfare between a no-bailout and a bailout policy, where \( k = \{dc, mc\} \) denotes the market structure at the second period. Also, we denote by \( \lambda_{mc} = \min \{\lambda_m, \lambda_m^c\} \) the total demand served under monopoly depending on whether it is constrained on its outsider market or not.

The difference \( \Delta W_k \) can be decomposed as a function of \( \Delta u_k \) the welfare gain of serving the consumers who only borrow under a bailout and \( \Delta t_k \) (resp., \( \Delta s_k \)) the differences in transportation (resp., switching) costs, that is we have

\[
\Delta W_k = \Delta u_k + \Delta t_k + \Delta s_k, \tag{10}
\]

where the expressions of \( \Delta u_k \), \( \Delta t_k \) and \( \Delta s_k \) are given in Appendix G. At the equilibrium of the game, we have

\[
\begin{array}{c|c|c}
\Delta u_k & k = dc & k = mc \\
\hline
(2\lambda_d - 1)(R - c) & (\lambda_{mc} - 1)(R - c) \\
\hline
(11t^2 - 4st + 2s^2) / (36t) - t\lambda_d^2 & (11t^2 - 4st + 2s^2) / (36t) - t\lambda_{mc}^2 / 2 \\
\hline
s(t-s) / 2t & s(t-s) / 2t - s(\lambda_{mc} - 1)/2
\end{array}
\]

From a welfare perspective, a systematic bailout has one certain advantage over a no-bailout alternative, namely it provides a full market coverage which increases welfare by \( \Delta u_k \) (the rationing effect). However, this effect may be offset by the variations of total switching and transportation costs. First, a systematic bailout may or may not increase the number of switching consumers and
therefore switching costs $\Delta s_k$ (the switching costs effect). Furthermore, even if a systematic bailout enables two banks to cover the market and the average transportation cost paid by a consumer is higher under monopoly, it may not lead to a decrease in transportation costs, so that the sign of $\Delta t_k$ is also ambiguous (the transportation cost effect). This situation may arise if the number of consumers switching under duopoly is high, whereas the surviving bank serves a limited number of outsiders under monopoly.

Following Proposition 5, a systematic bailout always increases welfare if it prevents the failure of one bank. Indeed, the switching costs and the transportation costs effects increase when the rationing effect decreases. A monopoly might for instance only serve a low proportion of the market, either because it is very constrained by its funding sources or because few outsiders are able to regain access to credit when the profitability of the credit market is low. In these cases, the rationing effect $\Delta u_{mc}$ is high, and it is not offset by gains in terms of lower switching ($\Delta s_{mc}$) or transportation costs ($\Delta t_{mc}$) related to this poor market coverage. On the contrary, if the monopoly’s financial capacity or the profitability of the credit market are high enough, most of the outsider market is covered. The no-bailout allocation is less efficient than the bailout one due to high switching and transportation costs. Therefore, a bailout enhances welfare because it preserves market coverage while limiting inefficient mobility.

If a systemic shock does not lead to a failure, a bailout no longer provides the most efficient allocation of credit. Because the financial capacity $\lambda_d$ constrains both banks to serve only their respective insider markets, no consumer switches, while transportation costs are minimized. Therefore, if the credit rationing effect is not too severe, a no-bailout policy improves welfare ex post. First, this holds if each bank financial constraint $\lambda_d$ is high relatively to the profitability of the credit market (i.e $R < R_w$). The result also depends on the level of switching costs. If switching costs are low enough, transportation costs under a bailout increase because of inefficient poaching (see $\Delta t_{dc}$). On the contrary, if switching costs increase, poaching is reduced if there is a bailout, such that the no-bailout alternative remains welfare-enhancing if and only if the financial capacity $\lambda_d$ is also high enough relatively to this increase in switching costs (i.e $\lambda_d > \lambda_w$).

Comparative statics on the expected welfare effect of a systematic bailout In Lemma 2, we provide some comparative statics to the difference in expected social welfare between a no-bailout and a systematic bailout policy. Let $E(\Delta W)$ denote this difference, and $W_d$ the welfare under duopoly at the second period. We have

$$E(\Delta W) = \delta(1 - \phi)[(q_{dc}(\Delta W_{dc}) + 2q_{mc}(\Delta W_{mc}) - q_0W_d], \tag{11}$$

where $\Delta W_k$ and $\Delta W_{mc}$ are given in Eq. (10), and $W_d = R - c - (11t^2 + 8st - 10s^2)/(36t)$.

**Lemma 2.** $E(\Delta W)$ is increasing with the proportion of stable funding sources and decreasing with switching costs. It is also decreasing in credit profitability $R$ if $\lambda_m < \lambda_m^c$, and it is ambiguous otherwise.

**Proof.** Appendix H. \[31\]

31In our model, the transportation costs effect is minimized because the efficient rationing rule apply. However, even if the rule did not apply, the transportation costs effect is strictly beneficial to the no-bailout alternative as long as constrained banks only serve their insider market.

32In our model, the switching costs effect under a bailout are also maximized when switching costs are low. This counterintuitive result stems from the fact that we assume $s > t/2$ from (A2) and (A6), and that when $s = t$, no poaching occurs, such that the switching cost effect is also null under a bailout. More generally, $\Delta t + \Delta s$ is decreasing with switching costs as long as $s > 2t/5$. 

18
Lemma 2 states that whatever the nature of the shock and the resulting market structure, a no-bailout policy is more likely to be welfare-enhancing when switching costs are low. First, low switching costs have a negative effect under a bailout, because they lead to intense and inefficient poaching and increase transportation costs (see $\Delta t_{dc}$). By contrast, if switching costs are high, poaching is reduced under unconstrained competition. Second, low switching costs may have positive effects on welfare under a no-bailout, depending on the severity of the financial constraint.

If the financial constraint is really severe, any remaining bank only serves its insiders and no switching occurs, such that the level of switching costs plays no role.

If a monopoly if financially constrained, then the demand for credit by outsiders remains bounded by available funding sources $\lambda_m$. Their constrained demand is for that reason insensitive to switching costs, but the cost of serving this demand decreases with switching costs. In this case, a decrease in switching costs also enables a monopoly to serve outsiders at a lower cost.

Finally, if the monopoly is expected to be unconstrained, a decrease in switching costs also benefits welfare, but this time through a decrease in credit rationing. Indeed, low switching costs enable the monopoly to attract a large share of outsiders, so that the welfare advantage of a duopoly in terms of market coverage is reduced. Indeed, the demand for credit to outsiders $\lambda_m^c$ is decreasing in switching costs, such that the rationing effect $\Delta u_{mc}$ is moderate when $s$ is low. However, a decrease in switching costs exert a countervailing effect on the transportation and switching costs effects (see $\Delta s_{mc}$ and $\Delta t_{mc}$). Indeed, this increase in outsider market under monopoly comes at a cost, namely a large increase in sunk transportation costs, while this increase is more moderate under duopoly, as consumers are less sensitive to a variation in switching costs. Similarly, as low switching costs lead primarily to more switching under monopoly, its welfare advantage is reduced by higher total switching and transportation costs. The net effect of this market increase is positive for the monopoly.

A no-bailout policy is also more likely to be welfare-enhancing when market profitability is low. First, this is always the case under duopoly or under a constrained monopoly, since the only effect of high market profitability is to increase the marginal value of rationing $\Delta u$. Second, this may also or may not be the case when a monopoly is financially unconstrained. Under unconstrained monopoly, an increase in the market profitability enables more consumers to regain access to credit by switching to the remaining bank, such that market coverage $\lambda_m^c$ increases. Therefore, a high market profitability decreases rationing under monopoly and increases welfare, but it also increases the marginal value of the remaining rationing.

In Corollary 2, we analyze how the probability that both banks face an indiosyncratic shock (without facing a systemic shock) impacts the expected welfare effects of a systematic bailout policy.

**Corollary 2.** A no-bailout policy increases social welfare ex post if there is a high probability that both banks remain active with strong financial constraints and if the level of switching costs is sufficiently low.

From Eq.(11), the sign of $E(\Delta W)$ is independent of the probability that a systemic shock occurs. Proposition 3 states that if there are no costs of bailing out banks, $E(\Delta W)$ may be positive is the probability of a constrained duopoly is high enough, provided that the conditions of Proposition 3 ii) are fulfilled.

In Proposition 6, we derive the condition on the return on investment such that a policy of decreasing switching costs is most effective in increasing social welfare under a no-bailout with respect to a systematic bailout.

**Proposition 6.** If $\lambda_m > \lambda_m^c$, the positive marginal impact of reduced switching costs on the welfare effects of systematic bailout policies increases with the profitability of the credit market.
Proof. Appendix I. □

The result of Proposition 4 stems from the crossed impact of the credit profitability and the level of switching costs on the market expansion effect $\Delta u$, namely a marginal rationing effect. Indeed, from the perspective of a no-bailout policy, it becomes more valuable to decrease switching costs in order to gain additional outsider consumers when the marginal loss due to rationing is high.

Also, if the return on investment is high enough, the difference in switching costs $\Delta s$ is decreasing in the level of switching costs, while it is increasing otherwise. Indeed, from Lemma 8, if the return on investment is low, a no-bailout policy will benefit from lower aggregate switching costs, as few outsiders have incentives to ask for credit relative to a systematic bailout environment. Thus, any decrease in switching costs would lessen this advantage. On the contrary, when the return on investment is high, $\Delta s$ is negative and any decrease in switching costs improves social welfare and favors a no-bailout policy.

A small countervailing effect arises when one also considers the difference in transportation costs $\Delta t$. When the return on investment is high, many outsiders ask for credit to the remaining bank, and a decrease in switching cost increases this demand. However, when the return on investment is low, a decrease in switching cost mostly leads to an increase in the population switching under a systematic bailout.

5.4 Policy implications

We now derive some policy implications from the model, both ex ante and ex post.

Ex ante, a credible systematic bailout always minimizes interest rates. Indeed, the alternative liquidation policy no longer guarantees banks a future profit on their investments. An interesting issue is whether the introduction of complementary resolution instruments to liquidation (bail-in, long-term liquidity requirements) would change this statement. We showed that banks do not compete for a pool of new credit relationships if they expect to be strongly financially constrained in case of crisis, even if they survive the shock. The overall effect of a change in resolution policy on credit prices depends on whether it provides banks with sufficient liquidity buffers.

The model also implies that a removal of systematic bailout policies provides a government with incentives to design measures aiming at reducing switching costs. Under a no-bailout policy, the positive effect of high switching costs on discount pricing declines. Also, a decrease in switching costs benefits old borrowers under normal competition, because it enables more consumers to benefit from a poaching rate, while remaining insiders are less locked-in.33 Therefore, any policy aiming at reducing switching costs reduces the trade-off between contesting banks’ rents and preserving competition for new borrowers.

Ex post, the effect of a bank failure on interest rates and welfare depends heavily on a reliable access to stable funding sources, but also on credit market conditions, from profitability to borrowers’ mobility. First, the model shows that public support may be justified if the banking sector would otherwise cut access to credit. However, if remaining banks are not very constrained in their credit supply, the pro-competitive argument proves to be a poor defense of bailout, especially in markets where relationship lending prevails like lending to SMEs. In this case, the ex post positive spillovers of a bailout on competition may be overestimated. Borrowers may have little to gain from competition when there is little profitability of bank credit (for instance because of poor growth perspectives) or when the cost of switching is high. Under such circumstances, a decrease in competition in credit markets leaves a large number of borrowers unaffected or better-off. This intuition provides for instance a novel perspective on the current attempts to define the critical

$^{33}$Formally, we have $d(x_{k}^{i}r_{k}^{i})^{du}/ds > -d(x_{k}^{o}r_{k}^{o})^{du}/ds$ because $(r_{k}^{i})^{du} > (r_{k}^{o})^{du}$ and $(x_{k}^{i})^{du} > (x_{k}^{o})^{du}$. 

20
functions of a bank under resolution, such as lending to SMEs.

6 Conclusion

In this article, we analyzed the competitive externalities exerted ex ante and ex post by a systematic bailout on credit markets, using a no-bailout policy as counterfactual. A systematic bailout appears to benefit borrowers ex ante. Its impact on borrowers ex post depends the level of stable funding sources for banks, the profitability of the credit market and the level of switching costs. A bailout has ex post positive effects on the credit market, through a better market coverage and a larger set of choices for consumers. However, these advantages are reduced when switching costs are high and the profitability of the market is low. Under these conditions, in average, insider consumers have little to gain from the preservation of competition, because rationing on the credit market might decrease their interest rate when there are high switching costs.

In the future, further research is needed to understand how our results could be modified by other frictions on the credit market such as asymmetric information. Our work could also be enriched by analyzing a richer set of regulatory instruments to resolve bank failures. Finally, empirical tests of the competitive effects of bailout should include measures of switching costs and credit constraints in retail banking markets.

References


Appendix

Appendix A - Market coverage and profit-maximizing interest rates
Appendix A1 - Market coverage at \( t = 2 \) under unconstrained competition  
At the equilibrium at \( t = 2 \) under unconstrained competition, poaching occurs for both banks if \((\bar{x}_A)^{du} < (\bar{x}_B)^{du}\). Also, the market is covered if the indifferent consumer between staying with one bank and switching derives a positive utility from borrowing, \( i.e. \ p(\rho - (r^1_A)^{du}) - t(\bar{x}_A)^{du} > 0 \) and \( p(\rho - (r^1_B)^{du}) - t(1 - (\bar{x}_B)^{du}) > 0 \). Replacing for \((r^1_A)^{du}\) and \((r^1_B)^{du}\) given in (3) and (4), these two conditions are written as \(|r^1_B - r^1_A| < (t - s)/2p\) and \( R > c + t + s/2 + p(|r^1_A - r^1_B|)/2\). In a symmetric equilibrium, those two inequalities are true from (A1) and (A5).

Appendix A2 - Market coverage at \( t = 1 \) at equilibrium  
From Proposition 3, the first-period interest rates are higher under a no-bailout policy than under a systematic bailout. To prove the market coverage at \( t = 1 \) under a no-bailout or a systematic bailout policy, it is sufficient to show that the indifferent consumer \( \bar{x}_D \) between both banks at \( t = 1 \) derives a positive utility from borrowing under a no-bailout.

From Eq. (1) and since \( r^1_A = r^1_B \) from Proposition 2, the indifferent consumer derives a utility from borrowing at \( t = 1 \) equal to \( p(\rho - r^1_A) - t/2 \). From (A1)-(A6), \( \pi_{mc} \) the discount given in Proposition 3 is positive for any \( \lambda_m \in (1/2, 1) \). Also, from (A4), we have \( \delta \phi > 1/2 \). Therefore, \( r^1_A \) is always lower than \( (c + t)/p - \sigma_{du}/2 \), where \( \sigma_{du} = 2s/3p \). The utility of the indifferent consumer at \( t = 1 \) is higher than \( R - (c + 3t/2 - s/3) \), which is positive from (A5).

To conclude, the indifferent borrower derives a positive utility at \( t = 1 \) under a no-bailout policy for all \((R, t, s, \phi, \delta, \pi_{mc}, \lambda_m)\) satisfying to Assumptions (A1)-(A6). This implies that at \( t = 1 \) the market is covered under both policies.

Appendix A3 - Equilibrium under financial constraints at the second stage - Preliminaries  
From Assumption (A2), we know that if both firms are active and constrained, they always act as local monopolies, such that the only difference between the constrained duopoly and the monopoly case is the severity of the financial constraint. Therefore, we will first provide conditions for a (local) monopoly \( A \) with a general financial constraint \( \lambda \in (0, 1) \) to be constrained on its insider and outsider markets. We will then apply our results to the relevant cases under constrained duopoly (Appendix A3.a) and monopoly (Appendix A3.b).

We denote by \( (\bar{x}^i_A) = p(\rho - r^i_A)/t \) (resp. \( (\bar{x}^o_A) = (p(\rho - r^o_A) - s)/t \)) the insider (resp. outsider) consumer indifferent between borrowing from bank \( A \) and not borrowing. Let \( (\bar{x}^i_A) \) (resp. \( \lambda_A \)) denote the profit-maximizing value of \( (\bar{x}^i_A) \) (resp. \( (\bar{x}^o_A) \)) if a bank does not face any constraints on its insider (resp. outsider) market.

Let assume first that \( (\bar{x}^i_A) < \min \{\bar{x}_D, \lambda\} \).

In that case, bank \( A \) is able to maximize its profit on its insider market given by

\[
(\pi^i_A) = (\bar{x}^i_A)(p r^i_A - c).
\]  
(12)

Taking the derivative of (12) with respect to \( r^i_A \), we find that bank \( A \) maximizes its profit by setting \( r^i_A = (R + c)/2p \). Replacing for \( r^i_A \) into \( (\bar{x}^i_A) \), we find that \( (\bar{x}^i_A) = (R - c)/2t \). Therefore, the inequality characterizing this case is given by \( (R - c)/2t < \min \{\bar{x}_D, \lambda\} \). This is impossible for \( \lambda = \lambda_d \) from (A2) and (A5). Moreover, we show in Appendix B2 that Case 1 is impossible at the equilibrium of the game. In what follows, we derive the equilibrium if \( (R - c)/2t > \min \{\bar{x}_D, \lambda_m\} \).

Let now assume that \( \lambda_A < \max \{\bar{x}_D, \lambda\} \).

In that case, bank \( A \) is able to maximize its profit on its outsider market given by

\[
(\pi^o_A) = (\bar{x}^o_A - \bar{x}_D)(p r^o_A - c).
\]  
(13)
Taking the derivative of (13) with respect to \( r_A^i \), we find that it sets \( r_A^i = (R + c - t\bar{x}_D - s)/2p \) at the equilibrium. Replacing for \( r_A^i \) into \( \hat{\lambda}_A \), we find that \( \hat{\lambda}_A = (R - c + t\bar{x}_D - s)/2t \).

We assume that banks use poaching at the second period. From Appendix A1, a necessary and sufficient condition for banks to use poaching at the second period is that \( \pi_2^A - r_A^1 < (t - s)/2p \). First, this implies that \( \hat{\lambda}_A > \lambda_d \) from (A3) and (A5), such that only a monopoly may be unconstrained on its outsider market. Second, this implies that \( \hat{\lambda}_A > \bar{x}_D \) from (A5).

**Appendix A3.a - Equilibrium at the second stage under constrained duopoly** If the constraint is severe (i.e., \( \lambda_d \leq \bar{x}_D \)), since the efficient credit rationing rule applies, bank \( A \) serves only its insiders. Bank \( A \) sets \( (r_A^2)^{dc} = (R - t\lambda_d)/p \), and it makes a profit

\[
(\pi_A^2)^{dc} = \lambda_d(R - c - t\lambda_d). \tag{14}
\]

If the constraint is softer (i.e., \( \lambda_d \geq \bar{x}_D \)), bank \( A \) may serve both its insiders and outsiders. The bank chooses the interest rate \( (r_A^2)^{dc} = (R - t\bar{x}_D)/p \) for its insiders and the interest rate \( (r_A^2)^{dc} = (R - s - t\lambda_d)/p \) for its outsiders, respectively. In that case, bank \( A \) makes a profit

\[
(\pi_A^2)^{dc} = \lambda_d(R - c - t\lambda_d) + (\lambda_d - \bar{x}_D)(t\bar{x}_D - s). \tag{15}
\]

**Appendix A3.b - Equilibrium at the second stage under monopoly A** If the constraint is tight (i.e., \( \lambda_m \leq \bar{x}_D \)), bank \( A \) is constrained to serve only its insiders at a price \( (r_A^i)^{mc} = (R - t\lambda_m)/p \). In that case, it makes a profit given by

\[
(\pi_A^2)^{mc} = (\lambda_m)(R - c - t\lambda_m). \tag{16}
\]

If \( \lambda_m \geq \bar{x}_D \), bank \( A \) serves its entire insider market \( \bar{x}_D \) at a rate \( (r_A^i)^{mc} = (R - t\bar{x}_D)/p \). Two cases may be defined depending on whether the monopoly is financially constrained or not on its outsider market, i.e. if it cannot serve optimally its outsider market. Bank \( A \) is constrained by the number of outsiders it can serve in the market if \( \lambda_m \leq \hat{\lambda}_A \), where \( \hat{\lambda}_A \equiv (R + t\bar{x}_D - c - s)/2t \). In that case, it chooses a price \( (r_A^i)^{mc} = (R - s - t\lambda_m)/p \) for outsiders. It makes a profit given by

\[
(\pi_A^2)^{mc} = \lambda_m(R - c - t\lambda_m) - (\lambda_m - \bar{x}_D)(s - t\bar{x}_D). \tag{17}
\]

If \( \lambda_m \geq \bar{x}_D \) and \( \lambda_m \geq \hat{\lambda}_A \), bank \( A \) is unconstrained on its insider and outsider markets. In that case, it chooses a price \( (r_A^i)^{mu} = (R + c - t\bar{x}_D - s)/2p \) for its outsider consumers. Its profit is given by

\[
(\pi_A^2)^{mu} = \bar{x}_D(R - c - t\bar{x}_D) + (R - c - t\bar{x}_D - s)^2/4t. \tag{18}
\]

**Appendix B - First period competition under uncertainty**

**Appendix B1 - Profit maximization at the first period under a systematic bailout policy** At the first period, each bank \( k \) maximizes the expected discounted value of its profit given by (6) where \( (\pi_k^2)^{du} \) is given by Eq. (5) and \( \pi_k^1 \) is given by Eq. (7). Solving for the first-order conditions of profit-maximization, we find that the profit-maximizing interest rates are \( (r_k^1)^{du} \), \( (r_k^2)^{du} \) and \( r_k^1 \) given in Proposition 1.  

\footnote{By symmetry, the constraint for bank \( B \) is given by \( \lambda_m \leq \hat{\lambda}_B \), with \( \hat{\lambda}_B = (R + t(1 - \bar{x}_D) - c - s)/2t \).}
Replacing for the interest rates of Proposition 1, we find that the symmetric equilibrium profits are given by

\[
\pi_k^2 = \frac{5t^2 - 4st + 2s^2}{18t},
\]

\[
\pi_k^1 = \frac{t - \delta \sigma_d}{2},
\]

and

\[
\pi_k = \frac{(t - \delta \sigma_d)}{2} - \delta \left(\frac{[s + 2t]^2 + [t - s]^2}{18t}\right).
\]

**Appendix B2 - Profit maximization at the first period if there is no bailout**

In i), we determine banks’ profits if they play a symmetric strategy at stage 1. In ii) we study whether banks have incentives to deviate from the symmetric strategy and derive the conditions under which there is a symmetric equilibrium.

i) If banks set symmetric interest rates at the first period, it must be that \( \bar{x}_D = 1/2 \) at the second period. There are five cases at \( t = 2 \). Either both banks remain active with financial constraints, both banks remain active without financial constraints, only bank A remains active, only bank B remains active or both banks exit the market.

At the first period, each bank \( k \) maximizes the expected discounted value of its profit given by

\[
\pi_k = \pi_k^1 + \delta E(\pi_k^2),
\]

where \( \pi_k^2 \) is bank \( k \)'s profit at the second period. The profit of each bank \( k \) at the second period depends on the scenario. With probability \( \phi \), both banks remain active at the second period without financial constraints. In that case, their profit is given by Eq. (5). With probability \( (1 - \phi)q_{dc} \), both banks remain active at the second period with financial constraints and make a profit \( (\pi_k^2)_{dc} \) given in Eq. (14). With probability \( (1 - \phi)q_{mc} \), only bank A remains active at the second period. In that case, since \( \lambda_m > 1/2 \), in a symmetric equilibrium, bank A cannot serve its entire outsider market. If \( \lambda_m \leq \lambda^c_m \), its profit is given by \( (\pi_A^2)_{mc} \) in Eq. (15). If \( \lambda_m \geq \lambda^c_m \), bank A makes a profit equal to \( (\pi_A^2)^m \) in Eq. (16). In cases where only bank A remains active, bank B makes zero profit. Similarly, the symmetric case holds if only bank B remains active at the second period. Finally, if both banks exit the market, they make zero profit.

If \( \lambda_m \leq \lambda^c_m \), solving for the first-order conditions of banks’ profit-maximization, we find that if there is a symmetric equilibrium, each bank \( k \) charges the interest rate given by

\[
r_k^1 = \frac{c + t}{p} - \delta (\phi \frac{2s}{3p} + (1 - \phi)q_{mc} \frac{s - t(1 - \lambda_m)}{p}).
\]

If \( \lambda_m \geq \lambda^c_m \), solving for the first-order conditions of banks’ profit-maximization, we find that if there is a symmetric equilibrium, each bank \( k \) charges the interest rate given by

\[
r_k^1 = \frac{c + t}{p} - \delta (\phi \frac{2s}{3p} + (1 - \phi)q_{mc} \frac{R - c - 3t/2 + s}{2p}).
\]

Replacing for the interest rates given by Proposition 1 in Eq. (7) and (9), we find the profits at the equilibrium of stage 1 under symmetric equilibrium. If \( \lambda_m \leq \lambda^c_m \), each bank \( k \in \{A, B\} \) makes a profit at the first period given by

\[
\pi_k^1 = \frac{t}{2} - \delta (\phi \frac{s}{3} + (1 - \phi)q_{mc} \frac{s - t(1 - \lambda_m)}{2}).
\]
The total expected profit is given by

$$\pi_k = \frac{t}{2} + \delta [\phi_s \left(5t^2 - 4st + 2s^2\right) - \frac{18}{t} + (1 - \phi_s)(q_{dc}\lambda d(R - c - \lambda dt) + q_{mc}\lambda mR - c - t(\lambda m - 1/4\lambda m) - s)]].$$

Also, at the second period, from (15), if bank $k$ is a monopoly, its profit is given by

$$(\pi_A^{\mu}) = \lambda m(R - c - t\lambda m) + (\lambda m - 1/2)(t/2 - s).$$

If $\lambda m \geq \lambda^c$, each bank $k \in \{A, B\}$ makes a profit at the first period given by

$$\pi_k = \frac{t}{2} - \delta(\phi_s + (1 - \phi)q_{mc}\left(\frac{R - c - 3t/2 + s}{4}\right)).$$

The total expected profit is given by

$$\pi_k = \frac{t}{2} + \delta \left(\frac{5t^2 - 4st + 2s^2}{18} + (1 - \phi)(q_{dc}\lambda d(R - c - \lambda dt) + q_{mc}\left(\frac{(R - c - s)^2 + 3)t^2}{4}\right))\right].$$

Also, at the second period, from (16), if bank $k$ is a monopoly, its profit is given by

$$(\pi_A^{\mu}) = (R - c - t/2)/2 + (R - c - t/2 - s)^2/4t.$$

ii) We now determine whether banks have incentives to deviate from the symmetric strategies in which $r_A^1 = r_B^1$. Suppose that bank $A$ charges $r_A^1 < r_B^1$ such that $\bar{x}_D > 1/2$. In this case, a Necessary Deviation Condition "NDC" for an asymmetric equilibrium to exist is that bank $A$ has more incentives than bank $B$ to lower marginally its first period interest rate at the symmetric equilibrium, that is we have

$$\left.\frac{\partial \pi_A}{\partial r_A^1}\right|_{r_A^1=r_B^1} < \left.\frac{\partial \pi_B}{\partial r_B^1}\right|_{r_A^1=r_B^1}. \quad \text{(NDC)}$$

The asymmetric market structure at $t = 2$ is possible if the NDC holds. In what follows, we analyze the different cases that may arise under asymmetric prices, depending on the expected competition at $t = 2$. Since $\bar{x}_D > 1/2$, for a given shock and a level of funding $\lambda d$ or $\lambda m$, bank $B$ always faces either an equal or a lower financial constraint than bank $A$. For instance, if, in expectation of monopoly, bank $A$ is constrained on some insiders (i.e $\lambda m < \bar{x}_D$), then bank $B$ is either constrained on insiders (i.e $\lambda m < 1 - \bar{x}_D$), on outsiders (i.e $\lambda m \in (1 - \bar{x}_D, \bar{\lambda}_B)$) or none (i.e $\lambda m > \bar{\lambda}_B$). Since no profitable deviation exists if the expected market structure at $t = 2$ is symmetric, we focus on situations where the asymmetric pricing lowers the financial constraint of bank $B$ relative to $A$ at $t = 2$.

Case 1. Let assume $(R - c)/2t > \bar{x}_D$, such that both banks are always constrained on their insider markets by their market shares.

Case 1a: Bank $A$ expects to face a constraint $\lambda d$ on its insider market (i.e $\lambda d < \bar{x}_D$). If bank $B$ expects to be constrained on its outsider market (i.e $\lambda d \in (1 - \bar{x}_D, \bar{\lambda}_B)$), the NDC is equivalent to $s < t(1 - \lambda d)$. This is impossible if $s \geq t(1 - \lambda d)$. If bank $B$ expects to be unconstrained (i.e $\lambda d > \bar{\lambda}_B$), the NDC is equivalent to $R \leq c + 3t/2 - s$, which contradicts (A5).

Case 1b: Bank $A$ expects to face a constraint $\lambda d$ on its outsider market (i.e $\lambda d < \bar{\lambda}_A$). If bank $B$ expects to be unconstrained (i.e $\lambda d > \bar{\lambda}_B$), the NDC is equivalent to $R \leq c + 2t\lambda d - t - s - (r_B^1 - r_A^1)/2$. Since $r_A^1 < r_B^1$, the NDC contradicts the condition on $\lambda d < \bar{\lambda}_A$: in this case, the constraint for bank $A$ to be constrained on its outsider market is binding. When $\lambda d = \bar{\lambda}_A$, both banks are unconstrained, and the equilibrium is symmetric.
The same results hold in expectation of a financial constraint $\lambda_m$. Since $\lambda_m > \lambda_d$ from (A2) and (A3), the condition $s \geq t(1 - \lambda_d)$ written in 1.a is more constraining than $s \geq t(1 - \lambda_m)$.

To conclude for case 1, we showed that the NDC fails in case 1.a from (A6), i.e $s \geq t(1 - \lambda_d)$. In case 1.b, we showed that if the NDC is feasible, then the market structure of case 1.b is impossible. Alternatively, if this market structure is possible, it does not provide incentives for bank $A$ to set lower interest rates. Therefore, this market structure is feasible if and only if its combination with other expected market structure provides incentives for $r^1_A < r^1_B$. By case 1.a, no other expected market structure does provide incentives for bank $A$ to set lower interest rates, such that the market structure in case 1.b is impossible.

Case 2. Let assume $(R - c)/2t < \bar{x}_D$, such that bank $A$ is not constrained on its insider market by its first-period market share. In that case, a monopoly can always maximize its profit on its insider market, such that its profit on its insider market no longer depends on $\bar{x}_D$. Therefore, in expectation of a monopoly, bank $A$’s profit is either a negative function of $\bar{x}_D$ (through its negative effect on profit on the outsider consumers) or it is independent of $\bar{x}_D$ (if no outsider is served), while the incentives for bank $B$ and in expectation of other market structures remain unchanged. Therefore, the NDC is strictly harder to meet than if $(R - c)/2t > \bar{x}_D$. Therefore, the necessary condition for an asymmetric equilibrium to exist is never verified. Finally, since $(R - c)/2t > 1/2$ from (A5), Case 2 does not arise in a symmetric equilibrium.

We conclude. Under (A1)-(A5), if $s \geq t(1 - \lambda_d)$, there do not exist a combination of expected market structures which is both possible and gives bank $A$ incentives to set lower interest rates than bank $B$. Therefore, only a symmetric equilibrium exists.

Appendix C: Comparison of interest rates under both bailout regimes at $t = 1$ and $t = 2$

Appendix C1 - Comparison of interest rates at $t = 1$ From Proposition 1 and 2, the difference between the first period interest rates under a systematic bailout policy and a no-bailout is given by

$$\Delta_1 = (\phi - 1)\sigma_d + (1 - \phi)q_{mc}\sigma_{mc}.$$ 

Replacing for $\sigma_d$ and $\sigma_{mc}$ given in Proposition 1 and 2, if $\lambda_m \leq \lambda_m^c$, we have $\Delta_1 \leq 0$ if and only if $s(2 - 3q_{mc}) > -3q_{mc}(1 - \lambda_m)t$. Since $2q_{mc} + qdc + qo = 1$, we have $q_{mc} \leq 1/2$. Therefore, since $2 - 3q_{mc} < 0$ and $1 - \lambda_m > 0$, the condition $\Delta_1 \leq 0$ is always verified. Hence, if $\lambda_m \leq \lambda_m^c$, a systematic bailout policy reduces the interest rate at the first period.

If $\lambda_m > \lambda_m^c$, we have $\Delta_1 \geq 0$ if and only if $R \geq c + 3t/2 + s(4 - 3q_{mc})/3q_{mc}$. Note that $\lambda_m > \lambda_m^c$ can be written as $R \leq c + 2t\lambda_m - t/2 + s$. Therefore, $\Delta_1 \geq 0$ is compatible with the inequality $\lambda_m > \lambda_m^c$ if and only if $c + 4t\lambda_m - t/2 \geq c + 3t/2 + s(4 - 3q_{mc})/3q_{mc}$, that is, if and only if $s(1 - 2/3q_{mc}) > t(1 - \lambda_m)$. Since $q_{mc} \in (0,1/2)$, we have $s(1 - 2/3q_{mc}) < 0$. Therefore, the inequality $\Delta_1 \geq 0$ is not compatible with $\lambda_m > \lambda_m^c$. Hence, if $\lambda_m > \lambda_m^c$, we have $\Delta_1 < 0$ and a systematic bailout policy also reduces the interest rate at the first period.

Appendix C2 - Comparison of interest rates at $t = 2$. We compare the interest rates on the credit market under a no-bailout policy and a systematic bailout policy.

i) If both banks remain active at the second period, from Propositions 1 and 2, we have

$$(r^i_k)^{dc} - (r^i_k)^{du} = (R - c - t(2 + 3\lambda_d))/3 - s/3)/p. \quad (17)$$

Therefore, we have $(r^i_k)^{dc} \leq (r^i_k)^{du}$ if and only if $\lambda_d \geq \lambda_i \equiv (R - c - 2t/3 - s/3)/t$. From (A3) and (A6), we assume $\lambda_d \in (1 - s/t, 1/2)$. We have $\lambda_i$ increasing in $R$, such that $\lambda_i \in (1 - s/t, 1/2)$ is
equivalent to \( R \in I \) with \( I = (c + 5t/3 - 2s/3, c + 7t/6 + s/3) \). We now determine if this condition is possible under \( R \geq R \) given in (A5).

Case 1. \( s \in (t/2, 3t/5) \)
In this case, we have \( R = c + 3t/2 - s/3 \), and \( R \in I \) equivalent to \( 2s > t \). This is always the case from (A2) and (A6). This implies that \( R > c + 5t/3 - 2s/3 \) under (A1)-(A6).

Case 2. \( s \in (3t/5, t) \)
In this case, we have \( R = c + t + s/2 \), and \( R < c + 7t/6 + s/3 \) equivalent to \( t > s \) which is exactly (A1). Therefore, \( R \in I \) under (A1)-(A6).

For convenience, we finally write \( R < c + 7t/6 + s/3 \) as \( s \geq s_r \), with \( s_r = 3(R - c) - 7t/2 \).
To conclude, we have \( (r_k^i)_{dc} \) if \( s \geq s_r \) and \( \lambda_d \geq \lambda_i \) for all \( R, \lambda_d, s, t \) satisfying to (A1)-(A6). In all other cases, we have \( (r_k^i)_{dc} \) if \( s < s_r \).

ii) If only one bank remains on the market, our previous comparisons on prices for insider consumers hold, except that now \( \lambda_d = 1/2 \). Replacing for \( \lambda_d = 1/2 \) into (17), we have that \( (r_k^i)^{mc} - (r_k^i)_{dc} = (R - c - 7t/6 - s/3) \). Therefore, if \( s \geq s_r \), we have \( (r_k^i)^{mc} < (r_k^i)_{dc} \) for all \( R, \lambda_d, s, t \) satisfying to (A1)-(A6), and \( (r_k^i)^{mc} > (r_k^i)_{dc} \) otherwise.

iii) We now turn to the comparison of the interest rates for outsiders if only one bank remains in the market with financial constraints. Let \( \lambda_m \equiv (R - c + t/2)/2t \) defined in paragraph 4.2.2. From Propositions 1 and 2, we have

\[
(r_k^o)_{mcr} - (r_k^o)_{du} = (R - c - t(1 + 3\lambda_m)/3 - 2s/3)/p,
\]
where \( \lambda_m \in (1/2, \min \{1, \lambda_m^c\}) \) from (A3). We have \( (r_k^o)_{mcr} \leq (r_k^o)_{du} \) if and only if \( \lambda_m > \lambda_o \), where \( \lambda_o \equiv (R - c - t/3 - 2s/3)/t \). Therefore, there are three cases. In case (iii-a), we have \( \lambda_o \leq 1/2 \). In case (iii-b), we have \( \lambda_o \in (1/2, \min \{1, \lambda_m^c\}) \). In case (iii-c), we have \( \lambda_o \geq \min \{1, \lambda_m^c\} \).

(iii-a) We have \( \lambda_o \leq 1/2 \) if and only if \( R \geq c + 3t/2 + s \). We have \( \lambda_o \leq 1 \) if and only if \( R \leq c + 4t/3 + 2s/3 \). This contradicts \( R \geq c + 3t/2 + s \), such that this case is impossible. We have \( \min \{1, \lambda_m^c\} = \lambda_m^c \) if and only if \( R \leq c + 3t/2 + s \). We have \( \lambda_o \leq \lambda_m^c \) if and only if \( R \geq c + 7t/6 + s/3 \). In Appendix C2), we proved that this inequality does not contradict (A5).

(iii-b) If \( \lambda_o \geq \min \{1, \lambda_m^c\} \), then \( \lambda_o > \lambda_m \) and we have \( (r_k^o)_{mcr} \geq (r_k^o)_{du} \).
For convenience, we finally write \( R < c + 7t/6 + s/3 \) as \( s \geq s_r \), with \( s_r = 3(R - c) - 7t/2 \).
To conclude, if \( s \geq s_r \) and \( \lambda_m > \lambda_o \), we have \( (r_k^o)_{mcr} < (r_k^o)_{du} \) for all \( R, \lambda_m, s, t \) satisfying to (A1)-(A5). In all other cases, we have \( (r_k^o)_{mcr} > (r_k^o)_{du} \).

iv) Finally, if the monopoly is not financially constrained, from Propositions 1 and 2, we have

\[
(r_k^o)^{mu} - (r_k^o)_{du} = (R - c - 7t/6 - s/3)/2p.
\]
We note that \( 2((r_k^o)^{mu} - (r_k^o)_{du}) = (r_k^o)^{mc} - (r_k^o)_{du} \). Therefore, if \( s \geq s_r \), we have \( (r_k^o)^{mu} \leq (r_k^o)_{du} \) for all \( R, \lambda_m, s, t \) satisfying to (A1)-(A6). Otherwise, we have \( (r_k^o)^{mu} \geq (r_k^o)_{du} \).

Appendix D : Comparison of the effect of a systematic bailout between insiders and outsiders Let \( \Delta^{i/o} = \Delta^i/\Delta^o \), with \( \Delta^i = (r_k^i)^{mc} - (r_k^i)_{du} \) and \( \Delta^o = (r_k^o)^{mc} - (r_k^o)_{du} \), assuming \( \Delta^o \neq 0 \).

From Proposition 2, if \( \lambda_m \geq \lambda_m^c \), we have \( \Delta^{i/o} = 2 \), such that the effect of a systematic bailout impacts twice as more the interest rates paid ex post by insider consumers as the interest rates paid by outsiders.
From Proposition 2, if $\lambda_m < \lambda^c_m$ and $R \neq c + t\lambda_m + t/3 + 2s/3$, we have

$$\Delta^{i/o} = \frac{\Delta^i}{\Delta^o} = \frac{R - c - 7t/6 - s/3}{R - c - t\lambda_m - t/3 - 2s/3}.$$

In what follows, we determine the conditions such that $|\Delta^{i/o}| < 1$ if $\lambda_m < \lambda^c_m$.

Case a. If $\Delta^i > 0$, we have $|\Delta^{i/o}| < 1$ if either $\Delta^o > \Delta^i$ or $\Delta^o < -\Delta^i$. The inequality $\Delta^i > 0$ is equivalent to $R > c + 7t/6 + s/3$. From (A1), (A2) and (A6), we have $c + 7t/6 + s/3 > \max\{c + t + s/2, c + 3t/2 - s/3\}$. Since $\lambda_m^c = (R - c + t/2 - s)/(2t)$, the inequality $\lambda_m < \lambda^c_m$ implies that $R > c - t/2 + s - 2t\lambda_m$. Therefore, if $c - t/2 - 2t\lambda_m + s > c + 7t/6 + s/3$, we can conclude that $\Delta^i > 0$ is always true for all $R$ satisfying to $\lambda_m < \lambda^c_m$. Since $c - t/2 - 2t\lambda_m + s > c + 7t/6 + s/3$ is equivalent to $\lambda_m > 5/6 - s/3t$, we conclude that $\Delta^i > 0$ for all $R$ satisfying to $\lambda_m < \lambda^c_m$ if $\lambda_m > 5/6 - s/3t$. If $\lambda_m < 5/6 - s/3t$, we have $\Delta^o > \Delta^i$. We now show that when $\Delta^i > 0$, then $\Delta^o < -\Delta^i$ is impossible. The inequality $\Delta^o < -\Delta^i$ is equivalent to $R < c + 3t/4 + t\lambda_m/2 + s/2$. This condition is compatible with $\Delta^i > 0$ if $\lambda_m > 5/6 - s/3t$. Also, it is compatible with $\lambda_m < \lambda^c_m$ if $\lambda_m < 5/6 - s/3t$. Therefore, the inequality $\Delta^o < -\Delta^i$ never holds. To conclude for this case, we have $|\Delta^{i/o}| < 1$ if $R > c + 7t/6 + s/3$ and $\lambda_m < 5/6 - s/3t$.

Case b. We have $|\Delta^{i/o}| < 1$ if $\Delta^i < 0$ and either $\Delta^i > \Delta^o$ or $\Delta^o > -\Delta^i$. The inequality $\Delta^i < 0$ is equivalent to $R < c + 7t/6 + s/3$. We already showed that it can only hold if $\lambda_m < 5/6 - s/3t$. Therefore, we cannot have both $\Delta^i < 0$ and $\Delta^i > \Delta^o$. The inequality $\Delta^o > -\Delta^i$ is equivalent to $\lambda_m < 2(R - c - 3t/4 - s/2)/t$. The latter inequality is compatible with $\Delta^i < 0$ if $\lambda_m < 5/6 - s/3t$.

To conclude for case b, we have $|\Delta^{i/o}| < 1$ if $R < c + 7t/6 + s/3$, and either $\lambda_m < 3/2 - 5s/(3t)$ or both $\lambda_m < 5/6 - s/(3t)$ and $R > c + 3t/4 + t\lambda_m/2 + s/2$, i.e. $\lambda_m < (R - c - 3t/4 - s/2)/2t$.

To sum-up cases a and b, if $\lambda_m < \min\{5/6 - s/(3t), (R - c - 3t/4 - s/2)/2t\}$, we have $|\Delta^{i/o}| < 1$.

Appendix E: Comparison of consumer surplus under a systematic and a no-bailout policy at $t = 2$. We denote by $u(x, r) = p(\rho - r) - tx$ the utility obtained by a borrower located at a distance $x$ from its bank who borrows at an interest rate $r$.

- Both banks remain active at $t = 2$ with no bailout:

Assume that both banks remain in the market at $t = 2$ if there is no bailout. In a symmetric equilibrium, we have $CS_2^{dc} - CS_2^{du} =$

$$2\int_0^{\lambda_d} u(x, (r^i_k)^{dc}) dx - 2\int_0^{\tilde{\lambda}_d} u(x, (r^i_k)^{du}) dx - \int_{(\tilde{\lambda}_d)^{du}}^{\tilde{\lambda}_d} u(1 - x, (r^o_k)^{du}) - s) dx.$$

Replacing for $(r^i_k)^{du}$ and $(r^o_k)^{du}$ given by Proposition 1 and $(r^i_k)^{dc}$ given by Proposition 2, we have

$$CS_2^{dc} - CS_2^{du} = -R + c + t\lambda_d^2 + \frac{31t^2 + 16st - 2s^2}{36t}.$$

Therefore, we have $CS_2^{dc} - CS_2^{du} \geq 0$ if and only if $R \leq R_{cs} = c + t\lambda_d^2 + (31t^2 + 16st - 2s^2)/36t$.

Case a: $R = c + 3t/2 - s/3$ (i.e., $s \leq 3t/5$)

We have $R_{cs} \geq R$ if and only if $t^2(36\lambda_d^2 - 23) + 28st - 2s^2 \geq 0$. From (A1), this polynomial function of degree 2 in $\lambda_d$ is positive if and only if $\lambda_d \in [0, \lambda_d^c]$, where $\lambda_d^c = \sqrt{23t^2 - 28st + 2s^2}/6t$. We have $\lambda_d^c \in [0, 1/2]$ if $s \in [t(7 - \sqrt{42}), 3t/5]$. Otherwise, we have $\lambda_d^c < 0$.  

30
Case b: $R = c + t + s/2$

We have $R_{cs} \geq R$ if and only if $t^2 (36\lambda_d^2 - 5) - 2st - 2s^2 \geq 0$. From (A1), this polynomial function of degree 2 in $\lambda$ is positive if and only if $\lambda_d \in [0, \lambda_{cs}^2]$, where $\lambda_{cs}^2 = \sqrt{5t^2 + 2st + 2s^2}/6t$. We have $\lambda_{cs}^2 \in [0, 1/2]$.

To conclude, we have $CS_{2c}^{du} \geq CS_{2d}^{du}$ only if $R \leq R_{cs}$, $s \geq t(7 - \sqrt{42})$ and $\lambda_d \geq \lambda_{cs}$, where $\lambda_{cs} = \max \{\lambda_{cs}^1, \lambda_{cs}^2\}$. Otherwise, we have $CS_{2c}^{du} \leq CS_{2d}^{du}$.

- One bank remains active at $t = 2$ if there is no bailout.

In a symmetric equilibrium, we have

$$CS_{2c}^{mc} - CS_{2d}^{du} = 2 \int_0^{\bar{x}_D} u(x, (r_k^i)^{mc}) dx + \int_{\bar{x}_D}^{\lambda_{mc}} u(x, (r_k^0)^{mc}) - s) dx$$

$$-2 \int_0^{(\bar{x}_A)^{du}} u(x, (r_k^i)^{du}) dx - 2 \int_{(\bar{x}_A)^{du}}^{\bar{x}_D} u(1 - x, (r_k^0)^{du}) - s) dx,$$

where $(r_k^i)^{du}$ and $(r_k^0)^{du}$ given in Proposition 1, $(r_k^i)^{mc}$ given in Proposition 2, $(r_k^0)^{mc} = (R - t\lambda_{mc} - s)/p$ and $\lambda_{mc} = \min \{\lambda_m, \lambda_m^c\}$ depending on whether or not the monopoly is constrained or not on its outsider market. From (A3), we have $\lambda_{mc} \leq \min \{1, \lambda_m^c\}$. We have $\partial(CS_{2c}^{mc} - CS_{2d}^{du})/\partial \lambda_{mc} = t(\lambda_{mc} - 1/2) \geq 0$ from (A3) and (A5). We now prove that even at the maximum of $\lambda_{mc}$ given by $\min \{1, \lambda_m^c\}$, we have $CS_{2c}^{mc} < CS_{2d}^{du}$. There are two cases.

Case a: $\min \{1, \lambda_m^c\} = 1$ (i.e. $R > c + 3t/2 + s$).

Replacing $\lambda_{mc}$ by 1 in (18), we have $CS_{2c}^{mc} - CS_{2d}^{du} |_{\lambda_m = 1} = c + 10t/9 + 4s/9 - s^2/18t - R$. This is always negative since we assume $R > c + 3t/2 + s$.

Case b: $\min \{1, \lambda_m^c\} = \lambda_m^c$ (i.e. $R < c + 3t/2 + s$). Replacing $\lambda_{mc}$ by $\lambda_m^c$ in (18), we have

$$CS_{2c}^{mc} - CS_{2d}^{du} = \frac{(R - c - 11t/2 - s)(R - c - 7t/2 - s) - (4/9)(5t + s)^2}{8t}.$$

Therefore, $CS_{2c}^{mc} - CS_{2d}^{du}$ is a convex polynomial function of degree 2 in $R$. Let $\{R_{csm}^1, R_{csm}^2\}$ the two roots of the equation $CS_{2c}^{mc} - CS_{2d}^{du} = 0$, where

$$R_{csm}^1 = c + 9t/2 + s - (1/3)\sqrt{109t^2 + 40st + 4s^2},$$

and

$$R_{csm}^2 = c + 9t/2 + s + (1/3)\sqrt{109t^2 + 40st + 4s^2}.$$ Let first prove that $R_{csm}^1 < R$. This is true if $R_{csm}^1 < c + t + s/2$, which simplifies to $(7s - t)(5t + s) > 0$. This inequality is always true from (A1) and (A6). Also, we observe that $R_{csm}^2 > R$. However, we have $R_{csm}^2 > c + 3t/2 + s$. Therefore, in Case b, we have $R \in (R_{csm}^1, R_{csm}^2)$. To conclude, we have $CS_{2c}^{mc} < CS_{2d}^{du}$ for all possible financial constraints faced by a monopoly.

Appendix F: Comparison of expected social welfare

Assume that both banks always remain active in the market. Difference in welfare $\Delta W$ is given by

$$\Delta W_{dc} = 2 \int_0^{\lambda_d} (R - c - tx) dx - \int_0^{(\bar{x}_A)^{du}} (R - c - tx) dx - \int_{(\bar{x}_A)^{du}}^{\bar{x}_D} (R - c - t(1 - x) - s) dx.$$
Replacing for \((r^i_k)du\) and \((r^o_k)du\) given in Proposition 1 into \((\bar{x}_A)du\), we have

\[
\Delta W_{dc} = (2\lambda_d - 1)(R - c) - t\lambda_d^2 + \frac{11t^2 + 8st - 10s^2}{36t}.
\]

Therefore, \(\Delta W_{dc} \) is positive if and only if

\[
R \geq R_w = c - (36t^2\lambda_d^2 - 11t^2 - 8st + 10s^2)/(36t - 72t\lambda_d).
\]

We now compare \(R_w\) to \(R\) given by (A5).

Case a: if \(s < 3t/5\), we have \(R = c + 3t^2/2 - s/3\). Therefore, the inequality \(R_w \geq R\) is equivalent to \(36t^2\lambda_d^2 - \lambda_d(108t^2 - 24st) + 43t^2 - 20st + 10s^3 \geq 0\). This inequality holds if \(\lambda_d \in [\lambda_m, 1/2] \) from (A3). Indeed, the polynomial function of degree 2 in \(\lambda_d\) admits two roots denoted by \(\lambda_w^1 = 3/2 - (1/3t)(s + (\sqrt{2}/2)\sqrt{19t^2 - 8st - 3s^2}) \in [0, 1/2] \) from (A1) and \(\lambda_w^0 > 1/2\). Also, we always have \(\lambda_d > \lambda_w^0\) given (A6) if \(\lambda_w^1 < 1 - s/t\), i.e., \(s < t(\sqrt{519/2} - 10)/11 < 3t/5\).

Case b: if \(s > 3t/5\), \(R = c + t + s/2\). The inequality \(R_w \geq R\) is equivalent to \(36t^2\lambda_d^2 - \lambda_d(72t^2 + 36st) + 25t^2 + 10st + 10s^2 \geq 0\). It admits two roots denoted \(\lambda_w^2\) and \(\lambda_w^0\), with \(\lambda_w^2 = 1 + s/2t - (\sqrt{111t^2 + 26st - s^2})/6t \in [0, 1/2] \) and \(\lambda_w^0 > 1/2\) from (A1). Therefore, \(R_w \geq R\) is equivalent to \(\lambda_d \in \{\lambda_w^2, 1/2\}\). Also, we have \(\lambda_d > 1 - s/t\) from (A6), and \(\lambda_w^2 < 1 - s/t\) if \(s < t(13 + 3\sqrt{119})/82 < 3t/5\), such that \(\lambda_w^2 > 1 - s/t\) in Case b.

To conclude, \(\Delta W_{dc} \geq 0\) only if \(R \geq R_w\) if \(s < t(\sqrt{519/2} - 10)/11\), and if both \(R \geq R_w\) and \(\lambda_d \geq \lambda_w\) with \(\lambda_w = \max\{\lambda_w^1, \lambda_w^2\}\) otherwise.

Now consider the case where there is only one bank remaining. At a symmetric equilibrium, difference in welfare \(\Delta W_{mc}\) is given by

\[
\int_0^{\lambda_{mc}} (R-c-tx)dx + \int_{1/2}^{\lambda_{mc}} (-s)dx - 2\int_0^{(\bar{x}_A)du} (R-c-tx)dx - \int_0^{(\bar{x}_D)du} (R-c-t(1-x)-s)dx,
\]

where \((r^i_k)du\) and \((r^o_k)du\) into \((\bar{x}_A)du\) given by Proposition 1 and \(\lambda_{mc} = \min\{\lambda_m, \lambda_m^c\}\). From (A3), we have \(\lambda_{mc} \leq \min\{1, \lambda_m^c\}\). We have \(\partial(\Delta W_{mc})/\partial \lambda_{mc} = R - c - s - t\lambda_{mc}\). We now prove that even at the maximum of \(\lambda_{mc}\) given by \(\min\{1, \lambda_m^c\}\), we have \(\partial(\Delta W_{mc})/\partial \lambda_{mc} > 0\) and \(\Delta W_{mc} < 0\). There are two cases.

Case a: \(\min\{1, \lambda_m^c\} = 1\) (i.e., \(R > c + 3t/2 + s\)).

We have \(\partial(\Delta W_{mc})/\partial \lambda_{mc} = R - c - t - s > 0\) since \(R > c + 3t/2 + s\). From (19), we have \(\Delta W_{mc}|_{\lambda_{mc} = 1} = -(7t^2 + 10(st + s^2))/36t < 0\). Therefore, \(\Delta W_{mc}\) always negative when a monopoly is constrained on its outsider market.

Case b: \(\min\{1, \lambda_m^c\} = \lambda_m^c\) (i.e., \(R < c + 3t/2 + s\)).

We have \(\partial(\Delta W_{mc})/\partial \lambda_{mc} = R - c - t - s > 0\) from (A5). Replacing \(\lambda_{mc}\) by \(\lambda_m^c\) in (19), we have

\[
\Delta W_{mc}|_{\lambda_{mc} = \lambda_m^c} = \frac{27(R - c - 7t/6 - s)^2 - 17t^2 - 20(st + s^2)}{36t}.
\]

Solving for \(R\), we have \(\Delta W_{mc}|_{\lambda_{mc} = \lambda_m^c} > 0\) if and only if

\[
R > R_{Wm} = c + 7t/6 + s \sqrt{17t^2 + 20(st + s^2)/(3\sqrt{3})}.
\]

We have \(R_{Wm} > c + 3t/2 + s\) equivalent to \(42t^2 + 60(st + s^2)\), such that \(R_{Wm} > c + 3t/2 + s\). Therefore, in Case B, \(R < R_{Wm}\) and \(\Delta W_{mc}\) is always negative when a monopoly is not constrained on its outsider market.
Appendix I : Cross effect of return on investment and switching costs on given in (11) equal to
\[ \Delta \text{R}_{c} \]
This equality is positive if and only if

Appendix H : Comparative statics on the expected welfare. If \( \lambda_{m} < \lambda_{m}^{c} \), we have \( E(\Delta W) \) given in (11) equal to
\[ E(\Delta W) = E(\Delta W) = \delta(1-\phi)[(\Delta W_{dc}) + 2q_{mc}(\Delta W_{mc}) - q_{0}W_{d}], \]
where \( \Delta W_{dc}, \Delta W_{mc} \) given in (10) and \( W_{d} \) is given in the comparative statics of Section 5.3. Since \( 0 < q_{dc} + 2q_{mc} < 1 \) under (A2) and (A3), we have
\[ \frac{\partial E(\Delta W)}{\partial R} = \delta(1-\phi)(-1 + 2q_{dc}\lambda + 2q_{mc}\lambda_{m}) \leq 0, \]
and from (A1), (A3) and (A6)
\[ \frac{\partial E(\Delta W)}{\partial s} = \delta(1-\phi)[-q_{mc}(2\lambda_{m} - 1) - (5s - 2t)/9t] \leq 0. \]
If \( \lambda_{m} > \lambda_{m}^{c} \), the expression of \( E(\Delta W) \) given in (11) is equal to
\[ E(\Delta W) = E(\Delta W) = \delta(1-\phi)[(\Delta W_{dc}) + 2q_{mc}(\Delta W_{m}) - q_{0}W_{d}], \]
where \( \Delta W_{m} \) given in (10). Therefore, we have
\[ \frac{\partial E(\Delta W)}{\partial R} = \delta(1-\phi)[6q_{mc}(R - c - s) - t(4 - 8q_{11}\lambda_{d} - q_{mc})]/4t. \]
This is equality is positive if and only if \( R > c - t/6 + s + 2t(1 - 2q_{dc}\lambda_{d})/3q_{mc} \). This is possible if at least \( R > c + 7t/6 + s \). From (A1), (A5) and (A6), we have
\[ \frac{\partial E(\Delta W)}{\partial s} = \delta(1-\phi)[-3q_{mc}(R - c - t/2 - s)/2t - (5s - 2t)/9t] \leq 0. \]

Appendix I : Cross effect of return on investment and switching costs on \( \Delta W_{mc} \). From the expression of \( \Delta W_{mc} |_{\lambda_{mc} = \lambda_{m}^{c}} \) obtained in Appendix F, we have
\[ \frac{\partial}{\partial R} \frac{\partial}{\partial s} \Delta W_{mc} |_{\lambda_{mc} = \lambda_{m}^{c}} = -\frac{3}{4t} \leq 0. \]
such that it is more effective to decrease switching costs when \( R \) is high.