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Platform-Merchant Competition for Selling Services

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Abstract

In this paper, we study whether a monopolistic platform prefers to impose price parity when it competes with merchants for selling services. The platform and the direct sales channel are differentiated in quality on the consumer side and in terms of efficiency. We show that the platform imposes price parity when it is highly differentiated in quality on the consumer side and that this restriction lowers the total transaction fee paid by consumers and merchants. Price parity increases the total buying price of consumers who buy from merchants who receive high benefits of selling on the platform and decreases it otherwise.

Keywords: Two-Sided Markets, Selling Channel, Price Parity, Direct Sales.

JEL Codes: E42; L1; O33.

1 Introduction

In several markets, selling services impact transaction costs and consumer perception of product quality. Retailers may strategically decide to sell directly to consumers or to outsource their selling services to a platform. For example, a florist may sell flowers directly at his physical shop or via an online marketplace. However, platforms may have enough market power to impose restrictions to retailers, such as price parity. Price parity clauses are agreements whereby the price of the product sold on a platform cannot be higher than the price available on a merchant's website. A key policy question is whether this restriction increases retail prices and reduces consumer surplus.

In this paper, we analyze a platform's incentives to impose price parity when merchants trade off between direct sales and platform sales.[1] The sales channel impacts the quality of the product sold to consumers and the (net) selling costs incurred by merchants. The two sales channels are differentiated in quality on the consumer side and in terms of efficiency on the retailer side. We show that the platform prefers to impose price parity if it is sufficiently differentiated from the direct sales channel on the consumer side. Price parity reduces the total transaction fee paid by consumers and merchants for making a transaction on the platform. However, it increases the total price buying price for consumers who buy from merchants who receive high benefits of selling on the platform.

Recently, in various industries (i.e., hotel booking, e-book, payments) and several countries, competition authorities have examined price parity clauses imposed by platforms.[2] Such restrictions may reduce consumer and merchant surplus through different mechanisms: inflation of transaction fees or retail prices, constraints on consumer choice, restrictions of merchants' strategic options. For example, several hotels like Accor have decided to renounce to sell rooms through Online Travel Agencies (OTA) given the amount of transaction fees paid to online reservation platforms.[3] Our paper enriches the existing literature by introducing the role of the quality differentiation of selling services and competition between direct sales and platform sales.

We build a model to study competition between a platform and a continuum of monopolistic merchants to market a service. The platform and the merchants do not compete on the (main) retail market but only on selling services.[4] Our decision to model monopolistic

[1] Chevalier et al. (2001) look at the opposite case, that is when brick and mortar retailers impose restrictive clauses to online platforms to prevent free riding on the merchant's site.

[2] For the online hotel booking case and price parity clauses: see the District Court Judgment in 2013 (US), the decisions by the European Commission in 2012 and 2013, national cases in the UK (OFT, 2014), Germany (BKartA,2013), and other countries in 2015 (Decision 15-D-06 France of the Competition Authority).

[3] The percentage grew of about 28 % on each transaction in the last 4 years. See the article on Le Figaro (October, 2014) : "La parade d'Accor pour résister à Booking.com".

[4] The literature on online commerce platforms refers to this model as the agency model. By contrast, when the platform buys the primary product directly from the retailer and resells it to consumers, the vertical

merchants is motivated by our focus on competition between merchants and the platform. We analyze how the degree of differentiation between selling channels impacts the total buying price of consumers when the platform imposes restrictions to merchants. The platform is differentiated from the direct sales channel both in terms of value added to the consumer buying experience and in terms of cost savings offered to merchants. On the consumer side, we assume that the platform adds value to a consumer's purchase by offering a selling service of higher quality than the direct sales channel.[5] On the seller side, the platform brings heterogeneous selling benefits to merchants. Indeed, in some markets, merchants enjoy higher benefits of direct sales because they collect insightful information on their customers or incur lower accounting costs. In other markets, they enjoy higher benefits of selling on the platform thanks for instance to a better security of transactions or quicker delivery services.

At the first stage of the game, the platform decides whether or not to impose price parity. Such a restriction implies that merchants are forbidden to price discriminate across sales channels. The platform also determines the level of transaction fees charged to consumers and merchants, respectively. At the second stage of the game, merchants have to decide whether to offer their product on the platform or sell it directly to consumers, or on both channels. At the last stage of the game, consumers decide whether or not to buy the product and choose their preferred sales channel. If two sales channels are available, consumers trade off between two versions of the same product (i.e., one version for each selling mode) that they perceive as differentiated in quality.

In the first part of the paper, we determine consumer demand and merchant profits according to the number of sales channels that are available. If the platform does not impose any restriction, merchants are allowed to price discriminate across selling channels and obtain higher profits of doing so. However, if the latter obtain very high benefits of selling through

structure is said to be organized according to the wholesale model. Nevertheless, even if we adopt an agency model, in our paper, sales revenue is not split between suppliers and retailers according to endogenously pre-determined shares and the merchant keeps the entire revenue coming from the product sales.

[5]In some markets, it is not easy to determine whether the platform offers a higher quality of service to consumers. For example, Jin et al. (2007) study the trade-off faced by sportscard sellers on their decision of selling online or offline according to the features of the good. Online platform sales reduce consumers' search costs of rare cards, whereas physical sales offer consumers the ability to test in person the quality of the card. In our framework, we show in Appendix I that the platform has no incentives to impose price parity if the direct sales channel delivers higher benefits to consumers and merchants.

the platform, their customers always prefer to buy through the platform because they pay a lower price for a higher quality of service. Therefore, both selling channels are used only if a merchant obtains low benefits of selling through the platform. Under price parity, merchants are constrained to choose the same retail price for the high and the low quality of service. If the platform chooses a low consumer fee, price parity eliminates competition between selling channels. Indeed, all consumers buy on the platform as they obtain a higher quality of service for the same retail price. If the platform chooses a higher consumer fee, some consumers prefer to buy on the platform and other directly from merchants. Therefore, some merchants compete with the platform for selling services.

Then, we analyze a merchant's decision to accept the platform's service according to the platform's decision to impose price parity. If the platform does not impose any restriction, merchants always prefer to accept the platform's service. Offering an additional selling channel enables them to price discriminate across consumers according to their preference for the quality of selling services. If the platform imposes price parity, some merchants prefer not to sell on the platform either because it is too costly or because it lowers consumer demand for the product. If the platform chooses a low consumer fee, competition between selling channels is eliminated. Therefore, a merchant trades off between selling directly or on the platform. Selling on the platform is better if the merchant obtains high selling benefits and the total transaction fee is sufficiently low. Indeed, the merchant is able to internalize the consumer's cost of buying the high quality as both qualities do not compete against each other. If the platform chooses a higher fee, the direct sales channel is not eliminated. A merchant sells on the platform if his selling benefit is high with respect to his selling cost. Since the merchant is unable to internalize the consumer's cost of buying the high quality when he is not allowed to price discriminate, the merchant's decision to accept the platform's service does not depend on the consumer fee.

Later, we study the platform's pricing strategy in two scenarios: when there are no restrictions and when the platform imposes price parity. If there are no restrictions, the platform's profit depends on the total transaction fee. If both selling modes are very differentiated on the consumer side, the platform chooses a total transaction fee such that both selling channels are used by consumers at all merchants'. If the degree of differentiation is lower, the platform prefers to set a transaction fee that eliminates the direct sales channel at some merchants'. For very low degrees of differentiation, the platform sets a total transaction fee at its marginal cost and makes zero profit.

By contrast, under price parity, the platform's profit depends on how the total transaction fee is split between consumers and merchants. The imposition of price parity suppresses merchant internalization of consumer benefits and merchant pass-through of their selling benefits to consumers. Therefore, the platform chooses the structure of fees that maximizes its profit. When the degree of differentiation increases on the consumer side, the platform extracts more surplus from consumers by increasing the consumer fee and lowering the merchant fee. When the platform brings higher selling benefits to merchants, it extracts more surplus from merchants by increasing the merchant fee and lowering the consumer fee.

Subsequently, we determine whether the platform prefers to impose price parity. We show that a platform prefers to impose price parity if the degree of differentiation on the consumer side is high enough. A higher degree of differentiation has two effects on a platform's incentives to impose price parity. First, the platform is able to extract more surplus from consumers when selling modes are more differentiated. Second, it becomes less difficult for the platform to attract merchants under price parity when the degree of differentiation increases, because it chooses the optimal price structure when it imposes this restriction. Consequently, it is able to reduce the merchant fee when it increases the consumer fee. Hence, the platform becomes all the more efficient for merchants since it adds value to the consumer purchase. Finally, we analyze the impact of price parity on the transaction fees and retail prices. Price parity decreases the total fee paid by a consumer and a merchant for making a transaction on the platform. However, the total buying price is increased under price parity for consumers who buy from merchants who enjoy high benefits of selling on the platform. For merchants who enjoy low benefits of selling on the platform, the total buying price is reduced under price parity.

The reminder of the paper is as follows. In Section 2, we survey the literature that is related to our study. In Section 3, we introduce the model and our assumptions. In Section 4, we analyze a merchant's incentives to sell its product on the platform. In Section 5, we study whether the platform prefers to impose price parity and determine the profit-maximizing fees chosen by the platform. We also compare the total buying price under price parity and no restrictions. Finally, we conclude. All proofs are in the Appendix.[6]

2 Related literature

Our work complements a strand of the literature on platform markets that examines the impact of contractual restrictions on consumer surplus and social welfare. In most papers (as in ours), the intermediary operates with an agency model, which allows the retailer to control final prices when the product is sold by the intermediary.

A first type of restriction labelled as price parity or price coherence aims at forbidding merchants to price discriminate according to the sales channel used by the consumer. Narrow price parity clauses forbid merchants to charge a lower price for the products sold directly than on the platform. When there are multiple intermediaries, wide price parity clauses forbid merchants to charge a lower price when they sell through a competing intermediary. Whether or not such clauses reduce consumer surplus depends on the assumptions on consumer and merchant preferences, on the modelling of competition between sales channels and on the retail market structure.

Johnson (2017) compares the wholesale and the agency models. He shows that price parity clauses may raise industry profits but lower consumer surplus. However, when profit-sharing rather than revenue-sharing contracts are used, price parity clauses may have a procompetitive effect by encouraging retailer entry.

Edelman and Wright (2015) show that a monopolistic intermediary prefers to impose price coherence on symmetric competing retailers. This restriction increases retail prices and causes an overconsumption of the intermediary's services, an over-investment in benefits to buyers, a reduction in consumer surplus and sometimes welfare. In their main model, the platform and sellers do not compete for selling services because all consumers that join the platform prefer to use it. All symmetric sellers join the platform provided that the latter charges a fee that is sufficiently low. In our work, the main difference is that the platform and the merchants compete to sell to consumers. Merchants may strategically decide to refuse

[6] A mathematica file giving all the computations in detail is available upon authors' request.

to sell on the platform to increase their profits. Since some merchants may delist from the platform, the latter faces a reduction in merchant demand when it raises its transaction fees under price parity.

Boik and Corts (2016) analyze a vertical relationship model in which a monopolistic merchant can sell his products via two differentiated platforms, that may impose wide price parity clauses. They show that such clauses increase both retail prices and platform fees. Moreover, they have a negative impact on entry of low quality platforms in the market. Wide price parity clauses reduce the elasticity of implied demand for a platform because the fee increase will be subject to less pass-through by a multi-platform seller. In their work, they focus on multi-homing across different platforms, whereas we focus on multi-homing between platform sales and direct sales. Furthermore, they consider that the platform does not charge any fee to consumers. In our setting, the pass-through of the platform's (merchant) fee to consumers who buy through the platform sales channel will be also different under single-homing and multi-homing (with direct sales). However, the degree of pass-through is not systematically lower under multi-homing. It depends in our model on the marginal impact of an increase in quality differentiation on consumer demand.

Johansen and Vergé (2017) model competition between two platforms and direct sales with multiple competing sellers. If seller participation is exogenous, like Boik and Corts (2016) and Johnson (2017), they find that platforms tend to raise the fees charged to merchants when they impose wide price parity clauses. However, with endogenous seller participation, platforms may reduce the fees charged to sellers under price parity. This reduction depends on the degree of interbrand competition. In the absence of interbrand competition, the platform raises the merchant fee under price parity. When the degree of interbrand competition increases, this reduces the platform's incentives to raise its fees and price parity clauses may even benefit consumers. In our setting, price parity clauses may also benefit consumers, because the latter are able to buy a higher quality at a lower total price.

Another strand of the literature assumes that price comparisons are costly and models the role of search costs. Wang and Wright (2018) build a framework in which consumers search for firms directly or through a platform. Consumers may use platforms as showrooms to learn and compare prices without concluding the transaction through them. They find that price coherence has several anticompetitive effects. Firstly, it eliminates competition between the merchant and the platform on the sales channel. Secondly, it soften competition between platforms. In our setting, price coherence does not always eliminate competition between the merchant and the platform on the sales channel, because we assume that both sales channel are differentiated in quality. This effect arises only if the platform charges a very low transaction fee to consumers, such that consumers always prefer the sales channel of high quality.

Our article is also related to a vast literature on selling modes. This strand of the literature examines merchants' incentives to market a product or a service through a platform and whether the presence of a platform increases efficiency and social welfare (see Baye and Morgan (2001), Galeotti and Morgan (2008), Hagiu and Wright (2014), Einav et al. (2016)). However, the literature modelling competition between a platform and merchants is scarce. Bourreau and Verdier (2010) study competition between a platform and merchants in the market for payment cards. They show that the platform can adjust its price structure to deter merchants from entering the market. Also, one part of this literature looks at whether in multi-channel retailing environments, sales in one channel impact sales in other channels. For example, Abhishek et al. (2016) look at whether e-tailers should use the agency model and sell via a platform or directly via their own website. However, in their model, merchants cannot multi-home and sell both via the direct and the intermediated channel. They find that the choice depends on the demand spillovers between the platform channel and the direct channel, as well as on the degree of competition between merchants in the retail market. Our paper is also related to the literature on one-way complements (See Broos and Gautier (2017) and Flores-Fillol et al. (2018)) and bundling in two-sided markets (See Amelio and Jullien, (2012) Chao and Derenger (2013)). Indeed, consumers only buy the selling service on the platform if they buy the primary product from a merchant (one-way complements). Furthermore, the selling service is bundled with the primary product on the direct sales channel.

The empirical literature on merchant-platform competition is scarse. Several papers analyze the impact of competition between online and offline sales on retail prices. Jin et al. (2007) examine the trade-off of selling sportcards online vs. offline. Goolsbee (2001) estimates the relative price sensitivity of individuals' choice of whether to buy computers online or in retail stores. Hu and Smith (2013) look at the e-book industry and study whether the digitalization of books cannibalizes the sales of physical books. Nevertheless, in our model, when merchants decide to sell directly their product, they may choose to sell it either online or offline. Therefore, in our paper the nature of competition does not rely on the offline characteristics of the sales channel. Lastly, Hunold et al. (2018) empirically investigate the effects of the abolition of Booking.com narrow price parity clause in Germany on market outcomes. They find that after the abolition, hotels were more frequently able to establish the direct channel as the cheapest channel. Also, consistent with our model, they do not find evidence for inflated transaction fees charged by the online travel agents following the imposition of price parity clauses.

3 The model

We build a model to study whether a monopolistic platform prefers to impose price parity to merchants when the latter may sell directly to consumers.

The platform The platform is a marketplace (e.g., a payment platform, a booking platform for hotels, a platform delivering meals for restaurants) that acts as intermediary between consumers and merchants. Consumers and merchants pay respectively the fees f_B and f_S to use the platform.[7] The platform's total intermediation cost is c_P and the total transaction fee is $f_P = f_B + f_S$. The letter B stands for buyers and the letter S stands for sellers.

We consider two cases. In case $k = nr$, the platform does not impose any restriction to merchants. In case $k = pp$, the platform imposes price parity. This implies that the merchant is constrained to choose the same retail price for all sales channels. The platform's profit is Π^k for all $k \in \{nr, pp\}$.

[7] As in Wang and Wright (2018), Johansen and Vergé (2017), Boik and Corts (2016), Edelman and Wright (2015) we use linear fixed per transaction fees. Platforms may also adopt a revenue-sharing business model and set fees that are proportional to merchants' retail prices, as in Johnson (2017).

Merchants A continuum of monopolistic merchants offer a product/service to consumers and may decide to sell it also through a platform. For example, a hotel may offer rooms either directly on its website or through its website and Booking.com.

Each merchant decides on whether to sell through the platform and on the retail price of the product on each sales channel. It the platform imposes price parity, a merchant is not allowed to price discriminate across sales channels. When consumers buy through the direct sales channel, the selling service is bundled to the purchase of the product. When consumers buy through the platform, the latter only pay to merchants the price of the product and pay a fee to the platform for the selling service. The retail price on sales channel j is p_j^k, where $k \in \{nr, pp\}$ represents the platform's rules. If a merchant sells only through the direct sales channel, he makes a profit π_S. If a merchant sells through two channels with price discrimination, he makes profit π_{pd}^{2sc}. If a merchant sells through two channels without price discrimination (i.e., price parity), he makes profit π_{pp}^{2sc}.

We assume that merchants differ across their total net benefit of selling through the platform. The benefit of selling through the platform is $\sigma_I(b_S) \equiv \sigma_I b_S$, where $\sigma_I > 0$ and b_S is drawn from a continuously differentiable uniform distribution H_S on $[0, 1]$ with a density of $h_S \equiv 1$.[8] The marginal cost of selling the product is normalized to zero. When a merchant sells through the platform, he pays a fee f_S to the platform. Therefore, the marginal net cost of selling through the platform is $c_I = f_S - \sigma_I b_S$ and the marginal cost of selling through the direct sales channel is $c_S = 0$.

Buyers Each merchant faces a continuum of buyers. A buyer gives a value y to the basic version of the product that is drawn independently from b_S on the support $[0, v]$ from a continuously differentiable uniform distribution F with a density of $f \equiv 1/v$, where $v > 0$. The survival function is $D(.) = 1 - F(.)$.

The consumer's perception of the product quality depends on the quality of the selling service, which differs for the platform and the direct sales channel. For example, the quality of

[8] In our setting, the benefit σ_S of selling directly to consumers is normalized to zero. In Appendix H, we show that the platform never imposes price parity when merchants obtain strictly higher benefits of selling directly to consumers. Furthermore, our results hold for more general functions of $\sigma_j(b_S)$ but we choose a linear multiplicative model to obtain simple expressions of the platform's prices.

a meal produced by a restaurant increases with the speed of the delivery service.[9] We assume that a consumer of type y who buys through sales channel j obtains a value $\lambda_j(y) \equiv \lambda_j y$, where $\lambda_j > 0$. The difference $\Delta\lambda \equiv \lambda_I - \lambda_S$ represents the degree of differentiation between the selling service on the platform and the direct sales channel. We also consider that a consumer perceives the quality of service on the platform as higher than the direct sales channel, that is we have $\Delta\lambda > 0$.[10] We assume that the functions λ_I, λ_S and $\Delta\lambda$ are continuous and strictly increasing.[11]

The net utility of buying through sales channel $j \in \{I, S\}$ in case $k \in \{nr, pp\}$ for a consumer of type y is therefore

$$u_j^k(y) = \lambda_j y - p_j^k - (f_B^j)^k, \tag{1}$$

where $\lambda_j y$ is the fixed utility of consuming the service, p_j^k the retail price paid to the merchant in case $k \in \{nr, pp\}$, and $(f_B^j)^k$ the transaction fee paid for buying through sales channel j in case k.[12] If he does not buy, the consumer utility is equal to zero. We assume that a consumer pays no additional transaction fee when he buys directly from the merchant and that he pays a transaction fee to the platform, that is, we have $f_B^I \equiv f_B$ and $f_B^S \equiv 0$. The selling service is therefore bundled to the product on the direct sales channel.

Additional Assumptions:

We make a set of technical assumptions to ensure that there may be a solution in which there is competition between sales channels (i.e., we avoid market tipping by the platform or platform exit).

(A1) $v\sqrt{\lambda_I \lambda_S} \geq \sigma_I$.

[9] Consumers prefer to order on the platform deliveroo if they believe they can obtain their meal rapidly. In the hotel booking market, the perception of the quality of a hotel room depends on the accuracy of the information offered by the seller. In the payments market, the perception of the safety of a payment method depends on the fraud prevention technologies used by the payment processor.

[10] In some markets, the platform can deliver higher information, reduce search costs, decrease the expected time for delivery. In other markets, consumers prefer to trust merchants for selling services.

[11] We do not study the case in which the two qualities are sold through the same selling channel. For example, Amazon uses different qualities of delivery services (depending on the speed) as a price discrimination device.

[12] The specification of the fixed utility follows McAfee (2007).

Assumption (A1) implies that if a merchant has no benefit of selling through the platform (i.e., $b_S = 0$), he does not accept the platform's service.

(A2) $c_P \leq \min(\sigma_I(\sqrt{\lambda_I} - \sqrt{\lambda_S})/\sqrt{\lambda_S}, \sigma_I/2)$.

Assumption (A2) implies that the platform's marginal cost is low enough with respect to the selling benefit experienced by merchants, that is, that $c_P \leq \sigma_I$. We will show that such a condition is necessary for the platform to obtain a positive profit in equilibrium if it does not impose restrictive rules to merchants. Assumption (A2) implies that $\sigma_I(\sqrt{\lambda_I} - \sqrt{\lambda_S}) \geq c_P\sqrt{\lambda_S}$. This condition means that the degree of differentiation between selling channels is sufficiently high on the consumer side with respect to the benefit of selling through the direct sales channel. Assumption (A2) also implies that $c_P \leq \sigma_I/2$, which enables us to show that the interior solution under price parity is always better than the corner solutions if the degree of differentiation on the consumer side is very low.[13]

(A3) $v(\Delta\lambda) \geq 2\sigma_I - 2c_P$.

Assumption (A3) ensures that the platform is sufficiently differentiated on the consumer side. This simplifies our comparisons between the platform's profit if it imposes price parity and if there are no restrictions.[14]

Finally, we assume that a merchant's marginal revenue of selling through sales channel j is strictly increasing in p_j^k. Furthermore, the difference between the marginal revenue of selling through the platform and the direct sales channel is strictly increasing. This enables us to find an interior solution when each merchant chooses the retail price that maximizes his profit.

Timing of the game:

The timing of the game is as follows:

[13] While the last two conditions are not necessary to resolve our model, it enables us to restrict our study of the platform's profit-maximizing strategy to a limited number of cases. More precisely, Assumption (A2) also enables us to limit the number of corner solutions that need to be studied under price parity when the degree of differentiation between selling channels is sufficiently high.

[14] We will discuss how the profit-maximizing prices change under price parity if we question this assumption in Appendix I.

1. The platform sets the consumer fee f_B and the merchant fee f_S and decides whether or not to impose price parity to merchants.

2. Merchants learn their transaction benefit b_S. They decide on how many sales channels to offer to consumers and on the price of the service.

3. Consumers learn their value for the product y, decide whether or not to consume and which version of the product to buy.

We look for the subgame perfect equilibrium of this game, and solve the game by backward induction.

4 The number of sales channels

In this section, we study whether a merchant prefers to sell through one or two sales channels to consumers according to the quality and the net cost of selling services.

4.1 The merchant's profit at the equilibrium of stage 3

We determine the merchant's profit at the equilibrium of stage 3 according to the fees charged by the platform and the number of selling channels offered to consumers.

If the platform sets a very low fee to consumers, all consumers prefer to buy on the platform under price parity when this selling mode is available. Let $f_0 \equiv v(\Delta\lambda)/2$ denote the maximum consumer fee such that the direct sales channel is eliminated under price parity. Let also

$$\widetilde{b_S} \equiv (\Delta\lambda(f_S + v\lambda_I) - f_B(\lambda_I + \lambda_S))/(\sigma_I \Delta\lambda) \tag{2}$$

denote the maximum transaction benefit such that the merchant is able to obtain the monopoly profit if all consumers buy through the platform under price parity. Finally, if the direct sales channel is not eliminated, the merchant may prefer to steer consumers towards the platform if its benefit of selling through this sales channel is sufficiently low.[15]

[15] The intuition for this result is be explained in the analysis of Lemma 1.

We denote the threshold such that this situation happens by

$$\widehat{b_S} \equiv \widetilde{b_S} - 2\sqrt{\lambda_I \lambda_S}(f_B - f_0)/(\sigma_I \Delta\lambda). \tag{3}$$

In Lemma 1, we determine the merchant's profit-maximizing price at the equilibrium of stage 3 according to the platform's price.

Lemma 1 *The merchant's profits and prices according to the number of selling channels offered to consumers.*

i) ***Single sales channel:*** *If a merchant sells only directly to consumers, he sets a price* $p_S = (v\lambda_S)/2$ *and makes a profit* $\pi_S = (v\lambda_S)/4$*. If a merchant sells only on the platform, he sets a price* $p_I = (v\lambda_I - \sigma_I b_S - f_B + f_S)/2$ *and makes a profit*

$$\pi_I = (\lambda_I v - f_P + \sigma_I b_S)^2/(4v\lambda_I).$$

ii) ***Multi-channels and price parity:*** *Suppose that the platform sets a consumer fee such that* $f_B \leq f_0$*. Consumers always prefer to buy on the platform. If* $b_S \leq \widetilde{b_S}$*, a merchant sets a price* p_I *and makes a profit* π_I*. If* $b_S \geq \widetilde{b_S}$*, a merchant sets a price* $\overline{p} = \lambda_S f_B/\Delta\lambda$ *and makes a profit*

$$\overline{\pi} \equiv (v\Delta\lambda - f_B)((b_S\sigma_I - f_S)\Delta\lambda + f_B\lambda_S)/(v\Delta\lambda).$$

Suppose that the platform sets a consumer fee such that $f_B > f_0$*. If* $b_S \leq \widehat{b_S}$*, a merchant prefers to set a price* p_I *such that all consumers buy on the platform. He makes a profit* π_I*. If* $b_S \geq \widehat{b_S}$*, a merchant prefers to set a price* p_S *such that both selling channels are used by consumers. He makes a profit*

$$\pi_{pp}^{2sc} = \pi_S + (v\Delta\lambda - f_B)(b_S\sigma_I - f_S)/(v\Delta\lambda).$$

iii) ***Multi-channels without restrictions****: If* $b_S \leq f_P/\sigma_I$*, a merchant sets the prices* p_S *and* p_I *for the direct sales and the platform sales channel, respectively. Both selling channels* *are used by consumers and his profit is equal to*

$$\pi_{pd}^{2sc} = \pi_S + (f_B + f_S - v\Delta\lambda - b_S\sigma_I)^2/(4v\Delta\lambda).$$

If $b_S \geq f_P/\sigma_I$*, a merchant sets a price equal to* p_I*. Consumers only buy on the platform and he makes a profit* π_I*.*

Proof. See Appendix A. ■

If a merchant offers only one selling channel to consumers, he obtains the monopoly profit at the equilibrium of stage 3. He chooses the retail price so as to extract the surplus that the marginal consumer obtains of buying a given quality. A merchant's monopoly profit decreases with the total cost of making a transaction for the consumer and the merchant. If the merchant sells only on the platform, this total (net) cost is equal to $f_B + f_S - \sigma_I b_S$. If the merchant sells only directly, this total cost is equal to zero. The merchant's profit also increases with the quality perceived by consumers.

Under price parity, if a merchant sells through two selling channels, consumers trade off between buying the high quality through the platform, the low quality through the direct sales channel and not consuming. If the platform sets a very low fee (i.e., lower than $f_0 \equiv v(\Delta\lambda)/2$), the imposition of price parity suppresses competition between selling channels because consumers always prefer to buy through the platform. If the merchant's benefit of selling through the platform is low, he sets the monopoly price p_I and prefers that consumers only buy through the platform. When the merchant's benefit of selling becomes higher than $\widetilde{b_S}$, the retail price becomes low enough such that the merchant is constrained to set $p = \overline{p}$. At this price, consumers are indifferent between the high and the low quality of service.

If the platform sets a higher fee (i.e., higher than $f_0 \equiv v(\Delta\lambda)/2$), some consumers wish to buy on the platform and other on the direct sales channel. Merchants trade off between setting a price such that all consumers buy on the platform or such that both selling channels are used. If both selling channels are used, merchants set the same price p_S for the high and the low quality. The imposition of price parity has two consequences on the retail price that is set on the platform. First, merchants do not internalize the consumer's surplus of buying the high quality. Second, they do not pass through to consumers the benefits of selling on the platform. The first effect reduces merchants' profits, whereas the second effect increases them. If the merchant's benefit of selling on the platform is higher than $\widehat{b_S}$, the second effect is dominant. Therefore, the merchant prefers to set a price equal to p_S such that both selling channels are used by consumers. Otherwise, the merchant prefers to set a price p_I such that all consumers buy on the platform. Therefore, he steers consumers towards the platform by choosing a high retail price.

If a merchant multi-homes and is allowed to price discriminate, he is able to internalize the consumer's net benefit of buying through the platform rather than through the direct sales channel. Furthermore, he passes through his opportunity cost of selling through the platform instead of selling directly. Therefore, the merchant has no incentive to eliminate the direct sales channel himself. If both selling channels are used by consumers, the merchant's profit depends on the difference in the total opportunity cost incurred by a consumer and the merchant for making a transaction through the platform rather than directly. This difference equals $f_P - \sigma_I b_S$. If this opportunity cost is positive, both selling channels are used by consumers. If this opportunity cost is negative, all consumers prefer to buy through the platform.

4.2 The impact of price parity on retail prices and consumer demand

We analyze the impact of price parity on retail prices and consumer demand for the platform for given fees charged by the platform at the equilibrium of stage 3.

The impact of price parity on retail prices: If a merchant accepts the platform's service under price parity, Lemma 2 gives the comparison of retail prices for a given level of fees f_B and f_S chosen by the platform. The no restrictions case refers to the case in which merchants sell through both selling channels with price discrimination.

Lemma 2 *Assume that a merchant accepts the platform's service when the latter imposes price parity. The retail price on the direct sales channel does not vary under price parity* *compared to the no restrictions case. If $f_B \leq f_0$ and $b_S \leq \widetilde{b_S}$, the retail price on the platform is the same under price parity and no restrictions (i.e., $p_I^{pp} = p_I^{nr} = p_I$). If $f_B \leq f_0$ and $b_S \geq \widetilde{b_S}$, the retail price on the platform is higher under price parity (i.e., $p_I^{pp} = \overline{p} \geq p_I^{nr} = p_I$). If $f_B > f_0$ and $b_S \leq \widehat{b_S}$, the retail price on the platform is the same under price parity and no restrictions (i.e., $p_I^{pp} = p_I^{nr} = p_I$). If $f_B > f_0$ and $b_S \in (\widehat{b_S}, (v(\Delta\lambda) - f_B + f_S)/\sigma_I)$, the retail price on the platform is higher under no restrictions than under price parity (i.e., $p_I^{nr} = p_I \geq p_I^{pp} = p_S$). If $b_S \geq (v(\Delta\lambda) - f_B + f_S)/\sigma_I)$, the retail price on the platform is higher under price parity than under no restrictions (i.e., $p_I^{nr} = p_I \leq p_I^{pp} = p_S$).*

Proof. See Appendix B-1. ■

A merchant chooses the same retail price for the direct sales channel in all cases. Indeed, the price of the direct sales channel only depends on the merchant's cost of selling the low quality. Therefore, the platform's restrictive rule has no impact on consumers' trade-off between the low quality and not consuming.

By contrast, price parity affects the retail price paid by consumers who buy through the platform as it restricts competition between selling channels. However, merchants react differently to this restriction, depending on their benefit of selling through the platform and on the transaction fee chosen by the platform for consumers.

If the platform sets a consumer fee such that the direct sales channel is eliminated, merchants with low transaction benefits (i.e., lower than $\widetilde{b_S}$) choose the same retail price under price parity and no restrictions.[16] Merchants with high benefits of selling through the platform prefer to set a higher retail price under price parity than under no restrictions in order to steer consumers towards the platform.

If the platform sets a higher fee, merchants with high transaction benefits set a price such that both selling channels are used by consumers under price parity. However, they are constrained to choose the same price for both selling channels. Therefore, the price on

[16] This result is caused specifically by our assumption that consumer demand for the platform is linear and does not hold with general distributions. Indeed, in that specific case, the price on the platform when the merchant sells only through the platform is identical to the price when the merchant sells through the platform and the direct sales channel with price discrimination (i.e., $p_I^{pp} = p_I^{nr} = p_I$). Since the merchant chooses the price p_I under price parity if his benefits of selling through the platform are lower than $\widetilde{b_S}$, the prices are identical under price parity and no restrictions.

the platform sales channel only depends on the merchant's cost of selling the low quality. On the contrary, if there are no restrictions, the transaction costs incurred by consumers and merchants have two effects on the retail price. First, the merchant passes through his opportunity cost of selling through the platform to consumers. If this opportunity cost is negative, the merchant has an incentive to reduce the retail price on the platform. Second, he internalizes the consumers' net benefit of buying through the platform. Since consumer demand increases with the degree of differentiation between sales channels, the merchant has an incentive to increase the retail price on the platform. Whether the retail price on the platform increases or decreases under price parity compared to the case in which there are no restrictions depends on how both effects compensate for each other. As explained before, if the merchant's benefit of selling through the platform is high, the first effect is dominant and the retail price is lower under no restrictions than under price parity. Otherwise, the second effect is dominant and the retail price is higher under price parity than under no restrictions.

Therefore, the imposition of price parity does not necessarily generate an inflation of the retail price paid by consumers for given fees charged by the platform. It depends on merchants' benefits of selling through the platform and on whether the direct sales channel is eliminated.

Consumer demand for the platform's sales channel: We denote consumer demand for the platform's service at the equilibrium of stage 3 by D_I^k for $k \in \{nr, pp\}$. Furthermore, we denote consumer demand for the platform if all consumers buy through the platform by D_I. We have $D_I^{nr} = (v\Delta\lambda - f_P + \sigma_I b_S)/(2v\Delta\lambda)$, $D_I^{pp} = (v\Delta\lambda - f_B)/(v\Delta\lambda)$ and $D_I = (v\lambda_I + \sigma_I b_S - f_P)/(2v\lambda_I)$.

Lemma 3 gives the comparison of consumer demand for the platform's sales channel in our linear case for given platform fees.

Lemma 3 *Suppose that the platform sets $f_B \leq f_0$. If $b_S \leq f_P/\sigma_I$, consumer demand for the platform is higher under price parity than under no restrictions (i.e., $D_I \geq D_I^{nr}$). If $b_S \in (f_P/\sigma_I, \widetilde{b_S})$, the demand for the platform is identical under price parity and no restrictions and equal to D_I. If $b_S \geq \widetilde{b_S}$, the demand for the platform is higher under no* *restrictions than under price parity (i.e., $D_I^{nr} \geq D_I$). Suppose that the platform sets $f_B > f_0$. If $b_S \leq f_P/\sigma_I$, consumer demand for the platform is higher under price parity than under no restrictions (i.e., $D_I^{pp} \geq D_I^{nr}$). If $b_S \in (f_P/\sigma_I, \widehat{b_S})$, the demand for the platform is identical under price parity and no restrictions and it is equal to D_I. If $b_S \geq \widehat{b_S}$, the demand for the platform is higher under no restrictions than under price parity (i.e., $D_I \geq D_I^{nr}$).*

Proof. See Appendix B-2. ■

The impact of price parity on consumer demand for the platform depends on merchants' benefits of selling through the platform. For merchants who perceive high selling benefits, consumer demand for the platform is higher if there are no restrictions. By contrast, for merchants who perceive low benefits of selling through the platform, consumer demand for the platform is higher under price parity.

The result of Lemma 3 is caused by the impact of price parity on consumers' trade-off between both selling channels and on the retail prices chosen by merchants. If the platform imposes price parity, competition with the direct sales channel is restricted on the consumer side. If the consumer fee is very low, the direct sales channel is eliminated. When the merchant's benefit of selling through the platform is low, there is no difference between retail prices on both sales channels. Therefore, consumer demand for the platform is higher under price parity than under no restrictions, because consumers trade off between buying the high quality and not consuming, instead of choosing between both qualities. If the merchant's benefit of selling through the platform is intermediary, all consumers buy through the platform under price parity and no restrictions. Therefore, consumer demand for the platform is identical in both cases. If the merchant's benefit of selling through the platform is high, the retail price on the platform is higher under price parity than under no restrictions. Consumer demand for the platform may therefore become lower under price parity.

If the platform sets a higher price, the direct sales channel is not always eliminated. If the merchant's benefit of selling through the platform is low, the retail price on the platform is identical under price parity and no restrictions. Since consumers only buy on the platform under price parity, consumer demand for the platform is higher under price parity than under no restrictions. When the merchant's benefit of selling through the platform is higher than $\widehat{b_S}$, both selling channels are used by consumers. However, merchants are forced to sell the two qualities at the same retail price under price parity, which prevents them from passing through their selling benefits to consumers. For high values of b_S, the retail price is therefore higher under price parity than no restrictions and consumer demand is lower under price parity.

In what follows, we restrict our analysis to cases in which the platform sets a total transaction fee f_P such that $f_P \leq v\lambda_I + \sigma_I$, that is, such that $D_I \geq 0$. Otherwise, the demand for the platform would always be equal to zero if all consumers buy on the platform. We also focus on the case in which the platform sets a consumer fee f_B such that $f_B \leq v\Delta\lambda$ under price parity, that is, such that $D_I^{pp} \geq 0$. Otherwise, the demand for the platform under price parity would be equal to zero.

4.3 The merchant's choice of the number of sales channels

In Lemma 4, we analyze a merchant's decision to offer the platform's service according to fees charged by the platform and the degree of differentiation between sales channels under price parity and no restrictions. Let $\widehat{f_B} \equiv v(\lambda_I - \sqrt{\lambda_I \lambda_S})$ denote the minimal consumer fee such that both selling channels are used at all merchants' if the latter accept the platform's service.

Lemma 4 *The merchant's decision to offer the platform's service:*
i) Price parity: *All merchants offer the direct sales channel. If $f_B \geq \widehat{f_B}$, a merchant sells on the platform if and only if $b_S \geq f_S/\sigma_I$. If $f_B < \widehat{f_B}$, a merchant sells on the platform if and only if $b_S \geq b_S^1 \equiv (f_P - v\lambda_I + v\sqrt{\lambda_I \lambda_S})/\sigma_I$.*
ii) No restrictions: *All merchants offer both selling channels. Both selling channels are used if and only if $b_S < f_P/\sigma_I$. Otherwise, if $b_S \geq f_P/\sigma_I$, consumers only buy on the platform.*

Proof. See Appendix C. ■

A merchant's decision to sell on two different sales channels depends on how much surplus he can extract from consumers by offering an additional sales channel and on the opportunity cost of selling on the platform.

If the platform imposes price parity, there are two cases. In the first case, the platform sets a high consumer fee (i.e., $f_B \geq \widehat{f_B}$), such that all merchants prefer that consumers buy through both selling modes under price parity. In that case, a merchant's incentives to offer the platform's service only depend on his relative net benefit of selling through the platform compared to the direct sales channel (i.e., $\sigma_I b_S - f_S$). If the platform is perceived as less costly than the direct sales channel, merchants prefer to sell also on the platform. Otherwise, they sell only directly. Merchants' incentives to sell on the platform increase with the selling benefit σ_I and decrease with the merchant fee f_S. The minimum consumer fee $\widehat{f_B}$ such that both selling channels are used at all merchants' who sell on the platform increases with the degree of differentiation between selling channels.

In the second case, the platform sets a lower consumer fee, such that some merchants sell only on the platform. Those merchants compare the monopoly profit with the direct sales channel to the monopoly profit with the platform channel. They prefer to sell only on the platform if their selling benefit if sufficiently high (i.e., higher than b_S^1). Interestingly, the elasticity of merchant acceptance of the platform depends on the total transaction fee f_P because monopolistic merchants internalize the consumer's cost of buying on the platform if they only sell one version of the product.

If the platform does not impose any restrictive rule, all merchants offer the direct sales channel. Indeed, when a monopoly sells only via the platform, it can increase its profit by selling the low quality as well because of a market expansion effect.[17] If a monopolist sells the low quality, it increases his profit by offering also the high quality because he is able to internalize the consumer's benefit of buying the high quality and to pass through the opportunity cost of selling the high quality in his pricing decision. Offering two versions of the service with price discrimination is always better for a merchant than offering only the low quality.[18]

[17] There is a market expansion effect that increases the merchant's profit by $\pi_S(\widetilde{y}_S) - \pi_S(\widetilde{y}_I)$ (i.e., the profit of selling via his own channel to all consumers plus the loss of not selling via his own channel to $D(\widetilde{y}_I)$ consumers). The merchant can further increase its profit by adjusting the price of the product sold via the platform. In the above equation $\widetilde{y}_k$ denotes the indifferent consumer between buying through sales channel k and not consuming.

[18] This result is caused by our assumption that the marginal impact of an increase in quality differentiation on consumer demand is positive.

Therefore, if there are no restrictions, consumers are always able to buy on both selling channels, but if the merchant's selling benefit exceeds some threshold, they all prefer to buy on the platform. Both selling channels are used only at merchants' who obtain low selling benefits. Under price parity, consumers may not be able to buy through the platform if the merchant's selling benefit is low because some merchants do not sell on this channel. However, both selling channels are used only at merchants' who obtain high selling benefits.

5 The platform's fees

In this section, we determine the fees that maximize the platform's profit and analyze whether the platform prefers to impose price parity.

5.1 No restrictions

We denote by V_I^{nr} the volume of transactions that takes place on the platform and by Π^{nr} the platform's profit if there are no restrictions. The platform's profit is given by $\Pi^{nr} = (f_P - c_P)V_I^{nr}$. From Lemma 1 and 2, the volume of transactions that takes place on the platform depends on the total transaction fee. If $b_S \leq f_P/\sigma_I$, both selling channels are used by consumers and the demand for the platform's service is D_I^{nr}. If $b_S \geq f_P/\sigma_I$, all consumers buy through the platform and the demand for the platform's service is D_I. Therefore, we have

$$V_I^{nr}(f_P) = \int_0^{\min(f_P/\sigma_I,1)} \min(\max(D_I^{nr}, 0), 1) db_S + \int_{\min(f_P/\sigma_I,1)}^1 \min(\max(D_I, 0), 1) db_S.$$

If $f_P \geq \sigma_I$, since b_S belongs to $[0, 1]$, both selling channels are used by consumers at all merchants'. If $f_P \leq \sigma_I$, some merchants sell only through the platform. In Proposition 1, we give the profit-maximizing total transaction fee chosen by the platform if there are no restrictions.

Proposition 1 *Suppose that the platform does not impose any restrictive rule to merchants. If $v(\Delta\lambda) \geq 3\sigma_I/2 - c_P$, the platform chooses a total transaction fee such that both selling* *channels are used at all merchants' (Full Competition), that is* $(f_P^{nr})^{FC} = (2v\Delta\lambda + 2c_P + \sigma_I)/4$. *The platform makes a profit equal to*

$$(\Pi^{nr})^{FC} = \frac{(2v\Delta\lambda - 2c_P + \sigma_I)^2}{32v\Delta\lambda}.$$

If $v(\Delta\lambda) \in ((c_P^2\lambda_S - \sigma_I\Delta\lambda + 2\sigma_I c_P\Delta\lambda)/(2\sigma_I\lambda_I), 3\sigma_I/2 - c_P)$, *the platform chooses a total transaction fee such that some merchants only sell through the platform (Partial Competition). The total transaction fee is equal to*

$$(f_P^{nr})^{PC} = \frac{1}{3\lambda_S}(c_P\lambda_S - 2\sigma_I\Delta\lambda + \sqrt{c_P^2\lambda_S^2 + 2\lambda_S\sigma_I(c_P + 3v\lambda_I)\Delta\lambda + (4\lambda_I^2 - 5\lambda_I\lambda_S + \lambda_S^2)\sigma_I^2}).$$

The platform makes a profit equal to $(\Pi^{nr})^{PC} = ((f_P^{nr})^{PC} - c_P)V^{nr}((f_P^{nr})^{PC})$.
If $v(\Delta\lambda) < (c_P^2\lambda_S - \sigma_I\Delta\lambda + 2\sigma_I c_P\Delta\lambda)/(2\sigma_I\lambda_I)$, *the platform makes zero profit and chooses* $\underline{f_P^{nr}} = c_P$.

Proof. See Appendix D. ■

If there are no restrictions, the monopolistic platform chooses the total price such that its mark-up on its marginal cost is invertly proportional to the elasticity of the volume of transactions. If the degree of differentiation is high on the consumer side (i.e., higher than $3\sigma_I/2 - c_P$), competition for selling services is weak. Both selling channels are used by consumers because the platform sets a relatively high fee. Merchants have no incentives to steer consumers towards using only the platform. Therefore, the profit-maximizing transaction fee only depends on the elasticity of consumer demand for the platform's service. If the degree of differentiation is lower on the consumer side, competition for selling services becomes fiercer. When the platform reduces its total transaction fee, it eliminates the direct sales channel at some merchants' because all consumers prefer to buy through the platform. It also increases consumer demand for the platform services when both selling channels are used by consumers. The platform chooses a total transaction fee such that the marginal benefits from an increase in the transaction volume are equal to the marginal costs that are due to a lower margin. The lower the total transaction fee $(f_P^{nr})^{PC}$, the higher the number of merchants who do not sell directly to consumers because of competition with the platform.

When the platform is not very efficient (i.e., if c_P is high), if the degree of differentiation on the consumer side is small, the platform may even have no market power and make zero profits because of competition with merchants.

5.2 Price parity

We denote by V_I^{pp} the volume of transactions that takes place on the platform and by Π^{pp} the platform's profit under price parity. We have

$$\Pi^{pp} = (f_B + f_S - c_P)V_I^{pp}.$$

The transaction volume depends on the price structure chosen by the platform. From Lemma 1 and 2, if $f_B \geq \widehat{f_B}$, merchants such that $b_S \geq f_S/\sigma_I$ accept the platform's service. Both selling channels are used by consumers and the demand for the platform's service is given by D_I^{pp}. Therefore, the transaction volume is given by

$$V_I^{pp} = \int_{f_S/\sigma_I}^{1} D_I^{pp} db_S.$$

If $f_B \in (f_0, \widehat{f_B})$ and $b_S \geq \widehat{b_S}$, both selling channels are used by consumers and the platform obtains a demand given by D_I^{pp}. If $b_S \in (b_S^1, \widehat{b_S})$, all consumers buy on the platform and the platform obtains a demand given by D_I. The transaction volume is then given by

$$V_I^{pp} = \int_{\widehat{b_S}}^{1} D_I^{pp} db_S + \int_{b_S^1}^{\widehat{b_S}} D_I db_S.$$

If $f_B \leq f_0$, and $b_S \geq \widetilde{b_S}$, both selling channels are used by consumers and the platform obtains a demand given by D_I^{pp}. If $b_S \in (b_S^1, \widetilde{b_S})$, all consumers buy on the platform and the platform obtains a demand given by D_I. the platform's profit is then given by

$$V_I^{pp} = \int_{\widetilde{b_S}}^{1} D_I^{pp} db_S + \int_{b_S^1}^{\widetilde{b_S}} D_I db_S.$$

In Proposition 2, we give the profit-maximizing prices chosen by the platform under price parity.[19]

Proposition 2 *If $v\Delta\lambda \in (2\sigma_I - 2c_P, c_P + 2\sigma_I)$, there is an interior solution to the platform's profit-maximization problem. In that case, the platform maximizes its profit by choosing a consumer fee that is equal to $f_B^I = (c_P + 2v\Delta\lambda - \sigma_I)/3$ and a merchant fee that is equal to $f_S^I = (c_P - v\Delta\lambda + 2\sigma_I)/3$. The platform makes a profit*

$$(\Pi^{pp})^I = \frac{(v\Delta\lambda - c_P + \sigma_I)^3}{27\sigma_I v\Delta\lambda}.$$

If $v\Delta\lambda > c_P + 2\sigma_I$, there is a corner solution. The consumer fee and the merchant fee are equal to $f_S^c = 0$ and $f_B^c = (v\Delta\lambda + c_P)/2$, respectively. The platform makes a profit

$$(\Pi^{pp})^c = \frac{(v\Delta\lambda - c_P)^2}{4v\Delta\lambda}.$$

Proof. See Appendix E. ■

Under price parity, the platform is able to use the price structure to maximize its profit. It can even subsidize the demand on one side of the market to increase it on the other side. When the degree of differentiation on the consumer side increases, the platform prefers to extract more surplus from consumers by increasing the consumer fee and decreasing the merchant fee. By contrast, when the selling benefit that the platform brings to merchants increases, the platform prefers to extract more surplus from merchants by increasing the merchant fee and reducing the consumer fee. Note that in the special case in which $v\Delta\lambda = \sigma_I$, the fees on the consumer side and the merchant side are both equal to $c_P + \sigma_I$.

It is interesting to compare our setting to Edelman and Wright (2015). In their model, the platform does not face a reduction in demand when it raises seller fees under price parity, provided that sellers continue to participate. In their model, all merchants are symmetric. They all decide to join the platform provided that the merchant fee is sufficiently low. This fee is completely passed through to consumers and the buyers who join the platform always use it. Therefore, there is no competition between merchants and the platform on selling services in their model. Since we model asymmetric monopolistic merchants, the platform

[19] Our results if $v\Delta\lambda < 2\sigma_I - 2c_P$ are discussed in Appendix I.

reduces merchant acceptance of its service when it raises the merchant fee.[20] The reduction in merchant acceptance depends on the degree of differentiation between selling modes (in terms of cost efficiency) on the merchant side.

5.3 The platform's profit-maximizing strategy

The platform chooses to impose price parity if it enables it to reach a higher profit. In Proposition 3, we show that the platform prefers to impose price parity if it is sufficiently differentiated.

Proposition 3 *If $v\Delta\lambda > 2\sigma_I - 2c_P$, the platform always prefers to impose price parity.*

Proof. See Appendix F. ■

Proposition 3 shows that the platform prefers to impose price parity when the degree of differentiation between selling channels is high. The platform's decision to impose price parity depends on a trade-off between extracting surplus from consumers and merchants. On the consumer side, the platform's ability to extract surplus through the consumer fee depends on the retail prices and the elasticity of consumer demand for the high quality. If the platform imposes price parity, merchants are constrained to set the same retail price for both qualities. The price of the high quality is reduced, while a consumer's benefits of buying the high quality remains unchanged. The platform can therefore extract a higher share of the consumer surplus of buying the high quality through the consumer fee, to the detriments of merchants. All else being equal, a higher degree of differentiation between selling modes increases the platform's incentives to extract consumer surplus by imposing price parity.

On the merchant side, under price parity, merchant acceptance of the platform's sales channel is elastic to the merchant fee. Some merchants prefer to refuse the platform's sales channel to eliminate competition between selling services. This reaction to the platform's restriction reduces the volume of transactions on the platform. As shown in Lemma 4, the elasticity of merchants' acceptance depends on the relative efficiency of both selling modes. If

[20] Edelman and Wright (2015) model asymmetric sellers in the extension of their model. However, they consider asymmetric sellers from the perspective of consumers. They assume that one merchant brings no transactional benefit to consumers.

the direct sales channel becomes relatively more efficient (i.e., because σ_I is low), merchants become less reluctant to accept the platform's service for a given level of the fee charged by the platform (see Lemma 4). As a consequence, all else being equal, the platform suffers from a lower reduction of the volume of transactions when it imposes price parity. If the platform increases the fee charged to merchants under price parity, their participation is reduced. Interestingly, the merchant fee decreases with the degree of differentiation on the consumer side. Therefore, a higher degree of differentiation on the consumer side also increases the platform's relative efficiency, and therefore, merchants' participation to the platform. If the degree of differentiation is very high, the platform even reaches full acceptance of its service by merchants under price parity.

Therefore, our analysis shows that a platform may decide to impose price parity in order to remain a two-sided market when merchants have market power. The imposition of price parity suppresses the internalization of the consumer fee by merchants and the pass-through of merchants' marginal selling benefits to consumers. Furthermore, this restriction also reduces the impact of double marginalization on consumer demand.[21] The platform's incentives to impose price parity increase with the degree of differentiation on the consumer side, which also impacts positively merchant participation.

5.3.1 The impact of price parity on the platform's fees and total price

In Proposition 3, we examine the impact of imposing price parity on the total fee paid by consumers and merchants who make a transaction on the platform.

Proposition 4 *If $v\Delta\lambda > 2\sigma_I - 2c_P$, the total transaction fee is lower under price parity than under no restrictions.*

Proof. See Appendix F. ■

The result of Proposition 3 shows price parity does not generate an inflation of the total transaction fee paid by consumers and merchants if the degree of differentiation is

[21] This effect is due to the fact that we only model usage fees. If we considered that some consumers cannot use the platform because it is too costly for them, the price structure would also impact the platform's profit under no restrictions or if all consumers buy through the platform (See Mariotto and Verdier, 2017).

sufficiently high. The platform's market power is limited by the consumers' option to buy through another sales channel and the merchants' ability to refuse this selling mode.

It is interesting to note however that in our setting, the platform is indifferent as regards to the way the total fee is split between consumers and merchants in the no restrictions case. In Proposition 5, we compare however the total buying price paid by consumers under price parity and no restrictions.

Proposition 5 *Consumers who pay at a merchant who receives high benefits of selling through the platform pay a higher total buying price to buy through the platform under price parity. If* $v\Delta\lambda$ *belongs to* $[\max(2\sigma_I - 2c_P, 3\sigma_I/2 - c_P), c_P + 2\sigma_I]$*, the total buying price is higher under price parity than under no restrictions if* $b_S \geq \overline{b_S}$*, where* $\overline{b_S} \equiv (2v\Delta\lambda - 2c_P + 11\sigma_I)/\sigma_I$*, and lower otherwise. If* $v\Delta\lambda > c_P + 2\sigma_I$*, the total buying price is higher under price parity than under no restrictions if* $b_S \geq \overline{\overline{b_S}}$*, where* $\overline{\overline{b_S}} = (-2c_P + 2v\Delta\lambda + \sigma_I)/(4\sigma_I)$*, and lower otherwise.*

Proof. See Appendix G. ■

Merchants who receive high benefits of selling through the platform do not pass them through to consumers under price parity. Therefore, even if consumers pay lower fees to buy through the platform, this is not sufficient to compensate for the increase in the retail price. Consumers pay a higher total price to buy through the platform under price parity than under no restrictions. If merchants receive lower benefits of selling through the platform, the fall in the fees chosen by the platform under price parity compensates for the increase in the retail price. Therefore, consumers pay a lower total price to buy through the platform under price parity than under no restrictions.

6 Conclusion

In this article, we contribute to the debate on price parity clauses by analyzing competition between a platform and merchants for selling services. A platform prefers to impose price parity when it is very differentiated from merchants on the consumer side. This restriction enables the platform to use the price structure to attract consumers and merchants. We find that price parity clauses may reduce the total price paid by consumers who buy from merchants who receive low benefits of selling on the platform. In such a case, the reduction in the transaction fee charged by the platform compensates for a higher retail price. On the contrary, consumers who buy from merchants who receive high benefits of selling through the platform pay a higher total price under price parity. Indeed, price parity prevents those merchants from passing through high selling benefits to consumers.

In the future, it would be also interesting to study the case in which the platform also offers two qualities of the service, that is a high and low version of the service. Moreover, another interesting case would be to look at a situation where there are three sales channels competing, that is the case in which the merchant can market the product either by its own website, directly in the physical shop or via the platform. Finally, it would be also relevant to endogenize investments in quality.

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6.1 Appendix

Appendix A: proof of Lemma 1 Notations:

We denote by y_j the indifferent consumer between buying and not buying if the merchant single-homes with sales channel j. If the merchant multi-homes, in case $k \in \{nr, pp\}$, we denote by y_I^k the consumer who is indifferent between buying the high and the low quality and by y_S^k the consumer who is indifferent between buying the low quality and not consuming. The indifferent consumers at the profit-maximizing prices are denoted respectively by $\widetilde{y}_j^k$ for $k \in \{nr, sh, pp\}$ and $j \in \{I, S\}$.

As in McAfee (2007), we denote the merchant's marginal revenue of selling quality j through selling channel k by

$$MR_j^k(p) = p - (\lambda_j^k)'((\lambda_j^k)^{-1}(p))D((\lambda_j^k)^{-1}(p))/f((\lambda_j^k)^{-1}(p)).$$

Since $(\lambda_j^k)'(p) = \lambda_j^k$, $f((\lambda_j^k)^{-1}(p)) = 1/v$, and $D((\lambda_j^k)^{-1}(p)) = 1 - p/(v\lambda_j^k)$ in our linear case, we have $MR_j^k(p) = 2p - \lambda_j^k v$. Recall that we assumed that MR_j^k is strictly increasing. Since MR_j^k and λ_j^k are strictly increasing, the function $g_j(.) = MR_j^k(\lambda_j^k(.))$ is strictly increasing. We also assumed that the function $g_2(.) = MR_I(\lambda_I(.)) - MR_S(\lambda_S(.))$ is strictly increasing.

i) **Single sales channel:** For $j \in \{I, S\}$, if a merchant sells only through sales channel j, he makes profit

$$\pi_j = D(y_j)(p_j - c_j),$$

where y_j is the consumer who is indifferent between consuming or not the product. From (1), since the reservation utility of the consumer is zero if he does not consume, the indifferent consumer is given by $u_j(y_j) = 0$, that is, we have $\lambda_j(y_j) = p_j + f_B^j$. Therefore, the merchant's profit of selling quality j can be rewritten as $\pi_j = D(y_j)(\lambda_j(y_j) - f_B^j - c_j)$. The choice of the profit-maximizing price is therefore equivalent to the choice of the indifferent consumer. Taking the derivative of π_j with respect to y_j and replacing for $MR_j(\lambda_j(y_j))$ into $(\pi_j)'$, we find that $(\pi_j)'(y_j) = -f(y_j)(MR_j(\lambda_j(y_j)) - f_B^j - c_j)$. Replacing for $y_j = \widetilde{y}_j$ into the previous equality and since $(\pi_j)'(\widetilde{y}_j) = 0$, we have that $MR_j(\lambda_j(\widetilde{y}_j)) = f_B^j + c_j$. Therefore, the merchant chooses his price such that his marginal revenue equals his marginal cost.

Replacing for $MR_j(\lambda_j y_j) = 2\lambda_j y_j - \lambda_j v$ in our linear case, we find that the profit-maximizing indifferent consumer is given by $y_j = (v\lambda_j + c_j + f_B^j)/(2\lambda_j)$. The profit-maximizing price is given by $p_j = (v\lambda_j + c_j - f_B^j)/2$ and the merchant's profit when he sets p_j is given by

$$\pi_j = (\lambda_j v - c_j - f_B^j)^2/(4v\lambda_j).$$

Since $c_S = f_B^S = 0$, the merchant's maximum profit if he sells only directly to consumers is given by $\pi_j = (v\lambda_S)/4$. Since $c_I = f_S - \sigma_I b_S$ and $f_B^I = f_B$, the merchant's maximum profit if he sells only through the platform is given by $\pi_I = (\lambda_I v - f_P + \sigma_I b_S)^2/(4v\lambda_I)$.

ii) **Multi-channels and price parity:** Suppose that the merchant accepts the platform's service. In ii-a), we study the case in which the merchant sets $p \leq \lambda_S f_B/\Delta\lambda$. In ii-b), we study the case in which the merchant sets $p > \lambda_S f_B/\Delta\lambda$. In ii-c), we conclude.

ii-a) Under price parity, if the merchant sets $p \leq \lambda_S f_B/\Delta\lambda$, consumers trade off between both selling channels. A consumer of type y obtains a utility $\lambda_I y - p - f_B$ if he buys through the platform and $\lambda_S y - p$ if he buys through the direct sales channel. The indifferent consumer $\widetilde{y}^{pp}$ between both selling channels is given by $\widetilde{y}^{pp} = f_B/\Delta\lambda$. Therefore, if $p \leq \lambda_S f_B/\Delta\lambda$, the indifferent consumer obtains a positive utility of consuming through the direct sales channel. The profit-maximizing price is then given by p_S if $p_S \leq \lambda_S f_B/\Delta\lambda$, that is if $f_B \geq f_0$, where $f_0 \equiv (\Delta\lambda)v/2$. The merchant obtains a profit equal to π_{pp}^{2sc} given in Lemma 1. If $p_S \geq \lambda_S f_B/\Delta\lambda$, the constraint $p \leq \lambda_S f_B/\Delta\lambda$ is binding and the merchant sets $\overline{p} = \lambda_S f_B/\Delta\lambda$. In that case, all consumers buy through the platform and the merchant makes the profit $\overline{\pi}$ given in Lemma 1.

ii-b) If the merchant sets $p > \lambda_S f_B/\Delta\lambda$, a consumer buys through the the direct sales channel if $\lambda_S y > p$ and $y < f_B/\Delta\lambda$. Since $p > \lambda_S f_B/\Delta\lambda$, it is impossible to have $y > f_B/\Delta\lambda$ and $y < f_B/\Delta\lambda$. Therefore, all consumers buy through the platform. The profit-maximizing price is then given by p_I if $p_I > \lambda_S f_B/\Delta\lambda$. We have $p_I > \lambda_S f_B/\Delta\lambda$ if and only if $\Delta\lambda(\lambda_I v - \sigma_I b_S - f_B + f_S) > 2\lambda_S f_B$, that is if and only if $b_S < \widetilde{b_S}$ given in (2). If $b_S \geq \widetilde{b_S}$, the constraint $p > \lambda_S f_B/\Delta\lambda$ is binding and the merchant sets $\overline{p} = \lambda_S f_B/\Delta\lambda$.

ii-c) To determine the merchant's profit-maximizing strategy, we compare his profit if he sets $p > \lambda_S f_B/\Delta\lambda$ or $p \leq \lambda_S f_B/\Delta\lambda$ according to its transaction benefit b_S. If $f_B < f_0$, it is impossible that both selling channels are used by consumers. All consumers buy through the platform. If $b_S < \widetilde{b_S}$, the merchant sets a price equal to p_I and makes a profit π_I. If $b_S \geq \widetilde{b_S}$, the merchant sets a price equal to $\overline{p}$ and obtains a profit $\overline{\pi}$. If $f_B \geq f_0$, the merchant trades off between setting a price such that all consumers buy through the platform or such that both selling channels are used by consumers. If $b_S < \widetilde{b_S}$, the merchant compares π_I to π_{pp}^{2sc}. Using the function reduce of mathematica, we find that $\pi_{pp}^{2sc} - \pi_I > 0$ if and only if $b_S \geq \widehat{b_S}$, where $\widehat{b_S}$ is given by (3). As $\widehat{b_S} \leq \widetilde{b_S}$, the merchant prefers to set a price equal to p_S if $b_S \in (\widehat{b_S}, \widetilde{b_S})$. If $b_S \leq \widehat{b_S}$, the merchant prefers to set a price equal to p_I. If $b_S > \widetilde{b_S}$, the merchant compares $\overline{\pi}$ to π_{pp}^{2sc}. Since $\pi_{pp}^{2sc} - \overline{\pi} > 0$, the merchant always prefers to set a price equal to p_S if $b_S > \widetilde{b_S}$.

ii-d) The merchant's profit-maximizing strategy can be therefore summarized as follows. If $f_B < f_0$, all consumers buy through the platform. If $b_S < \widetilde{b_S}$, the merchant sets a price equal to p_I and makes a profit π_I. If $b_S \geq \widetilde{b_S}$, the merchant sets a price equal to $\overline{p}$ and obtains a profit $\overline{\pi}$. If $f_B \geq f_0$ and $b_S \geq \widehat{b_S}$, the merchant sets a price equal to p_S, both selling channels are used by consumers and the merchant makes a profit π_{pp}^{2sc}. Otherwise, if $f_B \geq f_0$ and $b_S \leq \widehat{b_S}$, the merchant sets a price equal to p_I and all consumers buy through the platform.

iii) **Multi-channels without restrictions**: Suppose that the merchant chooses to price discriminate across sales channels. Under no restrictions, if the merchant sets a price p_I for the high quality and p_S for the low quality such that consumers buy through both selling channels, the indifferent consumer between the high and the low quality is given by $\widetilde{y}_I^{mh} = (p_I - p_S + f_B)/(\Delta\lambda)$, whereas the indifferent consumer between the low quality and not consuming is given by $\widetilde{y}_S^{mh} = p_S/\lambda_S$. If there is an interior solution, the merchant sets a price p_I and p_S for the high and the low quality, respectively. The indifferent consumer between the high and the low quality obtains a positive utility of buying if and only if $\sigma_I b_S \leq f_P$. Therefore, if $\sigma_I b_S \leq f_P$, the merchant obtains the profit π_{pd}^{2sc} given in Lemma 1. If $\sigma_I b_S > f_P$, consumers trade off between buying through the platform and not consuming. The merchant sets a price p_I for the high quality and makes a profit π_I. It is also useful to note that in our setting, a merchant who sells through both selling modes always prefers to price discriminate when the platform allows him to do so.[22]

Appendix B: prices and consumer demand

Appendix B-1: comparison of prices From Lemma 1, if $f_B \leq f_0$ and $b_S \geq \widetilde{b_S}$, the merchant sets $\overline{p}$ under price parity and p_I under no restrictions. If $b_S \geq \widetilde{b_S}$, we have $p_I \leq \overline{p}$. Therefore, the retail price increases under price parity. If $f_B > f_0$ and $b_S \leq \widehat{b_S}$, the merchant sets p_I both under price parity and no restrictions. If $f_B > f_0$ and $b_S \geq \widehat{b_S}$, the merchant sets p_I for the high quality under no restrictions and p_S under price parity. We have $p_I \geq p_S$ if and only if $b_S \leq (v(\Delta\lambda) - f_B + f_S)/\sigma_I$. If $f_B > f_0$, we have $(v(\Delta\lambda) - f_B + f_S)/\sigma_I \geq \widehat{b_S}$. Therefore, the price is reduced if $b_S \in (\widehat{b_S}, (v(\Delta\lambda) - f_B + f_S)/\sigma_I)$ under price parity and increased otherwise.

Appendix B-2: comparison of consumer demands Suppose that the platform sets $f_B \leq f_0$. If $b_S \leq f_P/\sigma_I$, consumer demand for the platform is higher under price parity than under no restrictions (i.e., $D_I \geq D_I^{nr}$). Indeed, if $b_S \leq f_P/\sigma_I$, we have

$$D_I - D_I^{nr} = \frac{(f_P - b_S\sigma_I)\lambda_S}{2v\lambda_I\Delta\lambda} \geq 0.$$

If $b_S \in (f_P/\sigma_I, \widetilde{b_S})$, the demand for the platform is identical under price parity and no restrictions and equal to D_I. If $b_S \geq \widetilde{b_S}$, the demand for the platform is higher under no restrictions than under price parity (i.e., $D_I^{nr} \geq D_I$). Indeed, since $b_S \geq \widetilde{b_S} \geq f_P/\sigma_I$, we have

$$D_I - D_I^{nr} = \frac{(f_P - b_S\sigma_I)\lambda_S}{2v\lambda_I\Delta\lambda} \leq 0.$$

Suppose that the platform sets $f_B > f_0$. If $b_S \leq f_P/\sigma_I$, consumer demand for the platform is higher under price parity than under no restrictions (i.e., $D_I^{pp} \geq D_I^{nr}$). Indeed, we have

$$D_I^{pp} - D_I^{nr} = \frac{v\Delta\lambda - f_B + f_S - b_S\sigma_I}{2v\Delta\lambda}.$$

[22] Anderson and Dana (2008) provide general conditions under which price discrimination is a profitable strategy for a monopolist.

Since $D_I^{pp} - D_I^{nr}$ is decreasing in b_S, this function is positive for $b_S \leq (v\Delta\lambda - f_B + f_S)/\sigma_I$ and negative otherwise. We have $(v\Delta\lambda - f_B + f_S) - f_P = v\Delta\lambda - 2f_B$. Since $f_B > f_0$, we have $(v\Delta\lambda - f_B + f_S) - f_P < 0$. Therefore, if $b_S \leq f_P/\sigma_I$, we have $b_S \leq (v\Delta\lambda - f_B + f_S)/\sigma_I$. This implies that $D_I^{pp} - D_I^{nr} \geq 0$ for all $b_S \leq f_P/\sigma_I$. If $b_S \in (f_P/\sigma_I, \widehat{b_S})$, the demand for the platform is identical under price parity and no restrictions and it is equal to D_I. If $b_S \geq \widehat{b_S}$, the demand for the platform is higher under no restrictions than under price parity (i.e., $D_I \geq D_I^{pp}$). Indeed, if $b_S \geq \widehat{b_S}$, we have

$$D_I - D_I^{pp} = \frac{\sigma_I(b_S - \widehat{b_S})}{2v\lambda_I\Delta\lambda} \geq 0.$$

Appendix C: Merchant acceptance of the platform's service **i) Price parity:** In i-a), we start by studying the case in which the platform sets $f_B \leq f_0$. In i-b), we study the case in which the platform sets $f_B > f_0$.

i-a) Suppose that the platform sets $f_B \leq f_0$. From Lemma 1, if the merchant accepts the platform's service, consumers always prefer to buy through the platform. If $b_S \leq \widetilde{b_S}$, the merchant makes a profit π_I. If $b_S \geq \widetilde{b_S}$, the merchant makes a profit $\overline{\pi}$ given in Lemma 1. We need to compare the merchant's profit if he accepts the platform's service to the profit of selling only directly to consumers given by π_S.

If $b_S \leq \widetilde{b_S}$, the function $\pi_I - \pi_S$ is a polynomial function of degree two in b_S. It is convex in b_S because the coefficient of b_S^2 is given by $\sigma_I^2/(4v\lambda_I) > 0$. It admits two roots that we denote by b_S^1 and b_S^0, where $b_S^1 \geq b_S^0$. We have $b_S^1 = (f_P - v\lambda_I + v\sqrt{\lambda_I\lambda_S})/\sigma_I$ and $b_S^0 = (f_P - v\lambda_I - v\sqrt{\lambda_I\lambda_S})/\sigma_I$. We have $\pi_I - \pi_S \geq 0$ if and only if $b_S \geq b_S^1$ or $b_S \leq b_S^0$. However, we show below that the case in which $b_S \leq b_S^0$ is impossible. Indeed, the maximum total price that the platform can set such that consumer demand D_I is positive when all consumers buy through the platform is $f_P = v\lambda_I + \sigma_I$. Therefore, we have $b_S^0 \leq (\sigma_I - v\sqrt{\lambda_I\lambda_S})/\sigma_I$. Under Assumption (A1), we have $\sigma_I - v\sqrt{\lambda_I\lambda_S} \leq 0$. This implies that $b_S^0 \leq 0$. Since $b_S \geq 0$, the case in which $b_S \leq b_S^0$ is impossible. Therefore, we have $\pi_I - \pi_S \geq 0$ if and only if $b_S \geq b_S^1$

and $b_S \leq \widetilde{b_S}$. We have

$$\widetilde{b_S} - b_S^1 = \frac{1}{\sigma_I \Delta\lambda}(-2f_B\lambda_I + v\Delta\lambda(2\lambda_I - \sqrt{\lambda_I\lambda_S})).$$

Since $f_B \leq v\Delta\lambda$ and $2\lambda_I \geq \sqrt{\lambda_I\lambda_S}$, if $f_B \leq f_0$, we have $\widetilde{b_S} - b_S^1 \geq 0$. Therefore, a merchant accepts the platform's service if $b_S \in (b_S^1, \widetilde{b_S})$.

If $b_S \geq \widetilde{b_S}$, we compare $\overline{\pi}$ to π_S. We have that $\pi_S - \overline{\pi}$ is a polynomial function of degree 1 in b_S. The coefficient of b_S is equal to $(-v\Delta\lambda + f_B)/(v\Delta\lambda)$ and it is negative because $f_B \leq v\Delta\lambda$. The value of $\pi_S - \overline{\pi}$ at $\widetilde{b_S}$ is negative. Therefore, for all $b_S \geq \widetilde{b_S}$, we have $\pi_S \leq \overline{\pi}$ and merchants accept the platform's service.

To sum up case i-a), if $f_B \leq f_0$, merchants accept the platform's service if $b_S \in (b_S^1, \widetilde{b_S})$ and $b_S \geq \widetilde{b_S}$. Therefore, if $f_B \leq f_0$, they accept the platform's service if $b_S \geq b_S^1$.

i-b) Suppose that the platform sets $f_B > f_0$. If $b_S \leq \widehat{b_S}$, a merchant makes a profit π_I if he accepts the platform's service. In i-a), we proved that $\pi_I - \pi_S \geq 0$ if and only if $b_S \geq b_S^1$. We have

$$\widehat{b_S} - b_S^1 = \frac{1}{\sigma_I \Delta\lambda}(2\lambda_I v(\Delta\lambda) - 2f_B(\lambda_I + \sqrt{\lambda_I\lambda_S})).$$

The function $\widehat{b_S} - b_S^1$ is decreasing in f_B and equal to zero for $f_B = \widehat{f_B}$. Therefore, we have $\widehat{b_S} - b_S^1 \leq 0$ for $f_B \geq \widehat{f_B}$ and $\widehat{b_S} - b_S^1 \geq 0$ for $f_B \leq \widehat{f_B}$. Therefore, if $f_B \geq \widehat{f_B}$, we have $b_S \leq \widehat{b_S} \leq b_S^1$ and all merchants refuse the platform's service for $b_S \leq \widehat{b_S}$. If $f_B \in (f_0, \widehat{f_B})$, we have $b_S \leq b_S^1 \leq \widehat{b_S}$ and a merchant accepts the platform's service if $b_S \in (b_S^1, \widehat{b_S})$.

If $b_S \geq \widehat{b_S}$, a merchant makes a profit $\pi_{pp}^{2sc} = \pi_S + (v\Delta\lambda - f_B)(b_S\sigma_I - f_S)/(v\Delta\lambda)$. We compare the merchant's profit if he accepts the platform's service to the profit of selling only directly to consumers given by π_S. Since $v\Delta\lambda - f_B \geq 0$, we have that $\pi_{pp}^{2sc} - \pi_S \geq 0$ if and only if $b_S \geq (f_S/\sigma_I)$. Therefore, a merchant accepts the platform's service if $b_S \geq f_S/\sigma_I$ and $b_S \geq \widehat{b_S}$. We have $\widehat{b_S} \geq f_S/\sigma_I$ if and only if $f_B \leq \widehat{f_B}$ and $\widehat{b_S} \leq f_S/\sigma_I$ otherwise. Therefore, if $f_B \in (f_0, \widehat{f_B})$ and $b_S \geq \widehat{b_S}$, a merchant accepts the platform's service if and only if $b_S \geq \widehat{b_S}$. If $f_B \geq \widehat{f_B}$ and $b_S \geq \widehat{b_S}$, a merchant accepts the platform's service if and only if $b_S \geq f_S/\sigma_I$.

To sum up case i-b), if $f_B \in (f_0, \widehat{f_B})$, a merchant accepts the platform's service if $b_S \geq \widehat{b_S}$ and $b_S \in (b_S^1, \widehat{b_S})$. Therefore, if $f_B \in (f_0, \widehat{f_B})$, a merchant accepts the platform's service if $b_S \geq b_S^1$. If $f_B \geq \widehat{f_B}$, merchants accept the platform's service if $b_S \geq f_S/\sigma_I$ and refuse it otherwise.

Combining case i-a) and i-b), we find that if $f_B \leq \widehat{f_B}$, a merchant accepts the platform's service if $b_S \geq b_S^1$ and refuses it otherwise. If $f_B \geq \widehat{f_B}$, a merchant accepts the platform's service if $b_S \geq f_S/\sigma_I$ and refuses it otherwise.

ii) No restrictions: If a merchant accepts the platform's service and $b_S \leq f_P/\sigma_I$, he makes a profit $\pi_{pd}^{2sc} = \pi_S + (f_B + f_S - v\Delta\lambda - b_S\sigma_I)^2/(4v\Delta\lambda)$. This equality implies that $\pi_{pd}^{2sc} - \pi_S \geq 0$. Therefore, if $b_S \leq f_P/\sigma_I$, a merchant always makes more profit by offering both selling channels than offering only the direct sales channel. Furthermore, if $b_S \leq f_P/\sigma_I$, we have $\pi_{pd}^{2sc} - \pi_I = \lambda_S(f_P - b_S\sigma_I)^2/(4v\lambda_I\Delta\lambda) \geq 0$. Therefore, if $b_S \leq f_P/\sigma_I$, a merchant always makes more profit by offering both selling channels than offering only the selling channel of high quality. If $b_S \geq f_P/\sigma_I$, if a merchant offers the platform's service, all consumers buy through the platform. Therefore, we compare the merchant's profit if he offers the platform's service and if he offers only the direct sales channel. In i-a), we proved that $\pi_I - \pi_S \geq 0$ if and only if $b_S \geq b_S^1$. Since $b_S^1 = (f_P - v\lambda_I + v\sqrt{\lambda_I\lambda_S})/\sigma_I$, we have $b_S^1 - f_P/\sigma_I = v\sqrt{\lambda_I}(\sqrt{\lambda_S} - \sqrt{\lambda_I})/\sigma_I$. Since $\sqrt{\lambda_S} \leq \sqrt{\lambda_I}$, we have $b_S^1 \leq f_P/\sigma_I$. Therefore, if $b_S \geq f_P/\sigma_I$, we have $b_S \geq b_S^1$ and a merchant such that $b_S \geq f_P/\sigma_I$ accepts the platform's service. To conclude ii), if there are no restrictions, all merchants accept the platform's service.

Appendix D: Proof of Proposition 1 If $f_P > \sigma_I$, if there is an interior solution, the profit-maximizing total transaction fee is given by $(f_P^{nr})^{FC}$ of Proposition 1. We have $(f_P^{nr})^{FC} \geq c_P$ if and only if $v\Delta\lambda \geq c_P - \sigma_I/2$. We have $(f_P^{nr})^{FC} > \sigma_I$ if and only if $v(\Delta\lambda) > 3\sigma_I/2 - c_P$. Futhermore, the second-order condition holds at $f_P = (f_P^{nr})^{FC}$. Since $c_P \leq \sigma_I$, we have $3\sigma_I/2 - c_P \geq c_P - \sigma_I/2$. Therefore, there is an interior solution to the platform's profit-maximization problem and it satisfies to the constraint $(f_P^{nr})^{FC} \geq c_P$.

If $f_P < \sigma_I$, if there is an interior solution, the profit-maximizing total transaction fee is given by $(f_P^{nr})^{PC}$ of Proposition 1. We have $(f_P^{nr})^{PC} < \sigma_I$ if and only if $v(\Delta\lambda) < 3\sigma_I/2 - c_P$. We have $(f_P^{nr})^{FC} \geq c_P$ if and only if $2\sigma_I\Delta\lambda(v\lambda_I - c_P) > c_P^2\lambda_S - \sigma_I\Delta\lambda$. This condition amounts to

$$v\Delta\lambda > (c_P^2\lambda_S - \sigma_I\Delta\lambda + 2\sigma_I c_P\Delta\lambda)/(2\sigma_I\lambda_I).$$

The latter condition is compatible with the condition $v(\Delta\lambda) < 3\sigma_I/2 - c_P$ if $c_P \leq \sigma_I$, which is true under Assumption (A2).

Appendix E: Proof of Proposition 2 The platform's profit is defined by parts. It is twice continuously differentiable on each of the compact sets on which it is defined. In i), we start by determining the transaction fees that can maximize the platform's profit if there is an interior solution on each of the three segments. Then, in ii), we determine the possible corner solutions. Lastly, in iii), we try to compare the platform's profit in all cases.

i) If $f_B > \widehat{f_B}$, solving for the first-order condition of the platform's profit-maximization problem, we obtain that if there is an interior solution, it must be that $f_B^I = (c_P + 2v\Delta\lambda - \sigma_I)/3$ and $f_S^I = (c_P - v\Delta\lambda + 2\sigma_I)/3$. The second-order conditions hold if and only if $v\Delta\lambda - c_P + \sigma_I > 0$. This inequality is true under Assumption (A2). In that case, the platform's profit is given by $(\Pi^{pp})^I$ as defined in Proposition 2. We have $f_B^I + f_S^I \geq c_P$ if and only if $\sigma_I \geq c_P$, which is true under Assumption (A2).

If $f_B \in \left(f_0, \widehat{f_B}\right)$, there may also be an interior solution given by (f_B^I, f_S^I). We have $f_B^I \geq f_0$ if and only if $v\Delta\lambda \geq 2\sigma_I - 2c_P$. We have $f_S^I \geq 0$ if and only if $v\Delta\lambda \leq 2\sigma_I + c_P$.

If $f_B < f_0$, if there is an interior solution, this solution satisfies to the constraints $f_B < f_0$ and $f_S > 0$ if and only if $v\Delta\lambda < 2\sigma_I - 2c_P$. The solution is given by

$$f_B^{I2} = \frac{\Delta\lambda}{6\lambda_I}(c_P + 5v\lambda_I - \sigma_I - \sqrt{((v\lambda_I + \sigma_I) - c_P)^2 - 3v^2\lambda_I\lambda_S}),$$

and

$$f_S^{I2} = \frac{-1}{\lambda_I}(v\lambda_I^2 - 5\lambda_I(v\lambda_S + \sigma_I) - c_P(\lambda_I + \lambda_S) + \lambda_S\sigma_I + (\lambda_I + \lambda_S)\sqrt{(v\lambda_I + \sigma_I - c_P)^2 - 3v^2\lambda_I\lambda_S}).$$

Therefore, if $v\Delta\lambda \in [2\sigma_I - 2c_P, c_P + 2\sigma_I]$, if there is an interior solution, it is given by (f_B^I, f_S^I). If $v\Delta\lambda < 2\sigma_I - 2c_P$, if there is an interior solution, it is given by (f_B^{I2}, f_S^{I2}). If $v\Delta\lambda > c_P + 2\sigma_I$, there is no interior solution.

ii) We now examine the different corner solutions on each segment.

ii-a) We start by the segment for which $f_B \in \left[\widehat{f_B}, v\Delta\lambda\right]$. Note that the platform never chooses $f_B = v\Delta\lambda$, otherwise it makes zero profit. If $v(\sqrt{\lambda_I} - \sqrt{\lambda_S})^2 \leq c_P$ (i.e., $v \leq v_{C1}$), there may be a corner solution such that $f_S^{C1} = 0$ and $f_B^{C1} = (c_P + v\Delta\lambda)/2 > \widehat{f_B}$. In that case, the platform makes a profit

$$(\Pi^{pp})^{C1} = (\Pi^{pp})^{I}.$$

If $v(\sqrt{\lambda_I} - \sqrt{\lambda_S})\sqrt{\lambda_I} < c_P + \sigma_I$ (i.e., $v \leq v_{C2}$) and $v(\lambda_I - \sqrt{\lambda_S \lambda_I}) \geq c_P - \sigma_I$ (which is always true under Assumption A2), there may be also a corner solution such that $f_B^{C2} = \widehat{f_B}$ and $f_S^{C2} = (c_P - v\lambda_I + v\sqrt{\lambda_I \lambda_S} + \sigma_I)/2 > 0$. In that case, the platform makes a profit

$$(\Pi^{pp})^{C2} = \frac{\sqrt{\lambda_S}(c_P - \sigma_I + v(-\lambda_I + \sqrt{\lambda_S \lambda_I}))(-v\Delta\lambda\sqrt{\lambda_I} + (\sqrt{\lambda_I} + \sqrt{\lambda_S})(c_P - \sigma_I))}{4\sigma_I(\sqrt{\lambda_I} + \sqrt{\lambda_S})^2}.$$

There is another corner solution such that $f_B^{C3} = \widehat{f_B}$ and $f_S^{C3} = 0$. In that case, the platform makes a profit

$$(\Pi^{pp})^{C3} = \frac{\sqrt{\lambda_S}(c_P - \sigma_I + v(-\lambda_I + \sqrt{\lambda_S \lambda_I}))}{\sqrt{\lambda_I} + \sqrt{\lambda_S}}.$$

ii-b) If $f_B \in \left[f_0, \widehat{f_B}\right]$, there may be two corner solutions identical to "C2" and "C3". There may be another corner solution if $v\Delta\lambda < 2c_P + 2\sigma_I$ (i.e., $v \leq v_{C4}$) such that $f_B^{C4} = f_0$ and $f_S^{C4} = (2c_P - v\Delta\lambda + 2\sigma_I)/4 > 0$. We have $f_B^{C4} + f_S^{C4} \geq c_P$ if and only if $\sigma_I \geq 0$. In that case, the platform makes a profit

$$(\Pi^{pp})^{C4} = \frac{(2c_P - v\Delta\lambda - 2\sigma_I)^2}{32\sigma_I}.$$

There may be also a corner solution such that $f_B^{C5} = f_0$ and $f_S^{C5} = 0$. We have $f_B^{C5} + f_S^{C5} \geq c_P$ if and only if $v\Delta\lambda \geq 2c_P$. In that case, the platform makes a profit

$$(\Pi^{pp})^{C5} = (v\Delta\lambda - 2c_P)/4.$$

If $v\Delta\lambda \geq c_P$, there is also another corner solution that we denote by $C4bis$. This corner solution is identical to C1 (i.e., $f_S = 0$ and $f_B = (c_P + v\Delta\lambda)/2$). The platform makes a profit

$$(\Pi^{pp})^{C4bis} = \frac{(c_P - v\Delta\lambda)^2}{4v\Delta\lambda}.$$

ii-c) If $v\lambda_S \leq 4(c_P - \sigma_I)$, there may be a corner solution such that $f_B^{C6} = 0$ and $f_S^{C6} = (4c_P - v\lambda_S + 4\sigma_I)/8$. The platform makes a profit

$$(\Pi^{pp})^{C6} = \frac{(4c_P + v\lambda_S - 4\sigma_I)^2}{64\sigma_I}.$$

Finally, as shown in our mathematica file, the corner solution such that $f_B^{C7} < f_0$ and $f_S^{C7} = 0$ is impossible.

iii) In the last step, we compare the platform's profit in each case.

iii-a) If $v\Delta\lambda \in [2\sigma_I - 2c_P, c_P + 2\sigma_I]$, it is possible to show that the interior solution given by (f_B^I, f_S^I) maximizes the platform's profit. We compare the platform's profit with each of our corner solutions to the platform's profit with the interior solution I. We have $(\Pi^{pp})^{C1} = (\Pi^{pp})^I$. We have $(\Pi^{pp})^{C2} \geq (\Pi^{pp})^{C3}$ and $(\Pi^{pp})^{C4} \geq (\Pi^{pp})^{C5}$. Therefore, we can eliminate corner solutions $C3$ and $C5$ from our comparisons. Let us start by corner solution $C2$. We have $(\Pi^{pp})^I \geq (\Pi^{pp})^{C2}$ if and only if $v(\lambda_S + 3\sqrt{\lambda_S\lambda_I} - 4\lambda_I) \leq 4(\sigma_I - c_P)$. Using the function reduce of mathematica, we find that this is always true under our assumptions. We continue our series of comparisons with corner solution $C4$. We have $(\Pi^{pp})^I \geq (\Pi^{pp})^{C4}$ if and only if $v\Delta\lambda \geq 8(c_P - \sigma_I)/5$. This latter inequality is always true because $c_P \leq \sigma_I$ under Assumption (A2). We now compare $(\Pi^{pp})^I$ to $(\Pi^{pp})^{C4bis}$. We have

$$(\Pi^{pp})^I - (\Pi^{pp})^{C4bis} = \frac{(4v\Delta\lambda + \sigma_I - 4c_P)(c_P - v\Delta\lambda + 2\sigma_I)^2}{108\sigma_I v\Delta\lambda}.$$

Therefore, if $v\Delta\lambda \geq c_P$, we have $(\Pi^{pp})^I \geq (\Pi^{pp})^{C4bis}$. Finally, corner solution C6 cannot exist if $v\Delta\lambda \in [2\sigma_I - 2c_P, c_P + 2\sigma_I]$. Therefore, the platform makes a higher (or equal) profit with the interior solution I than with the interior solutions given by $C1, C2, C3, C4, C4bis, C5$ and $C6$. This implies that if there is an interior solution to the platform's profit-maximization problem, the platform's profit reaches a maximum at (f_B^I, f_S^I).

iii-b) If $v\Delta\lambda > c_P + 2\sigma_I$, there is a corner solution. Therefore, we need to compare the platform's profit at all the possible corner solutions and find the maximum value that the platform can obtain. If $\sigma_I(\sqrt{\lambda_I} - \sqrt{\lambda_S}) \geq c_P\sqrt{\lambda_S}$, we rank the thresholds on v such that each corner solution is possible. Corner solution j is possible if and only if $v \leq v_{Cj}$ for $j \in \{1, 2, 4\}$. Corner solution 3 is possible if $\widehat{f_B}$ is greater than c_P, that is if $v(\lambda_I - \sqrt{\lambda_I\lambda_S}) \geq c_P$. Corner solution $C5$ is possible if $v\Delta\lambda \geq 2c_P$. Corner solution $C4bis$ is possible if $v\Delta\lambda \geq c_P$. We have $(c_P + 2\sigma_I)/\Delta\lambda \geq \max(v_{C1}, v_{C2}, v_{C4})$. Indeed, we have $(c_P + 2\sigma_I)/\Delta\lambda \geq v_{C1}$ if and only if $\sigma_I(\sqrt{\lambda_I} - \sqrt{\lambda_S}) \geq c_P\sqrt{\lambda_S}$. The latter inequality is true under Assumption (A2). We have $(c_P + 2\sigma_I)/\Delta\lambda \geq v_{C2}$ if and only if $c_P \leq \sigma_I$, which is true under Assumption (A2). Furthermore, we have $(c_P + 2\sigma_I)/\Delta\lambda \geq v_{C4}$. Therefore, if $v\Delta\lambda > c_P + 2\sigma_I$, corner solutions $C3$, $C4bis$ and $C5$ are possible and they are always possible because $c_P+2\sigma_I \geq 2c_P$, $c_P+2\sigma_I \geq c_P$ and $(c_P+2\sigma_I)/\Delta\lambda \geq c_P/(\lambda_I - \sqrt{\lambda_I\lambda_S})$. We show in our mathematica file that $(\Pi^{pp})^{C4bis} \geq (\Pi^{pp})^{C3}$ and $(\Pi^{pp})^{C4bis} \geq (\Pi^{pp})^{C5}$. Finally, if $\lambda_I \geq (3\lambda_S)(c_P - 2\sigma_I)/(4c_P - 4\sigma_I)$, corner solution $C6$ is possible. In that case, we have $(\Pi^{pp})^{C4bis} \geq (\Pi^{pp})^{C6}$. Therefore, the corner solution is $C4bis$.

Appendix F: Proof of Proposition 3 In i) we determine whether the platform prefers price parity if there is an interior solution under price parity and if all merchant accept the platform's service under no restrictions. In ii), we study the case in which the services are very differentiated on the consumer side.

i) From Proposition 1 and 2, if $v\Delta\lambda$ belongs to $[\max(2\sigma_I - 2c_P, 3\sigma_I/2 - c_P), c_P + 2\sigma_I]$, we have

$$(\Pi^{nr})^{FC} - (\Pi^{pp})^{I} = \frac{(8c_P - 8v\Delta\lambda - 5\sigma_I)(2c_P - 2v\Delta\lambda + \sigma_I)^2}{864\sigma_I v\Delta\lambda}.$$

We have $(\Pi^{nr})^{FC} \leq (\Pi^{pp})^{I}$ if and only if $v\Delta\lambda \geq (8c_P - 5\sigma_I)/8$. If $c_P < \sigma_I/2$, we have $\max(2\sigma_I - 2c_P, 3\sigma_I/2 - c_P) = 2\sigma_I - 2c_P$. Therefore, we have $(8c_P - 5\sigma_I)/8 < 2\sigma_I - 2c_P$. Therefore, if $v\Delta\lambda$ belongs to $[2\sigma_I - 2c_P, c_P + 2\sigma_I]$, the platform always prefers to impose price parity. We have $f_P^{pp} - f_P^{nr} \leq 0$ if and only if $v\Delta\lambda \geq c_P + \sigma_I/2$. We have $c_P + \sigma_I/2 \leq 2\sigma_I - 2c_P$ if and only if $c_P \leq 3\sigma_I/4$. This is always true if $c_P < \sigma_I/2$. Therefore, if $v\Delta\lambda$ belongs to $[\max(2\sigma_I - 2c_P, 3\sigma_I/2 - c_P), c_P + 2\sigma_I]$, we have $f_P^{pp} - f_P^{nr} \leq 0$.

ii) If $v\Delta\lambda > c_P + 2\sigma_I$, there is a corner solution under price parity that we denoted by C4bis. We compare $(\Pi^{pp})^{C4bis}$ to $(\Pi^{nr})^{FC}$. If $v\Delta\lambda > c_P + 2\sigma_I$, we have

$$(\Pi^{pp})^{C4bis} - (\Pi^{nr})^{FC} = \frac{8(v\Delta\lambda - c_P)^2 - (2v\Delta\lambda - 2c_P + \sigma_I)^2}{32v\Delta\lambda} \geq 0.$$

Therefore, the platform prefers price parity to no restrictions. We have $f_P^{C4bis} - f_P^{nr} =$

$-\sigma_I/4 \leq 0$. Therefore, the total transaction fee is lower under price parity than no restrictions.

iii) If $v\Delta\lambda < \max(2\sigma_I - 2c_P, 3\sigma_I/2 - c_P)$ and if there is a corner solution under price parity. If $c_P < \sigma_I/2$, we have $\max(2\sigma_I - 2c_P, 3\sigma_I/2 - c_P) = 2\sigma_I - 2c_P$.

We start by studying the case in which $v \leq v_{C1}$. If $v\Delta\lambda$ belongs to $(3\sigma_I/2 - c_P, 2\sigma_I - 2c_P)$, we need to compare $(\Pi^{nr})^{FC}$ to $(\Pi^{pp})^{C1}$. Since $(\Pi^{pp})^{C1} = (\Pi^{pp})^{I}$, we have $(\Pi^{nr})^{FC} - (\Pi^{pp})^{C1} \leq 0$. For $v\Delta\lambda < 3\sigma_I/2 - c_P$, the profit under no restrictions is given by $(\Pi^{nr})^{PC}$ and we compare $(\Pi^{nr})^{PC}$ to $(\Pi^{pp})^{C1}$. We have $(\Pi^{nr})^{PC} \leq (\Pi^{pp})^{C1}$. Therefore, the platform prefers price parity. We have $(f_P^{nr})^{PC} \geq (f_P^{nr})^{C1}$. Therefore, the total price is reduced under price parity.

Appendix G: In this Appendix, we examine whether price parity reduces the total cost incurred by a consumer for buying the product and the selling service.

i) From Proposition 1 and 2, if $v\Delta\lambda$ belongs to $[\max(2\sigma_I - 2c_P, 3\sigma_I/2 - c_P), c_P + 2\sigma_I]$, the platform sets $(f_P^{nr})^{FC}$ under no restrictions and f_B^I under price parity.

Under price parity, we have $f_B^I \geq f_0$. Therefore, from Lemma 1, the merchant sets a common price on both selling channels equal to p_I if $b_S \leq \widehat{b_S}$ and equal to p_S if $b_S > \widehat{b_S}$. Replacing for f_B^I and f_S^I given in Proposition 2 into $p_I + f_B^I$ given in Lemma 1, we have that $p_I + f_B^I = (4v\lambda_I - v\lambda_S - 3\sigma_I b_S + \sigma_I + 2c_P)/6$ and $p_S + f_B^I = (4v\lambda_I - v\lambda_S - 2\sigma_I + 2c_P)/6$. We denote the total price paid by consumers under price parity by TP_B^{pp}.

Under no restrictions, the merchant sets p_I on the platform sales channel in all cases. Since $p_I + (f_B^{nr})^{FC} = (v\lambda_I - \sigma_I b_S + (f_P^{nr})^{FC})/2$, we have

$$p_I + (f_B^{nr})^{FC} = (2v\Delta\lambda + 2c_P + \sigma_I + 4(v\lambda_I - \sigma_I b_S))/8.$$

We denote the total price under no restrictions by TP_B^{nr}. We now compute the difference in the total price for consumers under price parity and no restrictions. If $b_S > \widehat{b_S}$, we have

$$TP_B^{pp} - TP_B^{nr} = \frac{1}{24}(2c_P - 2v\Delta\lambda + (12b_S - 11)\sigma_I),$$

and if $b_S \geq \widehat{b_S}$, we have $p_S + f_B^I = (c_P + v\Delta\lambda)/2$. Therefore, we have

$$TP_B^{pp} - TP_B^{nr} = \frac{1}{24}(2c_P - 2v\Delta\lambda + \sigma_I).$$

If $b_S > \widehat{b_S}$, we have $TP_B^{pp} - TP_B^{nr} \geq 0$ if and only if $b_S \geq \overline{b_S}$, where

$$\overline{b_S} \equiv \frac{2v\Delta\lambda - 2c_P + 11\sigma_I}{\sigma_I}.$$

Note that $\overline{b_S} \geq \widehat{b_S}$ (where $\widehat{b_S}$ is computed at f_B^I and f_S^I). Therefore, the total price is higher under price parity if $b_S \geq \overline{b_S}$, lower under price parity if $b_S \in (\widehat{b_S}, \overline{b_S})$. If $b_S \leq \widehat{b_S}$, we have $TP_B^{pp} - TP_B^{nr} \leq 0$ if and only if $v\Delta\lambda \geq c_P + \sigma_I/2$. Since $c_P \leq \sigma_I/2$, we have $3\sigma_I/2 - c_P \geq c_P + \sigma_I/2$. Since $v\Delta\lambda \geq 3\sigma_I/2 - c_P$, this implies that $v\Delta\lambda \geq c_P + \sigma_I/2$. Hence, the total price is lower under price parity if $b_S \leq \widehat{b_S}$.

ii) If $v\Delta\lambda > c_P + 2\sigma_I$, the platform sets $(f_P^{nr})^{FC}$ under no restrictions and $f_S^c = 0$ and $f_B^c = (v\Delta\lambda + c_P)/2$ under price parity. We use the same reasoning as above to compare the total buying price of the consumer under price parity and no restrictions (replacing for the retail price p_S under price parity if $b_S > \widehat{b_S}$ and p_I otherwise). If $b_S > \widehat{b_S}$, we have

$$TP_B^{pp} - TP_B^{nr} = \frac{1}{8}(2c_P - 2v\Delta\lambda + \sigma_I(-1 + 4b_S)).$$

Therefore, we have $TP_B^{pp} - TP_B^{nr} \geq 0$ if and only if $b_S \geq \overline{\overline{b_S}}$, where $\overline{\overline{b_S}}$ is given by

$$\overline{\overline{b_S}} = \frac{-2c_P + 2v\Delta\lambda + \sigma_I}{4\sigma_I}.$$

Note that $\overline{\overline{b_S}} \geq \widehat{b_S}$. If $b_S < \widehat{b_S}$, we have $TP_B^{pp} - TP_B^{nr} = -\sigma_I/8 \leq 0$. Therefore, the total buying price under price parity is higher if $b_S \geq \overline{\overline{b_S}}$ and lower otherwise.

Appendix H: Higher benefits of direct sales In this Appendix, we assume that a merchant obtains a benefit $\sigma_S > 0$ of selling directly to consumers and no benefits of selling through the platform (i.e., $\sigma_I = 0$). If the merchant accepts the platform's service, he trades off between setting a price such that all consumers buy through the platform or such that consumers use both selling channels as in Lemma 1. If $b_S \leq b_S^2 \equiv (-2f_B + \lambda_S v \Delta\lambda)/(\sigma_S \Delta\lambda)$, the merchant may set the price p_S such that both selling channels are used by consumers under price parity. If $b_S \geq b_S^2$, the merchant is constrained to set $\overline{p}$. If $f_B \leq \widetilde{f_B} \equiv (f_S + v\lambda_I)\Delta\lambda/(\lambda_I + \lambda_S)$, the merchant may set p_I such that all consumers buy through the platform or is constrained to set $\overline{p}$ if $f_B > \widetilde{f_B}$. We determine the profit-maximizing strategy for the merchant. We have $\pi_{pp}^{2sc} - \pi_I \geq 0$ for $b_S \leq b_S^2$ and $f_B \leq \widetilde{f_B}$ if and only if $b_S \leq b_S^3$ where

$$b_S^3 = \frac{f_B(\sqrt{\lambda_I} - \sqrt{\lambda_S})^2 - \Delta\lambda(f_S + v(\lambda_I - \sqrt{\lambda_I \lambda_S}))}{\sigma_S(\Delta\lambda)(\sqrt{\lambda_I/\lambda_S})}.$$

For $b_S \leq b_S^2$, we have $b_S^3 \leq b_S^2$.

If $f_B \leq \widetilde{f_B}$ and $b_S \leq b_S^3$, the merchant prefers to set a price such that both selling channels are used by consumers. If $f_B \leq \widetilde{f_B}$ and $b_S \in (b_S^3, b_S^2)$, the merchant obtains a higher profit with π_I than π_{pp}^{2sc}. If $b_S \geq b_S^2$, the merchant obtains a higher profit with π_I than $\overline{\pi}$. If $f_B > \widetilde{f_B}$ and $b_S \leq b_S^2$, the merchant obtains a higher profit with π_{pp}^{2sc} than $\overline{\pi}$. If $f_B > \widetilde{f_B}$ and $b_S > b_S^2$, the merchant obtains $\overline{\pi}$.

We now compare in each case whether the merchant makes a higher profit with π_S than by accepting the platform's service using the reduce function of mathematica. We find that in all cases, according to the value of f_B and the merchant's benefit of selling through the direct sales channel, the merchant prefers to sell only through the direct sales channel because this selling mode is more efficient than the platform.

It follows from this reasoning that the platform never imposes price parity, otherwise, all merchants refuse its service.

Appendix I: Low differentiation on the consumer side If $v\Delta\lambda < 2\sigma_I - 2c_P$, we need to determine whether the interior solution given by (f_B^{I2}, f_S^{I2}) maximizes the platform's profit. In our mathematica file, we prove that we have $\min(v_{C1}, v_{C2}, v_{C3}, v_{C4}) = v_{C3}$. Using the function reduce of mathematica, we find that $(\Pi^{pp})^{I2} \geq (\Pi^{pp})^{C5}$ and that $(\Pi^{pp})^{I2} \geq (\Pi^{pp})^{C4}$. If $v \leq v_{C1}$, we find that $(\Pi^{pp})^{I2} \leq (\Pi^{pp})^{C1}$. Therefore, if $v \leq v_{C1}$, the platform makes a higher profit with the corner solution C1 than with the interior solution I2. From our analysis in iii-a), we have $(\Pi^{pp})^{C1} = (\Pi^{pp})^{I} \geq (\Pi^{pp})^{C2}$ and $(\Pi^{pp})^{C2} \geq (\Pi^{pp})^{C3}$. We also have

$(\Pi^{pp})^{C1} = (\Pi^{pp})^{I} \geq (\Pi^{pp})^{C4Bis}$. Therefore, if $v < v_{C1}$, the platform maximizes its profit by choosing the first corner solution C1, unless it makes a higher profit with $C6$. However, we have $(\Pi^{pp})^{C1} \geq (\Pi^{pp})^{C6}$.

It remains to determine the platform's maximum profit if $v \in [v_{C1}, (2\sigma_I - 2c_P)/(\Delta\lambda)]$. Corner solutions 1 is not possible. Since $(\Pi^{pp})^{C4Bis} \geq (\Pi^{pp})^{C3}$, $(\Pi^{pp})^{C4Bis} \geq (\Pi^{pp})^{C5}$ and $(\Pi^{pp})^{C4Bis} \geq (\Pi^{pp})^{C6}$, we need to compare $(\Pi^{pp})^{C4}$, $(\Pi^{pp})^{C4Bis}$, $(\Pi^{pp})^{C2}$ and $(\Pi^{pp})^{I2}$. The results are complex with our set of parameters. We choose therefore to analyze various cases.

- c_P close to zero (very efficient platform)

If c_P is close to zero, the numerator of $(\Pi^{pp})^{C4Bis} - (\Pi^{pp})^{C4}$ has the sign of $-v\Delta\lambda(-v\Delta\lambda + 2\sigma_I)^2 \leq 0$. Therefore, $(\Pi^{pp})^{C4} \geq (\Pi^{pp})^{C4Bis}$. We have $(\Pi^{pp})^{I2} \geq (\Pi^{pp})^{C4} \geq (\Pi^{pp})^{C2}$. Therefore, the platform's profit is maximal at corner solution $C4Bis$.

- λ_I very close to λ_S

The numerator of $(\Pi^{pp})^{C4Bis} - (\Pi^{pp})^{C4}$ has the sign of $8c_P^2\sigma_I \geq 0$ and the denominator is positive. The numerator of $(\Pi^{pp})^{C4Bis} - (\Pi^{pp})^{I2}$ has the sign of $27c_P^2\sigma_I$ and is positive.

- λ_S very small and close to zero

The numerator of $(\Pi^{pp})^{C4Bis} - (\Pi^{pp})^{C4}$ has the sign of $-(v\lambda_I - 2\sigma_I)((-2c_P + v\lambda_I)^2 - 2v\lambda_I\sigma_I)$ if λ_S is close to zero.

- c_P close to $\sigma_I/2$

We have that $(\Pi^{pp})^{I2} \geq (\Pi^{pp})^{C4} \geq (\Pi^{pp})^{C2}$. We find that $(\Pi^{pp})^{C4Bis} - (\Pi^{pp})^{I2} \geq 0$ if v is close to $(2\sigma_I - 2c_P)/(\Delta\lambda)$ and $(\Pi^{pp})^{C4Bis} - (\Pi^{pp})^{I2} \leq 0$ otherwise.