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Accuracy and Preferences for Legal Error

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Abstract

Legal procedures used to determine liability trade-off type-1 errors (e.g., false convictions) against type-2 errors (e.g., false acquittals). After noting that people's relative preferences for type-1 errors (compared to type-2 errors) appear to be negatively correlated with technological advancements, we study how the accuracy of evidence collection methods may affect the trade-off between these two errors. Counter-intuitively, we find that under some conditions greater accuracy may result in a higher probability of type-1 error (or type-2 error) maximizing deterrence. Then, assuming both errors are decreasing in accuracy, we characterize the type-1 error that emerges under electoral pressures (when the median voter's preferences are implemented): convictions occur more often than is socially optimal, but less often than is necessary to maximize deterrence. Moreover, as the harm from crime increases, the median voter becomes less tolerant of type-1 errors as the legal system's accuracy increases. We also show that, because the median voter is less averse towards type-1 errors than the average citizen, an increase in accuracy may reduce welfare.

Keywords: Crime, deterrence, legal errors, accuracy, standard of proof, election.

JEL classification: K4.

1 Introduction

How do people's preferences over legal error change as technological advances enable the collection of more reliable evidence and thereby enhance accuracy? The history of many legal institutions, which have evolved from employing high type-1 error (false findings of liability) procedures that came close to employing a presumption of guilt, to requiring proof beyond a reasonable doubt for all elements of a crime in the United States (*In Re Winship* 1970), suggests that

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enhanced accuracy may tilt people's preferences towards lower type-1 errors.[1] Moreover, a simple casual observation suggests that there is correlation in the direction that supports this hypothesis between a country's technological development and its citizens' preferences over type-1 versus type-2 errors (failures to find liability).[2] This intuitive relationship between accuracy and people's relative preference for type-1 errors is illustrated in figure 1.[3] The horizontal axis measures the ICT Development Index (IDI), which is a composite index designed to measure countries' information and communication technologies. On the vertical axis, we have the fraction of people (among those who have reported a strict preference) who believe type-1 errors are worse than type-2 errors.[4] These preferences are taken from the 2006 International Social Survey Program Role of Government survey. The survey includes 35 countries.[5]

In this article, we present a simple law enforcement model to investigate the relationship between increased accuracy and the composition of optimal as well as the median voter's most preferred legal errors. Legal systems are imperfect, and they necessarily produce both type-1 and type-2 errors. At least two factors that can impact these errors have been analyzed in the literature. First, the accuracy of the legal system, which pertains to the informativeness of legal evidence, can presumably reduce both types of errors. Second, given any level of accuracy of the legal system, governments can impact the distribution of type-1 and type-2 errors by changing procedural rules, including the standard of proof applicable in various legal proceedings.[6] For instance, a higher standard of proof, by making it harder to convict a defendant, will lower the probabilities of convicting offenders (higher probability of type-2 error) as well as those who comply with the law (lower probability of type-1 error). Thus, there exists a trade-off between type-1 and type-2 errors.

We study the interplay between these two sources of variation in judicial errors by questioning how legal procedures would likely respond to changes in

[1] See, *e.g.*, Smith (2005), arguing that even as recently as in the early nineteenth century "many English criminal defendants ... did not benefit from a presumption of innocence but, rather, struggled against a statutory presumption of guilt."

[2] We assume that accuracy in evidence gathering in the criminal justice system increases in the general rate of development in communication and information technologies.

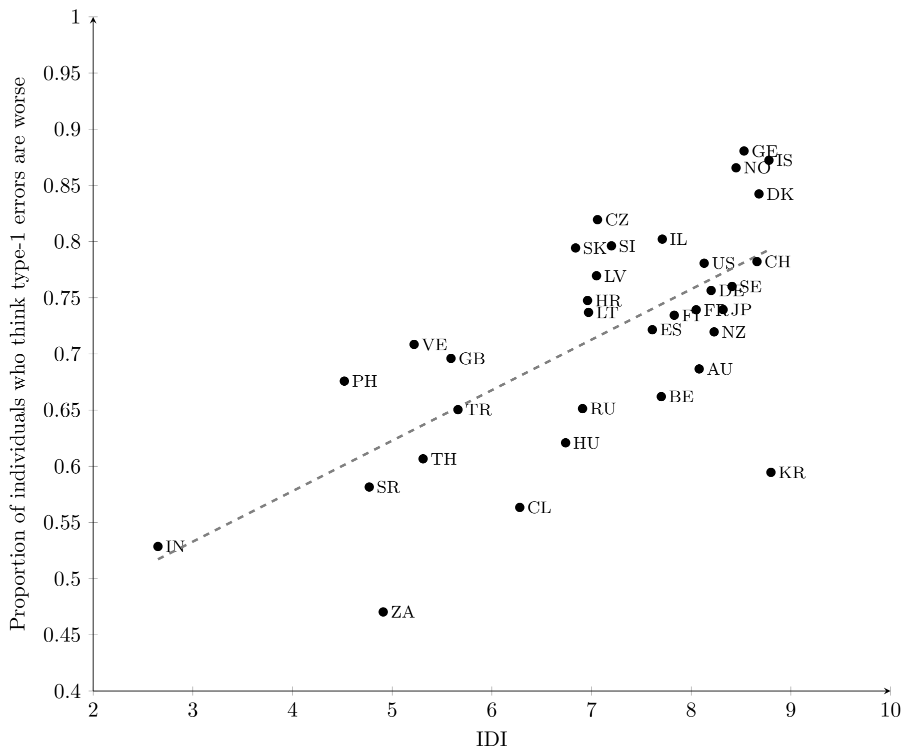
[3] Of course, many additional factors can contribute to cross-country differences as well changes over time in people's preferences over legal errors (see, *e.g.*, Givati 2019 and Johnson and Koyama 2014). We present figure 1 only to highlight an intuitive relationship which motivates our research question, rather than claiming that it alone demonstrates any kind of causal relationship.

[4] Observing the correlation between the IDI and preferences using a scatter plot does not eliminate the potential role of omitted variables such as the violent crime rate which differs across countries may depend upon a country's level of information and communication technologies.

[5] Note that Taiwan does not appear on the scatter diagram since we have no IDI index for this country.

[6] In the economics literature, various standards of proof (including *preponderance of the evidence*, and *proof beyond a reasonable doubt*, which are the most frequently used standards in the civil and criminal settings, respectively) are often conceived of as threshold levels of evidence which must be met for a finding of liability. See, *e.g.*, the references in footnote 8, below.

Figure 1: ICT Development Index and relative preference for type-1 errors



Sources: IDI Index: Measuring the Information Society - Report 2017. Opinions on legal errors are taken from the 2006 International Social Survey Program Role of Government survey.

the accuracy of the legal system. To answer this question, we show that when the state can adjust its legal procedures in response to changes in accuracy, it is possible that it may do so in a way that increases one type of error while reducing the other. Focusing on the case of a monetary sanction (*i.e.*, the sanction is fully transferable), we start by identifying the technical properties that legal procedures must possess for increases in accuracy to result in reductions in both type-1 and type-2 errors. With monetary sanctions, the socially optimal type-1 error also maximizes deterrence (because the policymaker will try to minimize the external harm from crime, which is achieved by maximizing deterrence), and an increase in accuracy has an ambiguous effect on the trade-off between the two types of errors.[7] To exemplify, assume that following an increase in accuracy, increasing the probability of type-1 error by a small amount leads to a very large increase in the probability of convicting an offender. This

[7]More specifically, it has an ambiguous effect on the marginal effect of an increase in the type-1 error on the probability to convict the guilty.

small increase would then enhance deterrence and would therefore be welfare increasing. Conversely, if after the increase in accuracy, a large reduction in type-1 error is associated with only a very modest reduction in the probability of convicting the guilty, it would be socially desirable to implement reductions in type-1 errors. In this latter case, increased accuracy can cause a reduction in both types of errors. This is because an increase in accuracy increases the probability of correct convictions for a given type-1 error, and this effect can off-set the reduction in the probability of convicting the guilty caused by the small reduction in type-1 errors (see figure 4, below, for a graphical illustration of this case).

Next, assuming this technical property holds (the deterrence maximizing type-1 and type-2 errors are decreasing with accuracy), we question how citizens' preferences for legal procedures may respond to technological advancements which increase the accuracy of legal proceedings. More specifically, we focus on the preferences of the median voter (who we assume complies with the law). Consistent with the observations related to figure 1, our model suggests that the median voter's preferred type-1 error is decreasing in accuracy under a broad range of circumstances. However, quite importantly, although this relationship is likely to hold true for crimes which result in relatively large harms, they need not hold for infractions where the harm is relatively small. This is because, when the external harm is relatively large, the median voter is mainly concerned with deterrence, and the deterrence maximizing level of type-1 error is decreasing with accuracy. This is no longer true when the harm is relatively small, in which case the median voter also cares about the cost of punishment. Consequently, even if the deterrence maximizing type-1 error decreases with accuracy, this may not be true for the median voter's preferred type-1 error. Thus, our model supplies a rationale for the intuitive relationship between accuracy and people's most preferred legal procedures pertaining to crimes that cause relatively large harms, while also explaining why, even modern and well developed nations may not place much of a value on mitigating type-1 errors pertaining to minor infractions.

Although the primary focus of our article is the relationship between accuracy and legal standards, in conducting our analysis we uncover results pertaining to closely related issues as well as the prior literature. In particular, we show that the median voter's most preferred legal procedures lie in between the welfare maximizing and deterrence maximizing procedures. Moreover, we show that an increase in accuracy can counter-intuitively reduce welfare when the median voter's preferences are implemented. These results relate to the small but growing literature on the political economics of law enforcement, which questions how public enforcers who face election pressures may deviate from optimal policies.[8] Our results add to that first line of research by showing that

[8]Regarding the theoretical literature, see for instance Daughety and Reinganum 2021, Dittmann 2004, Friehe and Mungan 2020, Langlais and Obidzinski 2007, Mungan 2017, Obidzinski 2019, Yahagi 2021 and 2022. Regarding the empirical evidence, more generally on over-punishment associated with political competition, see for instance Berdejó and Yuchtman 2021, Dušek 2012, Dyke 2007, LaFree and Tseloni 2006, Levitt 1997 and Makowsky

if legal systems are shaped by electoral pressures, they are likely to result in over-punitive institutions, as in the prior literature. Our results also contribute to the extensive literature identifying deterrence and welfare maximizing legal standards.[9] They complement this second line of research by showing that, although welfare maximizing legal procedures may require sacrificing a large degree of deterrence, standards that emerge in systems prone to electoral pressures may not do so. Our primary focus relates these extensive two strands of the literature to the less studied issue of accuracy (Kaplow and Shavell 1994, Kaplow 2012, and Obidzinski 2019). In doing so, in addition to uncovering implicit technical assumptions made in the prior literature, we provide a positive explanation for the intuitive –and arguably observed– relationship between accuracy and citizens' preferred legal standards. Additionally, we show that the presence of political pressures can cause increased accuracy to reduce rather than enhance welfare as is assumed in prior work analyzing accuracy.

As we have noted, there are economic analyses which formalize the standard of proof as a mechanism through which one can trade one type of legal error against another (see the references in footnote 8). Here, we build on this literature by taking these previously identified mechanisms and studying how they may change as a function of the accuracy of the legal system. Subsequently, we use the resulting model to study how the optimal; deterrence maximizing; and democratically determined type-1 errors may change as a function of the accuracy of the legal system. To the best of our knowledge, neither of these two tasks have been previously undertaken in the literature.

In the next section, we give an overview of our model and discuss some examples to convey the intuition behind some of our results and also to familiarize the reader with the technical concepts that we use in our subsequent formal analyses. Then, in section 3, we derive conviction probabilities as a function of legal standards employed and accuracy. In section 4, we study how accuracy affects the median voter's most preferred legal procedures. Finally, section 5 discusses the implications of our findings and concludes.

2 Model Overview

Before presenting detailed explanations of the various components in our model, we first provide an intuitive explanation of how it functions along with examples to demonstrate our key results. Our objective is to study the relationship between the legal system's accuracy (denoted a) and legal errors that would emerge through political processes, as well as errors which minimize crime and maximize welfare. To do so, we investigate (in section 3) how legal standards

and Stratmann 2009. In particular, McCannon 2013 provides some empirical evidence that political pressure affects legal errors, as it may lead to more false convictions.

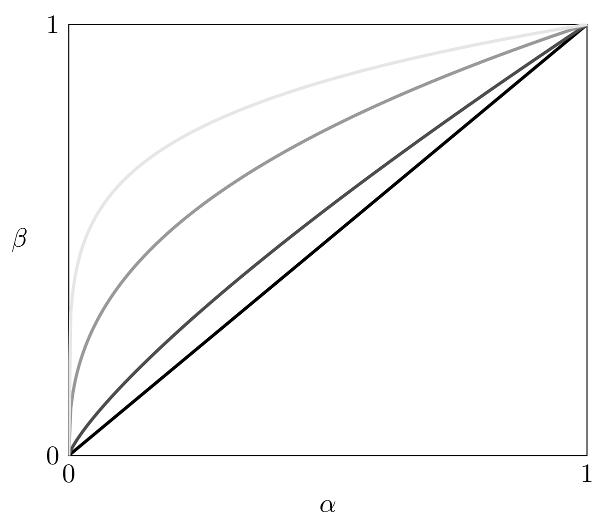
[9] This literature is quite extensive and has been expanding rapidly recently. See, *e.g.*, Demougin and Fluet 2005, 2006, Fluet and Mungan 2022, Garoupa 2018, Kaplow 2011, 2012, Kaplow and Shavell 1994, Lando 2009, Lando and Mungan 2018, Miceli 1990, Mungan 2011, 2020a, 2020b, Mungan and Samuel 2019, Mungan and Wright 2022, Obidzinski and Oytana 2019, 2020, Rizzolli and Saraceno 2013, Yilankaya 2002.

can be used to affect the relationship between the probability of incorrectly finding an innocent person liable (defined as type-1 errors and denoted α) and the probability of correctly convicting a guilty person (denoted β, which implies that the probability of type-2 errors is $1 - \beta$).

The relationship between α and β is very similar to those between 'hit rates' and 'false alarm rates' previously studied in the field of Receiver Operation Characteristic (ROC) analysis (Egan, 1975). In fact, holding the level of accuracy constant, β can be expressed as a function of α. The resulting function, $\beta(\alpha)$, can then be interpreted as the maximum probability of convicting a guilty person given a type-1 error probability of α, and represented through ROC curves as in figure 2, below.

When the degree to which a legal system can distinguish between offenders and law abiding citizens is enhanced, we say that the legal system has become more accurate. This corresponds to the legal system being able to select a higher β given any α (or, alternatively, selecting a lower α, given any β). Thus, the impact of increasing a is to shift the ROC curve outwards (*i.e.*, to the northwest of the graph) as depicted in figure 2, which depicts an example of a family of ROC curves that are obtained under various level of accuracy. We use the notation $\beta(\alpha, a)$ to reflect the dependency of the ROC curve on accuracy. More accurate systems lead to β functions which circumscribe less accurate systems. Thus, the lighter gray curves represent higher accuracy than darker gray curves.

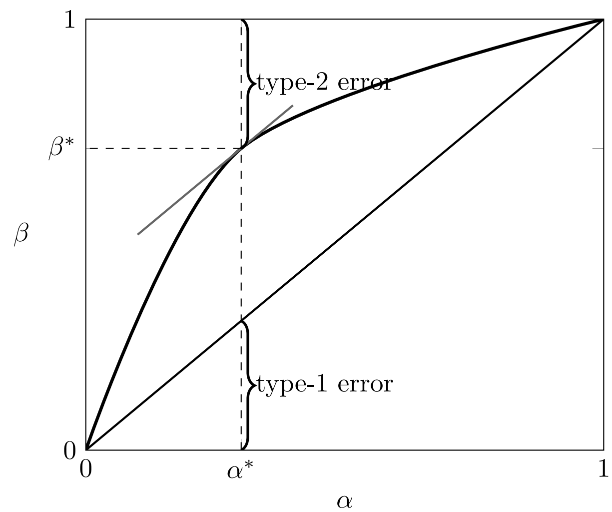
Figure 2: Relationship between the level of accuracy, type-1 and type-2 errors



Given any level of accuracy, the government can select an error pair along the ROC curve that it faces. The error pair that is selected naturally impacts people's incentives to comply with the laws. Polinsky and Shavell (2007, p.

427) summarize these incentive effects as follows: “both types of error reduce deterrence ... Mistaken acquittal diminishes deterrence because it lowers the expected fine if an individual violates the law. Mistaken conviction also lowers deterrence because it reduces the difference between the expected fine from violating the law and not violating it.” In this framework, the two errors dilute incentives to the same extent, and it therefore follows that the deterrence maximizing error pair is that which minimizes the sum of these two errors (*i.e.*, minimizing $\alpha + (1 - \beta)$), which corresponds to maximizing the discriminatory power of the legal system (*i.e.*, $\beta - \alpha$). This naturally occurs at that point where the ROC curve has a slope of 1, which is depicted in figure 3, below.

Figure 3: The optimal type-1 and type-2 errors



A strand of the law and economics research has questioned how these error pairs –and specifically the error pair which maximizes deterrence– relate to standards of proof used in legal systems. Demougin and Fluet (2005, 2006), in particular, have related the *preponderance of the evidence* standard to the error pair which maximizes deterrence by noting that it is obtained when the law convicts if there is evidence which is more likely to be produced by a guilty person than an innocent person.

These observations from the existing literature allow us to investigate how error-pairs change as the legal system becomes more accurate. While investigating optimal investments in legal accuracy, Kaplow and Shavell (1994) assume that greater accuracy reduces both types of errors.[10] We demonstrate here that

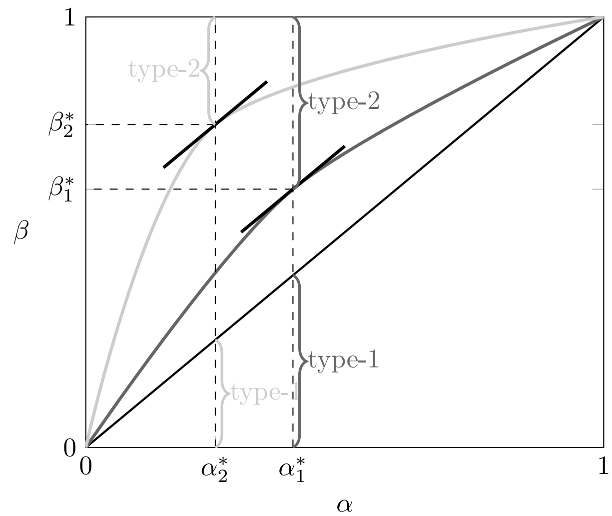
[10]Kaplow and Shavell (1994) is silent on whether, in their setting, legal standards are exogenously fixed or whether they optimally respond to changes in accuracy. They simply state that “$q_i'(k) < 0$ and $q_i''(k) > 0$, for $i = 0, 1$” where k is their measure of accuracy and

this assumption does not generally hold. Moreover, this assumption can fail to hold regardless of whether the legal standards in place are unresponsive to legal accuracy, or whether they are chosen to serve a specific objective (*e.g.*, maximize deterrence, welfare, or the median voter's utility).

This is best demonstrated by focusing on how the deterrence-maximizing error pairs respond to changes in accuracy. This is because these error-pairs are implemented through the same standard, namely preponderance of the evidence, regardless of the legal system's accuracy. Moreover, the deterrence-maximizing error pair is both optimal and maximizes the median voter's utility when sanctions are monetary, as formally proven in section 4 (see proposition 1). Thus, we illustrate, through the help of graphs depicting three illustrative cases, how increased accuracy may impact the deterrence-maximizing error pair.

Figure 4 depicts a case where the Kaplow-Shavell assumption holds: an increase in accuracy (moving from the darker gray to the lighter gray curve) causes the deterrence-maximizing error-pair to move to the north-west of the graph. This corresponds to a reduction in both types of errors, as depicted in the graph.

Figure 4: Case 1 - Both types of errors decrease

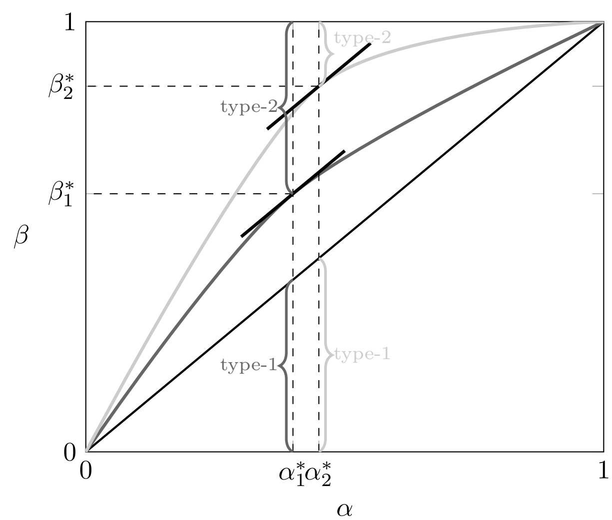


Figures 5 and 6, on the other hand, depict cases where the Kaplow-Shavell assumption is violated, because the two errors move in opposite directions in response to an increase in accuracy. In figure 5, an increase in accuracy causes

q_i denote the two types of errors (Kaplow and Shavell 1994 p.4). Thus, it is unclear whether Kaplow and Shavell (1994) implicitly assume that legal procedures that affect these two errors are also adjusted as the degree of accuracy is increased. As we explain here, their assumption can be violated regardless of the underlying assumption.

the deterrence-maximizing pair to shift to the north-east of the graph, this corresponds to an increase in type-1 errors and a reduction in type-2 errors. The errors move in the opposite directions in figure 6. We note however that deterrence is increasing with the level of accuracy in both cases, as the difference between the probabilities of punishing the guilty and the innocent rises.

Figure 5: Case 2 - Higher type-1 error

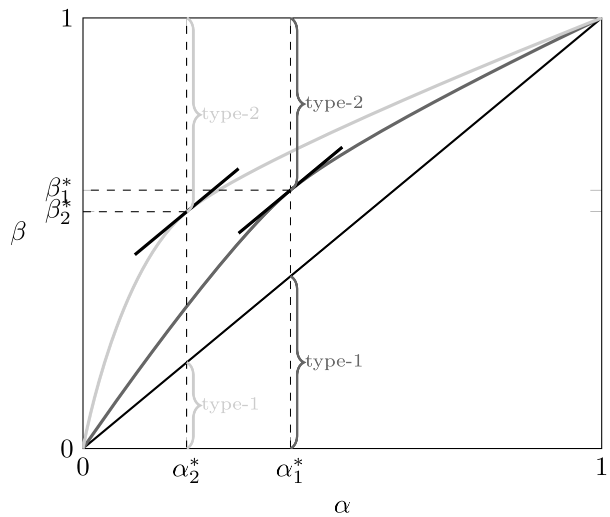


Through these observations we illustrate that one of the two types of errors may increase as a result of enhanced accuracy. In doing so, we used an intuitive and general definition of accuracy wherein more accurate systems are characterized by a more informative evidence generating process. Given the intuitive appeal of the case depicted in figure 4, where both types of errors are decreasing in accuracy, we question what type of restrictions would need to be imposed on how accuracy affects evidence generation for this case to be obtained.

Specifically, we show in Proposition 2 that a necessary condition for the deterrence maximizing probability of a type-1 error to be decreasing in accuracy is that, following an increase in the level of accuracy, an increase in that type-1 error has to be less effective in reducing the probability of type-2 error than it was prior to the increase in accuracy. Given this property, when sanctions are fully transferable monetary fines, it follows that the crime-minimizing type-1 error also maximizes social welfare and would also be chosen by the median voter (see proposition 1, below) and these are all decreasing in a.[11] However, when sanctions are not purely transferable, a divergence between these three standards emerge, and therefore it is ambiguous, *a priori*, how the median

[11] In this case where sanctions are fully transferable (such as monetary sanctions), sanctions generate a surplus that is distributed back to the population.

Figure 6: Case 3 - Higher type-2 error



voter's preferred legal standard responds to changes in accuracy. To explain our results in this case, we first briefly explain how non-transferable sanctions cause a divergence between the three standards.

For analytical convenience, consider the case where punishment triggers private costs to those who are convicted, but no additional (public) costs.[12] In this case, we note in proposition 1 that the three type-1 errors of interest, α^b (which mazimizes deterrence), α^u (which is chosen by the median voter), and α^w (which maximizes welfare), rank as: $\alpha^b > \alpha^u > \alpha^w$, which reflects the divergence between them. We provide an intuitive explanation of this result. First, $\alpha^b > \alpha^u$. This is because the median voter cares not only about the possibility of being the victim of a crime, but also avoiding conviction. The probability of conviction is minimized by setting $\alpha = 0$ whereas criminal harms are minimized by α^b; thus the median voter prefers something in between these two values, and therefore $\alpha^b > \alpha^u$. Second, $\alpha^u > \alpha^w$. This is due to two reasons. The median voter does not internalize criminal benefits, which enter the social welfare function. Additionally, the median voter, who complies with the law in equilibrium, internalizes a smaller portion of expected punishment costs, because her likelihood of being punished is smaller than the average probability of puinshment. Both considerations cause the median voter to under-internalize the costs associated with increases in type-1 errors, resulting in her preferring a greater than optimal type-1 error.

[12]This would be the case, for instance, when the punishment is partially monetary and partially non-monetary, and the monetary portion exactly covers the public cost of implementing the non-monetary portion of the sanction.

Quite importantly, our explanation above highlights that the median voter's objective diverges from deterrence maximization, because the median voter fears being falsely convicted, and thus chooses a lower than deterrence maximizing type-1 error. The 'weight', loosely speaking, that the median voter attaches towards avoiding false convictions is naturally smaller when the harm from victimization is larger. Therefore, as the harm from crime increases relative to other considerations, the median voter's objective becomes more closely aligned with deterrence maximization. Thus, for crimes involving relatively high harms, the median voter's preferred type-1 error closely tracks α^b. Therefore, when α^b is decreasing with accuracy (as in figure 4), so does the type-1 error chosen by the median voter (the result formalized by proposition 3). However, when harms from crime are small relative to other considerations, the weight that the median voter attaches to false conviction avoidance can be large, and thus her most preferred type-1 error and α^b as well as α^w can move in opposite directions in response to increases in accuracy (results formalized by propositions 4 and 5). Thus, counterintuitively, an increase in accurracy can cause reductions in welfare by increasing the divergence between the procedures implemented (a function of α^u) and those that are optimal (a function of α^w).

In the remaining sections we formalize these results an provide a more complete analysis, starting with a derivation of conviction probabilities.

3 Conviction Probabilities

We first explain the relationship between β (the probability of correct conviction) and α (the probability of false conviction) in detail. Then, we explain how accuracy affects this relationship.

3.1 Probabilities of Conviction

The probabilities of conviction, β and α, refer to the overall conviction probabilities associated with the legal system (as opposed to conviction probabilities conditional on adjudication, for instance), because our objective is to identify a relationship between the overall accuracy of the legal system and citizens' tolerance levels towards false convictions. We assume that various legal procedures in the legal system, including the standard of proof and rules pertaining to legal searches and seizures, can be adjusted to impact both α and β.

We adapt the approach in Fluet and Demougin (2005) and (2006) to explain how legal procedures affect probabilities of conviction. Specifically, we assume that each person emits a signal x with density $g(x)$ if he commits a crime and with density $\tilde{g}(x)$ if he does not commit crime, such that $L(x) \equiv \frac{g(x)}{\tilde{g}(x)}$ is the likelihood ratio with which signal x is produced by individuals who commit the crime versus individuals who refrain from crime. Both density functions have support $X \subseteq \mathbb{R}$ where X is an interval and L satisfies the Monotone Likelihood

Ratio Property (MLRP) such that

$$\frac{dL(x)}{dx} < 0 \text{ (MLRP)} \tag{1}$$

which implies that small signals are more inculpatory than large signals. Thus, the legal system sets a threshold signal, $\overline{x}$, and convicts individuals who emit signals $x < \overline{x}$. This choice implies that individuals who do not commit crime are convicted with probability

$$\alpha(\overline{x}) \equiv \tilde{G}(\overline{x}) \tag{2}$$

where $\tilde{G}$ is the cumulative distribution function (CDF) associated with $\tilde{g}$. Since $\tilde{G}$ is increasing, the inverse relationship can be noted as follows:

$$\overline{x}(\alpha) = \tilde{G}^{-1}(\alpha) \tag{3}$$

Thus, we may express the probability of convicting people who have committed crimes, as a function of α instead of a function of $\overline{x}$, as follows:

$$\beta(\alpha) = G(\tilde{G}^{-1}(\alpha)) \tag{4}$$

where G is the CDF associated with g. Expressing β as a function of α instead of $\overline{x}$ simplifies the analysis by highlighting the trade-off between type-1 and type-2 errors in a very compact manner.

By utilizing MLRP, we can make the following observations regarding the mechanics of this trade-off. First, by differentiating (4), we can note how much a marginal increase in type-1 error reduces type-2 errors, as follows:

$$\beta_\alpha(\alpha) = \frac{g(\tilde{G}^{-1}(\alpha))}{\tilde{g}(\tilde{G}^{-1}(\alpha))} = L(\overline{x}(\alpha)) > 0 \text{ for all } \alpha \tag{5}$$

Next, we can note that increasing false convictions leads to diminishing reductions in type-2 errors since

$$\beta_{\alpha\alpha}(\alpha) = \frac{dL(\overline{x}(\alpha))}{dx}\frac{d\overline{x}(\alpha)}{d\alpha} < 0 \tag{6}$$

due to MLRP and the fact that $\overline{x}$ is increasing in α.

Finally, it follows from the definition of β and α that $\beta(0) = 0$, $\beta(1) = 1$, and $\beta(\alpha) > \alpha$ for all $\alpha \in (0, 1)$. Figure 2, in the preceding section, depicts the properties of $\beta(\alpha)$ under systems with differing levels of accuracy. Thus, figure 2 also illustrates that the relationship between α and β naturally hinges on the accuracy of the legal system, which is a consideration which we have thus far neglected. Next, we define a general concept of accuracy.

3.2 Accuracy and Probabilities of Conviction

We conceive of accuracy as the ability of the legal system to distinguish between offenders and non-offenders, which can be accomplished, for instance, by better evidence collection methods. Thus, the impact of increased accuracy is a reduction in the likelihood of type-2 error that must be produced for any targeted probability of type-1 error. Formally, if we let a denote accuracy, we may note this relationship by letting β depend on a, as follows:

$$\beta_a(\alpha, a) > 0 \text{ for all } \alpha \in (0, 1)$$

As explained in section 2, it is not possible to pin down how exactly accuracy impacts the marginal reductions in type-2 error caused by increases in type-1 error, *i.e.*, the sign of $\beta_{\alpha a}(\alpha, a)$ cannot be ascertained. Because this relationship is crucial for our analysis, in the next section, we seek to identify intuitive restrictions which are consistent with assumptions made in the prior literature pertaining to accuracy. This requires explaining the remaining components of our model.

4 Preferences and Type-1 Errors

In order to investigate the impact of accuracy on the trade-off between type-1 and type-2 errors, we first explain how these errors affect the incentives of potential offenders. Subsequently, we use these observations to derive optimal type-1 errors, which allows us to identify relationships between accuracy and these policies which are consistent with previous modeling assumptions. Finally, we analyze how the median voter's preferences are impacted by accuracy when it possesses these properties.

4.1 Potential Offenders' Behavior and Deterrence

Following a large body of law enforcement literature, we focus on a single crime (see Polinsky and Shavell 2007). Potential offenders commit that crime when their expected net-benefits from doing so exceed their expected net-benefits from refraining from crime (Becker 1968). To deter the commission of crime, the state imposes a punishment whose cost to offenders is normalized to 1. Thus, the potential pay-off from committing crime to a person is $b - \beta(\alpha, a)$, where b denotes his criminal gains. These benefits differ from person to person, and we assume they are distributed with positive density f with support $[\underline{b}, \overline{b}]$, and F is the cumulative distribution function associated with f. When a person refrains from crime, he forgoes the option to gain b, and is nevertheless convicted with probability α. Thus, a person commits crime if his benefit b is such that

$$b > \beta(\alpha, a) - \alpha \equiv b^*(\alpha, a) \tag{7}$$

Here, $b^*(\alpha, a)$ refers to the deterrence threshold obtained by a given α, a pair. We assume that $\underline{b} \leq 0$ and $\overline{b} > 1$, such that some people never commit a crime, and others cannot be deterred.[13] This implies that the crime rate can be expressed as $1 - F(b^*(\alpha, a))$.

[13] An individual can obtain a negative net benefit b from committing a crime if for instance the criminal gains obtained are lower than the resource costs of committing the crime or if committing crime conflicts with their personal values (see, *e.g.*, Benabou and Tirole 2011).

Note that due to the MLRP we have that

$$\beta_\alpha(0, a) > 1 > \beta_\alpha(1, a) \tag{8}$$

and $\beta_{\alpha\alpha}(\alpha, a) < 0$ for all α, a pairs. Thus, we can implicitly define $\alpha^b(a)$, the type-1 error which maximizes deterrence for any given a, as $\beta_\alpha(\alpha^b, a) = 1$. We note that $\alpha^b(a)$ plays a critical role in the analysis to follow and that we will refer to it frequently.

4.2 Welfare and Utility Maximizing Type-1 Errors

Given the responses of potential offenders to different legal procedures, we next investigate both the socially optimal type-1 error as well as the type-1 error that maximizes the utility of different groups of people. This requires specifying the costs of crime, which come in two varieties. First, the commission of crime causes losses of $h > \bar{b}$ to others, which we call the external cost of crime.[14] Second, the punishment of an individual generates both a *private* and a *public* cost, and their sum represents the social cost of punishment. The *public* cost per detection equals $\sigma \in [-1, \overline{\sigma}]$. The *private* cost is normalized to 1. Thus, the social cost of punishment is $1 + \sigma$. Note that when $\sigma = -1$, there is no social cost of punishment: the sanction is fully transferable, which is assumed frequently in the analysis of monetary sanctions.

Punishment costs, when they exist, are financed through lump sum taxes. We assume that both taxes and the harm induced by crime are born equally (or are equally likely to be shared) by each citizen. Conversely, when punishment leads to surplus, *e.g.*, in the case $\sigma = -1$, they are distributed back to the public. Thus, the 'tax' $\tau(\alpha, a)$ keeps track of the tax burden or surplus associated with law enforcement. Following Polinsky and Shavell (2007), we assume that the tax "is such that the government breaks even", with $\tau(\alpha, a) = \sigma n(\alpha, a)$, where $n(\alpha, a)$ is the proportion of convictions (we assume that the size of the population is normalized to 1)[15] which equals

$$n(\alpha, a) = F(b^*(\alpha, a))\alpha + (1 - F(b^*(\alpha, a)))\,\beta(\alpha, a) \tag{9}$$

The first and second terms on the right hand-side respectively represent the proportion of falsely and correctly convicted individuals.

[14] The assumption $h > \bar{b}$ does not affect the median voter's preferences regarding the type-1 error. However, if this assumption is violated ($h \leq \bar{b}$), then the optimal policy may not be well defined. Indeed, in this case, there are some efficient crimes, in the sense that the private benefit of these crimes (b) is higher than the external harm from committing them (h). If the expected cost of punishment is not too high, the policymaker does not wish to deter such crimes. For instance, if $\sigma = -1$ and $h < \bar{b}$, a socially optimal type-1 error should satisfy the FOC: $\beta(\alpha, a) - \alpha = h$. However, the solution to this FOC is generally not unique. Moreover, we are interested in studying acts that are inefficient.

[15] This normalization is in addition to the normalization of the size of the sanction (which also equals 1). These two normalizations, combined, imply that the volume of maximum punishment is normalized to 1. This normalization causes no loss in generality, because welfare is measured in the same units as this volume.

Next, we express the expected net-benefit of a person who complies with the law as

$$u(\alpha, a) = -(1 - F(b^*(\alpha, a)))h - (\alpha + \tau(\alpha, a)) \tag{10}$$

The first term of the expected utility (10) represents the expected *gross* external harm from crimes (we use the term *gross* because they exclude private benefits from crimes). The second term represents the (private) expected cost of sanctions, which equal the sum of the disutility from being punished in the event of a type-1 error (α) and the lump sum tax used to finance the punishment costs ($\tau(\alpha, a)$).

We assume that u is single peaked in α.[16] Thus, the type-1 error that maximizes the expected net-benefit from compliance, denoted $\alpha^u(a)$, is characterized by $u_\alpha(\alpha^u(a), a) = 0$.

We may similarly express the expected net-benefit from violating the law as:

$$v(\alpha, a, b) = b - (1 - F(b^*(\alpha, a)))h - (\beta(\alpha, a) + \tau(\alpha, a)) \tag{11}$$

Expressions (10) and (11) reflect our assumption that people have no intrinsic preferences over errors, since their utilities are not dependent on α or β directly.[17] Instead, they prefer one policy over another, based on the impact that policy will have on their well-being. We note that in our analysis, these preferences play a role only in the determination of the median voter's behavior, to which we will turn to in section 4.4. Next, we express social welfare, which consists of the sum of all individuals utilities, by making use of (10) and (11), as follows:

$$W(\alpha, a) = \int_{\underline{b}}^{b^*(\alpha,a)} u(\alpha, a) f(b) \mathrm{d}b + \int_{b^*(\alpha,a)}^{\overline{b}} v(\alpha, a, b) f(b) \mathrm{d}b \tag{12}$$

Substituting (10), (11) and then (9) in (12), we obtain:

$$W(\alpha, a) = \int_{b^*(\alpha,a)}^{\overline{b}} f(b)(b - h) \mathrm{d}b - (1 + \sigma) n(\alpha, a) \tag{13}$$

which implies that social welfare is the difference between the net expected benefit from crime (which is always negative since $h > \overline{b}$) and the social cost of punishment. Note that for purely monetary sanctions ($\sigma = -1$), the latter term equals zero: from a benevolent policy maker's perspective, sanctions are a mere transfer. We assume that W is single-peaked in α such that its

[16]We impose this assumption to simplify the analysis by eliminating the possibility of multiple maximizers. Whether u in fact has a unique maximizer naturally depends on the functional forms of f and β. The examples we have constructed by using the uniform distribution for criminal gains and the power function for β reveal that both u and welfare defined in (12) can in fact be single-peaked in α.

[17]It is of course possible for people to have intrinsic preferences over type-1 errors. However, assuming stable intrinsic preferences, *i.e.*, preferences which are not responsive to institutional changes, these factors cannot explain changes in voting behavior. Thus, we focus exclusively on the impact of type-1 errors on people's well-being.

unique maximizer, denoted $\alpha^w(a)$, is characterized by the first order condition $W_\alpha(\alpha^w(a), a) = 0$.

Next, we consider the differences between expected social welfare and the expected utility from compliance, because this comparison will play a key role in our analysis of divergences between policies selected by the median voter and optimal policies. First, the expected compliance utility is negatively affected by the expected *gross* external harm, while the benevolent policy maker cares about the expected *net* external harm (which includes the private benefits of offenders). Second, expected compliance utility only incorporates private sanction costs due to type-1 errors, whereas social welfare incorporates the expected sanction faced by all citizens (given by the proportion of convicted individuals $n(\alpha, a)$). The consequence of these two differences is that, except for particular values of the model parameters, we may expect the socially optimal type-1 error ($\alpha^w(a)$) to be different from the type-1 error that maximizes compliance utility ($\alpha^u(a)$).

To further investigate these differences, we first identify a sufficient condition for both $\alpha^w(a)$ and $\alpha^u(a)$ to be interior, *i.e.*, $\alpha^w(a), \alpha^u(a) \in (0,1)$. Otherwise, it is possible, for instance, for non-enforcement of laws (*i.e.*, $\alpha = 0$) to be optimal and maximize the expected compliance utility. The next lemma reveals that these possibilities can be ruled out when the external harm from crime is not too small.

Lemma 1 *For any given level of accuracy, $\alpha^u(a)$, $\alpha^v(a)$, and $\alpha^w(a)$ are interior for sufficiently large h.*

Proof. The proof follows directly from the facts that (i) u, v and w are linear in h, and (ii) any arbitrary interior α generates more deterrence than $\alpha = 0$ or $\alpha = 1$. Thus, $\alpha^u(a)$, $\alpha^v(a)$, and $\alpha^w(a)$ are interior for sufficiently large h. ■

The rationale behind lemma 1 follows from the fact that as the external harm from crime increases, its minimization becomes a more important concern relative to the minimization of punishment costs. The former objective is achieved by the type-1 error which maximizes deterrence (*i.e.*, $\alpha^b(a)$), which is interior, whereas the latter objective is achieved through the corner solution $\alpha = 0$. Thus, for sufficiently large external harms, the relative importance of the deterrence objective pulls both $\alpha^u(a)$ and $\alpha^w(a)$ closer to $\alpha^b(a)$, making them both interior.

Since we are interested in cases where some enforcement takes place, unless otherwise specified, we consider values of h and a such that $\alpha^u(a)$, $\alpha^v(a)$, and $\alpha^w(a)$ are interior. With this assumption in place, we proceed by investigating when and how optimal legal procedures differ from those that maximize compliance utility. Proposition 1, below, summarizes our findings.

Proposition 1 *(i) If private punishment costs are completely transferable, the deterrence maximizing type-1 error is both optimal and maximizes expected compliance utility, i.e. if $\sigma = -1$, then $\alpha^w(a) = \alpha^u(a) = \alpha^b(a)$. (ii) If punishment costs are not completely transferable, then the optimal type-1 error is smaller than that which maximizes compliance utility, which itself is smaller than the deterrence maximizing type-1 error, i.e., if $\sigma > -1$, then $\alpha^w(a) < \alpha^u(a) < \alpha^b(a)$.*

Proof. (i) If $\sigma = -1$, the FOCs characterizing $\alpha^u(a)$ and $\alpha^w(a)$ can be expressed as

$$b^*_\alpha(\alpha^u(a), a)\left[f\left(b^*(\alpha^u(a), a)\right)\left(h - b^*(\alpha^u(a), a)\right) + \left(1 - F\left(b^*(\alpha^u(a), a)\right)\right)\right] = 0 \tag{14}$$

and

$$f(b^*(\alpha^w(a), a))b^*_\alpha(\alpha^w(a), a)(h - b^*(\alpha^w(a), a)) = 0, \tag{15}$$

respectively. Both conditions are satisfied only if $b^*_\alpha = 0$, which implies that $\alpha^u(a) = \alpha^w(a) = \alpha^b(a)$.

(ii) First, we show (through contradiction) that $\alpha^u(a, \sigma) < \alpha^b(a)$ where we express $\alpha^u(a) = \alpha^u(a, \sigma)$ to note the dependency of α^u to the public cost of punishment. The FOC characterizing $\alpha^u(a, \sigma)$ is

$$u_\alpha(\alpha, a, \sigma) = -1 + f(b^*(\alpha, a))b^*_\alpha h - \sigma n_\alpha(\alpha, a) \tag{16}$$

Thus,

$$\alpha^u_\sigma(a, \sigma) = -\frac{u_{\alpha\sigma}(\alpha^u(a, \sigma), a, \sigma)}{u_{\alpha\alpha}(\alpha^u(a, \sigma), a, \sigma)} < 0 \text{ iff } n_\alpha > 0 \tag{17}$$

since $u_{\alpha\alpha}(\alpha^u(a, \sigma), a, \sigma) < 0$.

Next, suppose that there exists $\sigma' > -1$ such that $\alpha^u(a, \sigma') \geq \alpha^b(a)$. This implies that $b^*_\alpha(\alpha^u(a, \sigma'), a) = \beta_\alpha(\alpha^u(a, \sigma'), a) - 1 \leq 0 = \beta_\alpha(\alpha^b(a), a) - 1 = b^*_\alpha(\alpha^b(a), a)$ since β is concave in α due to the MLRP. Thus, from

$$n_\alpha(\alpha, a) = \beta_\alpha(\alpha, a) - F\left(b^*(\alpha, a)\right)b^*_\alpha(\alpha, a) - f\left(b^*(\alpha, a)\right)b^*_\alpha(\alpha, a)b^*(\alpha, a), \tag{18}$$

it follows that $n_\alpha(\alpha^u(a, \sigma'), a) > 0$ which implies via (17), that $\alpha^u_\sigma(a, \sigma') < 0$. This, in turn implies that $\alpha^u(a, \sigma) > \alpha^u(a, \sigma') > \alpha^b(a)$ for all $\sigma \in [-1, \sigma')$, which is a contradiction with the fact that $\alpha^u(a, -1) = \alpha^b(a)$. Thus, $\alpha^u(a, \sigma') < \alpha^b(a)$ for all $\sigma > -1$.

Second, we compare $\alpha^u(a)$ to $\alpha^w(a)$ (where we drop σ as an argument, since it no longer plays a role in the proof). We have

$$W_\alpha(\alpha, a) = u_\alpha(\alpha, a) - (1 - F(b^*(\alpha, a)))b^*_\alpha(\alpha, a) \tag{19}$$

Note that for all $\alpha < \alpha^b(a)$, we have that $b^*_\alpha(\alpha, a) = \beta_\alpha(\alpha, a) - 1 > 0 = \beta_\alpha(\alpha^b(a), a) - 1 = b_\alpha(\alpha^b(a), a)$ since β is concave in α due to the MLRP. Thus,

$$W_\alpha(\alpha, a) < u_\alpha(\alpha, a)\ \forall\alpha \in \left[0, \alpha^b(a)\right) \tag{20}$$

This implies that $W_\alpha(\alpha^u(a), a) < u_\alpha(\alpha^u(a), a) = 0$ since $\alpha^u(a) < \alpha^b(a)$ as shown in the first step. This, in turn, implies that $\alpha^w(a) < \alpha^u(a)$ since W is single peaked. ■

First, we explain the intuition behind the result that $\alpha^w(a) = \alpha^b(a)$ when $\sigma = -1$. In this case, the sanction is a pure transfer, and therefore there is no direct social cost associated with punishment. Thus, since the external harm is higher than the maximal benefit from crime (*i.e.*, $h > \bar{b}$), the objective of the benevolent policy maker is to maximize deterrence. As a result, the socially optimal type-1 error ($\alpha^w(a)$) equals the type-1 error that maximizes deterrence ($\alpha^b(a)$).

Second, we explain the rationale behind the result that $\alpha^u(a) = \alpha^b(a)$ when $\sigma = -1$. Type-1 errors affect the crime rate as well as the per-capita surplus received net of expected punishment costs by people who comply with the law. A reduction in the crime rate always increases the expected compliance utility, since (i) this reduces the criminal harms expected to be inflicted on people and (ii) it also increases the surplus redistributed to the public. Indeed, regarding (ii), the per-capita surplus net of the expected private punishment costs associated with compliance is the difference between $n(\alpha)$ and α, which is equal to the crime rate times the difference $\beta(\alpha, a) - \alpha$. This product, and thus the net surplus, is maximized by $\alpha^b(a)$. As a result, $\alpha^b(a)$ minimizes the external harm from crimes while maximizing the net surplus: $\alpha^b(a)$ maximizes the returns from compliance.

Third, we explain why $\alpha^w(a) < \alpha^u(a)$ when $\sigma > -1$. To do so, we consider the impact of lowering α slightly below $\alpha^b(a)$. This leads to (i) a change in the criminal harms inflicted as well as (ii) a change in the expected disutility from punishment. To explain the impact of (i), recall that part of the benevolent policy maker's objective is to minimize the expected *net* external harm while, in comparison, expected compliance utility is only affected by the expected *gross* external harm. Following a small decrease in the type-1 error below $\alpha^b(a)$, deterrence is decreased, which increases the total *gross* external harm by more than the *net* external harm. Thus, compliance utility incorporates a greater impact related to changes in criminal behavior than social welfare. The effect of (ii) occurs through differences in the impact on the expected disutility from punishment. Compliance utility only incorporates the costs of mistaken punishments (of magnitude α) whereas social welfare incorporates all punishment costs. Following a small decrease in the type-1 error below $\alpha^b(a)$, both disutilities are decreased (since convictions occur less frequently). However, the reduction in the conviction probability is greater for an offender than for a person who complies with the law. As a consequence, the positive impact of the small decrease in α is larger on the average person's expected utility (and thus on society) than on the expected compliance utility. Combining these two effects, we find that reducing α below $\alpha^b(a)$ generates a larger net increase in $W(\alpha, a)$ than in $u(\alpha, a)$, which causes the socially optimal type-1 error to be lower than the type-1 error which maximizes compliance utility.

Finally, we explain why $\alpha^w(a) < \alpha^b(a)$ and $\alpha^u(a) < \alpha^b(a)$ when $\sigma > -1$. The expected cost of punishment is positive, since the total disutility from punishment is greater than any surplus from enforcement that may be obtained. Lowering α slightly below $\alpha^b(a)$ decreases the expected costs of punishment through a reduction in the probability of punishment at the cost of a negligible decrease in deterrence, and is therefore both socially beneficial and increases compliance utility. Thus, both the socially optimal type-1 error ($\alpha^w(a)$) and the type-1 error that maximizes compliance utility ($\alpha^u(a)$) are lower than the type-1 error which maximizes deterrence ($\alpha^b(a)$).

4.3 Impact of Accuracy on Optimal Errors

Here, we study the impact of accuracy on optimal type-1 errors and note two important insights that are revealed from the analysis. First, we point out that absent restrictive assumptions one cannot generally ascertain the direction towards which optimal type-1 and type-2 errors move in response to increases in accuracy. This is important, because the narrow literature on accuracy, most notably Kaplow and Shavell (1994), make the assumption that increased accuracy reduces both type-1 and type-2 errors. However, as we demonstrate below (and as illustrated in section 2), for optimal type-1 and type-2 errors to be monotonically decreasing in accuracy, the trade-off between type-1 and type-2 errors must be affected in a particular direction through an increase in accuracy around the optimal error pair. Specifically, the reduction in one type of error that can be achieved through the increase in the other type of error must be decreasing in accuracy, *i.e.*, $\beta_{\alpha a}(\alpha^w(a), a) < 0$.

To demonstrate these insights, we first note that the manner in which accuracy affects the first order condition characterizing the optimal type-1 error, and therefore the sign of $W_{\alpha a}(\alpha^w(a), a)$, cannot be ascertained. We illustrate this fact by focusing on the simplest and most frequently analyzed case in the literature where the sanction is completely transferable, *i.e.*, $\sigma = -1$. In this case, the first order condition simplifies to

$$W_\alpha(\alpha, a) = f(b^*(\alpha, a))b^*_\alpha(\alpha, a)\left(h - b^*(\alpha, a)\right) \tag{21}$$

Thus, the socially optimal type-1 error is characterized by

$$\beta_\alpha(\alpha^w(a), a) - 1 = 0 \tag{22}$$

This leads to two important observations. First, this condition is the same as the one used to characterize $\alpha^b(a)$. Indeed, as shown in proposition 1, $\alpha^w(a) = \alpha^b(a)$ if $\sigma = -1$. Second, as explained in Demougin and Fluet (2005) and Mungan (2020a), $\alpha^b(a)$ can be interpreted as the *preponderance of the evidence* standard of proof. Using the implicit function theorem reveals that

$$\alpha^b_a(a) = -\frac{\beta_{\alpha a}(\alpha^b(a), a)}{\beta_{\alpha\alpha}(\alpha^b(a), a)} \tag{23}$$

From MLRP, the denominator of (23) is negative and thus

$$\alpha^b_a(a) < 0 \;\Leftrightarrow\; \beta_{\alpha a}(\alpha^b(a), a) < 0 \tag{24}$$

A similar result follows for type-2 errors, *i.e.*, $1 - \beta(\alpha^b(a), a)$:

$$\frac{\mathrm{d}\left(1 - \beta(\alpha^b(a), a)\right)}{\mathrm{d}a} < 0 \;\Leftrightarrow\; \frac{\beta_{\alpha a}(\alpha^b(a), a)}{\beta_a(\alpha^b(a), a)} > \beta_{\alpha\alpha}(\alpha^b(a), a) \tag{25}$$

Thus, the effect of increasing accuracy on the type-1 and type-2 errors that maximizes deterrence (and expected welfare when $\sigma = -1$) crucially depends on how accuracy affects the evidence generating and interpretation process. Therefore, as illustrated in section 2, caution is required when arguing that errors decrease with accuracy, since this need not always be true.[18] Note however that both types of errors cannot simultaneously increase following an increase in accuracy. Indeed, a simple implication of the envelope theorem, applied to the first order condition (22), is that deterrence is increasing with the level of accuracy. This means that the increase in one type of error is more than compensated by the decrease in the other type of error, resulting globally in a lower sum of the probabilities of error, and thus greater deterrence (*i.e.*, a lower crime rate).

Our discussion reveals that, in general, one cannot unambiguously ascertain the direction towards which optimal type-1 and type-2 errors move in response to increases in accuracy.[19] Thus, we note a technical necessary condition for the intuitive relationship assumed in the literature to hold, as follows.

Proposition 2 *Suppose the optimal type-1 error is monotonically decreasing in accuracy (i.e.,* $\alpha_a^w(a) < 0$*) for all* $\sigma \in [-1, \overline{\sigma}]$*, then* $\beta_{\alpha a}(\alpha^b(a), a) < 0$.

Proof. As noted above, if $\sigma = -1$ then $\alpha^w(a) = \alpha^b(a)$. Thus, (24) is a necessary condition for $\alpha^w(a)$ to be monotonically decreasing in a. ■

Proposition 2 reveals that for optimal type-1 errors to be decreasing in accuracy, the trade-off between type-1 and type-2 errors must display a type of diminishing returns from accuracy around the deterrence maximizing standard, *i.e.*, $\beta_{\alpha a}(\alpha^b(a), a) < 0$. This property appears to be a natural and useful benchmark case as it holds for many different functional forms of β.[20] Moreover, we note that the property is necessary but not sufficient. That it is necessary becomes apparent when one considers the case where $\sigma = -1$. In this case, as noted in Proposition 1, the objectives of maximizing deterrence and welfare coincide. Therefore, from (24), the optimal type-1 error moves in the same di-

[18]As previously noted, Kaplow and Shavell (1994) assume that increased accuracy always reduces both type-1 and type-2 errors. As we explain in section 2.2., our definition of accuracy is more general and only suggests that more accuracy allows one to better separate an innocent person from a guilty person given any desired type-1 error (*i.e.*, that $\beta(\alpha, a) - \alpha$ is increasing with a for all $\alpha \in (0, 1)$).

[19]A functional form that produces this result is $\beta(\alpha, a) = \gamma\beta_1(\alpha, a) + (1 - \gamma)\beta_2(\alpha)$ where

$$\beta_1(\alpha, a) = \frac{2^a}{2^a - 1}\left(1 - \frac{1}{(1+\alpha)^a}\right)$$

and:

$$\beta_2(\alpha) = 1 - (1 - \alpha)^{\frac{1}{1-\epsilon}}$$

We adapt $\beta_1(\alpha, a)$ from Lundberg and Mungan (2020) where the functional form was used to illustrate possibilities in a different context. More specifically, examples consistent with cases 1-3 depicted in figures 4-6 are obtained by letting $a_1 = 1 < a_2 = 2$ and $\gamma = 1$ (for case 1), $\gamma = 0.25$ and $\epsilon = 0.9$ (for case 2), and $\gamma = 0.25$ and $\epsilon = 0.1$ (for case 3).

[20]It holds, for instance, when the signals (x) are generated through a pair of normal distributions with fixed variances and the accuracy parameter measures the distance between the means of the signal when produced through $g(x)$ versus $\tilde{g}(x)$. It also holds when β is a power function of α and becomes more informative as accuracy is increased, such that $\beta(\alpha, a) = \alpha^{\frac{1}{1+a}}$ with $a > 0$.

rection as $\beta_{a\alpha}(\alpha^b(a), a)$, and thus the condition in proposition 2 is necessary for the optimal type-1 error to be decreasing in accuracy.

The reasons why the condition is not also sufficient to guarantee that type-1 errors are decreasing in accuracy are more complicated, and relate to the misalignment between the objectives of maximizing deterrence versus welfare when $\sigma > -1$. In this case, deterrence is maximized by $\alpha^b(a)$ whereas imprisonment costs are naturally minimized by $\alpha = 0$ –which eliminates false convictions completely. This causes the optimal solution, *i.e.*, $\alpha^w(a)$, to be 'pulled' closer to 0 (see proposition 1). Because greater accuracy leads to a more informative adjudication system (*i.e.*, $a'' > a'$ implies that $\beta(\alpha, a'') > \beta(\alpha, a')$ for all $\alpha \in (0, 1)$), it follows that at small levels of α, the marginal impact of false convictions on correct convictions has to be positive (*i.e.*, $\beta_{\alpha a}(\alpha, a) > 0$ for α close to 0). Thus, when imprisonment costs bring the optimal type-1 error to very small levels, the marginal impact of type-1 errors on correct findings of liability is positive around the optimal policy. This can cause the optimal type-1 error to be increasing, rather than decreasing in accuracy. We provide an example of this case in the Appendix.

Unless otherwise specified, we will assume in the remaining of our analysis that the deterrence maximizing type-1 error is monotonically decreasing in accuracy, and thus that $\beta_{\alpha a}(\alpha^b(a), a) < 0$. Moreover, we also make the following technical assumption on the same cross-derivative to ease the derivation of results.

Assumption 1. $\beta_{\alpha a}(\alpha^b(a), a)$ is bounded, *i.e.*, there exists $c > 0$ such that $|\beta_{\alpha a}(\alpha^b(a), a)| \leq c$ for all a.

4.4 Impact of Accuracy on the Median Voter's Most Preferred Type-1 Error

Next, we investigate how the median voter's most preferred error is related to accuracy when there are diminishing returns from accuracy. We start by noting that, in general, one cannot rule out the possibility that more than half of the population commits the crime in equilibrium under the policy chosen through an electoral process. However, when the person with median gains from offending (*i.e.*, $b^m \equiv F^{-1}(0.5)$) has an ideal type-1 error which when implemented causes him to refrain from crime, it follows that less than half the population will commit crime in equilibrium. We focus on this case because it is more consistent with both intuition and empirics, and we formalize when this result will be attained. For this purpose, let $m(\alpha, a) \equiv \max\{u(\alpha, a), v(\alpha, a, b^m)\}$ and let $\alpha^m(a) \equiv \arg\max_{\alpha} m(\alpha, a)$. It follows that if

$$u(\alpha^m(a), a) > v(\alpha^m(a), a, b^m) \tag{26}$$

then the median voter will prefer a type-1 error which induces less than half the population to commit crime, in which case $\alpha^m(a) = \alpha^u(a)$. Moreover, (26) holds whenever the median gain from criminal activity is sufficiently small (or negative) since this causes the utility from criminal gains to be small. [21] In what follows, we assume that b^m is low enough such that (26) holds and therefore $\alpha^u(a)$ represents the median voter's ideal type-1 error. This assumption is both intuitive and conforms with more realistic cases where only a small proportion of individuals have large benefits from committing crime. We also note that previous analyses (Lanlais and Obidzinski 2017, Mungan 2017, and Obidzinski 2019) which study the median voter's preferences in different law enforcement contexts identify other reasonable conditions under which the majority refrains from committing crime in equilibrium.

With this assumption in place, our previous findings in proposition 1 imply that an electoral process[22] which implements the median voter's most preferred policy will tend to generate greater than optimal type-1 errors. Next, we summarize how the median voter's most preferred type-1 error is affected by increased accuracy.

Proposition 3 *There exists a threshold level of harm, $\overline{h}$, such that $h > \overline{h}$ implies that $\alpha_a^u(a) < 0$.*

Proof. We can rewrite the first order condition for $\alpha^u(a)$ as

$$k(\alpha, a) - \frac{1}{h} l(\alpha, a) = 0 \tag{27}$$

where $k(\alpha, a) = f(b^*(\alpha, a))b^*_\alpha(\alpha, a)$ and $l(\alpha, a) = 1 + \sigma n_\alpha(\alpha, a)$. It follows that $\lim_{h\to+\infty}[k(\alpha, a) - \frac{1}{h}l(\alpha, a)] = k(\alpha, a)$, and thus $\lim_{h\to+\infty} \alpha^u(a) = \alpha^b(a)$, since $k(\alpha, a) = 0 \Leftrightarrow b^*_\alpha(\alpha, a) = 0$. Therefore,

$$\lim_{h\to+\infty} b^*_\alpha(\alpha^u(a), a) = b^*_\alpha(\alpha^b(a), a) = 0 \tag{28}$$

We note that

$$\alpha_a^u(a) = -\frac{k_a(\alpha^u(a), a) - \frac{1}{h} l_a(\alpha^u(a), a)}{\frac{\mathrm{d}[k(\alpha,a) - \frac{1}{h} l(\alpha,a)]}{\mathrm{d}\alpha}} \tag{29}$$

with $\frac{\partial[k(\alpha,a)-\frac{1}{h}l(\alpha,a)]}{\partial\alpha} < 0$, since $\alpha^u(a)$ is a proper maximum. Thus,

$$\begin{aligned}
\operatorname{sign}\left[\lim_{h\to+\infty} \alpha_a^u(a)\right] &= \operatorname{sign}\left[\lim_{h\to+\infty}\left[k_a(\alpha^u(a), a) - \frac{1}{h} l_a(\alpha^u(a), a)\right]\right] &(30)\\
&= \operatorname{sign}\left[\lim_{h\to+\infty} k_a(\alpha^b(a), a)\right] &(31)\\
&= \operatorname{sign}\left[\beta_{\alpha a}(\alpha^b(a), a) f(b^*(\alpha^b(a), a))\right] < 0 &(32)
\end{aligned}$$

[21] For a formal proof of this claim, note that $u(\alpha^m(a), a) \geq u(\alpha^v(a), a) > v(\alpha^v(a), a, b^m)$ if $b^m < \beta(\alpha^v(a)) - \alpha^v(a)$ where $\alpha^v(a) = \arg\max_\alpha v(\alpha, a, b^m)$, since $u(\alpha^v(a), a) - v(\alpha^v(a), a, b^m) = \beta(\alpha^v(a), a) - \alpha^v(a)$. But, $\beta(\alpha^v(a), a) - \alpha^v(a)$ has a positive lower bound since $\alpha^v(a) \notin \{0, 1\}$, which follows from lemma 1 and the fact that $\alpha^v(a) < \alpha^u(a)$. Thus, there exists $\varepsilon > 0$, such that $b^m \leq \varepsilon$ implies that (26) holds.

[22] Previous articles have analyzed the choice of monetary and non-monetary sanctions, detection, and accuracy through Downsian electoral competition (Langlais and Obidzinski 2017, Obidzinski 2019) or through the median voter theorem (Mungan 2017).

where the second equality follows from (28) and the facts that $l_a(\alpha^b(a), a)$ is bounded (per assumption 1) and is independent of h. The inequality in (32) holds since we have assumed that $\beta_{\alpha a}(\alpha^b(a), a) < 0$. Thus, if h is large enough, $\alpha_a^u(a) < 0$. ■

Proposition 3 states that an increase in accuracy reduces the median voter's preferred type-1 error, as long as the harm induced by the crime is high enough. When the external harm from crime (h) increases, the citizen's concern for the expected external harm becomes more important relative to her concern for the expected cost of sanctions. As a result, the citizen's objective tends toward the maximization of deterrence. Recall from equation (23) that the deterrence maximizing type-1 error is lowered with greater accuracy, as long as assumption 1 (diminishing returns from increased accuracy) is satisfied. Thus, for large enough harms, the median voter's most preferred type-1 error is also decreased, since it mimics the properties of the deterrence maximizing type-1 error.

The dynamics we outline above, of course, need not hold for small harm crimes. This is because when harms are small and the enforcement mechanism is inaccurate, the median voter need not be as concerned about deterrence as she is with other objectives (*i.e.*, tax consequences and limiting her expected false conviction costs). This may cause her to prefer very low type-1 errors. However, for type-1 errors which are much below the deterrence maximizing type-1 error, increased accuracy enhances the effectiveness of type-1 errors in eliminating type-2 errors, *i.e.*, $\beta_{\alpha a}(\alpha, a) > 0$ for sufficiently small α. Thus, when harms are small, the median voter's most preferred type-1 error can increase with more accuracy. We formalize this result by constructing a simple example.

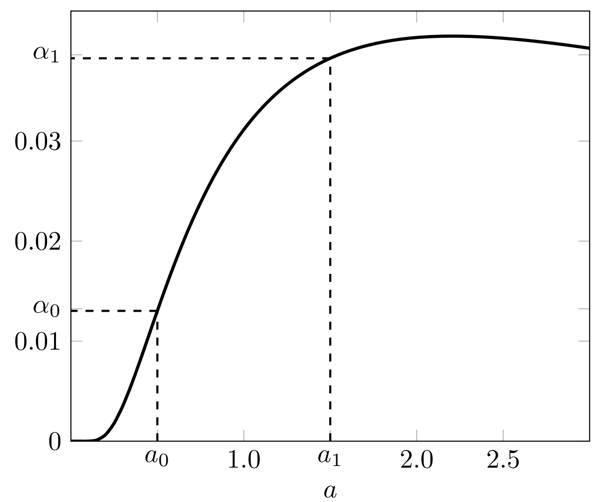
Example 1 *For $b \in [0, \bar{b}]$, let $f(b) = 0.5/\bar{b}$, such that $F(b) = 0.5(1 + \frac{b}{\bar{b}})$. Moreover, let $h = 1.2$, $\sigma = 0$, $\bar{b} = 1.1$ and $\beta(\alpha, a) = \alpha^{\frac{1}{1+a}}$.*

It is easy to verify that the specified β function satisfies all of the properties we have discussed in previous sections. Specifically, $\beta(0, a) = 0$, $\beta(1, a) = 1$, $\beta(\alpha, a) > 0 > \beta_{\alpha\alpha}(\alpha, a)$ for all $\alpha \in (0, 1]$, and $\beta_{\alpha a}(\alpha^b(a), a) < 0$ for $a > 0$.

In Figure 7, we plot the median voter's most preferred type-1 error $\alpha^u(a)$ obtained in our example as a function of accuracy. Since we take h as given when we vary accuracy, it follows that for small enough a, the harms from crime are insufficient to warrant enforcement from the median voter's perspective, thus $\alpha^u(a) = 0$ for small a. However, when a is not small, the median voter prefers some enforcement, and her most preferred type-1 error is increasing in accuracy. Some numerical values that emerge from our example are marked with dashed lines to illustrate this fact. Specifically, $\alpha_0 = \alpha^u(a = 0.5) \approx 0.01303 < \alpha_1 = \alpha^u(a = 1.5) \approx 0.03828$. This example is used to formalize the result of proposition 4.

Proposition 4 *For small h, an increase in accuracy may lead to an increase in the median voter's most preferred type-1 error (i.e., it is possible that $\alpha_a^u(a) > 0$).*

Figure 7: The median voter's most preferred type-1 error $\alpha^u(a)$

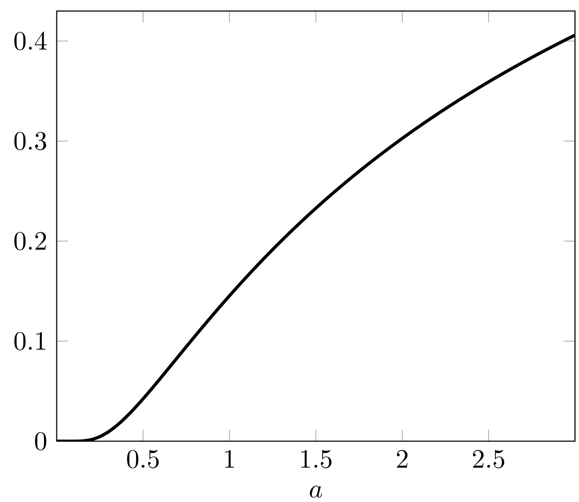


Next, we highlight an additional and counter-intuitive result that is revealed through Example 1 (and illustrated *via* figures 8 and 9).

Proposition 5 *An increase in accuracy may cause a reduction in welfare despite enhancing deterrence when the median voter's most preferred policy is implemented.*

Figures 8 and 9 graphically illustrate how an increase in accuracy may cause a reduction in social welfare despite enhancing deterrence. The figures plot the deterrence threshold (figure 8) and the expected welfare (figure 9) obtained when the median voter's most preferred policy is implemented. They illustrate that, when accuracy increases, the higher type-1 error implemented by following the median voter's preferences have a positive effect on deterrence, but a negative effect on expected welfare. The intuition behind the positive effect on deterrence is that, first, as shown in proposition 1, $\alpha^u(a) < \alpha^b(a)$ if $\sigma > -1$. Thus, an increase in the type-1 error above $\alpha^u(a)$ increases deterrence as long as $\alpha < \alpha^b(a)$. Second, *ceteris paribus*, an increase in the level of accuracy always has a positive impact on deterrence. The intuition behind the negative effect on expected welfare is that, as shown in proposition 1, $\alpha^u(a) > \alpha^w(a)$ if $\sigma > -1$. A rise in accuracy, by increasing the median voter's most preferred type-1 error, may exacerbate the discrepancy between the socially optimal type-1 error and the one preferred by the median voter. As a consequence, an increase in accuracy may counterintuitively lead to a loss of welfare.

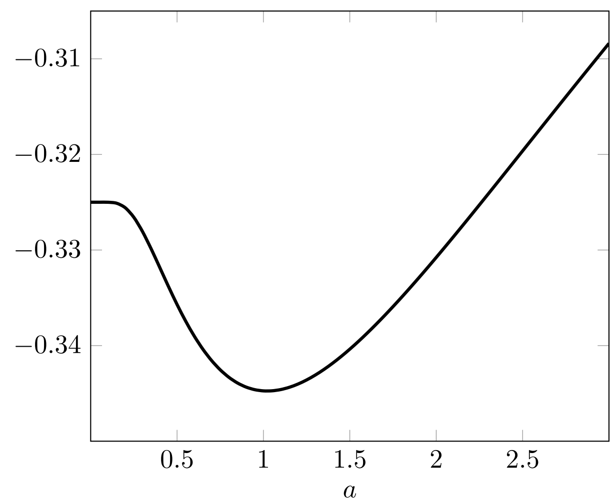
Figure 8: The deterrence threshold $b^*(\alpha^u(a), a)$



5 Conclusion

The evolution of many criminal law processes towards reducing type-1 errors is an interesting phenomenon. Here we have questioned whether this type of evolution is consistent with changes in the popular demand for legal institutions as a function of increased accuracy in the determination of guilt. Our findings suggest that for violations which generate relatively large social harms, the median voter's preferred type-1 error is decreasing with accuracy, which is consistent with historical trends. On the other hand, the same conclusion need not hold for infractions that cause relatively small harms. Our analysis also revealed that legal procedures that emerge under electoral pressures generate above optimal type-1 errors, *i.e.* falsely convict or impose liability more frequently than is optimal. Moreover, contrary to intuition, increases in accuracy can be welfare reducing. Another possible explanation behind the above-mentioned evolution of legal procedures is that socio-economic development (which may be expected to be positively correlated with accuracy) may be associated with a decline in violent crime or even a decrease in the prevalence of crime in general. Consequently, it can weaken the concern of the median voter (and of society) for crime deterrence, thus lowering the preferred type-1 error. Nonetheless, our analysis provides a new perspective through which one can study and interpret the evolution of legal institutions, and draws attention to specific ways in which electoral pressures can contribute to the emergence of inefficient legal procedures.

Figure 9: Expected welfare $W(\alpha^u(a), a)$



Appendix

Here, we provide an example illustrating that the condition we specify in proposition 2, *i.e.*, $\beta_{\alpha a}(\alpha^b(a), a) < 0$, is insufficient to guarantee that the optimal type-1 error is decreasing in a. We consider the case where $f(b) = \frac{1}{2\bar{b}}$; $F(b) = 0.5(1+\frac{b}{\bar{b}})$; and $\beta(\alpha, a) = \alpha^{\frac{1}{1+a}}$. It then follows that $\alpha^b(a) = (1+a)^{-\frac{1+a}{a}}$, and it can easily be verified that $\beta_{\alpha a}(\alpha^b(a), a) < 0$ for $a > 0$. Next, letting $\bar{b} = 1.1$, $h = 1.2$, and $\sigma = -0.95$, we plot the expected welfare (figure 10) as well as $\beta_{\alpha a}(\alpha, a)$ (figure 11) for $a \in \{0.2, 0.3\}$ where the thicker curves represent greater accuracy.

Figures 10 and 11 together illustrate two important facts. First, figure 10 illustrates that the optimal type-1 error increases in response to an increase in accuracy, from $\alpha_1^w = \alpha^w(0.2)$ to $\alpha_2^w = \alpha^w(0.3)$. Second, figure 11, which depicts $\beta_{\alpha a}(\alpha, a)$ as a function of α, reveals that for small type-1 errors, the marginal impact of type-1 errors on $\beta(\alpha, a)$ is increasing. Quite importantly, we see that the type-1 errors which maximize welfare are small enough such that $\beta_{\alpha a}(\alpha^w(a), a) > 0$ for $a \in \{0.2, 0.3\}$.

References

[1] Becker, G. (1968). Crime and Punishment: an Economic Approach. Journal of Political Economy, 76: 169-217.

Figure 10: Expected social welfare for $a = 0.2$ (thin curve) and $a = 0.3$ (thick curve)

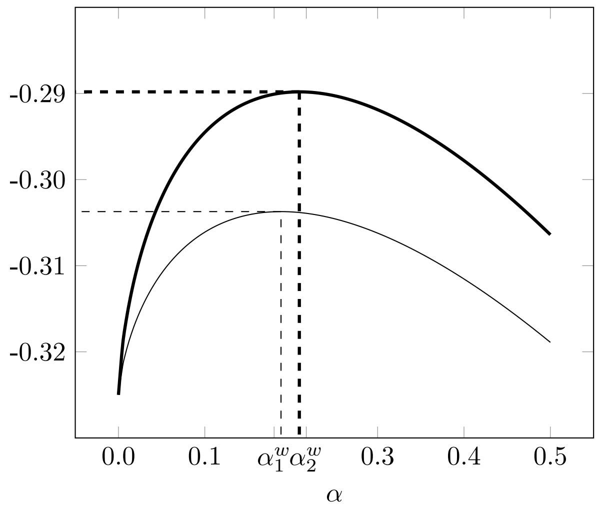
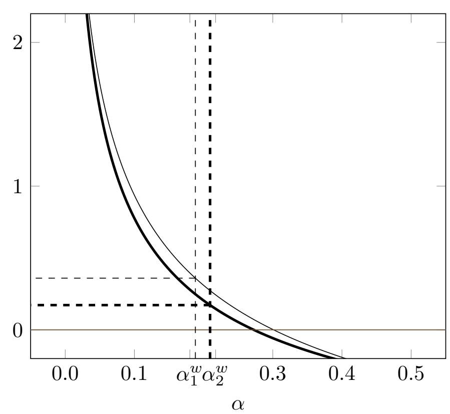


Figure 11: $\beta_{\alpha a}(\alpha, a)$ for $a = 0.2$ (thin curve) and $a = 0.3$ (thick curve)



[2] Benabou, R., and Tirole, J. (2011). Laws and norms. No. w17579. National Bureau of Economic Research.

[3] Berdejó, C. and Yuchtman, N. (2013). Crime, Punishment, and Politics: An Analysis of Political Cycles in Criminal Sentencing. Review of Economics and Statistics, 95(3): 741–756.

[4] Daughety, A. F., and Reinganum, J. F. (2021). Prosecutor Quality, Witness Participation, Crime, and Reform. American Economic Journal: Microeconomics, 13(4): 64-100.

[5] Demougin, D. and Fluet, C. (2005). Deterrence versus judicial error: A comparative view of standards of proof. Journal of Institutional and Theoretical Economics, 161(2): 267-295.

[6] Demougin, D. and Fluet, C. (2006). Preponderance of evidence. European Economic Review, 50: 963-976.

[7] Dittmann, I. (2004). The Optimal Use of Fines and Imprisonment if Government Do Not Maximize Welfare. Journal of Public Economic Theory, 8(4): 677–695.

[8] Dušek, L. (2012). Crime, Deterrence and Democracy. German Economic Review, 13(4): 447–469.

[9] Dyke, A. (2007). Electoral Cycles in the Administration of Criminal Justice. Public Choice 133(3/4): 417–437.

[10] Egan, J. P. (1975). Signal Detection Theory and Roc Analysis, Academic Press. ISBN-13 978-0122328503.

[11] Fluet, C. and Mungan, M. (2022). Laws and norms with (un)observable actions. European Economic Review, 145: 104129.

[12] Friehe, T. and Mungan, M. (2020). The political economy of enforcer liability for wrongful police stops. Journal of Public Economic Theory. https://doi.org/10.1111/jpet.12472.

[13] Garoupa, N. (2017). Explaining the standard of proof in criminal law: A new insight. Supreme Court Economic Review, 25: 111-122.

[14] Givati, Y. (2019). Preferences for criminal justice error types: Theory and evidence. The Journal of Legal Studies, 48: 307-339.

[15] Johnson, N. and Koyama, M. (2014). Taxes, lawyers, and the decline of witch trials in France. The Journal of Law and Economics, 57: 77-112.

[16] Kaplow, L. (2011). Optimal proof burdens, deterrence, and the chilling of desirable behavior. American Economic Review, 101: 277-80.

[17] Kaplow, L. (2012). Burden of proof. The Yale Law Journal, 121: 738-859.

[18] Kaplow, L. and Shavell, S. (1994). Accuracy in the determination of liability. Journal of Law and Economics, 37: 1-15.

[19] LaFree, G. and Tseloni, A. (2006). Democracy and crime: A multilevel analysis of homicide trends in forty-four countries, 1950-2000. Annals of the American Academy of Political and Social Sciences 605(1): 25–49.

[20] Lando, H. (2009). Prevention of crime and the optimal standard of proof in criminal law. Review of Law & Economics, 114: 33-52.

[21] Lando, H. and Mungan, M. (2018). The effect of type-1 error on deterrence. International Review of Law and Economics 53: 1-8.

[22] Langlais, E. and Obidzinski, M. (2016) Law Enforcement with a Democratic Government. American Law and Economics Review, 19: 162-201.

[23] Levitt, S.D. (1997). Using electoral cycles in police hiring to estimate the effect of police on crime. American Economic Review, 87(3): 270–90.

[24] Lundberg, A. and Mungan, M. (2020). The effect of evidentiary rules on conviction rates. George Mason Law & Economics Research Paper 20-17 (2020).

[25] Makowsky, M.D. and Stratmann, T. (2009). Political economy at any speed: What determines traffics citations? American Economic Review 99(1): 509–527.

[26] McCannon, B. C. (2013). Prosecutor elections, Mistakes, and appeals. Journal of Empirical Legal Studies 10(4): 696–714.

[27] Miceli, T. (1990). Optimal prosecution of defendants whose guilt is uncertain. Journal of Law Economics and Organization 6: 189-201.

[28] Mungan, M. (2011). A utilitarian justification for heightened standards of proof in criminal trials. Journal of Institutional and Theoretical Economics, 167: 352-370.

[29] Mungan, M. (2017). Over-incarceration and disenfranchisement. Public Choice, 172: 377-395.

[30] Mungan, M. (2020a). Justifications, excuses, and a affirmative defenses. The Journal of Law, Economics, and Organization, 36: 343-377.

[31] Mungan, M. (2020b). The optimal standard of proof with adjudication avoidance. Review of Law and Economics 16.

[32] Mungan, M., Obidzinski, M., and Oytana, Y. (2019). Standard of proof and accuracy. Working paper.

[33] Mungan, M. and Samuel, A. (2019). Mimicking, errors, and the optimal standard of proof. Economics Letters 174: 18-21.

[34] Mungan, M. and Wright, J. (2022). Optimal standards of proof in antitrust. International Review of Law and Economics, 71: 106083.

[35] Obidzinski, M. (2019). Accuracy in public law enforcement under political competition. Supreme Court Economic Review, 27: 195-212.

[36] Obidzinski, M. and Oytana, Y. (2019). Identity errors and the standard of proof. International Review of Law and Economics, 57: 73-80.

[37] Obidzinski, M. and Oytana, Y. (2020). Presumption of innocence and deterrence. Journal of Institutional and Theoretical Economics, 176: 377-412.

[38] Polinsky, A. M. and Shavell, S. (2007). The Theory of Public Enforcement of Law. Handbook of Law and Economics, edition 1, volume 1, chapter 6: 403-454, Elsevier.

[39] Rizzolli, M. and Saraceno, M. (2013). Better that ten guilty persons escape: punishment costs explain the standard of evidence. Public Choice, 155: 395-411.

[40] Smith, B. (2005) The presumption of guilt and the English law of theft, 1750-1850. Law and History Review 23: 133-199.

[41] Yilankaya, O. (2002) A model of evidence production and optimal standard of proof and penalty in criminal trials. Canadian Journal of Economics/Revue Canadienne d'économique 35: 385-409.

[42] Yahagi, K. (2021). Law enforcement with motivated agents. International Review of Law and Economics, 66 105982.

[43] Yahagi, K. (2022). Law enforcement with compensation statutes for legal error. Discussion Paper Series Institute of Social Science, University of Tokyo, WP E-21-008.