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Charitable Giving and Public Good Provision: an Optimal Tax Perspective

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Abstract

This paper studies the consequences of charitable giving for both the optimal tax system and the optimal provision of a public good. Through warm glow, taxpayers derive utility from their individual charitable contribution. Aggregate contributions then benefit to all individuals through the public good effect of charitable giving. The government has two sets of instruments to maximize social welfare : nonlinear taxes of both income and donations as well as direct contributions to the public good. First, I show that heterogeneity in altruism rather than heterogeneity in public good preferences advocate for a direct contribution to the public good by governments. Second I provide new optimal tax formulas able to match the actual tax treatment of giving and income in OECD countries. I show that the problems of setting the optimal subsidy to giving and the optimal income tax rates can be separated in a French-like tax credit system. This separation is no longer feasible in a US-like tax deduction system where optimal income tax rates necessarily depend on the externality associated to charitable giving: the stronger the externality, the higher should be the optimal income tax rate. These results are expressed in terms of empirically meaningful parameters and redistributive tastes of the government that can be taken to the data. I will rely on French taxpayer's data to provide a quantitative exploration of the optimal tax formulas for both income and charitable contributions.

Keywords: optimal taxation; charitable giving; public good; warm glow; multidimensional heterogeneity

JEL-Codes: H21, H41, D64

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I Introduction

Patterns in donations of individuals to charities vary substantially across countries. According to the Charities Aid Foundation, Indonesia has the world highest proportion of donors with 84% of Indonesian reporting a monetary gift to charities in 2021. This proportion of donors declines to 65% in the UK, 61% in the US, 30% in Italy and eventually falls to 3% in Georgia.[1] Both economics, political science and sociology provide potential explanations for these divergence in the importance of charitable giving across countries.[2] Among these explanations, the tax treatment of charitable giving can explain part of this cross-country heterogeneity. While most OECD countries do have some form of tax exemptions for charitable giving, the precise tools used to provide such incentives vary substantially. For instance, the US rely on a tax deduction system, where donations can reduce taxable income. Given the progressive nature of the income tax schedule, the deduction is more profitable for high income earners. In France, donations give right to a tax credit, which directly reduces tax liability in a uniform fashion. While there exist various positive explanations for these differences in the tax treatment of charitable giving, this paper provides conditions on both tax rates and public good provision for social welfare maximization, using the tools of optimal taxation theory.

I study an economy of heterogeneous taxpayers making consumption, donation and labor supply decision. Donation is both a private good, providing a warm-glow, and a public good as it creates a positive externality that benefits to all individuals in the economy. The government can tax or subsidize labor income and donation as well as directly providing a public good. In the most general version of my model, I assume that the utility derived from consuming the public good provided by the government can be different from the utility derived from consuming the public good funded by charitable giving. Integrating the interactions between donations, labor supply, taxes and public goods, I characterize both the optimal nonlinear tax schedule for labor income and donations as well as the optimal provision of public good by the government. My findings are threefold :

First, I show that charitable contributions affect optimal tax policy by amplifying standard behavioral responses to taxes, through both the externality and spillovers associated to donations. The reasoning is the following : tax policy can affect the amount individuals choose to give to charities, hence mechanically affecting the total amount of donations in the economy. Because all individuals derive some utility from the aggregate level of contributions, this has a direct impact on social welfare. In addition to this externality channel, a change in the aggregate level of contributions can affect the individual's optimal choice of giving which in turn changes the aggregate level of contributions and so on. This is the spillover effect of donations and I document its impact on social welfare hence its consequences for optimal tax policy. Compared to previous studies, the tax incidence exercise described here is performed in a general framework where tax instruments can depend nonlinearly on labor income and donations and where taxpayers can differ across many unobserved individual characteristics.

[1] See the annex table of the World Giving Index 2022 report for the complete list : https://www.cafonline.org/about-us/publications/2022-publications/caf-world-giving-index-2022

[2] Social Origins Theory for instance relates individual preferences for giving to the specific development of a modern State in a country (Salamon and Anheier (1998)).

Second, I derive optimal tax formulas in a realistic tax system able to replicate the properties of the actual fiscal treatment of charitable giving in OECD countries. This realistic tax instrument is the sum of an income tax schedule and of a specific schedule for donations. The income tax is a function of taxable income, which is defined as labor income net of the deduction for charitable giving. Both the income and donations tax schedules as well as the deduction function can be nonlinear. This family of tax instruments has been introduced by Jacquet and Lehmann (2021a) to study the optimal taxation of multiple income sources. Applying their methodology to the case of charitable giving allows me provide explicit formulas for both the income and donation tax schedules as well as deriving an optimality condition for the deduction rule for charitable giving. Following Saez (2001), these formulas are expressed in terms of meaningful empirical parameters so that they can be taken to the data to provide quantitative insights on the optimal tax treatment of charitable giving.

Third, in the tradition of Samuelson (1954), I derive a set of conditions on the optimal level of the public good in a context where both the government and taxpayers can contribute to its provision. Following standard practices in the literature, I first assume that the public good is simply the sum of the direct provision by the government and the aggregate amount of charitable giving. In this case, I show that under the appropriate separability assumptions, the public good should only be funded by charitable giving. This exploits the warm glow of giving: a public good financed by donations is at the same time a cost, it reduces the resources available for private consumption, but also a gain, through warm glow. This contrasts with a funding through taxes where only the cost dimension occurs. While Aronsson et al. (2021) derives such a result in an Atkinson and Stiglitz (1976) setting, I show that it holds in a more general framework with multidimensional heterogeneity. In particular, it holds with heterogeneous and non-separable preferences for the public good. In addition, I also study the optimal provision of the public good in the general framework with multidimensional heterogeneity and imperfect substitution between the public good provided by the government and the one funded by private contribution. I derive a modified Samuelson rule showing how behavioral responses of both labor income and donations to variation of the public good funded by the government have to be taken into account when setting its optimal level. In the specific case where taxpayers derive the same utility from the public good provided by the government and the one funded by private contributions, I show that optimal tax rates no longer depend on the externality parameter as soon as the government sets its public good contribution to its optimal level.

Related literature. This paper first relates to the literature on the optimal tax treatment of charitable contributions. Assuming a specific form of altruism where the rich care about the poor, Atkinson (1976) derives optimal subsidy formulas for donations. More recent contributions follow Andreoni (1989, 1990) and assume, as in the present paper, a warm glow motive of of giving. Using linear tax instruments, Saez (2004) derives optimal tax formulas in terms of sufficient statistics and redistributive preferences. By allowing for nonlinear tax instruments, I provide a more realistic description of the actual problem governments face when designing the fiscal treatment of charitable contribution. In particular, I emphasize the specific role of the nonlinearity of the income tax schedule when designing the deduction rule for charitable giving. In this case, both marginal tax rates as well as the curvature of the income tax affect the optimal deduction function. Since in practice, most OECD countries do rely on both a nonlinear income tax and deduction rules for giving, this channel has to be taken into account, although its quantitative importance has yet to be determined.

Using also nonlinear taxes, Aronsson et al. (2021) study a Mirrleesian economy where not only warm glow but also status considerations motivate charitable contributions. Moreover, they consider the problem of a non-welfarist government that does not directly value the utility derived from the act of giving. I explicitly relate to this paper in my mechanism design analysis of the unidimensional case where agents only differ in labor productivity. My main contribution with respect to Aronsson et al. (2021) is to allow for multidimensional heterogeneity of taxpayers. Using a two-type model with fixed hours of work and additive preferences, Diamond (2006) provides a simpler optimal policy analysis, describing how nonlinear subsidies of charitable giving can improve welfare by relaxing incentive compatibility constraints. Departing from the analysis of charitable contribution, Koehne and Sachs (2022) study the problem of optimal tax expenditure for work-related expenses. I show that in the case of a Mirrleesian economy the optimal tax treatment of giving as well as the contribution of the government to the public good do heavily depend on the precise form of labor separability assumed.

Second, I contribute to the extensive literature on the optimal provision of a public good. Since the seminal contribution of Samuelson (1954), the literature has studied the relationship between the provision of a public good and optimal tax issues. For instance, departing from the lump-sum tax assumption of Samuelson (1954), Atkinson and Stern (1974) investigate the public good provision problem in presence of distortive tax instruments. Introducing ex-ante heterogeneity between agents in a stylized two-type model, Boadway and Keen (1993) discuss how the standard Samuelson rule evolves in presence of optimal income taxation. The analysis of the tax treatment of charitable giving provided by Saez (2004), Diamond (2006) and Aronsson et al. (2021) also contribute to this literature by introducing voluntary contributions to the public good. In all those works, as in the present paper, agents are assumed small so that the public good is taken as given when taxpayers make their optimal consumption, donation and work decisions. Hence taxpayers donate to charity only because of warm glow, without taking into account the impact of their contributions on the public good. This is at odds with the Nash structure of the original problem studied by Samuelson (1954) and applied to the case of charitable giving by Warr (1982). In particular this neglects a potentially important aspect of public good provision which is free-riding. Yet, by carefully modeling the responses of individual contributions to changes in the aggregate level of contributions, I introduce a spillover parameter that can account for free-riding behavior: some individuals could reduce their donations in response to an increase of the donations of the others. To the best of my knowledge, this potential crowding out of individual contributions by aggregate contributions has not been studied in the optimal tax literature. My analysis shows that tax policy can trigger such free-riding patterns, as captured by the spillover parameter appearing in the optimal tax formulas. However, a quantitative exploration of the optimal tax formulas has yet to be performed to discuss the actual importance of this spillover effect.

This paper also falls within the multidimensional optimal tax literature. Using the tax perturbation approach initiated by Piketty (1997) and Saez (2001) and recently extended by Hendren (2019), Sachs et al. (2020) and Jacquet and Lehmann (2021b), I include a public good and charitable giving in a framework with multidimensional unobserved heterogeneity of taxpayers and nonlinear tax instruments. To the best of my knowledge, this paper is the first to feature these two elements in such a general optimal tax framework. In particular, I show that the assumed positive effect of charitable contributions on social welfare enters additively in the optimal nonlinear tax formulas for donations. This additivity is reminiscent of the result of Sandmo (1975) when studying optimal taxation in presence of externalities. Saez (2004) has already noted that this additive property is part of the optimal linear subsidy on charitable contribution. I therefore extend this result to the case of nonlinear tax instruments. This additive property can simplify tax incidence analysis in presence of an externality, even when tax instruments can take arbitrarily complex forms.

The paper is organized as follows. I introduce the general framework in Section II. I begin my analysis by studying optimal public good provision in the Mirrleesian economy in Section III. In Section IV, I characterize the optimal tax policy as well as the optimal provision of the public good in the general framework. Section V provides explicit formulas for the optimal tax schedule on income and donation, as well as optimality conditions for the deduction rule. Section VI concludes.

II General Framework

The economy consists of a unit mass of heterogeneous taxpayer and of a government. Taxpayers supply work and contributions to a public good G_1. The government can supply a public good G_0. Following Saez (2004), to distinguish the two forms of public good, I will refer to G_1 as the "contribution good" while G_0 will be labeled as the "government good".

II.1 Taxpayers' program

In the most general version of my model, taxpayers can differ in many individual characteristics summarized in a type vector $\theta = (\theta_1, \theta_2 ... \theta_n) \in \Theta$, where Θ is convex. Types are distributed according to a continuously differentiable density function $f : \theta \mapsto f(\theta)$. Importantly, types are only privately observed so that the government cannot directly target these individual characteristics with its policy instruments.

An individual with type θ chooses labor income y, private good consumption c and donations b to maximize a twice continuously differentiable utility function $\mathcal{U}$: $(c, y, b; G_1, G_0, \theta) \mapsto \mathcal{U}(c, y, b; G_1, G_0, \theta)$.

I assume that taxpayers enjoy private consumption (hence $U_c > 0$) and public good consumption (hence $U_{G_1}, U_{G_0} > 0$). Besides, they can enjoy the act of giving (hence $U_b \geq 0$), through the warm glow motive described in Andreoni (1989, 1990). On the other hand, earning labor income y requires an effort so that $U_y < 0$. Importantly, agents take both the government good G_0 and the contribution good G_1 as given when making their optimal individual decisions. The latter is equivalent to assume that taxpayers are small so that they neglect the impact of their individual contribution to G_1.[3] Eventually, note that preferences for the public goods can be heterogeneous so that for instance some agents could value more the government good G_0 while other could value more the contribution good G_1.

[3] This hypothesis is standard in the optimal tax literature on charitable contribution (see Saez (2004), Diamond (2006) or Aronsson et al. (2021)).

To both solve the optimal tax problem and clarify the role of assumptions on individual preferences on policy, my analysis heavily relies on marginal rates of substitution (MRS) between the private good consumption and both public good, donation and labor supply outcomes. Hence let :

$$\mathbf{S^x}\left(c,y,b;G_1,G_0,\theta\right) \stackrel{\text{def}}{\equiv} \frac{\mathcal{U}_x\left(c,y,b;G_1,G_0,\theta\right)}{\mathcal{U}_c\left(c,y,b;G_1,G_0,\theta\right)} \tag{1}$$

denote the MRS between private consumption and $x = \{b, G_1, G_0\}$. Besides one can define the MRS between private consumption and labor supply as :

$$\mathbf{S^y}\left(c,y,b;G_1,G_0,\theta\right) \stackrel{\text{def}}{\equiv} -\frac{\mathcal{U}_y\left(c,y,b;G_1,G_0,\theta\right)}{\mathcal{U}_c\left(c,y,b;G_1,G_0,\theta\right)} \tag{2}$$

The government can tax or subsidize labor income y and donations b through the non-linear tax schedule $T : (y,b) \mapsto T(y,b)$. The individual's budget constraint therefore implies $c + b = y - T(y,b)$. Hence an agent with type θ, taking $T(.)$, G_0 and G_1 as given, solves :

$$U(\theta, G_1, G_0) \stackrel{\text{def}}{\equiv} \max_{y,b} \quad \mathcal{U}\left(y - b - T(y,b), b, y; G_1, G_0, \theta\right) \tag{3}$$

The solution of (3) is denoted $\{y(\theta, G_1, G_0), b(\theta, G_1, G_0)\}$. Since the contribution good is the aggregate amount of donations, this implies a fixed-point condition :

$$G_1 \stackrel{\text{def}}{\equiv} \int_\theta b(\theta, G_1, G_0) f(\theta) d\theta \tag{4}$$

II.2 The Government's program

The government levies taxes to finance the public good G_0 and to redistribute resources across agents. Its budget constraint therefore takes the form :

$$\int_\theta T\left(y\left(\theta, G_1, G_0\right), b\left(\theta, G_1, G_0\right)\right) f(\theta) d\theta \geq G_0 \tag{5}$$

I suppose that the objective of the government is to maximize the sum over all types θ of a function $\Phi : (U, \theta) \mapsto \Phi\left(U\left(\theta, G_0, G_1\right), \theta\right)$.

$$SW \stackrel{\text{def}}{\equiv} \int\limits_{\theta \in \Theta} \Phi\left(U\left(\theta, G_1, G_0\right); \theta\right) f\left(\theta\right) d\theta \tag{6}$$

Hence the problem of the government is to maximize the generalized social welfare function define in (6) subject to the budget constraint (5). I constrain $\Phi(.)$ to be increasing in individual utility $U(.)$, and to be strictly increasing for at least one type θ. Assuming that $\Phi(.)$ can depend directly on θ allows me to cover a wide range of welfare criteria. For instance, $\Phi(U;\theta) \equiv \phi(\theta)U$, where weights $\phi(\theta)$ directly depend on type θ embeds *weighted utilitarists* views of justice in my framework. Hence standard *utilitarianism* is obtained when $\phi(\theta) = 1$ while a *Rawlsian* objective arises when $\phi(\theta) = 0$ except for the lowest type $\underline{\theta}$ with $\phi(\underline{\theta}) > 0$.

III Public Good Provision under Perfect Substitution

Before solving for the full problem of the government when individual preferences take the general form described in (3), I focus in this section on the optimal provision of the government good G_0, without characterizing the optimal tax schedule $T(y, b)$. Besides, I constrain the government good G_0 and the charity good G_1 to be perfect substitutes so that individuals only care about the sum of the two $G = G_0 + G_1$. This assumption of perfect substitution between G_0 and G_1 is standard in the analysis of the tax treatment of charitable giving.[4] Making this assumption rules out the channel through which donations could be more or less socially desirable depending on the nature of the public good they fund compared to the one funded by the government. Typically, donations should be more encouraged if G_1 is more valued by the poor compared to G_0. Such a mechanism is taken into account in Section IV where I derive optimal policy rules, for both G_0 and $T(y, b)$ without assuming perfect substitution between G_0 and G_1. Yet studying the perfect substitution case is still relevant to conceptually clarify the efficiency rationale for either fund a public good through taxes or through donations.

To tackle this question between a donation or a taxes based funding of G, I make another departure from the general framework by putting more structure on the individual utility function (3). Following Atkinson and Stiglitz (1976) and Gauthier and Laroque (2009), we know that separable preferences can deliver important theoretical insights in public finance problems. I therefore consider various forms of separability assumptions to assess the optimal provision of the public good G. Under the appropriate separability assumptions, it is not necessary to study the full problem of the government of maximizing the social welfare function (6) subject to the budget constraint (5) to characterize the optimal policy. Indeed, as noted by Gauthier and Laroque (2009), separability assumptions allow us to study subproblems of the welfare maximization program that depend only on efficiency issues, abstracting from the equity concerns associated to economies with unobserved individual heterogeneity.[5] More precisely, under separable preferences, it is possible to neutralize the impact alternative policies can have on individual utility and incentive constraints, so that the optimal policy can be found by analyzing only government revenue. A policy will therefore be optimal if, compared to the alternatives, it increases government revenue while leaving individual utility and incentive constraints unaffected.

To give an example on how explicit policy rules can be derived using separability assumptions, assume that preferences for private consumption and donations are separable from preferences for leisure and for the public good. Besides, assume that individuals have the same "taste" for donations, *i.e* the same degree of altruism.

[4] This is assumed in the main analysis of Saez (2004), Diamond (2006) and Aronsson et al. (2021).

[5] In other words, quoting Gauthier and Laroque (2009), "one can isolate in the second best program a part which has a first best shape: conditional on the values taken by some variables, the remaining ones are solutions of a first best program from which the incentive constraints are absent."

Formally, suppose that individual utility takes the form:

$$U(\theta, G_1, G_0) = \mathcal{U}\left(V(c,b), y; G, \theta\right) \tag{7}$$

with $V(c,b)$ a continuously differentiable function verifying $V_c, V_b > 0 > V_{bb}, V_{cc}$.

Proposition 1. *If individual preferences take the form of (7), then the government's contribution to the public good is nil at the optimum: $G_0^* = 0$.*

The proof of Proposition 1 is given in Appendix A.1. It follows from the proof of the Atkinson-Stiglitz theorem given by Konishi (1995), Laroque (2005) and Kaplow (2006). The idea is that that setting $G_0 = 0$ allows the government to increase its revenue net of its contribution to the public good, while maintaining individual utility unchanged and preserving incentive compatibility constraints. In other words, moving from $G_0 > 0$ to $G_0 = 0$ generates a Pareto-improvement, as this raise in government revenue can then be redistributed in a lump-sum fashion.

The intuition underlying Proposition 1 is twofold. First, because of separable preferences, the problem of setting the optimal level of G in an economy with heterogeneous agents boils down to finding the least costly way of funding the public good. Second, as already conjectured by Saez (2004), it is less costly to fund the public good through voluntary contributions because of the warm glow assumption: by making a donation, individuals lose utility because they have less money for private consumption c but make a utility gain from the warm glow attached to the donation. However, when the public good is financed by the government through taxes, only the utility loss from renounced consumption occurs, without any utility gain for paying a tax. Hence, for a same level of G, the resource cost of granting a certain level of individual utility is higher when G is funded through taxes than through donations.

Although this potential complete crowding-out of the government contribution by voluntary contributions because of warm glow has already been conjectured by Saez (2004) and formally derived in a Mirrleesian economy with unidimensional heterogeneity by Aronsson et al. (2021), the assumptions used to establish Proposition 1 allows us to clarify the role of government's provision of public goods in three ways.

First and perhaps most obvious point: the complete crowding out occurs because we assume perfect substitution between G_0 and G_1. As soon as we go back to the general framework described by (3), the complete crowding out of government's provision of public goods is unlikely to hold.[6]

Second and perhaps most important point: considering a utility function of the type of (7) does not put any restriction on the degree of heterogeneity for public good preferences nor on its degree of separability from work effort. Indeed, the parameter of heterogeneity θ in (7) can be a vector so that individuals can differ in their skills, as in Mirrlees (1971), but also in their valuation of the public good as well as in their responsiveness to tax incentives. To be clear, it is likely that differences in public good valuation, such as the poor getting more utility from G, and nonseparability, with the public good being for instance complement to leisure, would affect the optimal of G. What Proposition 1 emphasizes is that this preference heterogeneity and nonseparability would not change the efficiency rationale for funding G through donations instead of taxes.

[6] If for instance if we impose

$$\lim_{G_0 \to 0} \mathcal{U}(c, b, y; G_1, G_0) = -\infty$$

there should be a strictly positive provision of G_0 at the optimum.

Third, the assumptions underlying (7) however imposes that individuals have the same preferences for donations and that these preferences are separable from work effort. In other words, Proposition 1 is valid when individuals have the same degree of altruism and that this stricly positive gain from making donations ($V_b > 0$) does not interact with labor supply decisions. The intuition for this is that heterogeneous and nonseparable preferences for donation create efficiency issues when funding the public good. Then the efficiency rationale for only using private contributions becomes unclear as incentivizing giving, through tax policy, to reach the optimal level of G would generate distortions because of unobserved taste for making donations. In other words, such heterogeneity in altruism makes the problem of the optimal funding of the public good interfere with incentive constraints, while it was not the case for heterogeneous preferences for the public good.

Although it allows to establish the absence of government funding of the public good at the optimum, Proposition 1 does not characterize the optimal level of G. To provide some guidance on how this public good, funded only through charitable contributions, should be set at the optimal, assume that preferences for the public good are also homogeneous and separable from work effort. Formally, assume that individual utility now takes the form:

$$U(c, b, y; G_0, G_1, \theta) = \mathcal{U}\left(V(c, G, b), y; \theta\right) \tag{8}$$

This case roughly corresponds to what Aronsson et al. (2021) defines as "leisure separability", implying that both preferences for private consumption, donation and the public good are separable from work effort.[7]

Proposition 2. *If individual utility takes the form of* (8)*, then:*

- *There is no contribution to the public good by the government at the optimum:* $G_0^* = 0$
- *The optimal level of the public good* $G = G_1$ *should be such that:*

$$\int_\theta \left\{ S^G\left(c, b, y; G, \theta\right) + S^b\left(c, b, y; G, \theta\right) \right\} f(\theta) d\theta = 1 \tag{9}$$

The proof of Proposition 2 is given in Appendix A.2 and follows from the same logic as the proof of Proposition 1. Note that since the utility function (7) nests the case of leisure separability (8), the first part of Proposition 2 simply follows from applying Proposition 1. Yet the second part of Proposition 2 allows to pin down the actual level of public good, funded only through charitable giving, that should be implemented at the optimum. Ignoring the S^b term in (9), Proposition 9 coincides with the logic of Samuelson (1954) : the sum of the MRS between private and public goods should equal the MRT, which here is equal to 1. However, in presence of donations and warm glow, the Samuelson Rule should be amended to account for the private gain associated to the funding of the public good. This private, and homogeneous gain under (8), is captured by the S^b term in (9).

[7]The difference with Aronsson et al. (2021) is that θ can be a vector so individual could still differ in many dimensions, although preferences for both donations and the public good are identical under (8).

IV Tax Reforms

The objective of this section is to derive optimal policy prescription in the general framework described in Section II. Compared to the previous section on the optimal provision of public good, I will not constraint the degree of heterogeneity among taxpayers nor the substituability or complementarity between the two public goods G_0 and G_1.

Using the tax perturbation approach initiated by Piketty (1997), Saez (2001) and recently generalized by Sachs et al. (2020) and Jacquet and Lehmann (2021b), I use responses of taxpayers to tax reforms to characterize the optimal tax system in presence of charitable giving. A tax reform can be defined as followed :

Definition 1. *Starting from an initial tax schedule $T : (y, b) \mapsto T(y, b)$, a tax reform replaces $T(.)$ by a new schedule $\widetilde{T} : (y, b, t) \mapsto \widetilde{T}(y, b, t)$, with $t \in \mathbb{R}$ a scalar measuring the magnitude of the reform.*

Under a reformed tax schedule $\widetilde{T}$, a taxpayer with type θ enjoys utility :

$$\widetilde{U}(\theta, G_1, G_0, t) \overset{\text{def}}{\equiv} \max_{y,b} \quad \mathcal{U}\left(y - b - \widetilde{T}(y, b, t), b, y; G_1, G_0, \theta\right) \tag{10}$$

The first-order-condition of (10) with respect to y and b yields :

$$S^y\left(y - b - \widetilde{T}(y, b, t), y, b; G_1, G_0, \theta\right) = 1 - \widetilde{T}_y(y, b, t) \tag{11}$$

$$S^b\left(y - b - \widetilde{T}(y, b, t), y, b; G_1, G_0, \theta\right) = 1 + \widetilde{T}_b(y, b, t) \tag{12}$$

To derive behavioral responses to tax reforms, I use the implicit function theorem applied to taxpayer's first-order condition (11) and (12). To do so, I impose the following restriction on individual's preferences and the tax function :

Assumption 1.

- *The tax function $T(.)$ is twice continuously differentiable.*
- *The second-order conditions associated to (10) hold strictly.*
- *Problem (10) admits a unique global maximum.*

Assumption 1 corresponds to the sufficient conditions for the tax perturbation approach derived in Assumption 2 of Jacquet and Lehmann (2021b). It allows to apply the implicit function theorem to (11) and (12) and to prevent any jump in individual's choices after a small tax reform of magnitude t.

I show in Appendix B.1 that under Assumption 1, the total response of labor income and donations to any perturbation of magnitude t verifies :

$$\begin{pmatrix} \frac{dy}{dt} \\ \\ \frac{db}{dt} \end{pmatrix} = A^{-1}.\left\{ \begin{pmatrix} \frac{\partial \widetilde{T}}{\partial t} S_c^y - \frac{\partial \widetilde{T}_y}{\partial t} \\ \\ \frac{\partial \widetilde{T}}{\partial t} S_c^b + \frac{\partial \widetilde{T}_b}{\partial t} \end{pmatrix} - \begin{pmatrix} S_{G_1}^y \\ \\ S_{G_1}^b \end{pmatrix} \frac{dG_1}{dt} - \begin{pmatrix} S_{G_0}^y \\ \\ S_{G_0}^b \end{pmatrix} \frac{dG_0}{dt} \right\} \tag{13}$$

with $A = \begin{pmatrix} S^y\ S_c^y + S_y^y + \widetilde{T}_{y,y} & -S^b\ S_c^y + S_b^y + \widetilde{T}_{y,b} \\ \\ S^y\ S_c^b + S_y^b - \widetilde{T}_{b,y} & -S^b\ S_c^b + S_b^b - \widetilde{T}_{b,b} \end{pmatrix}$.

Formula (13) describes all the possible channel through which endogenous variables y and b can respond to a tax reform of magnitude t. One of this channel occurs through the response of the aggregate level of donation G_1, which is an endogeneous variable, to the tax reform, as captured by the $\frac{dG_1}{dt}$ term of (13). Depending on the complementarity or substitution between one's contribution and other's contributions, responses of individual donation b to taxes, which automatically trigger a change in G_1 by definition (4), can trigger responses of b to G_1 and so on. I will come back latter to this endogenous process between individual and aggregate donation, which has been neglected by former work on the optimal tax treatment of charitable giving. Eventually note that (13) is derived by differentiating first order conditions (11) and (12) so that a term $\frac{dG_0}{dt}$ appears in the formula. Yet, since a reform of the tax schedule does not trigger any endogenous response of G_0, which is a policy parameter that can be freely adjusted by the government, this $\frac{dG_0}{dt}$ term has not to be taken into account when measuring the incidence of a tax reform or designing optimal tax rates.

IV.1 Micro behavioral responses to tax reforms

To clarify the channel through which labor income y and donations b can be affected by a perturbation of magnitude t, it is useful to make the following two distinctions.

First, it is important to distinguish "micro responses" of y and b, which take the contribution good G_1 as given, from total responses or "macro responses", which include the reaction of y and b to changes in G_1. This distinction allows me to carefully deal with the circularity between responses of b, triggering changes in G_1, triggering changes in b and y through (13) and so on. Using (13), micro responses can be defined by ignoring the responses of G_1 (and of G_0) :

$$\begin{pmatrix} \frac{\partial y}{\partial t} \\ \\ \frac{\partial b}{\partial t} \end{pmatrix} = A^{-1}.\left\{ \begin{pmatrix} \frac{\partial \widetilde{T}}{\partial t} S_c^y - \frac{\partial \widetilde{T}_y}{\partial t} \\ \\ \frac{\partial \widetilde{T}}{\partial t} S_c^b + \frac{\partial \widetilde{T}_b}{\partial t} \end{pmatrix} \right\} \tag{14}$$

A second important and common distinction used in the literature is to disentangle responses of y and b driven by substitution effects from those driven by income effects. To do so, it is useful to study specific tax perturbations that allows to rewrite (14) in terms of compensated and income responses to tax reforms.

Lump sump tax reforms - Income effect: A lump-sum tax reform of magnitude ρ can be defined as :

$$\widetilde{T}(y,b,\rho) = T(y,b) - \rho \tag{15}$$

Such a reform changes tax liability uniformly without changing the marginal tax rate on y and b so that $\frac{\partial \widetilde{T}(y,b,\rho)}{\partial \rho} = -1$ and $\frac{\partial \widetilde{T_y}(y,b,\rho)}{\partial \rho} = \frac{\partial \widetilde{T}_b(y,b,\rho)}{\partial \rho} = 0$. Hence there would be no substitution effects in taxpayers responses so that I use $\frac{\partial y}{\partial \rho}$ and $\frac{\partial b}{\partial \rho}$ to measure the income responses of taxpayers to tax reforms.

Using (14) these income responses verify :

$$\begin{pmatrix} \frac{\partial y}{\partial \rho} \\ \\ \frac{\partial b}{\partial \rho} \end{pmatrix} = -A^{-1}. \begin{pmatrix} S_c^y \\ \\ S_c^b \end{pmatrix} \tag{16}$$

Compensated reform - Substitution effect:

For any choice $x = \{y, b\}$, let $X(\theta, G_1, G_0)$ denote the value of this choice measured at partial equilibrium, *i.e* taking G_1 as given. Then a compensated reform of the marginal net of tax rate of x is defined as :

$$\widetilde{T}(y,b,\tau_x) = T(y,b) - \tau_x \left(x - X(\theta, G_1, G_0)\right) \tag{17}$$

Hence such a reform leaves unchanged tax liability at the initial level $X(\theta, G_1, G_0)$. This implies for $x = y, b$: $\frac{\partial \widetilde{T}(y,b)}{\partial \tau_x} = \frac{\partial \widetilde{T}_{-x}}{\partial \tau_x} = \frac{\partial \widetilde{T}_x}{\partial \tau_{-x}} = 0$ and $\frac{\partial \widetilde{T}_x}{\partial \tau_x} = -1$. Hence compensated reforms only affect the marginal tax rate of x and thus can modify y and b only through substitution effects.

Using (14), the matrix of compensated responses is given by :

$$\begin{pmatrix} \frac{\partial y}{\partial \tau_y} & \frac{\partial y}{\partial \tau_b} \\ \\ \frac{\partial b}{\partial \tau_y} & \frac{\partial b}{\partial \tau_b} \end{pmatrix} = A^{-1}. \begin{pmatrix} 1 & 0 \\ \\ 0 & -1 \end{pmatrix} \tag{18}$$

Uncompensated response

An uncompensated reform of the marginal net of tax rate of $x \in \{y, b\}$ is defined as :

$$\widetilde{T}(y,b,\tau_x) = T(y,b) - \tau_x x \tag{19}$$

Hence an uncomponsated reform can be understood as a combination between the compensated reform (17) and a lump sum reform (15) with $\rho = t_x X\left(\theta, G_1, G_0\right)$. For $\{x_i, x_j\} \in \{y, b\}$, this yields the Slutsky equation :

$$\frac{\partial x_i^U}{\partial \tau_{x_j}} = \frac{\partial x_i}{\partial \tau_{x_j}} + X_j(\theta, G_1, G_0) \frac{\partial x_i}{\partial \rho} \tag{20}$$

Decomposition of micro responses between income and substitution effects.

Plugging (16) and (18) into (13), any micro response to a tax perturbation t can be rewritten in terms of substitution and income effects :

$$\begin{aligned}\begin{pmatrix}\frac{\partial y}{\partial t}\\ \\ \frac{\partial b}{\partial t}\end{pmatrix} &= A^{-}1\begin{pmatrix}S_c^y\\ \\ S_c^b\end{pmatrix}\frac{\partial \widetilde{T}}{\partial t} + A^{-}1\begin{pmatrix}1 & 0\\ \\ 0 & -1\end{pmatrix}\cdot\begin{pmatrix}\frac{-\partial \widetilde{T}_y}{\partial t}\\ \\ \frac{-\partial \widetilde{T}_b}{\partial t}\end{pmatrix}\\ &= -\begin{pmatrix}\frac{\partial y}{\partial \rho}\\ \\ \frac{\partial b}{\partial \rho}\end{pmatrix}\frac{\partial \widetilde{T}}{\partial t} - \begin{pmatrix}\frac{\partial y}{\partial \tau_y} & \frac{\partial y}{\partial \tau_b}\\ \\ \frac{\partial b}{\partial \tau_y} & \frac{\partial b}{\partial \tau_b}\end{pmatrix}\cdot\begin{pmatrix}\frac{\partial \widetilde{T}_y}{\partial t}\\ \\ \frac{\partial \widetilde{T}_b}{\partial t}\end{pmatrix}\end{aligned} \tag{21}$$

IV.2 From micro response of donations to macro responses of the contribution good

The question now is to translate micro behavioral responses to tax reforms, as described in (21), into response of the contribution good G_1 to a tax perturbation of magnitude t. Using (4), the level of the contribution good after a tax perturbation of magnitude t is defined by the fixed-point condition :

$$G_1(t) = \int_\theta b\left(\theta, G_1\left(t\right), G_0, t\right) f(\theta) d\theta \tag{22}$$

To measure the feedback between individual responses in donation behavior to aggregate donatation level, I introduce a spillover parameter, denoted Π. This parameter captures in a reduced-form way various channel through which individual donation motivated by a warm glow can interact with the actual level of the public good funded by charitable giving :

$$\Pi \overset{\text{def}}{\equiv} \frac{1}{1 - \int \frac{\partial b}{\partial G_1} f(\theta) d\theta} \tag{23}$$

Hence the impact of the perturbation t on G_1 is given by:

$$\frac{\partial G_1(t)}{\partial t} = \Pi \int_\theta \frac{\partial b\left(\theta, G_1(t), G_0, t\right)}{\partial t} f(\theta) d(\theta) \tag{24}$$

For instance, if preferences for making donations, through joy-of-giving, are separable from preferences for the contribution good G_1, then $\frac{\partial b}{\partial G_1} = 0$ and $\Pi = 1$. In this case, (24) implies that the response of aggregate contributions to tax reforms $\frac{\partial G_1}{\partial t}$ is just the sum of individual responses to the reform $\frac{\partial b(t)}{\partial t}$. However if individual and aggregate donations are substitute, so that $\frac{\partial b}{\partial G_1} < 0$, the spillover parameter Π is below 1 and micro responses are in this case lower than macro responses. This would no longer be true if individual and aggregate donations are complement. For instance, if $\frac{\partial b}{\partial G_1} > 0$ but strictly below 1[8], $\Pi > 1$ so that micro responses of donations to taxes are amplified at general equilibrium. There exists various microfoundation to predict either substitution or complementarity between individual and aggregate donations. Typically models of free-riding would predict substitution while models of reciprocity would predict complementarity. The vast empirical literature on the interactions among donors can then be used to accurately calibrate the parameter Π for future numerical exercises.[9]

To explicitly relate the macro response of aggregate contributions to the micro response of donation, one can use (21) to rewrite (24) in terms of income and substitution effects:

$$\frac{\partial G_1(t)}{\partial t} = -\Pi \int_\theta \left[\frac{\partial b}{\partial \rho} \frac{\partial \widetilde{T}}{\partial t} + \frac{\partial b}{\partial \tau_y} \frac{\partial \widetilde{T}_y}{\partial t} + \frac{\partial b}{\partial \tau_b} \frac{\partial \widetilde{T}_b}{\partial t} \right] f(\theta) d\theta \tag{25}$$

Now that the set of endogenous responses to a tax perturbation has been clarified by (21) and (25), I can study the problem of designing the optimal tax schedule $T(y, b)$ for any level of the government good G_0.

IV.3 Optimal Tax Schedule

First, I describe how the program of the government is affected by a tax perturbation of magnitude t, through micro responses. In other words, I study the government's problem taking the contribution good G_1 and government good level G_0 as given. Once these micro responses are carefully taken into account, I derive necessary conditions for social welfare maximization at general equilibrium, *i.e* including the endogenous responses of individual donations to aggregate ones.

The government's problem consists in maximizing (6) subject to budget constraint (5). For a perturbation of magnitude t, at a given level G_1 and G_0, I can form a partial equilibrium Lagrangian $\widehat{\mathscr{L}}$ to study the government's program:

$$\begin{aligned} \widehat{\mathscr{L}}(t, G_1, G_0) = \int_\theta \Bigg\{ & \widetilde{T}\left(y\left(\theta, G_1, G_0, t\right), b\left(\theta, G_1, G_0, t\right), t\right) - G_0 + \\ & \frac{1}{\lambda} \Phi\left(U\left(\theta, G_1, G_0, t\right); \theta\right) \Bigg\} f(\theta) d\theta \end{aligned} \tag{26}$$

with $\lambda > 0$ the Lagrange multiplier associated to the budget constraint (5).

Following Saez (2001), define marginal social welfare weight as :

$$g(\theta) \overset{\text{def}}{\equiv} \frac{\Phi_U\left(U\left(\theta, G_1, G_0\right); \theta\right)\ \mathcal{U}_c\left(c, b, y; G_1, G_0, \theta\right)}{\lambda}$$

[8] If $\frac{\partial b}{\partial G_1} = 1$ then Π goes to infinity. Further investigations on the existence of the fixed-point described in (22) could help discipline such extreme cases.

[9] Shang and Croson (2009) provides both a literature review of these models and experimental evidence to test substitution and complementarity.

The parameter $g(\theta)$ measures the welfare gain in money metric of giving an extra unit of consumption to taxpayers of type θ. Applying the envelope theorem to (10), the impact of a perturbation of magnitude t on the social welfare function verifies :

$$\frac{1}{\lambda} \left. \frac{\partial \Phi\left(U\left(\theta, G_1, G_0, t\right); \theta\right)}{\partial t} \right|_{t=0} = - \left. \frac{\partial \widetilde{T}\left(y, b, t\right)}{\partial t} \right|_{t=0} g(\theta) \tag{27}$$

Using the matrix of micro responses (21), the impact of the reform on tax liability measured at a given G_1 verifies :

$$\begin{aligned} \left. \frac{d\widetilde{T}\left(y\left(\theta, G_1, G_0, t\right), b\left(\theta, G_1, G_0, t\right), t\right)}{dt} \right|_{t=0} &= \left[1 - \frac{\partial y}{\partial \rho} T_y - \frac{\partial b}{\partial \rho} T_b\right] \left. \frac{\partial \widetilde{T}(y,b,t)}{\partial t} \right|_{t=0} \\ &\quad - \left[T_y \frac{\partial y}{\partial \tau_y} + T_b \frac{\partial b}{\partial \tau_y}\right] \left. \frac{\partial \widetilde{T}_y(y,b,t)}{\partial t} \right|_{t=0} \\ &\quad - \left[T_y \frac{\partial y}{\partial \tau_b} + T_b \frac{\partial b}{\partial \tau_b}\right] \left. \frac{\partial \widetilde{T}_b(y,b,t)}{\partial t} \right|_{t=0} \end{aligned} \tag{28}$$

Using (27) and (28), the impact of a perturbation t on the government's Lagrangian for a given G_1 verifies :

$$\begin{aligned} \left. \frac{\partial \widehat{\mathscr{L}}(t, G_1, G_0)}{\partial t} \right|_{t=0} = \int_\theta \Bigg\{ & \left[1 - g(\theta) - \frac{\partial y}{\partial \rho} T_y - \frac{\partial b}{\partial \rho} T_b\right] \left. \frac{\partial \widetilde{T}(y,b,t)}{\partial t} \right|_{t=0} \\ & - \left[T_y \frac{\partial y}{\partial \tau_y} + T_b \frac{\partial b}{\partial \tau_y}\right] \left. \frac{\partial \widetilde{T}_y(y,b,t)}{\partial t} \right|_{t=0} - \left[T_y \frac{\partial y}{\partial \tau_b} + T_b \frac{\partial b}{\partial \tau_b}\right] \left. \frac{\partial \widetilde{T}_b(y,b,t)}{\partial t} \right|_{t=0} \Bigg\} f(\theta) d\theta \end{aligned} \tag{29}$$

The objective now is to move from this partial equilibrium analysis to a measure of the impact of a perturbation t that includes the response of G_1. To do so, let η denote the impact of a variation of G_1 on the partial equilibrium Lagrangian $\widehat{\mathscr{L}}$:

$$\eta \stackrel{\text{def}}{\equiv} \frac{\partial \widehat{\mathscr{L}}(t, G_1, G_0)}{\partial G_1} = \int_\theta \left[\frac{\partial y}{\partial G_1} T_y + \frac{\partial b}{\partial G_1} T_b + g(\theta) S^{G_1}\right] f(\theta) d\theta \tag{30}$$

This parameter η captures the partial equilibrium externality of giving. It can be decomposed in two parts: a fiscal externality and a welfare externality. The fiscal externality reflects the impact on tax revenue of variations in the contribution good G_1, as captured by the $\frac{\partial y}{\partial G_1}$ and $\frac{\partial b}{\partial G_1}$ terms in (30). Indeed, depending on its interaction with labor supply, the contribution good can have an impact on income, which than has an impact on tax revenue proportional to the marginal income tax T_y. As already discussed when describing the spillover parameter Π, individual donation b can react to variation in the contribution good, *i.e* variations in aggregate donations, and this can also have an impact on tax revenue if voluntary contributions are taxed or subsidized. If individual reduce their donation in response to an increase in the aggregate level of donation, in other words if individual and aggregate donations are substitute, and if donations are subsidized, this would have a positive impact on tax revenue. This impact is captured by the $\frac{\partial b}{\partial G_1}$ term in (30) and is proportional to the marginal tax rate or the marginal subsidy to donation T_b. On top of this fiscal externality, variations in the contribution good has a direct welfare impact captured by the $g(\theta)S^{G_1}$ term in (30): depending on how strong are the preferences for the contribution good and who value the most this good, the welfare externality of giving will be more or less important.

The question is now to see how this partial equilibrium effect evolves when taking into account the general equilibrium effect (25). Let $\mathscr{L}$ denotes the Lagrangian of the government measured at general equilibrium, *i.e* taking into account the response of G_1 to a perturbation :

$$\mathscr{L}(t, G_0) \stackrel{\text{def}}{\equiv} \widehat{\mathscr{L}}(t, G_1(t), G_0)$$

Hence, using (30), the general equilibrium impact of a perturbation of magnitude t is given by :

$$\frac{\partial \mathscr{L}(t, G_0)}{\partial t} = \frac{\partial \widehat{\mathscr{L}}(t, G_1, G_0)}{\partial t} + \eta \frac{\partial G_1(t)}{\partial t} \tag{31}$$

Or using (26) and (24) :

$$\begin{aligned} \frac{\partial \mathscr{L}(t, G_0)}{\partial t} = \int_\theta \Bigg\{ & \left[1 - g(\theta) - \frac{\partial y(\theta)}{\partial \rho}\, T_y - (T_b + \eta\Pi)\, \frac{\partial b(\theta)}{\partial \rho}\right] \left.\frac{\partial \widetilde{T}(y, b, t)}{\partial t}\right|_{t=0} \\ & - \left[T_y \frac{\partial y(\theta)}{\partial \tau_y} + (T_b + \eta\Pi)\, \frac{\partial b(\theta)}{\partial \tau_y}\right] \left.\frac{\partial \widetilde{T}_y(y, b, t)}{\partial t}\right|_{t=0} \\ & - \left[T_y \frac{\partial y(\theta)}{\partial \tau_b} + (T_b + \eta\Pi)\, \frac{\partial b(\theta)}{\partial \tau_b}\right] \left.\frac{\partial \widetilde{T}_b(y, b, t)}{\partial t}\right|_{t=0} \Bigg\} f(\theta) d\theta \end{aligned} \tag{32}$$

Equation (32) describes the impact of a perturbation of magnitude t on social welfare and the tax liability, taking into account its impact on the contribution good G_1. Both the external and the spillover effect associated to individual contributions to a public good, as captured by the η and Π parameters constitute the main the departure of (32) from standard optimal tax formulas.

Hence the impact of a reform affect the government's program through mechanical and behavioral effect, on revenue and on welfare. The mechanical effect on tax revenue and on welfare is captured by the $1 - g(\theta)$ term and is proportional to the change in tax liability induced by the reform $\frac{\partial \widetilde{T}}{\partial t}$. As standard since Saez (2001), changes in labor supply create a behavioral response in tax revenue, through a substitution effect $\frac{\partial y}{\partial \tau_y} T_y$ proportional to the change in the marginal tax rate on income $\frac{\partial \widetilde{T}_y}{\partial t}$, and an income effect $\frac{\partial y}{\partial \rho} T_y$, proportional to $\frac{\partial \widetilde{T}}{\partial t}$. Compared to Saez (2001), the tax schedule can depend on a second outcome, which is donations b. Hence behavioral responses of donations also impact tax revenue, through the $\frac{\partial b}{\partial \tau_b}$ and $\frac{\partial b}{\partial p} T_b$ terms. The existence of a second tax base can create cross-base responses $\frac{\partial y}{\partial \tau_b}$ and $\frac{\partial b}{\partial \tau_b}$ that have also to be taken into account when measuring the impact of the reform on tax revenue. These effects are standard in optimal tax analysis in presence of multiple incomes.[10]

The specificity of (32) compared to the standard optimal tax framework arises when we take into account the external effect of donations. First, note that behavioral responses of donations b to taxes, as they trigger a mechanical change in the contribution good level G_1 which can than translate in a change in income y and donation b, introduce a new behavioral effect on tax revenue. Indeed combining (32) with the definition of η in (30), it appears for instance that a compensated response of donation to its net of tax rate $\frac{\partial b}{\partial \tau_b}$ creates an additional, partial equilibrium, impact on tax revenue proportional to $\frac{\partial y}{\partial G_1} + \frac{\partial b}{\partial G_1}$. A similar effect occurs with income responses $\frac{\partial b}{\partial \rho}$. Second, these behavioral responses of donations, and their impact on G_1, not only affect tax revenue, but also welfare. Using (30), this partial equilibrium impact is proportional to the welfare weighted marginal rate of substitution between the private and the contribution good $g(\theta) S^{G_1}$. Such a parameter captures the Pigovian correction that has to be made to the optimal tax system to account for the impact of donations on others well-being. As noted by Sandmo (1975) in a representative agent framework, and by Saez (2004) in the linear tax framework with charitable giving, this Pigovian term enters additively in the tax incidence formula (32).[11] Eventually, it appears from (32) that this partial equilibrium external effect of donations on both revenue and welfare can be amplified or dampened when the general equilibrium effects of G_1 are taken into account. Indeed, a behavioral response of donation to a reform triggers a change in the contribution good, or aggregate donation, G_1 which in turn can translate into a change in individual donation b and so on. As already mentioned, the spillover parameter Π captures this circularity process between individual and aggregate donation. Hence when analyzing the incidence of a reform, behavioral responses of donations have to be multiplied by the Π term to translate partial equilibrium effect into general equilibrium ones. It is this dampening or amplifying effect at general equilibrium, depending on substitution or complementarity between individual and aggregate donation, that has been neglected in previous analysis of the tax treatment of charitable giving.

While formula (32) can be used to determine the incidence of any tax reform, it does not indicate whether the reform generates a deficit or a surplus of the government's budget. It is therefore useful to study budget balanced perturbations to gauge the desirability of a reform. Following Sandmo (1998) and Jacobs (2018), an easy way to always balance budget after a reform is to use lump-sum transfers to offset loss or gains in revenue. Hence I normalize the multiplier on the government budget λ such that the lump-sum reform has no impact on the government's Lagrangian. It follows from the definition of a lump-sum reform given in (15) that $\frac{\partial \widetilde{T}}{\partial \rho} = -1$ and $\frac{\partial \widetilde{T}_y}{\partial \rho} = \frac{\partial \widetilde{T}_y}{\partial \rho} = 0$. Hence a budget-balanced reform necessarily verifies :

[10] See for instance Golosov et al. (2014), Jacquet and Lehmann (2021a) or Spiritus et al. (2022).

[11] This additive property will be clearer when we will derive explicit tax formulas in Section V.

$$\frac{\partial \mathscr{L}(t, G_0)}{\partial \rho} = 0$$
$$\Leftrightarrow \int_\theta \left\{ 1 - g(\theta) - \frac{\partial y}{\partial \rho} T_y - (T_b + \eta\Pi) \frac{\partial b}{\partial \rho} \right\} f(\theta) d\theta = 0 \tag{33}$$

with λ appearing in the $g(\theta)$ term.

Proposition 3. *If the shadow price of public fund λ verifies (33), a reform of magnitude $t > 0$ is socially desirable if the impact on the government's Lagrangian as defined in (32) is strictly positive, i.e $\frac{\partial \mathscr{L}}{\partial t} > 0$.*

Proposition 3 allows to assess the relevance of any reform of an initial tax schedule $T(y, b)$ as soon as welfare weights $g(\theta)$, micro responses of b and y and sufficient statistics Π and η are known.

IV.4 Optimal Government Good and Optimal Tax Schedule

To conclude this section on the incidence of tax reforms, I want to discuss how the provision of the government good G_0 interacts with the optimal tax system. First, as one can notice from (32) and from the definition of the externality and spillover parameters η and Π given in (30) and (23), the crowding out of individual contribution to changes in the level of the government good $\frac{\partial b}{\partial G_0}$ does not appear in the tax formula. Hence this crowding out parameter, which has been extensively studied in the empirical literature and put forward in the analysis of the tax treatment of charitable giving by Saez (2004) is not a sufficient statistic for the optimal tax schedule. The intuition for this is the following: in a tax perturbation approach, the necessary condition for optimality depends on how endogenous variables, chosen by individuals, react to tax reforms. Since a tax reform does not trigger any micro response, as already stressed in the discussion of (13), nor macro response of G_0, it is logic that such a crowding out is not relevant to determine the optimal tax rates. However, what my analysis shows is that because of the endogenous relationship between donations b and the contribution good G_1, the crowding out (or crowding in) of individual contributions by aggregate contributions $\frac{\partial b}{\partial G_1}$ does enter the optimal tax formulas, through the sufficient statistics Π and η. The only case where the crowding out of b by G_0 is relevant for the tax incidence analysis is when the contribution good and the government good are perfect substitutes. In this case, $\frac{\partial b}{\partial G_1} = \frac{\partial b}{\partial G_0}$, $\frac{\partial y}{\partial G_1} = \frac{\partial y}{\partial G_0}$ and $S^{G_1} = S^{G_0}$ so that the sufficient statistics η and Π can be computed by either measuring the response of individual donations to others donations or to the government's own contribution to the public good.[12]

To be clear, the fact that the crowding out effect $\frac{\partial b}{\partial G_0}$ does not appear in the tax incidence formula (32) does not mean that the level of the government good G_0 is not important for the optimal tax system. Indeed, all the sufficient statistics appearing in (32) are not policy invariant objects so that their value will vary depending on the level of G_0 at which they are evaluated. While (32) is valid for any level of G_0, it could

[12]The perfect substitution is actually implicitly assumed by Saez (2004). However, his analysis is conducted as if individuals could only adjust their donation to changes in the government provision G_0 and not to changes in the others donations G_1.

still be insightful to understand how these sufficient statistics evolve when evaluated at the optimal level of G_0. This requires first to characterize this optimal level G_0^*.

Proposition 4. *If the optimal provision of the government good G_0 is strictly positive at the optimum, then it should be set such that :*

$$\int_\theta \left(\frac{\partial y}{\partial G_0} T_y + (T_b + \eta \Pi) \frac{\partial b}{\partial G_0} + g(\theta) S^{G_0} \right) f(\theta) d(\theta) = 1 \tag{34}$$

The proof is given in Appendix B.2. Proposition 4 generalizes Proposition 2 to the case with potentially imperfect substitution between G_0 and G_1, taste heterogeneity and nonseparable preferences. The optimality condition (34) deviates from (9) and from the standard Samuelson Rule in three ways. First, in presence of potentially heterogeneous preferences for the government good G_0, the optimal provision of G_0 depend on the welfare weights of those having valuing the most the government good. This is why the MRS term S^{G_0} has to be welfare weighted by $g(\theta)$ in (34). Second, in the general utility function described in (3), there can be interactions between labor supply or donation behaviors and the provision of G_0. Hence y and b can react to changes in G_0 and this impacts optimal policy by affecting tax revenue. These effects are captured by the $\frac{\partial y}{\partial G_0} T_y$ and $\frac{\partial b}{\partial G_0} T_b$ terms. Third, responses of b to G_0 generate a change in G_1, through (4), which trigger a change in b and so on. Again the corrective term $\eta \Pi$ in (34) allows to translate the micro crowding out $\frac{\partial b}{\partial G_0}$ into a macro one $\pi \eta \frac{\partial b}{\partial G_0}$ that carefully takes into account the endogeneity of charitable contributions.

Now we can go a step further by analyzing the case of perfect substitution between G_0 and G_1. In this case, providing that $G_0^* > 0$, one can rewrite (34) as :

$$\int_\theta \left(\frac{\partial y}{\partial G_1} T_y + (T_b + \eta \Pi) \frac{\partial b}{\partial G_1} + g(\theta) S^{G_1} \right) f(\theta) d(\theta) = 1 \tag{35}$$

Using the definition of η given in (30), (35) can be rewritten as :

$$\eta \left(1 + \Pi \int_\theta \frac{\partial b}{\partial G_1} f(\theta) d(\theta) \right) = 1 \tag{36}$$

And using the definition of Π given in (23), (36) boils down to $\eta \Pi = 1$.

Proposition 5. *If the contribution and the government goods G_0 and G_1 are perfect substitutes and if the optimal provision of G_0 is strictly positive, the impact of a reform of magnitude t on any tax schedule $T(y, b)$ evaluated at G_0^* is given by :*

$$\frac{\partial \mathscr{L}(t, G_0^*)}{\partial t} = \int_\theta \Bigg\{ \left[1 - g(\theta) - \frac{\partial y(\theta)}{\partial \rho}\, T_y - (1 + T_b)\, \frac{\partial b(\theta)}{\partial \rho}\right] \frac{\partial \widetilde{T}(y, b, t)}{\partial t}\Bigg|_{t=0}$$

$$- \left[T_y \frac{\partial y(\theta)}{\partial \tau_y} + (1 + T_b)\, \frac{\partial b(\theta)}{\partial \tau_y}\right] \frac{\partial \widetilde{T}_y(y, b, t)}{\partial t}\Bigg|_{t=0} \tag{37}$$

$$- \left[T_y \frac{\partial y(\theta)}{\partial \tau_b} + (1 + T_b)\, \frac{\partial b(\theta)}{\partial \tau_b}\right] \frac{\partial \widetilde{T}_b(y, b, t)}{\partial t}\Bigg|_{t=0} \Bigg\} f(\theta) d\theta$$

Proposition 5 gives an example on how optimal policy rules on a specific instrument, expressed in terms of sufficient statistics can evolve when evaluated at the optimal level of other policy instruments. Here (37) shows that under perfect substitution, evaluating the optimal tax system $T(y, b)$ described by (32) at the optimal government good level G_0^* does not require to estimate the sufficient statistic $\eta\Pi$, which would necessarily be equal to 1. Yet, at this degree of generality, I can only describe the impact of reforming a tax schedule $T(y, b)$ on social welfare, without actually characterizing the optimal tax schedule. I therefore put additional constraints on the tax function $T(.)$ to actually derive optimal tax formulas, using less general but realistic tax instruments.

V Optimal Tax Treatment of charitable giving

V.1 Income tax, tax deduction and tax credit

In this section, I focus on a family of tax schedule $T(y, b)$ taking the following form :

$$T(y, b) = T_0\left(y - a\left(b\right)\right) + T_1(b) \tag{38}$$

with $T_0(.)$ the income tax schedule, $a(.)$ the deduction function and $T_1(.)$ the donation tax schedule. Both $T_0(.)$, $a(.)$ and $T_1(.)$ can be nonlinear, as long as $T(y, b)$ verifies Assumption 1.

Introduced by Jacquet and Lehmann (2021a) to study the optimal taxation of different sources of income, the tax function described in (38) provides a more tractable alternative to the fully nonlinear tax schedule while being more general than linear taxes. In the context of the tax treatment of charitable giving, two specific cases are worth considering: the pure tax deduction system and the pure tax credit system.

The tax treatment of giving follows a pure tax deduction rule when $T(.)$ takes the form :

$$T(y, b) = T_0\left(y - a\left(b\right)\right) \tag{39}$$

with $a(b) > 0$. In such a system, a gift of b reduces taxable income by an amount $a(b)$. [13]

The tax treatment of giving follows a pure tax credit rule when $T(.)$ takes the form :

$$T(y,b) = T_0(y) - T_1(b) \tag{40}$$

with $T_1(b) > 0$. Here a donation directly reduces tax liability.[15]

In the general case described in (38), the marginal tax rates on labor income y and gift b are given by :

$$T_y(y,b) = T_0'\left(y - a\left(b\right)\right) \tag{41a}$$

$$T_b(y,b) = -a'\left(b\right) T_0'\left(y - a\left(b\right)\right) + T_1'(b) \tag{41b}$$

For the optimal tax exercise, it can be useful to express tax formulas as a function of taxable income y_0, which in our setting in simply labor income net of the deduction for giving : $y_0 = y - a(b)$. In particular, I can define the compensated response of taxable income to a changes in the marginal net of tax rate x as :

$$\frac{\partial y_0}{\partial \tau_x} = \frac{\partial y}{\partial \tau_x} - a'(b)\frac{\partial b}{\partial \tau_x} \tag{42}$$

for $x = \{y, b\}$.

Conversely, it can be useful to derive the compensated responses of both y and b to reform of the marginal net of tax rate on taxable income t_0. To do so, consider the tax perturbation :

$$\widetilde{T}(y,b,\tau_0) = T_0\left(y - a\left(b\right)\right) + T_1(b) - \tau_0\left(y - a\left(b\right) - Y_0(\theta, G_1, G_0)\right)$$

This implies $\left.\frac{\partial \widetilde{T}(y,b,t)}{\partial \tau_0}\right|_{\tau_0=0} = 0$, $\left.\frac{\partial \widetilde{T}_y(y,b,t)}{\partial \tau_0}\right|_{\tau_0=0} = -1$ and $\left.\frac{\partial \widetilde{T}_b(y,b,t)}{\partial t}\right|_{t=0} = a'(b)$. Hence using the matrix of micro responses (21), we get :

$$\frac{\partial y}{\partial \tau_0} = \frac{\partial y}{\partial \tau_y} - a'(b)\frac{\partial y}{\partial \tau_b} \tag{43a}$$

$$\frac{\partial b}{\partial \tau_0} = \frac{\partial b}{\partial \tau_y} - a'(b)\frac{\partial b}{\partial \tau_b} \tag{43b}$$

Following Jacquet and Lehmann (2021b), I impose the following condition to derive optimal tax formulas :

[13] According to Peter and Lideikyte Huber (2022), this form of tax treatment of charitable giving is the most common in the OECD countries. In the US for instance, up to 60% of the donation can be deduced from taxable income. In this case, the deduction function would take the form $a(b) = 0,6\,b$.[14]

[15] In France for instance, the tax credit amounts to 66% of the gift, up to a limit of 20% of taxable income.

Assumption 2. *The mapping* $\Theta : \theta \mapsto \left(S^y\left(c,b,y;\theta,G_0,G_1\right), S^b\left(c,b,y;\theta,G_0,G_1\right)\right)$ *is invertible.*

To derive optimal tax formulas, one needs to rewrite the optimality condition (32) no longer in terms of types θ but in terms of labor income level y and donation level b. Assumption 2 allows me do so by moving from the type density function $f(\theta)$ to the labor income and donation densities $h_y(y)$ and $h_b(b)$, using the law of iterated expectations. For $x(\theta) \in \{y(\theta), b(\theta)\}$ and for $z \in \{y, b\}$, let $\overline{X}(z)$ denote the average of $x(\theta)$ among types θ for which $z(\theta) = z$. Formally, $\overline{X}(z) = \mathbb{E}\left[x\left(\theta\right)|z\left(\theta\right) = z\right]$. For instance $\overline{Y}(b)$ denotes the average labor income of θ-type taxpayers donating $b(\theta) = b$.

Eventually, as usual in the literature, I will express optimal tax formulas in terms of elasticity. So let $\epsilon(b)$ be the mean compensated elasticity of donations with respect to its own marginal net-of-tax rate. This mean is calculated among θ-types taxpayers who make donations $b(\theta) = b$.

$$\epsilon(b) \stackrel{\text{def}}{\equiv} \frac{1 - T_1'(b)}{b} \frac{\partial \overline{B(b)}}{\partial \tau_b} \tag{44}$$

V.2 Optimal Tax Schedules on Donation and Income

To characterize the optimal tax schedule donation $T_1(b)$, I study perturbation taking the form :

$$\widetilde{T}(y,b,t) = T_0\left(y - a\left(b\right)\right) + T_1(b) - tR_b(b) \tag{45}$$

where the function $R_b : b \mapsto R_b(b)$ describes the direction of the reform. A necessary condition for the optimality of $T_1(b)$ is that for every direction $R(b)$, a perturbation of magnitude t does not increase social welfare.

The reform (45) implies :

$$\left.\frac{\partial \widetilde{T}(y,b,t)}{\partial t}\right|_{t=0} = -R_b(b)$$

$$\left.\frac{\partial \widetilde{T}_y(y,b,t)}{\partial t}\right|_{t=0} = 0$$

$$\left.\frac{\partial \widetilde{T}_b(y,b,t)}{\partial t}\right|_{t=0} = -R_b'(b)$$

Using (32), the impact of the reform (45) on the government's Lagrangian is given by :

$$\frac{\partial \mathscr{L}(t, G_0)}{\partial t} = \int_\theta \left\{ \left[T_y \frac{\partial y(\theta)}{\partial \tau_b} + (T_b + \eta \Pi) \frac{\partial b(\theta)}{\partial \tau_b} \right] R_b'\left(b\left(\theta\right)\right) \right.$$

$$\left. - \left[1 - g(\theta) - \frac{\partial y(\theta)}{\partial \rho} T_y - (T_b + \eta \Pi) \frac{\partial b(\theta)}{\partial \rho} \right] R_b\left(b\left(\theta\right)\right) \right\} f(\theta) d\theta \tag{46}$$

Under Assumption 2, we can use the law of iterated expectation to rewrite (46) as :

$$\frac{\partial \mathscr{L}(t, G_0)}{\partial t} = \int_b \left\{ \int_\theta \left[\left(T_y \frac{\partial y(\theta)}{\partial \tau_b} + (T_b + \eta \Pi) \frac{\partial b(\theta)}{\partial \tau_b} \right) R_b'\left(b\left(\theta\right)\right) \right. \right.$$

$$\left. \left. - \left(1 - g(\theta) - \frac{\partial y(\theta)}{\partial \rho} T_y - (T_b + \eta \Pi) \frac{\partial b(\theta)}{\partial \rho} \right) R_b\left(b\left(\theta\right)\right) f(\theta | b) d\theta \right] \right\} h_b(b) db \tag{47}$$

Hence we can write (47) as :

$$\frac{\partial \mathscr{L}(t, G_0)}{\partial t} = \int_b \left\{ \left(T_y \frac{\partial \overline{Y(b)}}{\partial \tau_b} + (T_b + \eta \Pi) \frac{\partial \overline{B}(b)}{\partial \tau_b} \right) R_b'(b) \right.$$

$$\left. - \left(1 - \overline{G(b)} - \frac{\partial \overline{Y}(b)}{\partial \rho} T_y - \overline{(T_b + \eta \Pi) \frac{\partial B(b)}{\partial \rho}} \right) R_b(b) \right\} h_b(b) db \tag{48}$$

Now the impact of a tax reform is no longer a function of type θ but is expressed in terms of the donations density $h_b(b)$. To derive an explicit formula for the tax schedule on donations, I note that $\frac{\partial \mathscr{L}(t,G_0)}{\partial t}$ should be nil at the optimum for every direction $R_b(b)$. This yields the following result :

Proposition 6. *Given an income tax schedule $T_0(y_0)$ and a deduction rule $a(b)$, optimal or not, the tax schedule on donation $T_1(b)$ must verify at the optimum :*

$$\frac{T_1'(b) + \eta \Pi}{1 - T_1'(b)} \epsilon(b) b h_b(b) = \int_b^\infty \left(1 - \overline{G(z)} - \overline{T_0'(y_0) \frac{\partial Y_0(z)}{\partial \rho}} - \overline{(T_1'(z) + \eta \Pi) \frac{\partial B(z)}{\partial \rho}} \right) h_b(z) dz$$

$$- \overline{T_0'\left(y_0\right) \frac{\partial Y_0(b)}{\partial \tau_b}} h_b(b) \tag{49}$$

The proof is given in Appendix C.1. Except for the externality parameter, the optimality condition (49) is similar to the ABC tax formula derived by Diamond (1998) and Saez (2001).

First, it stresses the decreasing relationship between the marginal tax rate $T_1'(b)$ and the compensated elasticity $\epsilon(b)$. This is the standard inverse elasticity logic of Ramsey (1927). Applied to the case of charitable contributions, this implies that the higher the "price elasticity of giving" the less it should be taxed or the higher should it be subsidized. Estimates of this elasticity parameter provided for instance by Fack and Landais (2010, 2016) can be used to take (49) to the data.

Second, the integral term in (49) illustrates the decreasing relationship between $T_1'(b)$ and the average welfare weights of taxpayers with contributions above b. The logic is that these taxpayers see their tax liability mechanically increase after a rise of marginal tax rate at donation level b. So the more the planner value their welfare, the lower should be $T'(b)$. This should be balanced by the rise in tax revenue due to the mechanical effect and the income effect $1 - T_0' \frac{\partial y_0}{\partial \rho} - (T_1' + \eta\Pi)\frac{\partial b}{\partial \rho}$.

Third, as emphasized in Jacquet and Lehmann (2021b), in a context of multiple taxable outcome, potential cross-base responses have to be taken into account when setting optimal tax rates. Such cross-base responses are captured by the second line of (49). In our context there exists two tax base : the one on taxable income y_0 and the one on donation b. As one can clearly see from equation (87), the responses of y_0 is a mixture between cross-base response of labor income y and the response of donations through the deduction function $a(b)$.

Eventually, note that the main departure between Proposition 6 and standard ABC formulas is the externality parameter $\eta\Pi$. This externality makes the specificity of the tax treatment of charitable contribution and (49) shows that $\eta\Pi$ amplifies both compensated responses income responses. For instance, if the government and the contribution good are perfect substitute (so that $\eta = 1$ at the optimum) and if individual contributions are decreasing with aggregate contributions (so that $\Pi > 0$), this external effect pushes tax rates down through compensated responses and up through income responses.

Proposition 6 presents an optimality condition for the nonlinear tax credit, for a given (optimal or not) income tax and a given deduction function. In practice, most countries either rely on tax credit or on donation to subsidize charitable giving. It can therefore be interesting to study within system policy where the donation or the tax credit function is arbitrarily set to 0. First consider the pure tax credit system described by (40). In this case, (49) rewrites as:

$$\frac{T_1'(b) + \eta\Pi}{1 - T_1'(b)}\epsilon(b) b h_b(b) = \int_b^\infty \left(1 - \overline{G(z)} - \overline{T_0'(y)\frac{\partial Y(z)}{\partial \rho}} - \overline{\left(T_1'(z) + \eta\Pi\right)\frac{\partial B(z)}{\partial \rho}}\right) h_b(z) dz$$

$$- \overline{T_0'(y)\frac{\partial Y(b)}{\partial \tau_b}} h_b(b) \tag{50}$$

The only difference between (50) and (49) is simply that the income tax schedule, the income effect on labor supply and the cross-base effect depend on labor income y rather than income net of deduction y_0. This is simply due to the fact that in such a system, gross income equals taxable income. To get further intuitions on the optimal tax credit in this case, assume that first there is no income effect on labor supply: $\frac{\partial y)}{\partial \rho} = 0$. Such an hypothesis is standard in the literature where quasilinear utility is usually assumed since Diamond (1998) and is backed up by the empirical evidences suggesting little income effects (Saez et al. (2012),Kleven and Schultz (2014)). Second, assume that there is no cross-base response between labor income and the tax credit for donations: $\frac{\partial y}{\partial \tau_b} = 0$. To the best of my knowledge, the empirical literature does not provide evidence of such cross-base response so that this assumption does not seem too unrealistic. Under these two assumptions, the optimal tax credit formula simplifies to:

$$\frac{T_1'(b) + \eta \Pi}{1 - T_1'(b)} = \frac{1}{\epsilon(b)} \frac{1}{bh(b)} \int_b^{\infty} \left(1 - \overline{G(z)} - \overline{(T_1'(z) + \eta \Pi) \frac{\partial B(z)}{\partial \rho}} \right) h_b(z) dz \tag{51}$$

Hence under fairly realistic assumptions, the subsidy to charitable giving in a pure tax credit system does not depend on the shape of the income tax schedule. Tax formula (51) is simply an ABC-formula corrected by the externality created by charitable giving as measured by $\eta \pi$.

For the sake of future numerical exercises and because if fits the current tax treatment of giving in countries like France, it can still be useful to provide the optimal linear tax credit. I therefore derive in Appendix C.3 an optimality condition for such a linear tax credit.

Proposition 7. *Given an (arbitrary or optimal) income tax $T_0(.)$ and a deduction function $a(.)$, the optimal linear tax rate on donations t_b verifies :*

$$\frac{t_b + \eta \Pi}{1 - t_b} \int_\theta \epsilon^U(b(\theta)) b(\theta) f(\theta) d(\theta) + \int_\theta T_0'(y_0(\theta)) \frac{\partial y_0(\theta)}{\partial \tau_b} f(\theta) d(\theta) = \int_\theta [1 - g(\theta)] b(\theta) f(\theta) d\theta \tag{52}$$

$$with\ \epsilon^U(b(\theta)) \overset{def}{\equiv} \frac{1 - t_b}{b} \frac{\overline{\partial B(b)^U}}{\partial \tau_b}$$

Following exactly the same methodology it is possible to derive an optimality condition on the income tax schedule T_0. Let $\epsilon(y_0)$ denote the mean compensated elasticity of taxable income y_0 with respect to its marginal net of tax rate t_0 :

$$\epsilon(y_0) \overset{\text{def}}{\equiv} \frac{1 - T_0'(y_0)}{y_0} \frac{\overline{\partial Y_0(y_0)}}{\partial \tau_0} \tag{53}$$

Then the optimal tax formula for T_0 is given by :

Proposition 8. *Given a donation tax schedule $T_1(b)$ and a deduction rule $a(b)$, optimal or not, the income tax schedule $T_0(y_0)$ must verify at the optimum :*

$$T_0'(y_0)\ \epsilon(y_0)\ y_0\ h_0(y_0) = \int_{y_0}^{\infty} \left(1 - \overline{G}(z) - \overline{\frac{\partial Y_0(z)}{\partial \rho}\ T_0'(z)} - \overline{(T_1'(b) + \eta \Pi) \frac{\partial B(z)}{\partial \rho}} \right) h(z) dz$$
$$- (T_1'(b) + \eta \Pi) \frac{\overline{\partial B(y_0)}}{\partial \tau_0} h(y_0) \tag{54}$$

The proof is given in Appendix C.2. Notice that the total external effect $\eta\Pi$ of charitable giving still appears in the optimal tax formula of the income tax T_0. This implies that as soon as there exists a specific schedule for donations, such that $T_1'(b) \neq 0$, the externality has to be taken into account through income effects $\frac{\partial b}{\partial \rho}$ and the cross-base response $\frac{\partial b}{\partial \tau_0}$. Note eventually that this cross-base response would still matter even if there is no deduction for donations. In this case, $a(b) = 0$ and $y_0 = y$ so that the income tax is a standard labor income tax as studied in Mirrlees (1971). Yet the optimal tax formula would still be different from Diamond (1998) and Saez (2001) because the external effect would still occur through income effect and the compensated cross-base response of donations to the labor income tax $\frac{\partial b}{\partial \tau_y}$. The impact of the external effect on welfare hence on T_0 eventually depends on the relative strength of these income and substitution effects. Doerrenberg et al. (2017) provides an estimate of the uncompensated response of donations to reform of the income tax schedule $\frac{\partial b^u}{\partial \tau_0}$. Their study find a positive uncompensated elasticity of giving with respect to the net of tax rate on income, hence suggesting that in this case the income effect dominate the substitution effect.[16]

Again, the optimal income tax formula (54) can be used to describe within system optimal tax rates. Consider the same two assumptions that lead to the simplified formula for the tax credit system (50): no income effect on labor supply and no-cross base response of labor income to the giving subsidy. Besides assume no cross-base response of giving to the marginal income tax rate: $\frac{\partial b}{\partial \tau_y} = 0$. Than the optimal income tax rate in a pure tax credit system takes the form:

$$T_0'(y)\ \epsilon(y)\ y\ h_y(y) = \int_y^\infty \left(1 - \overline{G}(z) - \overline{(T_1'(b) + \eta\Pi)\frac{\partial B(z)}{\partial \rho}}\right) h_y(z)dz \tag{55}$$

Hence, absent income effect on charitable giving, the optimal income tax rate in a pure tax credit system is exactly the same as the ABC-formula of Diamond (1998) and Saez (2001). Now consider the same set of assumptions, in a pure deduction system as described by (39). Then the optimal income tax rates take the form:

$$\begin{aligned} T_0'(y_0)\ \epsilon(y_0)\ y_0\ h_0(y_0) = \int_{y_0}^\infty \left(1 - \overline{G}(z) - \overline{\frac{\partial Y_0(z)}{\partial \rho}\ T_0'(z)} - \eta\Pi\frac{\partial B(z)}{\partial \rho}\right) h(z)dz \\ + \eta\ \Pi\ a'(b)\overline{\frac{\partial B(y_0)}{\partial \tau_b}} h_{y_0}(y_0) \end{aligned} \tag{56}$$

Tax formula (56) departs from (55) in two ways. First, the relevant elasticity is no longer the elasticity of taxable income (ETI) but the elasticity of gross income. As documented by Chetty (2009), the ETI is not longer the relevant sufficient statistic in presence of deduction possibilities. Second, as soon as the price elasticity of giving is different from zero, as documented by a vast empirical literature[17], optimal income tax

[16] In most of his optimal tax exercise, Saez (2004) implicitly assume that compensated response of donations to the income tax is zero. Although their result on the cross-base response of charitable giving does not contradict this assumption, Doerrenberg et al. (2017) do find evidence of substitution effects when looking at deductible expenditures in general.

[17] see for instance Andreoni and Payne (2013) for a review

rates are linked to the marginal deduction for subsidy. All else equal and neglecting income effects $\frac{\partial b}{\partial \rho}$, the higher the external effect of giving, the higher should be the marginal income tax rate. The intuition is that in a pure deduction system, the subsidy to give is increasing with marginal income tax rate: it is more interesting to deduct donations for taxable income if the income tax rates are high.

V.3 Optimal Deduction Rule

After determining the key elements of the optimal tax schedules on income and donation, remains the question of the optimal deduction rule for charitable giving. In most OECD countries, it is through this deduction rule that donations are given a preferential tax treatment. So consider the following reform :

$$\widetilde{T}(y,b,t) = T_0\left(y - a\left(b\right) - tb\right) + T_1(b) \tag{57}$$

With $t > 0$ and $a(b) > 0$, this reform implies an increase in the deductibility of donations from taxable income. In other words, for a given donation b, taxable income is lower after the reform. With $t < 0$, taxable income at a given level b is higher after the reform. This is typically the reform considered by the Obama administration in 2010.[18]

Such a reform affect tax liability and marginal tax rates as follows :

$$\left.\frac{\partial \widetilde{T}(y,b,t)}{\partial t}\right|_{t=0} = -bT_0'(y_0) \tag{58a}$$

$$\left.\frac{\partial \widetilde{T}_y(y,b,t)}{\partial t}\right|_{t=0} = -bT_0''(y_0) \tag{58b}$$

$$\left.\frac{\partial \widetilde{T}_b(y,b,t)}{\partial t}\right|_{t=0} = -T_0'(y_0) + ba'(b)T_0''(y_0) \tag{58c}$$

Note that compared to reforms of the donation and income tax schedules described by (41b) and (41a), perturbation (57) affect marginal tax rates through T_0' and T_1' but also through the second-derivative of the income tax T_0''.

To see how this translates into the welfare impact of the reform, I plug (58) in the government's Lagrangian (32). Using the Slutsky equation (20), this yields the following proposition :

Proposition 9. • *The impact of a reduction of taxable income through the reform of the deduction function $a(b)$ described in* (57) *is given by :*

[18] See List (2011), p170.

$$\frac{\partial \mathscr{L}(t,G_0)}{\partial t} = \int_\theta \Bigg\{ \left[b(\theta)\left(g(\theta)-1\right) + T_0' \frac{\partial y_0^U(\theta)}{\partial \tau_b} + \left(T_1' + \eta\Pi\right) \frac{\partial b^U(\theta)}{\partial \tau_b} \right] T_0'\left(y_0\left(\theta\right)\right)$$

$$+ \left[T_0'\left(y_0\left(\theta\right)\right) \frac{\partial y_0(\theta)}{\partial \tau_0} + \left(T_1'\left(b\left(\theta\right)\right) + \eta\Pi\right) \frac{\partial b(\theta)}{\partial \tau_0} \right] b(\theta) T_0''\left(y_0\left(\theta\right)\right) \Bigg\} f(\theta) d\theta \tag{59}$$

- *A reform combining a change in the deduction rule* (57) *with* $t > 0$ *and a lump-sum reform balancing budget is welfare improving if* (59) *is strictly positive.*

To interpret proposition 9, first consider (59) without response of labor income y and donation b. Then the impact of a reform is simply :

$$\frac{\partial \mathscr{L}(t,G_0)}{\partial t} = \int_\theta b(\theta)\left(g(\theta)-1\right) T_0'\left(y_0\left(\theta\right)\right) f(\theta) d\theta$$

Absent behavioral responses, a reform such as (57) with $t > 0$ reduces taxable income hence reduces tax revenue while increasing individual welfare. For every dollar of charitable contribution of a θ-type taxpayer, this creates a mechanical loss in tax revenue of $-T_0'\left(y_0\left(\theta\right)\right)$. In the meantime, this reduction in tax liability yields an individual welfare gain of $g(\theta)T_0'\left(y_0\left(\theta\right)\right)$.

As usual, both labor supply and contributions can adjust to the tax change. Since a reform of the deduction rule changes the marginal tax rate on giving, these adjustments can either take the form of a direct response $T_1' \frac{\partial b_0^U(\theta)}{\partial \tau_b}$ or a cross-base response $T_0' \frac{\partial y_0^U(\theta)}{\partial \tau_b}$. The gain or loss in tax revenue will eventually depend on the sign of these uncompensated responses or in other words, on the relative strength of income and substitution effects. Again the main novelty here is that any behavioral response of giving is magnified by the total externality parameter $\eta\Pi$.

The main difference between reforming the deduction rule $a(b)$ and tax schedules $T_0(b)$ and $T_1(b)$ is the second line of (59). As one can seen from (58b) and (58c), a reform of the deduction rule affects both marginal tax rates on donations and labor income by changing the curvature of the income tax schedule. This therefore triggers compensated responses $T_0'(y_0)\frac{\partial y_0}{\partial \tau_0}$ and $T_1'(b)\frac{\partial b}{\partial \tau_0}$ that are proportional to the second derivative of the income tax $T_0''(y_0)$. This property of reforms of the deduction rule has been first noted by Jacquet and Lehmann (2021a) in the context of the taxation of multiple incomes. When dealing with charitable contribution, this incidence through T_0'' is amplified by the externality parameter $\eta\Pi$. Such an effect has to the best of my knowledge not been studied in previous analysis of the tax treatment of charitable contributions, as most of them relied on linear tax instruments and therefore could not account the incidence arising from the convexity of the income tax schedule.

VI Conclusion

In this paper I discuss how charitable giving affect both the optimal tax rates and the optimal level of the government's provision of a public good. From a conceptual point of view, I show that a social optimum where the public good is only funded through charitable contributions occur under fairly less restrictive assumptions than the ones usually assumed in the literature. In particular, I show that in a Mirrleesian economy, separability between preferences for the public and the private good and work effort is not required for this complete crowding out of the government's contribution to the public good to occur. From a policy point of view, I derive optimal tax formulas for both the donation and the income tax schedules. Besides, I show how deduction of donations from taxable income should be set at the optimum. Although the policy instruments considered here can match most of the OECD countries' tax treatment of charitable giving, the matching system used for instance in the UK is left out of the analysis. This system where the government tops up individual's contribution is an alternative to tax credits and deductions. Although studied by the empirical literature on giving[19], this mechanism has not been introduced in a formal optimal tax exercise.

[19]See for instance Peter and Lideikyte Huber (2022), chapter 9, part 3.2.1 for a brief review of the literature.

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A Proofs of the Results of Section III

A.1 Proof of Proposition 1

Consider an incentive compatible allocation of consumption, donation, labor income and public good $\{c'(\theta), b'(\theta), y(\theta), G\}$. Suppose also that this allocation verifies the nonnegativity constraint on the government's provision of the public good $G_0 \geq 0$.

It follows from (7) that a taxpayer of type θ enjoys a utility level:

$$\mathcal{U}\left(V\left(c'(\theta), b'(\theta)\right), y(\theta); G, \theta\right) = U(\theta) \tag{60}$$

Let $\mathcal{V}(\theta) = V\left(c'(\theta), b'(\theta)\right)$ denotes the subutility from consumption and donation enjoyed at the initial allocation. The initial allocation $\{c'(\theta), b'(\theta), y(\theta), G\}$ is incentive compatible so for any θ' :

$$\mathcal{U}\left(\mathcal{V}(\theta), y(\theta); G, \theta\right) \geq \mathcal{U}\left(\mathcal{V}(\theta'), y(\theta'); G, \theta\right) \tag{61}$$

Consider an alternative allocation $\{\hat{c}(\theta), \hat{b}(\theta), y(\theta), G\}$ with the same labor income, the same level of the public good but potentially different levels of donation and consumption. Suppose that this allocation yields the same subutility from consumption and donation so that:

$$\begin{aligned} V\left(\hat{c}(\theta), \hat{b}(\theta)\right) &= V\left(c'(\theta), b'(\theta)\right) \\ &= \mathcal{V}(\theta) \end{aligned} \tag{62}$$

In this case, we have:

$$\begin{aligned} \mathcal{U}\left(V\left(\hat{c}(\theta), \hat{b}(\theta)\right), y(\theta); G, \theta\right) &= \mathcal{U}\left(\mathcal{V}(\theta), y(\theta); G, \theta\right) \\ &= U(\theta) \end{aligned} \tag{63}$$

So that $\{\hat{c}(\theta), \hat{b}(\theta)\}$ does not change individual utility compared to the initial allocation.

Combining (63) with (61) yields :

$$\mathcal{U}\left(V\left(\hat{c}(\theta), \hat{b}(\theta)\right), y(\theta); G, \theta\right) \geq \mathcal{U}\left(\mathcal{V}(\theta'), y(\theta'); G, \theta\right)$$

so that $\{\hat{c}(\theta), \hat{b}(\theta)\}$ is incentive compatible. So moving from the initial allocation $\{c'(\theta), b'(\theta)\}$ to the candidate $\{\hat{c}(\theta), \hat{b}(\theta)\}$ will generate a strict Pareto-improvement if it increases government revenue.

For any $\{c(\theta), b(\theta), y(\theta), G\}$, government revenue net of public spending is given by:

$$
\begin{aligned}
R &= \int_\theta T\left(y(\theta), b(\theta)\right) f(\theta) d(\theta) - G_0 \\
&= \int_\theta y(\theta) - b(\theta) - c(\theta) f(\theta) d(\theta) - G_0 \\
&= \int_\theta \left(y(\theta) - c(\theta)\right) f(\theta) d(\theta) - G
\end{aligned}
\tag{64}
$$

where I used the taxpayer's budget constraint, the perfect substitution assumption $G = G_1 + G_0$ and the definition of $G_1 = \int_\theta b(\theta) f(\theta) d(\theta)$. Since $y(\theta)$ and G are fixed, our problem therefore takes the form of :

$$
\begin{aligned}
&\min_{c(\theta), b(\theta)} \int_\theta c(\theta) f(\theta) d(\theta) \\
&\text{subject to : } V\left(c(\theta), b(\theta)\right) = \mathcal{V}(\theta) \\
&\qquad\qquad G - \int_\theta b(\theta) f(\theta) d(\theta) \geq 0
\end{aligned}
\tag{65}
$$

The question is to know if at the optimum of (65), the nonnegativity constraint $G_0 \geq 0$ is binding or not. Assume by contradiction that the initial allocation $\{c'(\theta), b'(\theta)\}$ solves (65) with $G_0 > 0$. Suppose that the alternative allocation $\{\hat{c}(\theta), \hat{b}(\theta)\}$ verifies the nonnegativity constraint $G_0 \geq 0$ but with $\hat{b}(\theta) > b'(\theta)$, for all θ.

It follows from (62) that for all θ:

$$
\begin{aligned}
&V\left(\hat{c}(\theta), \hat{b}(\theta)\right) = V\left(c'(\theta), b'(\theta)\right) \\
&\Rightarrow \hat{c}(\theta) < c'(\theta)
\end{aligned}
\tag{66}
$$

where the last line follows from $\hat{b}(\theta) > b'(\theta)$ for all θ and $V(c, b)$ being strictly increasing in both c and b.

So $\{c'(\theta), b'(\theta)\}$ cannot solve (65). So at the optimum of (65) the constraint $G_0 \geq 0$ is necessarily binding and this proves Proposition 1.

A.2 Proof of Proposition 2

Consider an incentive compatible allocation of consumption, donation, labor income and public good $\{c'(\theta), b'(\theta), y(\theta), G'\}$. Suppose also that this allocation verifies the nonnegativity constraint on the government's provision of the public good $G_0 \geq 0$.

It follows from (8) that a taxpayer of type θ enjoys a utility level:

$$
\mathcal{U}\left(V\left(c'(\theta), b'(\theta), G'\right), y(\theta); \theta\right) = U(\theta) \tag{67}
$$

Let $\mathcal{V}(\theta) = V\left(c'(\theta), b'(\theta), G'\right)$ denotes the subutility from consumption, donations and the public good enjoyed by a type θ individual at the initial allocation. The initial allocation $\{c'(\theta), b'(\theta), y(\theta), G'\}$ is incentive compatible so for any θ' :

$$
\mathcal{U}\left(\mathcal{V}(\theta), y(\theta); \theta\right) \geq \mathcal{U}\left(\mathcal{V}(\theta'), y(\theta'); \theta\right) \tag{68}
$$

Consider an alternative allocation with the same labor income the same subutility from consumption, donation and the public good. In other words, consider an allocation $\{\hat{c}(\theta), \hat{b}(\theta), y(\theta), \hat{G}\}$ such that

$$\begin{aligned} V\left(\hat{c}(\theta), \hat{b}(\theta), \hat{G}\right) &= V\left(c'(\theta), b'(\theta), G'\right) \\ &= \mathcal{V}(\theta) \end{aligned} \tag{69}$$

In this case, we have:

$$\begin{aligned} \mathcal{U}\left(V\left(\hat{c}(\theta), \hat{b}(\theta), \hat{G}\right), y(\theta); \theta\right) &= \mathcal{U}\left(\mathcal{V}(\theta), y(\theta); \theta\right) \\ &= U(\theta) \end{aligned} \tag{70}$$

So that $\{\hat{c}(\theta), \hat{b}(\theta), \hat{G}\}$ does not change individual utility compared to the initial allocation.

Combining (70) with (68) yields :

$$\mathcal{U}\left(V\left(\hat{c}(\theta), \hat{b}(\theta), \hat{G}\right), y(\theta); \theta\right) \geq \mathcal{U}\left(\mathcal{V}(\theta'), y(\theta'); \theta\right)$$

so that $\{\hat{c}(\theta), \hat{b}(\theta), \hat{G}\}$ is incentive compatible. So moving from the initial allocation $\{c'(\theta), b'(\theta), G'\}$ to the candidate $\{\hat{c}(\theta), \hat{b}(\theta), \hat{G}\}$ will generate a strict Pareto-improvement if it increases government revenue. Using (64) and taking into account that the government also choose G to maximize government revenue, this problem takes the form:

$$\begin{aligned} &\min_{c(\theta), b(\theta), G} \int_\theta c(\theta) f(\theta) d(\theta) + G \\ &\text{subject to : } V\left(c(\theta), b(\theta), G\right) = \mathcal{V}(\theta) \\ &\qquad G - \int_\theta b(\theta) f(\theta) d(\theta) \geq 0 \end{aligned} \tag{71}$$

The Lagrangian associated to (71) is :

$$L = \int_\theta \left[c(\theta) - \phi(\theta)\left(V\left(c(\theta), b(\theta), G\right) - \mathcal{V}(\theta)\right)\right] dF(\theta) + G - \mu\left(G - \int_\theta b(\theta) dF(\theta)\right) \tag{72}$$

with $\phi(\theta)$ and μ the lagrange multipliers associated to the subutility and non-negativity constraints.

The F.O.C with respect to $c(\theta)$ yields :

$$\phi(\theta) = \frac{1}{V_c\left(c(\theta), b(\theta), G\right)} \tag{73}$$

The F.O.C with respect to $b(\theta)$ yields :

$$\phi(\theta) = \frac{\mu}{V_b\left(c(\theta), b(\theta), G\right)} \tag{74}$$

Combining (73) with leisure (74) yields :

$$\mu = \frac{V_b\left(c(\theta), b(\theta), G\right)}{V_c\left(c(\theta), b(\theta), G\right)} \tag{75}$$

The F.O.C with respect to G yields :

$$\int_\theta \left[\phi(\theta) V_G\left(c(\theta), b(\theta), G\right) + \mu\right] dF(\theta) = 1 \tag{76}$$

Combining (76) with (73) and (75) yields (9).

B Proofs of the Results of Section IV

B.1 Proof of equation 13

Differentiating (11) we get :

$$\left[\left(1-\widetilde{T}_y\right) S_c^y + S_y^y + \widetilde{T}_{y,y}\right] dy + \left[-(1+\widetilde{T}_b) S_c^y + S_b^y + \widetilde{T}_{y,b}\right] db = \left[\frac{\partial \widetilde{T}}{\partial t} S_c^y - \frac{\partial \widetilde{T}_y}{\partial t}\right] dt - S_{G_1}^y dG_1 - S_{G_0}^y dG_0$$

Using (11) and (12) this can be rewritten as :

$$\left[S^y\, S_c^y + S_y^y + \widetilde{T}_{y,y}\right] dy + \left[-S^b\, S_c^y + S_b^y + \widetilde{T}_{y,b}\right] db = \left[\frac{\partial \widetilde{T}}{\partial t} S_c^y - \frac{\partial \widetilde{T}_y}{\partial t}\right] dt - S_{G_1}^y dG_1 - S_{G_0}^y dG_0 \tag{77}$$

Differentiate (12) :

$$\left[\left(1-\widetilde{T}_y\right) S_c^b + S_y^b - \widetilde{T}_{b,y}\right] dy + \left[-(1+\widetilde{T}_b) S_c^b + S_b^b - \widetilde{T}_{b,b}\right] db = \left[\frac{\partial \widetilde{T}}{\partial t} S_c^b + \frac{\partial \widetilde{T}_b}{\partial t}\right] dt - S_{G_1}^b\, dG_1 - S_{G_0}^b\, dG_0$$

And using (11) and (12) :

$$\left[S^y\, S_c^b + S_y^b - \widetilde{T}_{b,y}\right] dy + \left[-S^b\, S_c^b + S_b^b - \widetilde{T}_{b,b}\right] db = \left[\frac{\partial \widetilde{T}}{\partial t} S_c^b + \frac{\partial \widetilde{T}_b}{\partial t}\right] dt - S_{G_1}^b\, dG_1 - S_{G_0}^b\, dG_0 \tag{78}$$

To sum up in matrix form :

$$\begin{pmatrix} S^y\, S^y_c + S^y_y + \widetilde{T}_{y,y} & -S^b\, S^y_c + S^y_b + \widetilde{T}_{y,b} \\ S^y\, S^b_c + S^b_y - \widetilde{T}_{b,y} & -S^b\, S^b_c + S^b_b - \widetilde{T}_{b,b} \end{pmatrix} \cdot \begin{pmatrix} dy \\ db \end{pmatrix} = \begin{pmatrix} \frac{\partial \widetilde{T}}{\partial t} S^y_c - \frac{\partial \widetilde{T}_y}{\partial t} \\ \frac{\partial \widetilde{T}}{\partial t} S^b_c + \frac{\partial \widetilde{T}_b}{\partial t} \end{pmatrix} dt - \begin{pmatrix} S^y_{G_1} \\ S^b_{G_1} \end{pmatrix} dG_1 - \begin{pmatrix} S^y_{G_0} \\ S^b_{G_0} \end{pmatrix} dG_0 \tag{79}$$

Let $A = \begin{pmatrix} S^y\, S^y_c + S^y_y + \widetilde{T}_{y,y} & -S^b\, S^y_c + S^y_b + \widetilde{T}_{y,b} \\ S^y\, S^b_c + S^b_y - \widetilde{T}_{b,y} & -S^b\, S^b_c + S^b_b - \widetilde{T}_{b,b} \end{pmatrix}$.

Assuming that the matrix A is invertible, one can rewrite (79) as (13).

B.2 Proof of Proposition 4

The impact of G_0 on G_1 can be measured through the fixed point condition :

$$G_1(G_0) = \int_\theta b(\theta, G_1(G_0), G_0)\, f(\theta) d\theta \tag{80}$$

Using (23), the general equilibrium response of G_1 to a change in G_0 is given by :

$$\frac{\partial G_1(G_0)}{\partial G_0} = \Pi \int_\theta \frac{\partial b(\theta, G_1(G_0), G_0)}{\partial G_0} f(\theta) d(\theta) \tag{81}$$

Differentiating (26), evaluated at $t = 0$, with respect to G_0 yields the partial equilibrium effect of a change in G_0 :

$$\frac{\partial \widehat{\mathscr{L}}(G_1, G_0)}{\partial G_0} = \int_\theta \left(\frac{\partial y}{\partial G_0} T_y + \frac{\partial b}{\partial G_0} T_b + g(\theta) S^{G_0} \right) f(\theta) d(\theta) - 1 \tag{82}$$

Then define a general equilibrium Lagrangian, taking into account the responses of G_1 to G_0 :

$$\widetilde{\mathscr{L}}(G_0) \overset{\text{def}}{\equiv} \widehat{\mathscr{L}}(G_1(G_0), G_0) \tag{83}$$

Differentiating (83) and using (30) yields :

$$\frac{\partial \widetilde{\mathscr{L}}(G_0)}{\partial G_0} = \frac{\partial \widehat{\mathscr{L}}(G_1, G_0)}{\partial G_0} + \eta \frac{\partial G_1(G_0)}{\partial G_0} \tag{84}$$

Combining (84), (81) and (82) yields:

$$\frac{\partial \widetilde{\mathscr{L}}(G_0)}{\partial G_0} = \int_\theta \left(\frac{\partial y}{\partial G_0} T_y + \frac{\partial b}{\partial G_0} (T_b + \eta \Pi) + g(\theta) S^{G_0} \right) f(\theta) d(\theta) - 1 \tag{85}$$

Equating (85) to 0 yields (34) and proves Proposition 4.

C Proofs of the Results of Section V

C.1 Proof of Proposition 6

Integrating (48) by parts yields :

$$\begin{aligned}
&\int_b \left(1 - \overline{G(b)} - \frac{\partial \overline{Y}(b)}{\partial \rho}\, T_y - (T_b + \eta \Pi)\, \frac{\partial \overline{B}(b)}{\partial \rho} \right) R_b(b) h_b(b) db = \\
&\int_b \left[\int_b^\infty \left(1 - \overline{G(b)} - \frac{\partial \overline{Y}(b)}{\partial \rho}\, T_y - (T_b + \eta \Pi)\, \frac{\partial \overline{B}(b)}{\partial \rho} \right) h_b(z) dz \right] R'(b) db \\
&+ \lim_{z \to +\infty} R(z) \int_0^b \left(1 - \overline{G(z)} - \frac{\partial \overline{Y}(z)}{\partial \rho}\, T_y - (T_b + \eta \Pi)\, \frac{\partial \overline{B}(z)}{\partial \rho} \right) h_b(z) dz
\end{aligned} \tag{86}$$

The condition for a balanced-budget reform (33) implies that the limit term in (86) is nil. So eventually (48) can be rewritten as :

$$\begin{aligned}
\frac{\partial \mathscr{L}(t, G_0)}{\partial t} = \int_b \Bigg\{ & \left(T_y \frac{\partial \overline{Y(b)}}{\partial \tau_b} + (T_b + \eta \Pi)\, \frac{\partial \overline{B}(b)}{\partial \tau_b} \right) h_b(b) \\
& - \int_b^\infty \left(1 - \overline{G(z)} - \frac{\partial \overline{Y}(z)}{\partial \rho}\, T_y - (T_b + \eta \Pi)\, \frac{\partial \overline{B}(z)}{\partial \rho} \right) h_b(z) dz \Bigg\} R_b'(b) db
\end{aligned} \tag{87}$$

Or using (41a) and (41b) :

$$\begin{aligned}
\frac{\partial \mathscr{L}(t, G_0)}{\partial t} = \int_b \Bigg\{ & \left(\overline{T_0'(y_0)\, \frac{\partial Y(b)}{\partial \tau_b}} + \overline{(T_1'(b) - a'(b) + \eta \Pi)\, \frac{\partial B(b)}{\partial \tau_b}} \right) h_b(b) \\
& - \int_b^\infty \left(1 - \overline{G(z)} - \overline{T_0'(y_0) \frac{\partial Y(z)}{\partial \rho}} - \overline{(T_1'(b) - a'(b) + \eta \Pi)\, \frac{\partial B(z)}{\partial \rho}} \right) h_b(z) dz \Bigg\} R_b'(b) db
\end{aligned} \tag{88}$$

At the optimum donation schedule $T_1(b)$, (87) should be zero for every direction $R_b(b)$ hence for every $R_b'(b)$. This implies :

$$h(b)\left(\frac{T_1'(b)+\eta\Pi}{1-T_1'(b)}\epsilon(b)b+\overline{T_0'\left(y_0\right)\frac{\partial Y(b)}{\partial\tau_b}}-\overline{a'(b)\frac{\partial B(b)}{\partial\tau_b}}\right)=$$
$$\int_b^{\infty}\left(1-\overline{G(z)}-\overline{T_0'(y_0)\frac{\partial Y(z)}{\partial\rho}}-\overline{\left(T_1'(b)-a'(b)+\eta\Pi\right)\frac{\partial B(z)}{\partial\rho}}\right)h_b(z)dz \tag{89}$$

Using (42) and (89) yields (49).

C.2 Proof of Proposition 8

A perturbation of the income tax schedule $T_0\left(y\left(0\right)\right)$ is given by

$$\tilde{T}(y,b,t)=T_0\left(y-a\left(b\right)\right)+T_1(b)-t.R_0\left(y-a\left(b\right)\right)$$

This implies :

$$\left.\frac{\partial\widetilde{T}(y,b,t)}{\partial t}\right|_{t=0}=-R_0(y_0)$$

$$\left.\frac{\partial\widetilde{T}_y(y,b,t)}{\partial t}\right|_{t=0}=-R'(y_0)$$

And

$$\left.\frac{\partial\widetilde{T}_b(y,b,t)}{\partial t}\right|_{t=0}=a'(b)R_0'(y_0)$$

Hence the impact of the reform on the government Lagrangian (32) is given by :

$$\frac{\partial\mathscr{L}(t,G_0)}{\partial t}=\int_\theta\Bigg\{\left[T_y\left(\frac{\partial y(\theta)}{\partial\tau_y}-a'(b(\theta))\frac{\partial y(\theta)}{\partial\tau_b}\right)+(T_b+\eta\Pi)\left(\frac{\partial b(\theta)}{\partial\tau_y}-a'(b(\theta))\frac{\partial b(\theta)}{\partial\tau_b}\right)\right]R_0'(y_0(\theta))$$
$$-\left[1-g(\theta)-\frac{\partial y(\theta)}{\partial\rho}\,T_y-(T_b+\eta\Pi)\frac{\partial b(\theta)}{\partial\rho}\right]R_0\left(y_0\left(\theta\right)\right)\Bigg\}f(\theta)d(\theta) \tag{90}$$

Applying the law of iterated expectations to (90) yields :

$$\frac{\partial \mathscr{L}(t,G_0)}{\partial t} = \int_{y_0} \left\{ \left[\overline{T_y \left(\frac{\partial Y(y_0)}{\partial \tau_y} - a'(b)\frac{\partial Y(y_0)}{\partial \tau_b} \right)} + \overline{(T_b + \eta\Pi) \left(\frac{\partial B(y_0)}{\partial \tau_y} - a'(b)\frac{\partial B(y_0)}{\partial \tau_b} \right)} \right] R_0'(y_0) \right.$$
$$\left. - \left[1 - \overline{G(y_0)} - \overline{\frac{\partial Y(y_0)}{\partial \rho}}\ T_y - (T_b + \eta\Pi)\frac{\partial B(y_0)}{\partial \rho} \right] R_0(y_0) \right\} h_0(y_0) dy_0 \tag{91}$$

Integrating (91) by parts and using (33) yields :

$$\frac{\partial \mathscr{L}(t,G_0)}{\partial t} = \int_{y_0} \left\{ \left[\overline{T_y \left(\frac{\partial Y(y_0)}{\partial \tau_y} - a'(b)\frac{\partial Y(y_0)}{\partial \tau_b} \right)} + \overline{(T_b + \eta\Pi) \left(\frac{\partial B(y_0)}{\partial \tau_y} - a'(b)\frac{\partial B(y_0)}{\partial \tau_b} \right)} \right] h(y_0) \right.$$
$$\left. + \int_0^{y_0} \left(1 - \overline{G}(z) - \overline{\frac{\partial Y(z)}{\partial \rho}\ T_y} - \overline{(T_b + \eta\Pi)\frac{\partial B(z)}{\partial \rho}} \right) h(z) dz \right\} R'(y_0) dy_0 \tag{92}$$

At the optimum, (92) should be zero, whatever the direction $R(y_0)$ hence whatever $R'(y_0)$. This implies at the optimum :

$$\int_{y_0}^{\infty} \left(1 - \overline{G}(z) - \overline{\frac{\partial Y(z)}{\partial \rho}\ T_y} - \overline{(T_b + \eta\Pi)\frac{\partial B(z)}{\partial \rho}} \right) h(z) dz =$$
$$\left[\overline{T_y \left(\frac{\partial Y(y_0)}{\partial \tau_y} - a'(b)\frac{\partial Y(y_0)}{\partial \tau_b} \right)} + \overline{(T_b + \eta\Pi) \left(\frac{\partial B(y_0)}{\partial \tau_y} - a'(b)\frac{\partial B(y_0)}{\partial \tau_b} \right)} \right] h(y_0)$$

Using (41a) and (41b) as well as (43a) and (43b), this can be rewritten as :

$$\int_{y_0}^{\infty} \left(1 - \overline{G}(z) - \overline{\frac{\partial Y_0(z)}{\partial \rho}\ T_0'(z)} - \overline{(T_1'(b) + \eta\Pi)\frac{\partial B(z)}{\partial \rho}} \right) h(z) dz =$$
$$\left[\overline{T_0'(y_0)\frac{\partial Y_0(y_0)}{\partial \tau_0}} + \overline{(T_1'(b) + \eta\Pi)\frac{\partial B(y_0)}{\partial \tau_0}} \right] h(y_0)$$

Plugging (53) in (C.2) yields (54) and ends the proof of Proposition 8.

C.3 Proof of Proposition 7

To derive the optimal linear tax formula for donation, consider the following perturbation :

$$\widetilde{T}(y,b,t) = T_0(y - a(b)) + t_b.b - t.b \tag{93}$$

with t_b the linear tax rate on donations. Such a reform implies:

$$\frac{\partial \widetilde{T}(y,b,t)}{\partial t} = -b$$

$$\frac{\partial \widetilde{T}_y(y,b,t)}{\partial t} = 0$$

$$\frac{\partial \widetilde{T}_b(y,b,t)}{\partial t} = -1$$

Using (32), the impact of the reform is therefore given by:

$$\frac{\partial \mathscr{L}(t,G_0)}{\partial t} = \int_\theta \left\{ b(\theta)\left(g(\theta)-1\right) + b(\theta)T_0'\frac{\partial y_0(\theta)}{\partial \rho} + b(\theta)(t_b+\eta\Pi)\frac{\partial b}{\partial \rho} + T_0'\frac{\partial y_0}{\partial \tau_b} + (t_b+\eta\Pi)\frac{\partial b}{\partial t_b} \right\} f(\theta)d\theta \tag{94}$$

At the optimum, (94) should be zero. Using the Slutsky equation (20), this implies:

$$\frac{t_b+\eta\Pi}{1-t_b}\int_\theta \epsilon^U\left(b\left(\theta\right)\right)b(\theta)f(\theta)d(\theta) + \int_\theta T_0'(y_0(\theta))\frac{\partial y_0(\theta)}{\partial \tau_b}f(\theta)d(\theta) = \int_\theta [1-g(\theta)]b(\theta)f(\theta)d\theta$$

C.4 Proof of Proposition 9

Using (32), the impact of the reform is given by :

$$\frac{\partial \mathscr{L}(t,G_0)}{\partial t} = \int_\theta \Bigg\{ \left[b(\theta)\left(g(\theta)-1\right) + T_y\left(b(\theta)\frac{\partial y(\theta)}{\partial \rho} + \frac{\partial y(\theta)}{\partial \tau_b}\right) + (T_b+\eta\Pi)\left(b(\theta)\frac{\partial b(\theta)}{\partial \rho} + \frac{\partial b(\theta)}{\partial \tau_b}\right)\right] T_0'$$
$$+ \left[T_y\left(\frac{\partial y(\theta)}{\partial \tau_y} - a'\left(b\left(\theta\right)\right)\frac{\partial y(\theta)}{\partial \tau_b}\right) + (T_b+\eta\Pi)\left(\frac{\partial b(\theta)}{\partial \tau_y} - a'\left(b\left(\theta\right)\right)\frac{\partial b(\theta)}{\partial \tau_b}\right)\right] b(\theta)T_0'' \Bigg\} f(\theta)d\theta \tag{95}$$

Using (43a),(43b) and the Slutsky equation (20), this can be rewritten as :

$$\frac{\partial \mathscr{L}(t,G_0)}{\partial t} = \int_\theta \Bigg\{ \left[b(\theta)\left(g(\theta)-1\right) + T_y\frac{\partial y^U(\theta)}{\partial \tau_b} + (T_b+\eta\Pi)\frac{\partial b^U(\theta)}{\partial \tau_b}\right] T_0'$$
$$+ \left[T_y\frac{\partial y(\theta)}{\partial \tau_0} + (T_b+\eta\Pi)\frac{\partial b(\theta)}{\partial \tau_0}\right] b(\theta)T_0'' \Bigg\} f(\theta)d\theta \tag{96}$$

Eventually using (41a), (41b) and (42) we can rewrite (96) as (59) and conclude the proof of Proposition 9.