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The Choice of Titling System in Land and the Blockchain*

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Abstract

Should the advent of the blockchain lead to the reorganization or even the replacement of traditional land title registration systems? In addressing this issue we first generalize the model developed by Arruñada and Garoupa (2005) to study optimal land titling systems. Instead of considering only recording and registration alone, we examine an a priori infinite set of systems, each characterized by its quality (the probability that there is no forfeiture for a given plot of land) and its unit transaction cost. In this respect, the blockchain is viewed as a cost-efficient mechanism, albeit one potentially jeopardized by hacking. We find that, despite the introduction of the blockchain, under some reasonable assumptions it is still socially optimal to maintain traditional public registration. In that case, the optimal quality of protection provided by traditional registration must be either sufficiently high (and higher than that of the blockchain), or low enough (and lower than that of the blockchain). Yet under another set of assumptions, it is optimal to rely on the blockchain alone and to abandon traditional registration.

Keywords: blockchain, land titling, forfeiture.

JEL Codes: O3, D23, L86, K25

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1 Introduction

Registering land titles is costly and time-consuming. This is a common picture worldwide. In France the cost of a real-estate transaction accounts for approximately 7.3% of the property value and takes an average of 42 days to complete, according to the World Bank (2019). In Germany the corresponding numbers are 6.6% of the property value and 52 days. In Estonia a real-estate transaction amounts to just 0.5% of the property value and it may be completed much more quickly (on average 17.5 days).[1] Plainly, there are differences between countries due to many factors: regulations, taxes, insurance costs, and real-estate agency fees. These differences make comparisons difficult.

Then there is the question of registration quality. Although the German and French land titling systems are characterized by numerous, costly, and lengthy registration procedures, the result in terms of the quality of land registration appears modest according to the World Bank's aggregated indicator of the quality of land registration (table 1).[2]

	Quality
OECD (high income)	23.2
China	24
Estonia	27.5
France	24.0
Germany	23.0
India	10.8
Sweden	27.5
U.K.	26.0
U.S. - Los Angeles	17.0
U.S.- NYC	18.0

Table 1: Data: World Bank (2019). The land registration quality index ranges from 0 to 30, with higher values indicating a higher quality of protection.

Both the cost (broadly speaking) and the quality of registration affect the functioning of the real-estate markets as well as the rest of the economy. For instance, costly and lengthy real-estate transactions slow workers' mobility and thus affect the labor market (Rupert and Wasmer 2012). Moreover, where property rights are not properly registered and are poorly protected, investments are deterred and growth becomes sluggish. More adequate registration of property titles may on the contrary facilitate access to credit markets and spur growth (Besley 1995; Alston, Libecap and Schneider 1996).[3]

The advent of the blockchain in land registration might challenge the design of contemporary titling systems. The blockchain, a particular type of distributed ledger technology (DLT), is a (theoretically)

[1] The cost of real-estate transactions includes fees, transfer taxes, stamp duties and any other payments to the property registry, notaries, public agencies or lawyers. The "time" index provides the median duration that property lawyers, notaries, or registry officials indicate is necessary to complete a property registration procedure.

[2] Note some limitations of the "quality of land registration" index. The index is the sum of scores for the "reliability of infrastructure", "transparency of information", "geographic coverage", "land dispute resolution", and "equal access to property" indices. The "reliability of infrastructure" index refers mainly to the use of digital tools (such as a database for cadastral plans, geographic information, and land title certificates), and so is only approximate.

[3] The idea that an efficient land registration system can facilitate access to credit through the use of property as collateral has been developed by De Soto (2000, 2001) and investigated by Besley, Burchardi, and Ghatak (2012).

secure, transparent information storage and transmission technology. Due to its advantages in terms of cost, duration, and production of immutable information, the blockchain is a candidate to replace or supplement traditional land registration systems (Arruñada 2019; von Wangenheim 2020). The reduction of transaction costs through the diminishing role of trusted intermediaries is a key proposition of blockchain proponents (Barbieri and Gassen 2017). Yet the blockchain encounters numerous legal and technical limits (Arruñada 2019; Barbieri and Gassen 2017; von Wangenheim 2020). For instance, running a blockchain like that associated with Bitcoin consumes vast amounts of electricity, which is likely to be detrimental to the environment, as indicated in the Bitcoin Energy Consumption Index. Furthermore both the quality of the data submitted to the blockchain and hijacking risks are instances of issues linked to the use of the blockchain in land titling. These limits might explain why the number of real-estate-related applications of the blockchain remains limited in practice.

The aim of this paper is to study how an optimal land titling system should use the blockchain. To perform this study, we develop a general version of the model introduced by Arruñada and Garoupa (2005). We first examine the properties satisfied by an optimal titling system (absent any blockchain). By contrast with Arruñada and Garoupa (*ibid*), we do not restrict ourselves to considering two alternative titling systems. In our setting, a large set of available protection levels is available. Any one of them can be associated with a given titling system and corresponds to a certain unit cost. The higher the protection level, the higher the cost, and the lower the share of owners who can afford to resort to the titling system. We then consider the blockchain, which is interpreted as a system providing a high level of protection but at a strictly lower cost than the traditional system. We do not assume, however, that the blockchain is the most secure titling system, since there is a certain risk of hijacking. Depending on the degree of protection and cost associated with traditional registration, we identify the conditions under which landowners prefer privacy (that is, no registration), traditional, or blockchain registrations. We find that, while resorting to the blockchain is always socially optimal, it is not always optimal to replace traditional public registration with the blockchain. But where it is optimal to keep traditional public registration, its level of protection should either be sufficiently high (and higher than that of the blockchain), or low enough (and lower than that of the blockchain), but should never take intermediate values. The actual outcome depends on the distribution of land values. While the location of the optimal quality of public registration is often indeterminate, there are conditions under which it is possible to state whether it should be high or not.

The remainder of the paper is organized as follows. In the next section, we briefly review the land title registration literature. In section 3, we lay out a general model of land titling and study the properties satisfied by optimal systems. In section 4, we present the pros and cons of blockchain land titling, and we introduce the blockchain in our theoretical framework. Section 5 investigates the choice of a landowner between privacy, blockchain and traditional registrations. Section 6 studies how the social welfare function depends on the quality of traditional registration and the blockchain. Section 7 presents our main results. First, we determine the conditions under which the advent of the blockchain makes traditional registration unnecessary at the first best. Second, when using traditional registration remains optimal, we study the conditions under which the optimal quality of traditional registration is high or low. Section 8 concludes. Appendix A sets outs the symbols used in the paper. All the proofs are relegated to an appendix B.

2 An Overview of the Law and Economics of Title Registration Systems

There are two main land title systems: the recording system and the Torrens or registration system. For instance, England, Australia, and Germany (Grunbuch) have adopted the Torrens system, while France, Spain, and the United States rely on the recording system. Under the recording system, the government keeps a public record of property transactions (the deeds). In the event of a dispute, the rightful claimant obtains the land, and the possessor may be financially compensated. On the contrary, in the Torrens system, there is a public register of land titles. Furthermore, the government certifies ownership rights at the time of the transaction. "The rightful claimant ordinarily cannot seek restoration of her title but only monetary compensation out of a public fund financed by registration fees" (Miceli et al. 2002, Miceli and Sirmans 2005).

Therefore, a first way to study land title systems is to focus on the way they address the following issue: who receives the land and who receives monetary compensation when a legitimate claim is made? From this perspective, Miceli and Sirmans (1995) study which system is better at promoting efficient exchange of land. They find that if transaction costs are low, both systems promote efficient exchange (a result in line with Coase's theorem). But if transaction costs are high, awarding the land to the true owner (rather than the possessor) may prevent it from ending up with the party who values it most. As the subjective value of land for its possessor is generally above its market price, and as transaction costs are generally high in a register system, this system is consistent in assigning title to the current property holder in the event of conflicting ownership claims.

Miceli et al. (2002) consider yet another issue. Because the Torrens (or registration) and the recording systems apply different principles to resolve conflicting claims to land title, the nature of the system affects both the value of the insurance premium paid by property owners and the land's value. They show that the price of land must be higher with the Torrens system. They also show that when the transaction costs associated with the Torrens system are higher than those of the recording system, the registration system is preferred by owners whose land commands a high value, whereas the recording system is preferred by owners whose land is of low value. In this regard, the distribution of equilibrium land prices matters for the choice of the title system.[4]

Arruñada and Garoupa (2005) present a model of land titling in order to study the different consequences of recording and registration. In contrast with the approach developed by Miceli and his co-authors, land prices are assumed to be fixed. On the other hand, Arruñada and Garoupa (2005) assume that people can buy and sell land without resorting to a titling system (this is the privacy option). They also assume that recording and registration differ in their costs and the degrees of protection offered against forfeiture. Registration is the costlier regime, but also the regime which provides the higher degree of protection. Recording comes at a lower cost, but is characterized by a higher probability of forfeiture. The third option, privacy, provides a low degree of protection, but implies no cost for the buyer and the seller. Which system is preferable depends on the distribution of land values.

Following Arruñada and Garoupa (2005), this paper assumes that a higher degree of protection goes with a higher cost. However, we consider a larger set of titling systems. Furthermore we introduce

[4] Miceli et al. (2011) study how title system characteristics affect developed real-estate assets and more generally different types of assets (see also, Miceli, Sirmans, and Turnbull 1998 and 2000). They show that when the risk of a claim is very high, the registration system is preferred over the recording system, despite higher transaction costs (and conversely).

the blockchain. Therefore we look at three kinds of regimes: privacy, traditional registration, and the blockchain. Because this paper is a generalization of Arruñada and Garoupa (2005)'s contribution, it shares its differences with other significant contributions to the Law-and-Economic analysis of title registration. In particular, we do not assume that the current owner values the land more than the claimant (as in Miceli and Sirmans 1995), nor do we consider the dynamic consequences of land titling systems, as in Miceli and Turnbull (1998, 2000).

3 A General Model of a Land Titling System

Following Arruñada and Garoupa (2005), we assume that title values V are distributed according to a distribution function F over a set $\mathcal{V} = [0, \bar{V}]$, which admits a density function f that takes strictly positive values.

The quality of a land titling system θ is defined by the probability that a landowner will not face forfeiture. Here, the term "quality" refers to the degree of protection, or security, provided by a land-titling system to the landowner. We denote the set of possible qualities for the land titling system by $\Theta = [\underline{\theta}, \bar{\theta}]$, with $0 < \underline{\theta}, \overline{\theta} \leq 1$. Thus the expected wealth of the owner of a piece of land of value V is θV.[5] Let $c(\theta)$ be the function giving the (constant) unit cost per transaction associated with a land-titling system whose quality is θ. Further, assume that $c : \Theta \to \mathbb{R}_+$ is a twice-differentiable strictly convex increasing function and $c(\underline{\theta}) = 0$.[6] That is, the unit cost increases more than proportionally with the quality of the land titling system.

We next consider whether an individual endeavoring to own a piece of land of value V would choose to register his transaction (provided by a public system, either through recording or registration) or rely on a pure private contract, an option we call privacy.[7] We will suppose privacy entails no transaction cost, but provides the lowest possible quality, namely $\underline{\theta}$ (with $\underline{\theta} < \theta$).

3.1 Registration versus privacy

Under registration, the expected value of a title V is θV, net of the registration cost $c(\theta)$. Under privacy, the expected value is $\underline{\theta} V$. Hence, a potential landowner would opt for registration if, and only if, the following condition holds:

$$V \geq \frac{c(\theta)}{\theta - \underline{\theta}} \equiv V_{T/P}(\theta).$$

$V_{T/P}(\theta)$ is the threshold land value above which traditional registration is preferred to privacy, where T stands for registration and P for privacy. In words, for a landowner, it is worth registering his title if the value added in terms of protection $(\theta - \underline{\theta})V$ is large enough to offset the cost $c(\theta)$ of such protection. This is possible only for land of the the highest value V.

The next Lemma establishes that this threshold value increases with the quality of the land titling system.

[5] Arruñada and Garoupa (2005) consider two values of θ corresponding to a recording system and a registration system respectively, and assume that the latter provides a greater degree of protection than the former.

[6] The function $c(\theta) = \frac{\alpha\left(\theta - \underline{\theta}\right)^2}{2}$, with $\alpha > 0$, satisfies these properties.

[7] Privacy can also refer to deregularized transactions, that is, informal exchanges undertaken by people who cannot afford the cost of formal titling procedures. See Galiani and Schargrodsky (2011) and Gutierrez and Moolina (2020) for studies of deregularized transactions.

Lemma 1. *$V_{T/P}(\theta)$ is increasing with θ.*

The interpretation of this result is clear: as the cost of protection $c(\theta)$ rises more than proportionally, the higher the quality θ, the higher the cost, and the lower the share of landowners registering their titles.

The following assumption will hold.

Assumption 1. $V_{T/P}(\overline{\theta}) = \frac{c(\overline{\theta})}{\overline{\theta}-\underline{\theta}} < \overline{V}$.

This assumption means that traditional registration might be chosen over privacy by some landowners even when traditional registration provides the highest degree of protection $\overline{\theta}$, and despite the high cost associated with this degree of protection.

3.2 Optimal quality of the registration system

We have seen above that the set of land values for which potential owners choose privacy is $[0, V_{T/P}(\theta)]$ and the set of land values for which registering is worthwhile is $[V_{T/P}(\theta), \overline{V}]$. Using the distribution of land values, we can compute the aggregate (expected) land values given a land titling system with quality θ. We use this expected value as a *measure* of social welfare. This social welfare function W is written:

$$W(\theta) = \int_0^{V_{T/P}(\theta)} \underline{\theta} V f(V) dV + \int_{V_{T/P}(\theta)}^{\overline{V}} \big[\theta V - c(\theta)\big] f(V) dV.$$

We define an optimal registration system as one associated with a value of θ that maximizes the social welfare function above.[8] The next proposition states a property satisfied by optimal registration systems:

Proposition 1. *When the quality of an optimal registration system θ^* is such that $\theta^* \in (\underline{\theta}, 1)$, it satisfies*

$$\int_{V_{T/P}(\theta^*)}^{\overline{V}} \big[V - c'(\theta^*)\big] f(V) dV = 0. \tag{1}$$

An increase in the quality of traditional registration reduces the proportion of owners who register their land traditionally. This is because the cost of registration increases more than proportionally with θ. Conversely the proportion of privately transacting landowners increases. Consequently, the effect at the margin is null, as the people involved are indifferent between privacy and traditional registration. Second, for those whose land value is high enough (i.e., greater than $V_{T/P}$) for them to register, the expected value of the land increases (by V), as does the cost of registration (by $c'(\theta)$). The condition tells us that the average value has to be equal to the average marginal cost for those who register.

More precisely, the condition above can be interpreted as follows. A marginal change in θ affects the land value for which a would-be landowner is indifferent between relying on a public officer or not. For this agent, the increase in the expected value of the land is offset by a corresponding increase in

[8]Note that we have ignored the fact that the individual benefit of public land titling for a would-be landowner (θV) is probably higher than its social value (since the owner's loss is at least in part gained by someone else in the economy). See Arruñada and Garoupa (2005, p. 716) for discussion.

the cost of the public officer. Therefore a marginal change in θ only affects the agents who strictly prefer to resort to the registration system. For those agents, the *aggregate* increase in the expected values of their land is exactly equal to the increase in the marginal cost of the land titling system (times the size of the set of those agents).

Specifically, according to condition (1), the net private marginal benefit of a registration quality of land V (*i.e.*, $V - c'(\theta)$)) is negative for land values close to the threshold $V_{T/P}(\theta^*)$ and positive for land values close to $\bar{V}$. To understand this property, let us focus on the value of the land $V_{T/P}(\theta^*)$ such that a landowner is indifferent between privacy and registration. The effect of a marginal increase in the quality θ on this agent's payoff reads $V_{T/P}(\theta^*) - c'(\theta^*)$. Given that $V_{T/P}(\theta^*) = \frac{c(\theta^*)}{\theta^* - \underline{\theta}}$, one can see that the net marginal payoff $V_{T/P}(\theta^*) - c'(\theta^*)$ is negative. To wit, what matters to the landowner indifferent between registration and privacy is only the unit cost of registration $c(\theta^*)$ and not the marginal cost $c'(\theta^*)$. Because of this, some of the registered land has values that are lower than the marginal cost of the registration quality. The reverse conclusion is obtained for high-value land. The optimal value of registration quality is such that the negative effects of a marginal increase in this quality on the expected net benefits of low land values are exactly compensated by the positive effects on the expected benefit of high land values.

Performing a static comparative analysis of optimal land titling systems, we get the following result:

Proposition 2. *The quality θ^* of an optimal registration system increases with respect to the highest value of land $\overline{V}$ and the quality of privacy $\underline{\theta}$. When the unit cost function c depends negatively on a productivity parameter a, an increase in a generally has ambiguous effect on θ^*.*

To understand the Proposition, recall that the net private marginal benefit of a registration quality of land V ($V - c'(\theta)$)) is negative for land values close to the threshold $V_{T/P}(\theta^*)$ and positive for land values close to $\bar{V}$. When $\bar{V}$ increases, the marginal social value of the quality θ increases since there are more land values for which the net private marginal benefit of registration is positive. It thus pays to improve the quality of the registration system. While there will be less registered land of low value, there will be more registered land of higher value, and there will be less forfeiture too.

When the degree of protection $\underline{\theta}$ provided by privacy improves, the threshold $V_{T/P}(\theta)$ increases, *ceteris paribus*, as privacy offers a higher degree of protection at no cost. The amount of registered land of low value decreases and thus the social marginal benefit of quality θ increases (since the net marginal private value of these qualities is negative). It is then worthwhile improving the quality of the titling system.

A rise in the productivity parameter a leading to a general decrease in the cost function $c(\theta, a)$ has in general an ambiguous effect on the optimal quality of the titling system. When a increases, the threshold $V_{T/P}(\theta)$ decreases and thus more plots of land are registered. But these plots of land have a negative net marginal private value of the registration quality. In turn, the marginal social value of the quality of the titling system is reduced. This negative effect, however, can be compensated by a general increase in the net marginal private values of registration quality. In that case, it is socially beneficial to increase the quality of the registration system. In the opposite case (which always occurs when the marginal cost of quality increases with a), reducing the quality of the registration system is the best decision.

4 Blockchain land title registration

In this section, we first briefly present the main characteristics of the blockchain technology. We also recall the conditions identified in the burgeoning literature on the law-and-economics of the blockchain under which the blockchain might efficiently replace or supplement more traditional land titling systems. Second, building on our presentation of the blockchain, we introduce this technology in our model. We view it as a cost-efficient mechanism albeit one potentially jeopardized by hacking.

4.1 Blockchain for land transactions

The blockchain technology is full of promises, notably in terms of disintermediation possibilities. This technology, however, has significant limitations, identified by recent literature.

4.1.1 What is the blockchain and how can it be used in land transactions?

The blockchain, a particular type of distributed ledger, is (*a priori*) a secure, transparent information storage and transmission technology. Each node of the blockchain has a copy of the ledger and participates in its updating through a consensus process. By comparison, the actual digital title registries use one central database. In a blockchain, each entry can be seen by all users, ensuring transparency. Once written, the added information is unmovable and time-stamped. The use of cryptographic procedures ensures that any attempt to change the recorded information will be detected. In principle, the blockchain should produce immutable information at a low cost.

Due to its advantages in terms of cost and security and its potential to disintermediate transactions, the blockchain appears to be a challenger to more traditional land title registration systems. In practice, several countries have introduced the blockchain technology in land registration. Blockchain applications can be found in both developing and advanced economies, such as Brazil, Georgia, Ghana, and Sweden. When adopting the blockchain, the public authorities in charge of land registration usually collaborate with private partners and seek to reduce delays and costs, or to increase trust. For instance, Sweden has adopted the blockchain technology to pre-register deeds in order to reduce delays associated with real-estate transactions (see Proskurova and Dörry 2018). This adoption is the result of cooperation between the Swedish land registration authority and private firms. Georgia has "anchored" (i.e. connected) its digital land registry to a blockchain, through cooperation between the National Agency of Public Registry (NAPR) and a bitcoin company called Bitfury. These two practical examples highlight that there are many ways to use the blockchain in land registration, as described in von Wangenheim (2020).[9]

Note also that there are three main types of blockchains: public, permissioned and private blockchains. With a public blockchain, anybody can access the ledger and validate the insertion of information. In a public blockchain, the system is completely disintermediated; only the parties to the transactions and the anonymous miners can validate blocks. In a permissioned blockchain, only a limited number of actors (the permissioned) may validate the transaction, and with a private blockchain, only one actor is in charge. Consortium blockchains differ slightly from permissioned blockchains. A consortium is a group of entities or individuals who wish to collaborate on a project. A consortium

[9] More precisely one can anchor digitized land registers in the blockchain, digitize deed registers in a blockchain, or tokenize property rights in the blockchain. Von Wangenheim (2020) explores in detail the advantages and limitations of these different possibilities.

blockchain consists in a generalized data-sharing platform for the members of the consortium who are all authorized to enter data.

4.1.2 Some limits to the use of the blockchain in land transaction registration

The blockchain, however, encounters numerous limits, both legal and technical (Arruñada 2019; Barbieri and Gassen 2017; von Wangenheim 2020). These limits might explain why the number of real-estate related applications of the blockchain remains limited in practice.

A significant obstacle is the control of the data submitted to the blockchain, which is a key function of a land titling system. For example, title registration goes with an extensive *ex ante* examination of rights. No such examination is expected in a public blockchain. A (public) blockchain induces risk associated with the lack of independent verification, lack of disclosure of the participant's identity, and the risk of irregularities resulting therefrom. Some argue that the cost of property transactions may even increase rather than decrease due to the risk induced by the blockchain, as one would need specialized intermediaries, extended due diligence exercise, and title insurances.

In addition, Kaczorowska (2019) draws attention to a risk of double selling due to a technical characteristic of the blockchain system, as "there is no guarantee that the order in which transactions are received by the nodes is the same order in which new blocks are added."

Furthermore, both the decentralization of power and the democratic functioning of consensus are jeopardized by the existence of "mining farms" which concentrate mining power (a power necessary to ensure transaction security). Actually, as was mentioned above, a blockchain can be hijacked if one acquires enough computing power. In the worst case, as was mentioned above, there may be a manipulation of priority in order to obtain ownership. Thus the blockchain does not eliminate the risk of capture by a group of miners, or by a central authority.

Arruñada (2019) also highlights the difficulties in switching from one register to another and the limits of a peer-to-peer blockchain where individuals become custodians of their cryptographic keys as well as of the legal integrity of their rights. Individuals can hardly do without intermediaries in this hypothetically peer-to-peer system.

Moreover, in practice, land registration systems differ widely from one country to another. Because of legal diversity across jurisdictions, the benefit of using the blockchain in land registration is country-dependent (Arruñada 2019; Kaczorowska 2019; von Wangenheim 2020).

In this connection, recall that proof of ownership may be produced either by deed registrations (as in France, Italy, the Netherlands, and in most US counties), or by title registration (as in Australia, England and Wales, Germany, Poland, Spain, and Sweden). Moreover, the examination of the rights may be made before or after registration. That is why some authors argue that the legal problems involved in the use of a public blockchain are less exacerbated in a deed-based system as opposed to a title-based system (Arruñada 2019; Kaczorowska 2019; von Wangenheim 2020). Indeed, title registration requires an extensive check of the correctness of the basis for entry, while the role of registrars seems to be limited more to formalities under deed registration. Yet, even with deed registration, there are significant limits to the use of the blockchain. Arruñada (*ibid*) makes clear that this replacement is unlikely to be total as contracts are necessarily incomplete and thus require a certain legal expertise.

As a matter of fact, in many civil law countries notaries advise, authenticate, and ascertain the legal capacities of the parties to a contract (Arruñada 1996). In other countries, lawyers represent the interests of their client during the land transaction. These activities cannot be fulfilled by the blockchain alone.

Therefore there seems to be a consensus with regard to the inadequacy of the blockchain in its pure form (meaning, public blockchain) to land titling registration. It does not seem plausible to design a public blockchain to replace a register. Potential challengers of traditional registration systems are private or consortium blockchains that allow governance to be set up and a leader to be identified (Legeais 2019). For example, in France notaries are invested with a *regalian* prerogative that cannot be replaced, and they are duty-bound to advise their clients. The national council of notaries intends to use the blockchain for copies of authentic instruments, through a consortium blockchain. A proof of concept could be set up for electronic authentic copies and enforceable electronic copies. The objective is to trace transactions among several actors.

To sum up, the blockchain appears to be a potential challenger to the traditional land registration system, with its proper limits. Yet legal frameworks differ from one to another with respect to their degrees of protection and unit costs. This diversity implies that the blockchain will more or less easily be able to “fit in”, adapt, and add value. In the remaining part of the section we rely on the above presentation to introduce the blockchain in our model of land titling.

4.2 Modeling blockchain registration

We interpret the blockchain as being a new registering technology that produces a certain level of quality $(1-h)\bar{\theta}$ at a unit cost per transaction of $\tilde{a}$. The blockchain is free of the traditional threats to other registering technology, like fire, human error, and so on. Yet as was seen above, the blockchain also entails its own risk, like hacking or wrongful entry. To account for such risks, we let h denote the probability of hacking ($0 < h < 1$). This amounts to assuming that blockchain cannot produce the best degree of protection potentially reached by traditional registration, as $h > 0$. Then, the expected gain of a transaction using the blockchain for land of value V is: $(1-h)\bar{\theta}V - \tilde{a}$. We make the following assumption.

Assumption 2.

1. $\tilde{a} < c\big((1-h)\bar{\theta}\big)$.
2. $\underline{\theta} < (1-h)\bar{\theta}$.

The first part of assumption 2 encapsulates the idea that the blockchain is a potential cost-saving mechanism. That is, for a quality of land titling equal to $(1-h)\bar{\theta}$, it is costlier to use the traditional registration system rather than the blockchain. If the inequality does not hold, the blockchain cannot develop. The second part states that the expected degree of protection provided by the blockchain is higher than that of privacy. Again, short of this condition, the blockchain cannot develop.

5 Individual Choices Between Blockchain, Traditional Registration, and Privacy

In this section, we study how a potential owner of a land title of value V, where V lies within $[0, \bar{V}]$, chooses between blockchain registration, traditional registration, and privacy.[10] We begin with the choice between blockchain and traditional registration. We contrast two cases. In the first one, as opposed to the second one, traditional registration offers more security than the blockchain. Then, we briefly consider the choice between the blockchain and privacy. We rely on the study of the registration choices to determine the set of land values associated with each of these choices. The determination of these choices will prove instrumental in the study of the optimal titling system when blockchain registration becomes available.

5.1 Blockchain vs traditional registration

Recall that the expected value of the land of value V under traditional registration is $\theta V - c(\theta)$, versus $(1-h)\bar{\theta}V - \tilde{a}$ with blockchain registration. Relying on traditional registration instead of blockchain registration is the best choice whenever:

$$\theta V - c(\theta) > (1-h)\bar{\theta}V - \tilde{a},$$

or,

$$V[\theta - (1-h)\bar{\theta}] > c(\theta) - \tilde{a}.$$

In order to discuss the choice between registering property in the blockchain or through traditional registration, we analyze the case separately where traditional registration provides a quality of protection at least as high as blockchain registration, and the case where traditional registration provides a lower quality of protection.

5.1.1 Traditional registration offers more security than the blockchain.

Let us first consider the case where traditional registration offers a quality of protection greater than the blockchain: $\theta > (1-h)\bar{\theta}$. People prefer traditional registration if

$$V > \frac{c(\theta) - \tilde{a}}{\theta - (1-h)\bar{\theta}} \equiv V_{T/B}(\theta). \tag{2}$$

$V_{T/B}(\theta)$ is the land value threshold above which traditional registration is preferred to the blockchain, where T stands for traditional and B for blockchain. Intuitively, it is worth using traditional registration (which provides better protection at a higher cost) when the difference between the expected values of the title under both systems $[\theta - (1-h)\bar{\theta}]V$ is greater than the difference in costs $c(\theta) - \tilde{a}$. When V is high, it may be worth using traditional registration to protect the property. On the contrary, the gain obtained by using traditional registration (which is a fraction of V) is negligible compared to the cost of relying on traditional registration when V is low.

[10] In practice, blockchain registration does not refer necessarily to a public blockchain. More generally, by "blockchain registration" we refer to title registration assisted by the blockchain, which may be a consortium blockchain.

Notice that some individuals may prefer traditional registration over blockchain only if the following necessary condition holds:

$$\bar{V} > V_{T/B}(\theta). \tag{3}$$

In words, the highest possible land value must exceed the threshold value $V_{T/B}(\theta)$ in order to have some landowners who might prefer to register their property through traditional registration than through the blockchain. This condition is less likely to be satisfied the closer θ is to the quality provided by the blockchain $(1-h)\bar{\theta}$. That is because, relying on traditional registration produces a difference in costs that exceeds the difference in expected values.

The next Proposition states the conditions under which some people would prefer traditional over blockchain registration when traditional registration offers a quality of protection at least as high as the blockchain ($\theta \in](1-h)\overline{\theta}, \overline{\theta}]$). As we shall see, it turns out that these conditions depend largely on how the threshold $V_{T/B}(\theta)$ changes in the vicinity of the highest quality $\bar{\theta}$ (formally, the sign of $V'_{T/B}(\bar{\theta})$). The Proposition is illustrated with five figures.

Proposition 3. *Consider a quality θ of the registration system, with $\theta \in](1-h)\overline{\theta}, \overline{\theta}]$*

1. *Suppose that $V'_{T/B}(\overline{\theta}) \leq 0$.*
   (a) *If $\overline{V} \leq V_{T/B}(\overline{\theta})$, all landowners prefer blockchain to traditional registration. (Figure 1)*
   (b) *If $V_{T/B}(\overline{\theta}) < \overline{V}$, then there exists $\theta' \in](1-h)\bar{\theta}, \bar{\theta}[$ such that $V_{T/B}(\theta') = \overline{V}$ and such that for all $\theta > \theta'$, $\bar{V} > V_{T/B}(\theta)$. All owners of land whose value lies within $[V_{T/B}(\theta), \overline{V}]$ prefer traditional registration to the blockchain when $\theta > \theta'$. (Figure 2)*
2. *Suppose $V'_{T/B}(\overline{\theta}) > 0$.*
   (a) *If $\bar{V} \leq \min_\theta V_{T/B}(\theta)$, all landowners prefer blockchain to traditional registration. (Figure 3)*
   (b) *If $\overline{V} < V_{T/B}(\overline{\theta})$ and $\bar{V} > \min_\theta V_{T/B}(\theta)$, there is an interval $[\theta'_1, \theta'_2]$ within $[(1-h)\overline{\theta}, \overline{\theta}]$ such that $V_{T/B}(\theta'_1) = V_{T/B}(\theta'_2) = \overline{V}$, and such that for all θ within this interval, $\bar{V} > V_{T/B}(\theta)$. All owners of land whose value lies within $[V_{T/B}(\theta), \overline{V}]$ prefer traditional registration to the blockchain when $\theta \in [\theta'_1, \theta'_2]$. (Figure 4)*
   (c) *If $V_{T/B}(\overline{\theta}) < \overline{V}$, there exists $\theta' \in](1-h)\bar{\theta}, \bar{\theta}[$ such that $V_{T/B}(\theta') = \overline{V}$, and for all $\theta > \theta'$, $\bar{V} > V_{T/B}(\theta)$. All owners whose land value V lies within $[V_{T/B}(\theta), \overline{V}]$ prefer traditional registration to the blockchain when $\theta > \theta'$. (Figure 5)*

The Proposition above shows that traditional registration can be discarded even when providing a higher quality than the blockchain. Indeed, it is more expensive to register the land through traditional registration than through the blockchain. Therefore, choosing traditional registration is advantageous only if the increase in expected value exceeds the increase in cost. This is possible only when the necessary condition in (3) is satisfied, that is when the maximal value of land $\bar{V}$ exceeds the threshold value $V_{T/B}$ (which makes traditional registration worthwhile). Proposition (3) depicts the different cases where this might happen, which are depicted in figures 1 to 5.

Figures 1 and 3 illustrate the cases where traditional registration is never preferred to the blockchain, as condition (3) is not satisfied. In these two graphs, we see that the threshold $V_{T/B}$ decreases when the quality θ of the traditional registration system is slightly higher than $(1-h)\bar{\theta}$. This is intuitive: as the quality of the traditional registration system increases with respect to the quality of the blockchain, it is less necessary to own land of great value to choose the former. Indeed, while the unit cost is always higher with traditional registration, the (expected) benefit of this system increases with the value of the land. Only with land of great value can the increase in quality associated with traditional registration be beneficial for a landowner. Now, it is not always true that the threshold value $V_{T/B}(\theta)$ invariably decreases. And even if that is the case, the minimum value of the threshold may well be greater than $\bar{V}$. In such cases (depicted in figures 1 and 3), no owner can actually benefit from the traditional system. The increase in quality remains too small for the traditional registration system to be worth it.

By contrast, figures 2 and 5 depict the case where traditional registration might be favored. This happens only if the degree of protection provided by traditional registration is sufficiently higher than that of the blockchain ($\theta > \theta'$). Figure 4 illustrates an alternative case, where the degree of protection takes an intermediate value ($\theta \in [\theta'_1, \theta'_2]$).

5.1.2 Traditional registration provides less security than the blockchain.

Let us now consider the second case where traditional registration provides a lower quality of protection than the blockchain: $\underline{\theta} \leq \theta < (1-h)\bar{\theta}$. That this assumption is empirically founded is, of course, disputable. Yet there is no denying that it is relevant when the traditional registration system is quite recent, *e.g.*, as in some Eastern European countries.

Let us furthermore denote by $\hat{\theta}$ the quality level such that the cost of traditional registration equals the cost of registering in the blockchain, $c(\hat{\theta}) = \tilde{a}$.[11]

Then we can see that blockchain registration is always preferred to traditional registration when $\theta \in [\hat{\theta}, (1-h)\bar{\theta}]$. Indeed, by assumption, we know that: $\tilde{a} \leq c\left((1-h)\bar{\theta}\right)$. Then when $\theta \in [\hat{\theta}, (1-h)\bar{\theta}[$, traditional registration provides a lower quality than that of the blockchain while being costlier ($c(\theta) < \tilde{a}$), and therefore *no one* prefers traditional registration to blockchain registration.

The previous analysis suggests that traditional registration could be chosen if its quality is relatively low, so that its cost becomes significantly lower than that of the blockchain. Indeed, whenever $\theta \in [\underline{\theta}, \hat{\theta}[$, so that $c(\theta) < \tilde{a}$, the individuals who prefer traditional registration to the blockchain are those for whom the land value V is such that:

$$V < \frac{\tilde{a} - c(\theta)}{(1-h)\bar{\theta} - \theta} \equiv V_{T/B}(\theta). \tag{4}$$

The threshold above is always well-defined. Indeed, there is always a landowner who prefers traditional registration because the savings allowed by traditional registration exceed the loss in expected value of the title and title values may be arbitrarily small.

[11] As the unit cost function $c(.)$ is continuous and monotonic, there exists a unique value $\hat{\theta}$ such that $c(\hat{\theta}) = \tilde{a}$.

5.2 Blockchain versus privacy

Let us now focus on the choice between registering the land title in the blockchain and not registering it. A potential landowner would rely on blockchain registration rather than privacy if, and only if the following condition holds:

$$V > \frac{\tilde{a}}{\bar{\theta}(1-h)-\underline{\theta}} \equiv V_{B/P}$$

$V_{B/P}$ is the threshold land value above which blockchain registration is preferred to privacy, where B stands for blockchain and P for privacy. Again, blockchain registration is favored when the gain in the land title expected value $[\bar{\theta}(1-h)-\underline{\theta}]V$ exceeds the cost of using the blockchain $\tilde{a}$.

We shall suppose that some people would always choose the blockchain rather than privacy. Thus, the following assumption will hold:

Assumption 3. $V_{B/P} = \frac{\tilde{a}}{\bar{\theta}(1-h)-\underline{\theta}} < \overline{V}$.

The interpretation of this assumption is straightforward. The value added by the blockchain compared to privacy for the most valuable land $\overline{V}$ must exceed the cost of blockchain registration $\tilde{a}$.

5.3 Sorting the threshold values $V_{T/B}(\theta), V_{T/P}(\theta)$, and $V_{B/P}$

Recall that an individual is indifferent between not registering and relying on traditional registration when the value of his property is:

$$V_{T/P}(\theta) = \frac{c(\theta)}{\theta-\underline{\theta}}.$$

Let us first consider the case where traditional registration offers a quality of protection higher than the blockchain ($\theta > (1-h)\bar{\theta}$). We shall hereafter make the following assumption.

Assumption 4. $V_{T/P}(\theta) \leq V_{T/B}(\theta), \forall\theta > (1-h)\bar{\theta}$.

This assumption, which can be rewritten $\frac{c(\theta)}{\theta-\underline{\theta}} \leq \frac{c(\theta)-\tilde{a}}{\theta-(1-h)\bar{\theta}}$, is certainly satisfied for θ slightly larger than $(1-h)\bar{\theta}$. This means that, whenever people prefer traditional registration to the blockchain, they also prefer it to privacy. This is a relevant assumption that greatly simplifies the analysis.

Assumption 4 also implies the following result:

Lemma 2. *Suppose that assumption 4 holds. Then, for all* θ *such that* $(1-h)\bar{\theta} \leq \theta$,

$$V_{B/P} \leq V_{T/B}(\theta). \tag{5}$$

To grasp the meaning of this Lemma, assume that its statement is false. That would imply that for a certain quality of the registration system, no one would find it advantageous to use the blockchain. Yet by assumption, $V_{T/P} \leq V_{T/B}$. Pick a value of land located in between $V_{T/P}$ and $V_{T/B}$. By definition of these thresholds, the owner of this land would either be indifferent between traditional registration, blockchain registration, and privacy, or he would prefer blockchain registration to traditional registration, and the latter to privacy. In this latter case, the owner of the land would thus prefer blockchain registration to privacy. But being indifferent between blockchain and privacy, or preferring blockchain to privacy is inconsistent with the assertion above that no one should choose blockchain registration.

Let us finally consider the case where traditional registration offers a lower quality of protection than the blockchain ($\theta < (1-h)\bar{\theta}$). The following Proposition gives a condition under which the different thresholds are all equal.

Proposition 4. *There is a quality $\check{\theta}$ of the traditional registration system such that $\underline{\theta} < \check{\theta} < (1-h)\bar{\theta}$ and*

$$V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta}) = V_{B/P} \tag{6}$$

if, and only if, $c'(\underline{\theta}) < \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}} \equiv V_{B/P}$.

To understand this Proposition, first observe that when the thresholds $V_{T/P}(\theta)$ and $V_{T/B}(\theta)$ are equal, then they are also equal to the threshold $V_{B/P}$. Indeed, whenever for a certain land value its owner is indifferent between traditional registration and privacy, as well as between traditional registration and blockchain registration, then this owner must be indifferent between the latter registration and privacy. Now, under the condition stated in the Proposition above, when the quality of traditional registration is close to its minimum value $\underline{\theta}$, one has $V_{T/B} > V_{T/P}$. That is because, since the quality provided by blockchain registration is greater than with traditional registration, and since the cost of the former is greater than the latter, to benefit from blockchain registration the value of land must be high enough. But as the quality of traditional registration increases, so does the threshold $T_{T/P}(\theta)$. Yet the threshold $V_{T/B}(\theta)$ must eventually go to zero as θ goes to $\hat{\theta}$. In plain English, when the quality of traditional registration improves, its cost rises more than proportionally and as the conditions provided by blockchain registration are unchanged, there are ever fewer land values for which traditional registration is advantageous. But then, there must be a quality of traditional registration which is such that the thresholds $V_{B/P}(\theta)$ and $V_{T/B}(\theta)$ are equal. Figure 6 illustrates the preceding argument.

6 Social Welfare and the Blockchain

Now we look at how social welfare, measured as the average of the expected land values, is modified by the advent of the blockchain. Recall that, before the advent of the blockchain, the social welfare function is written:

$$W(\theta) = \int_0^{V_{T/P}(\theta)} \underline{\theta} V f(V) dV + \int_{V_{T/P}(\theta)}^{\overline{V}} \big[\theta V - c(\theta)\big] f(V) dV.$$

The introduction of the blockchain strongly affects social welfare as landowners can now choose between three registration modes. As seen in the preceding section, it is natural to distinguish three different sets of quality for the traditional registration system. This quality may be higher than that of the blockchain. It may be lower than that of the blockchain, and its unit cost greater than the blockchain's (in that case, traditional registration is of no use). Finally, the quality and the unit costs of traditional registration may both be lower than their counterparts with the blockchain. We shall address the three cases in turn. In each case, we will look for the quality of the registration system that maximizes social welfare. In the next section, we will look for the quality that maximizes social welfare globally and not conditionally on belonging to one of the three sets.

6.1 Social welfare when the quality of traditional registration is high

In this subsection, we assume that the quality of traditional registration is higher than that of the blockchain ($(1-h)\bar{\theta} \leq \theta$).

Our first result refers to the case where no landowner ever uses traditional registration, whatever the quality of traditional registration.

Proposition 5. *Assume that either one of the following assumptions holds.*

1. $V'_{T/B}(\bar{\theta}) \leq 0$, $\bar{V} \leq V_{T/B}(\bar{\theta})$ *(figure 1),*
2. $0 < V'_{T/B}(\bar{\theta})$, *and the minimum of* $V_{T/B}$ *is achieved at a point* $\underline{\underline{\theta}}$ *such that* $\bar{V} \leq V_{T/B}(\underline{\underline{\theta}})$ *(figure 3).*

Then, for all θ *in* $](1-h)\bar{\theta}, \bar{\theta}]$, *no one uses the traditional registration and the social welfare function takes the constant value*

$$\mathcal{W}_B(\theta) = \int_0^{V_{B/P}} \underline{\theta} V f(V) dV + \int_{V_{B/P}}^{\bar{V}} [(1-h)V - \tilde{a}] f(V) dV. \quad (7)$$

In the cases referred to in the Proposition above, landowners do not use traditional registration despite the high degree of protection provided. Indeed, using traditional registration would imply bearing a cost increase that can never be compensated by a sufficiently large expected gain in holding land, as the minimum land value at which the increase in quality is worthwhile is above the maximum value of land in the economy considered. Hence, the landowners resort to either privacy or blockchain registration. This choice of registration is triggered by a land value being above the threshold $V_{B/P}$. This is reflected in the form taken by social welfare, with the two integrals, the first corresponding to the expected values of land not registered, and the second one pertaining to the values of land registered in the blockchain.

We now focus on cases where, by contrast with the Proposition above, the set of qualities for which some landowners rely on traditional registration is never empty. The next Proposition shows that, in these cases, the optimal quality of traditional registration is larger than a certain amount θ' (see figures 2, 4, and 5 for the different cases that may arise when the assumptions of the Proposition 5 do not hold).

Proposition 6.

1. *Assume that* $V'_{T/B}(\bar{\theta}) \leq 0$, $V_{T/B}(\bar{\theta}) < \bar{V}$. *Then social welfare is maximized by choosing a quality* θ *no lower than* θ', *where* θ' *is such that* $V_{T/B}(\theta') = \bar{V}$ *(figure 2).*
2. $V'_{T/B}(\bar{\theta}) \geq 0$, $\bar{V} < V_{T/B}(\bar{\theta})$, *and* $\bar{V} > \min_\theta V_{T/B}(\theta)$. *Then social welfare is maximized by choosing a quality* θ *in* $[\theta'_1, \theta'_2]$, *where* $V_{T/B}(\theta'_1) = V_{T/B}(\theta'_2) = \overline{V}$, $V'_{T/B}(\theta'_1) < 0$, $V'_{T/B}(\theta'_2) > 0$ *(figure 4).*
3. *Assume that* $V'_{T/B}(\bar{\theta}) \geq 0$, $V_{T/B}(\bar{\theta}) < \overline{V}$. *Then, social welfare is maximized by choosing a quality* θ *in* $[\theta', \overline{\theta}]$, *where* $V_{T/B}(\theta') = \overline{V}$ *(figure 5).*

In these three cases the social welfare function is written as follows

$$\mathcal{W}_B(\theta) = \int_0^{V_{B/P}} \underline{\theta} V f(V) dV + \int_{V_{B/P}}^{V_{T/B}(\theta)} \big((1-h)\bar{\theta}V - \tilde{a}\big) f(V) V + \int_{V_{T/B}(\theta)}^{\bar{V}} \big(\theta V - c(\theta)\big) f(V) dV. \quad (8)$$

As shown in the equation above, social welfare, in addition to the values of the plots that are not registered and the values of those that are registered on the blockchain, also includes the values of the plots that are registered traditionally. In all three cases, there is an optimal value for the quality of traditional registration. Except in the second case, the optimal value of the qualities belongs to the upper tail of the land value distribution. The optimal value of the registration quality balances the marginal increase in the average expected quality of land registered traditionally (net of the marginal (unit) cost), and the reduction in the size of the set of these plots (this trade-off is akin to that in section 3). In the second case, however, the optimal value of the quality of traditional registration is located differently. In this particular scenario, the threshold $V_{T/B}(\theta)$ lies above the maximum land value $\bar{V}$ either when the quality of traditional registration is close to that of the blockchain ($(1-h)\bar{h}$) or when it is close to its upper-bound $\bar{\theta}$. In both cases, that is because the unit cost of quality increases more than the expected benefit provided by registration. But there are some quality values that provide advantageous protection, neither too close to the blockchain's, nor the highest possible value $\bar{\theta}$. It is among these values that the optimal quality is located.

6.2 Social welfare when the quality of traditional registration is intermediate

In this subsection, we assume that the quality provided by the registration system is intermediate, namely that θ is in $[\hat{\theta}, (1-h)\bar{\theta}]$.[12] Under this assumption, not only is the quality provided by traditional registration lower than that provided by blockchain registration, it is also more costly. Therefore, traditional registration can never be preferred. The next result follows at once.

Proposition 7. *When* θ *belongs to* $[\hat{\theta}, (1-h)\bar{\theta}]$, *no one uses the traditional registration system. People use either the blockchain or resort to privacy, and the social welfare function takes the following constant value*

$$\mathcal{W}_B(\theta) = \int_0^{V_{B/P}} \underline{\theta} V f(V) dV + \int_{V_{B/P}}^{\bar{V}} [(1-h)V - \tilde{a}] f(V) dV, \quad (9)$$

[12] Recall that $\hat{\theta}$ is the quality such that $V_{T/B}(\hat{\theta}) = 0$ (equivalently $c(\hat{\theta}) = \tilde{a}$).

with $V_{B/P} = \frac{\tilde{a}}{\bar{\theta}(1-h)-\underline{\theta}}$.

Notice that social welfare is completely independent of traditional registration quality. This illustrates the fact that blockchain registration, whatever its shortcomings, may completely replace traditional registration when the latter is of intermediate quality. This replacement is merely the consequence of providing people with a larger set of choices (that is, the blockchain is not a constrained choice).

6.3 Social welfare when the quality of traditional registration is low

Let us now consider the case where the degree of protection θ provided by the traditional registration systems is lower than $\hat{\theta}$, where $\hat{\theta}$ is such that $V_{T/B}(\hat{\theta}) = 0$. Our first result refers to a case where, whatever its quality, traditional registration is, once again, of no use.

Proposition 8. *When* $\theta \leq \hat{\theta}$ *and* $V'_{T/B}(\underline{\theta}) < 0$*, or equivalently* $c'(\underline{\theta}) > \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}} \equiv V_{B/P}$*, people either use the blockchain or resort to privacy and the social welfare function takes the following constant value*

$$\mathcal{W}_B(\theta) = \int_0^{V_{B/P}} \underline{\theta} V f(V) dV + \int_{V_{B/P}}^{\bar{V}} [(1-h)V - \tilde{a}] f(V) dV. \tag{10}$$

Under the assumptions of the Proposition above, whenever a landowner prefers traditional registration to the blockchain (namely when $V \leq V_{T/B}(\theta)$) he also prefers privacy to the former. Thus, landowners actually choose between blockchain registration and privacy. Notice that, once again, social welfare does not depend on the quality of traditional registration. The reason for this is exactly the same as was seen when registration quality takes intermediate values: as no one uses traditional registration, its quality does not affect the (net) mean of the land values.

We now tackle settings where traditional registration will be used by some landowners, for some values of θ. To do this, recall that we have defined $\check{\theta}$ as the quality such that $V_{T/B}(\check{\theta}) = V_{T/P}(\check{\theta}) = V_{B/P}$.

Proposition 9. *Assume that* $c'(\underline{\theta}) \leq \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}} \equiv V_{B/P}$*. Then, the optimal quality of the traditional registration system is not greater than* $\check{\theta}$ *and the value of social welfare is*

$$W_B(\theta) = \int_0^{V_{T/P}(\theta)} \underline{\theta} V f(V) dV + \int_{V_{T/P}(\theta)}^{V_{T/B}(\theta)} \big(\theta V - c(\theta)\big) f(V) dV$$
$$+ \int_{V_{T/B}(\theta)}^{\bar{V}} \big((1-h)\bar{\theta}V - \tilde{a}\big) f(V) dV \tag{11}$$

A glance at figure 6 reveals that for some values of θ, some landowners will find it worthwhile to use traditional registration (the land values of these owners lie in between $V_{T/P}$ and $V_{T/B}(\theta)$). To understand why social welfare is greater when the traditional registration quality is lower than $\check{\theta}$, observe that a shift from a high quality (that is, larger than $\check{\theta}$) to low quality (that is, less than $\check{\theta}$) changes registration behaviors. There are now landowners who rely on traditional registration. Since they could have chosen blockchain registration or privacy, they are better off (that is, the expected net value of land of these owners increases). Actually, if one owner changes his behavior, his new decision must be welfare improving. The individual welfare of all the other owners, whose decisions are unchanged, are not affected by the change in the quality of the traditional registration system. Since the set of owners who rely on the traditional registration system is never negligible, the mean net value of the plots of land is greater when the quality of the traditional registration system is lower than $\check{\theta}$. Therefore, the best quality cannot be greater than this value.

7 Optimal quality of the traditional registration system with a blockchain

In the preceding section, we studied where the best quality of the traditional registration system is located, depending on the case: high quality (θ is above $(1-h)\bar{\theta}$), intermediate quality (θ is in between $\hat{\theta}$ and $(1-h)\bar{\theta}$), and low quality (θ is lower than $\hat{\theta}$). It remains for us to investigate where the optimal quality (assuming it is unique) is located. But we must also be mindful that it may be optimal to get rid of traditional registration, in which case, the choice of the optimal quality of the latter is simply meaningless. It is important to identify these cases since they seemingly imply a complete change in the registration system, with massive consequences for professionals acting in the field of land registration, such as notaries and lawyers. Because of their potential impact, we first focus on these cases. Secondly, we concentrate on the determining of the optimal quality of the traditional registration system when it remains socially profitable to keep it.

7.1 Blockchain ousts traditional registration

The next proposition sums up the conditions under which it is optimal to replace traditional registration with the blockchain.

Proposition 10. *Assume that $V'_{T/B}(\underline{\theta}) \leq 0$ or equivalently $V_{B/P} \leq c'(\underline{\theta})$, and either:*

1. *$V'_{T/B}(\bar{\theta}) \leq 0$, and $\bar{V} \leq V_{T/B}(\bar{\theta})$ (Figure 1), or*
2. *$V'_{T/B}(\bar{\theta}) > 0$, and $\min_\theta V_{T/B}(\theta) \geq \bar{V}$ (Figure 3).*

Then, whatever its quality, people will never use the traditional registration system. They will either use blockchain or resort to privacy.

In that case the social welfare function is as follows

$$\mathcal{W}_B(\theta) = \int_0^{V_{B/P}} \underline{\theta} V f(V)dV + \int_{V_{B/P}}^{\bar{V}} [(1-h)V - \tilde{a}]f(V)dV. \tag{12}$$

The statement of the proposition above summarizes what has been seen in Propositions 5, 7, and 8. We know that when the quality of traditional registration is intermediate, it is always socially optimal to register with the blockchain only. But when the quality cost of the traditional registration system grows more quickly than its benefit, whether the quality level is low or high, the blockchain is also the best choice from the social viewpoint. Therefore, while blockchain registration suffers from defects (that is, it is not free, and it is less secure than one might hope), it may still be the best alternative registration system.

7.2 Traditional registration remains

When do we need traditional registration to maximize social welfare? If the conditions of Proposition 10 are met, we know that no one ever relies on traditional registration. But when these conditions are not satisfied, there may be some qualities θ for which people will use the traditional registration system. However, we have to check whether there are any such people. We show that optimal quality is always such that some people rely on traditional registration. Moreover, it is also always optimal to rely on the blockchain (because some people will use it).

The analysis of the optimal quality of the traditional system leads us to identify two different situations. In the first situation, it is possible to locate the optimal quality of the traditional registration system.[13] The next two Propositions deal with two polar cases. Proposition 11 states conditions under which the optimal quality of the traditional registration system is low. By contrast, Proposition 12 gives some sufficient conditions under which the optimal quality of the traditional registration system is high.

Proposition 11. *Assume that*

1. $V'_{T/B}(\bar{\theta}) \leq 0$, $\bar{V} \leq V_{T/B}(\bar{\theta})$ *(Figure 1),*
2. $0 < V'_{T/B}(\bar{\theta})$, *and the minimum of* $V_{T/B}$ *is achieved at a point* $\underline{\underline{\theta}}$ *such that* $\bar{V} \leq V_{T/B}(\underline{\underline{\theta}})$ *(Figure 3).*

Then if $c'(\underline{\theta}) < V_{B/P}$*, it is optimal to use traditional registration with low quality and the optimal quality is lower than* $\check{\theta}$*.*

Proposition 12. *Assume either that*

1. $V'_{T/B}(\bar{\theta}) \leq 0$, $V_{T/B}(\bar{\theta}) < \bar{V}$ *(Figure 2).*
2. $V'_{T/B}(\bar{\theta}) \geq 0$ *and* $\bar{V} < V_{T/B}(\bar{\theta})$ *and* $V_{T/B}(\underline{\underline{\theta}}) < \bar{V}$*, where* $\underline{\underline{\theta}}$ *is the value of* θ *that minimizes* $V_{T/B}(\theta)$ *(Figure 4).*
3. $V'_{T/B}(\bar{\theta}) \geq 0$ *and* $\bar{V} > V_{T/B}(\bar{\theta})$ *and* $V_{T/B}(\underline{\underline{\theta}}) < \bar{V}$*, where* $\underline{\underline{\theta}}$ *is the value of* θ *that minimizes* $V_{T/B}(\theta)$ *(Figure 5).*

Then if $c'(\underline{\theta}) > V_{B/P}$*, it is optimal to use traditional registration with high quality and its optimal quality is greater than* θ'*.*

Under Proposition 11 and Proposition 12 the location of the optimal quality is clear. Under the assumptions of Proposition 11, it is optimal to keep traditional registration, and the degree of protection provided by traditional registration should be low (and cheap) to meet the registration needs of the individuals with land of low value. Individuals never choose traditional over blockchain registration if the degree of quality of traditional registration is high. On the contrary, under the assumptions of Proposition (12), while it is optimal to maintain traditional registration, the degree of protection provided by traditional registration is high (but costly). Traditional rather than blockchain registration increases the surplus of the individuals with the land of the highest value. Notice that in all cases, some landowners will rely on the blockchain.

[13] To determine this quality precisely, however, one needs to specify the unit cost function.

It turns out, however, that the location of the optimal quality of traditional registration is indeterminate. This is stated by the following result.

Proposition 13. *Assume that one of the following assumptions holds.*

1. $V'_{T/B}(\bar{\theta}) \leq 0$, $V_{T/B}(\bar{\theta}) < \bar{V}$ *(Figure 1).*
2. $V'_{T/B}(\bar{\theta}) \geq 0$ *and* $\bar{V} < V_{T/B}(\bar{\theta})$ *and* $V_{T/B}(\underline{\underline{\theta}}) < \bar{V}$, *where* $\underline{\underline{\theta}}$ *is the value of* θ *that minimizes* $V_{T/B}(\theta)$ *(Figure 4).*
3. $V'_{T/B}(\bar{\theta}) \geq 0$ *and* $\bar{V} > V_{T/B}(\bar{\theta})$ *and* $V_{T/B}(\underline{\underline{\theta}}) < \bar{V}$, *where* $\underline{\underline{\theta}}$ *is the value of* θ *that minimizes* $V_{T/B}(\theta)$ *(Figure 5).*

Then if $c'(\underline{\theta}) \leq V_{B/P}$, *it is optimal to use traditional registration and its quality is either lower than* $\check{\theta}$ *or higher than* θ'.

In all cases considered in the Proposition above, some landowners will use traditional registration, whether its quality is low or high (but not when this quality takes intermediate values). That is why one cannot precisely locate the optimal quality of traditional registration.[14] To understand what matters for the choice of the optimal quality it is useful to review in turn the expression of social welfare when the quality of the traditional registration system is either low or high.

When one uses the traditional registration system with the best high quality (higher than θ'), say θ^*, the social welfare function equals:

$$\mathcal{W}_B(\theta) = \int_0^{V_{B/P}} \underline{\theta} V f(V) dV + \int_{V_{B/P}}^{V_{T/B}(\theta^*)} [(1-h)\bar{\theta}V - \tilde{a}] f(V) dV$$
$$+ \int_{V_{T/B}(\theta^*)}^{\bar{V}} [\theta^* V - c(\theta^*)] f(V) dV. \quad (13)$$

On the other hand, when the best quality θ_* of traditional registration is among the lowest, that is $\theta_* \in [\underline{\theta}, \check{\theta}]$, then the social welfare function is written:

$$W_B(\theta) = \int_0^{V_{T/P}(\theta_*)} \underline{\theta} V f(V) dV + \int_{V_{T/P}(\theta_*)}^{V_{T/B}(\theta)} (\theta_* V - c(\theta_*)) f(V) dV$$
$$+ \int_{V_{T/B}(\theta_*)}^{\bar{V}} \left((1-h)\bar{\theta}V - \tilde{a}\right) f(V) dV \quad (14)$$

To determine the best quality of the traditional registration system one needs to compare the social welfare as expressed in (13) and in (14), when θ maximizes either (13) or (14). Unfortunately, in general nothing can be said about the comparison.

A few remarks, however, are in order, considering that we have the following inequalities[15]

$$V_{B/P} \leq V_{T/P}(\theta_*) < V_{T/B}(\theta_*) < V_{T/B}(\theta^*) \leq \bar{V}. \quad (15)$$

[14] We ignore the possibility of multiple optimal qualities.

[15] See the appendix for a proof that $V_{T/P}(\theta_*) < V_{T/B}(\theta^*)$.

1. All the owners for whom the land value V is higher than $V_{T/B}(\theta^*)$ are better off with a high-quality registration system.

2. All the owners whose land values V are in $]V_{B/P}, V_{T/B}(\theta_*)[$ are worse off with a high-quality registration system. They prefer a low-quality registration system to the blockchain, but they prefer the latter to the high-quality registration system.

3. All the owners whose land value V is $]V_{T/P}(\theta_*), V_{T/B}(\theta_*)[$ are worse off with a high-quality registration system. They prefer the low-quality registration system to privacy (or the blockchain), but they prefer privacy to the blockchain.

4. The other owners are indifferent between the two alternatives.

To sum up the previous analysis of Proposition 13, the optimal value of the quality θ depends on the distribution of land values. If the weights given to intermediate land values in the distribution function are greater than those given to high land values then it is optimal to choose a low quality land titling system for the traditional registration system (and conversely). The only thing we can be sure of is that it is never optimal to use the blockchain alone under the assumptions of Proposition 13.

To understand the thrust of this result, consider an economy where land values are often large. Introducing the blockchain benefits owners of land whose values are relatively low because it decreases the probability of forfeiture at a cost lower than that of the traditional land registration system. On the other hand, for the owners of highly valued land it is better to rely on a high-quality traditional registration rather than blockchain registration. That is because, the increase in the expected value of land with this system is higher than that obtained with the blockchain, and still more than compensates the rise in the unit cost. Since the owners of lowly valued land are few and the values of the latter are low, using the blockchain results at best in a slight increase in social welfare.

A comment may be added about the particular case where the quality achieved by the blockchain reaches the maximum potential quality $\bar{\theta}$ of traditional registration. Suppose indeed that the risk of hijacking h is nil. In such a case, it is intuitive to see that it is optimal to maintain traditional registration only if assumption $c'(\underline{\theta}) \leq V_{B/P}$ is fulfilled and if traditional registration provides a low degree of protection at a low cost, which happens when $\theta \in [\underline{\theta}, \check{\theta}]$. Under that assumption, it is obviously optimal for blockchain registration to supersede traditional registration.

8 Conclusion

In this paper, we have first generalized the model developed by Arruñada and Garoupa (2005) to study optimal land titling systems. In our setup, there is a priori an infinite set of systems, each characterized by its quality (namely, the probability that a given plot of land does not face forfeiture) and the unit transaction cost. We have used our model to study optimal titling systems when blockchain registration becomes available. In particular, we have addressed the following question: Can it be optimal to get rid of the traditional public registration system?

We have found that, despite the introduction of the blockchain, and under reasonable assumptions, it is still socially optimal to keep traditional public registration. This does not mean, however, that the traditional public registration system must not be adapted. We indeed show that the optimal quality of protection provided by traditional registration must be either sufficiently high (and higher than that of the blockchain), or low enough (and lower than that of the blockchain).

However, the advent of the blockchain in land titling may also render traditional registration unnecessary. This is the case when the expected additional value of traditional registration can never compensate for the additional cost, by comparison with the blockchain; and when the marginal cost of increasing the degree of protection beyond that of privacy is too high.

Interestingly, the conditions determining the optimal titling systems are in principle observable (*e.g.*, the distribution of land prices, the probability of hacking, etc.). Consequently, the theoretical approach developed in this paper can be useful in shaping the reform of titling systems.

A natural topic for further research is to study how notaries, for instance, are likely to adapt to the introduction of blockchain registration, and what public policies a country should follow to induce them to choose the optimal quality of the land titling system.

Another topic for further research would be to consider that changing a land titling system may be costly and this should be taken into account in devising optimal public titling systems when blockchain registration is available.

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A Notations

- V: land title value.
- $\overline{V}$: maximal land title value.
- θ: probability that a landowner will not face forfeiture. This term reflects the degree of protection provided by a land-titling system to the landowner, or equivalently the quality of the land titling system.
- $\underline{\theta}$: the lowest degree of protection. The lowest degree of protection is provided by privacy.
- $\overline{\theta}$: the highest degree of protection.
- h: probability of the blockchain being hacked.
- $c(\theta)$ unit cost per transaction associated with a land title system whose quality is θ.
- $V_{T/P}(\theta)$: threshold value of the title above which traditional registration if preferred to privacy, with $V_{T/P}(\theta) = \frac{c(\theta)}{\theta - \underline{\theta}}$.
- $V_{T/B}(\theta)$: threshold value of the title above (or below) which traditional registration if preferred to blockchain registration, with $V_{T/B}(\theta) = \frac{c(\theta) - \tilde{a}}{\theta - (1-h)\overline{\theta}}$.
- $V_{B/P}$: threshold value of the title above which blockchain registration if preferred to privacy, with $V_{B/P} = \frac{\tilde{a}}{(1-h)\overline{\theta} - \underline{\theta}}$.
- $W(\theta)$: social welfare function when blockchain technology is not available.
- $W_B(\theta)$: social welfare function when blockchain technology is available.
- $\tilde{a}$: unit cost per transaction associated with the blockchain.
- θ': degree of protection such that $V_{T/B}(\theta') = \overline{V}$ in case 1(b) of Prop. 3 (Fig. 2) and in case 2(c) of Prop. 3 (Fig. 5).
- θ'_1 and θ'_2: degrees of protection such that $V'_{T/B}(\theta'_1) = V'_{T/B}(\theta'_2) = \overline{V}$ in case 2(a) of Proposition 3, with $\theta'_{T/B} < \theta'_{B/P}$.
- $\underline{\underline{\theta}}$ is the value of θ that minimizes $V_{T/B}(\theta)$.
- $\hat{\theta}$: value of θ such that $c(\theta) = \tilde{a}$, the degree of quality of traditional registration such that the unit cost of transaction under traditional registration equals the unit cost of transaction under the blockchain.
- $\check{\theta}$: value of θ such that $V_{T/P}(\check{\theta}) = V_{T/P}(\check{\theta}) = V_{B/P}$, which exists under an assumption specified in Prop. 4.

B Proofs

Proof of Lemma 1

Proof. We have

$$\frac{d}{d\theta}V_{T/P}(\theta) = \frac{d}{d\theta}\left(\frac{c(\theta)}{\theta - \underline{\theta}}\right) = \frac{c'(\theta)(\theta - \underline{\theta}) - c(\theta)}{\theta - \underline{\theta}}.$$

Since $c(.)$ is strictly convex and $c(\underline{\theta}) = 0$, we get

$$0 \leq c'(\theta)(\theta - \underline{\theta}) - c(\theta).$$

The result follows. □

Proof of Proposition 1

Proof. Assume that the optimal registration system is such that $\theta \in (\underline{\theta}, 1)$. Then using Leibnitz's rule, this optimal system satisfies the following necessary condition

$$\underline{\theta}\frac{d}{d\theta}\left(V_{T/P}(\theta)\right)V_{T/P}(\theta)f\left(V_{T/P}(\theta)\right) - \frac{d}{d\theta}V_{T/P}(\theta)\left(\theta V_{T/P}(\theta) - c(\theta)\right)f\left(V_{T/P}(\theta)\right) + \int_{V_{T/P}(\theta)}^{\overline{V}}\left(V - c'(\theta)\right)f(V)dV = 0, \quad (16)$$

which reduces to

$$\int_{V_{T/P}(\theta)}^{\overline{V}}\left(V - c'(\theta)\right)f(V)dV = 0. \quad (17)$$

□

Proof of Proposition 2

Proof. Using the necessary condition (17) and the Implicit Function Theorem we can show that:

$$\frac{\partial\theta^*}{\partial\overline{V}} = -\frac{(\overline{V} - c'(\theta^*))f(\overline{V})}{W''(\theta^*)}. \quad (18)$$

From (17) we necessarily have $\overline{V} > c'(\theta^*)$ and the conclusion follows (since by the necessary second-order condition $W''(\theta^*) < 0$).

We also have

$$\frac{\partial\theta^*}{\partial\underline{\theta}} = [\frac{c(\theta^*)}{\theta^* - \underline{\theta}} - c'(\theta^*)]f(\frac{c(\theta^*)}{\theta^* - \underline{\theta}})\frac{c(\theta^*)}{(\theta^* - \underline{\theta})^2}\frac{1}{\frac{\partial^2 W}{\partial^2\theta}},$$

with $\frac{\partial^2 W}{\partial^2\theta} < 0$. Therefore, the sign of $\frac{\partial\theta^*}{\partial\underline{\theta}}$ is the same as that of $\frac{c(\theta^*)}{\theta^* - \underline{\theta}} - c'(\theta^*)$.
Since $c(.)$ is strictly convex, we have $c'(\theta^*) > \frac{c(\theta^*)}{\theta^* - \underline{\theta}}$. Therefore,

$$\frac{\partial\theta^*}{\partial\underline{\theta}} > 0.$$

Assume now that the cost function depends on an exogenous efficiency parameter a (*e.g.*, efficiency of the organization). Let us assume that $c(\theta, a)$, with $c(\underline{\theta}, a) = 0$, $\frac{\partial c(\theta,a)}{\partial a} < 0$, $\frac{\partial^2 c(\theta,a)}{\partial^2 a} < 0$ (c is strictly concave in a).
From (17) we have

$$\int_{V_{T/P}(\theta)}^{\overline{V}} \big(V - \frac{\partial c}{\partial \theta}(\theta, a)\big) f(V) dV = 0. \tag{19}$$

Using the necessary condition (19), the definition of $V_{T/P}(\theta)$ and the implicit function theorem we obtain

$$\frac{\partial \theta^*}{\partial a} = -\frac{\frac{\partial^2 W}{\partial \theta \partial a}}{\frac{\partial^2 W}{\partial^2 \theta}}$$

$$\frac{\partial^2 W}{\partial \theta \partial a} = \underbrace{-[\frac{c(\theta^*, a)}{\theta^* - \underline{\theta}} - \frac{\partial c}{\partial \theta}(\theta^*, a)]}_{(+)} \underbrace{\frac{d(\frac{c(\theta,a)}{\theta - \underline{\theta}})}{da}}_{(-)} f(\frac{c(\theta, a)}{\theta - \underline{\theta}}) \underbrace{- \int_{\frac{c(\theta,a)}{\theta-\underline{\theta}}}^{\overline{V}} \frac{\partial^2 c(\theta, a)}{\partial \theta \partial a} f(V) dV}_{(+)}$$

We have $\text{sign}(\frac{\partial \theta^*}{\partial a}) = \text{sign}(\frac{\partial^2 W}{\partial \theta \partial a})$ since $\frac{\partial^2 W}{\partial^2 \theta} < 0$. Furthermore, since $c(.)$ is strictly convex in θ, we have $\frac{\partial c}{\partial \theta}(\theta^*, a) > \frac{c(\theta^*,a)}{\theta^* - \underline{\theta}}$. Therefore, the sign of $\frac{\partial \theta^*}{\partial a}$ is ambiguous, unless $\frac{\partial c(\theta,a)}{\partial \theta \partial a} > 0$, since in this case $-\int_{\frac{c(\theta,a)}{\theta-\underline{\theta}}}^{\overline{V}} \frac{\partial^2 c(\theta,a)}{\partial \theta \partial a} f(V) dV < 0$. □

Proof of Proposition 3

Proof. Assume that $V'_{T/B}(\overline{\theta}) < 0$. If $\overline{V} \leq V_{T/B}(\overline{\theta})$, then no one ever prefers traditional registration over the blockchain. When $V_{T/B}(\overline{\theta}) < \overline{V}$, then, there exists θ' such that for all θ in $[\theta', \overline{\theta}]$, $V_{T/B}(\theta) < \overline{V}$. And then owners of land whose value lies in $[V_{T/B}(\theta), \overline{\theta}]$ prefer traditional registration over the blockchain.

Now assume that $V'_{T/B}(\overline{\theta}) > 0$.
Case 2 (a): If the function $V_{T/B}(\theta)$ reaches a local minimum at a point located in $[(1-h)\overline{\theta}, \overline{\theta}]$ such that $\bar{V} \leq \min_\theta V_{T/B}(\theta)$, all owners of land prefer the blockchain to traditional registration.

Case 2 (b): If the function $V_{T/B}(\theta)$ reaches a local minimum at a point located in $](1-h)\overline{\theta}, \overline{\theta}]$ such that $\bar{V} > \min_\theta V_{T/B}(\theta)$, and if $V_{T/B}(\overline{\theta}) > \overline{V}$, by continuity of $V_{T/B}(\theta)$, there are two values θ'_1 and θ'_2, such that the interval $[\theta'_1, \theta'_2]$ is included in $[(1-h)\overline{\theta}, \overline{\theta}]$ and such that for all θ in this interval, $V_{T/B}(\theta) < \overline{V}$, then all the owners of land with value in $[V_{T/B}(\theta), \bar{V}]$ prefer to rely on traditional registration.

Case 2 (c) If the function $V_{T/B}(\theta)$ reaches a local minimum at a point located in $](1-h)\overline{\theta}, \overline{\theta}]$ such that $\bar{V} > \min_\theta V_{T/B}(\theta)$ and if $V_{T/B}(\overline{\theta}) < \overline{V}$, then there is a quality θ' such that for all θ in $[\theta', \overline{\theta}]$, $V_{T/B}(\theta) < \overline{V}$.

□

Proof of Lemma 2

Proof. Assume the opposite. Then there exists θ, $(1-h)\bar{\theta} \leq \theta$, $V_{T/B}(\theta) < \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}}$. By definition, of $V_{T/B}(\theta)$, $(1-h)\bar{\theta}V_{T/B} - \tilde{a} = \theta V_{T/B} - c(\theta)$. But since $V_{T/B}(\theta) < \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}}$, this implies that $(1-h)\bar{\theta}V_{T/B} - \tilde{a} < \underline{\theta}V_{T/B}$. Therefore, we have $\theta V_{T/B} - c(\theta) < \underline{\theta}V_{T/B}$, or $V_{T/B} < \frac{c(\theta)}{\theta-\underline{\theta}} = V_{T/P}(\theta)$, which contradicts assumption 4. □

Proof of Proposition 4

Proof. (Necessity). Suppose that there exists a quality $\check{\theta} > \underline{\theta}$ such that $V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta}) = V_{B/P}$. But

$$V_{T/P}(\theta) = V_{T/B}(\theta) \iff \frac{c(\theta)}{\theta - \underline{\theta}} = \frac{\tilde{a}}{(1-h)\bar{\theta} - \underline{\theta}}. \tag{20}$$

Since $c\big((1-h)\bar{\theta}\big) > \tilde{a}$, this implies that the value $\check{\theta}$ such that (20) holds is such that $\check{\theta} < (1-h)\bar{\theta}$. It is even lower than $\hat{\theta}$ since $V_{T/B}(\theta)$ takes negative values when θ is in $[\hat{\theta}, (1-h)\bar{\theta}[$. Now, since $V_{B/P}(\theta)$ is increasing, this implies that $\lim_{\theta\to\underline{\theta}} V_{T/P}(\theta) = c'(\underline{\theta}) < \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}}$.
(Sufficiency). Assume $\lim_{\theta\to\underline{\theta}} V_{T/P}(\theta) = c'(\underline{\theta}) < \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}}$. Since $V_{T/B}(\underline{\theta}) = \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}}$, $V_{T/B}(\hat{\theta}) = 0$ and $V_{T/P}(\theta)$ is increasing there is a value $\check{\theta}$ such $V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta})$. It also follows that $V_{T/P}(\check{\theta}) = \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}}$.

□

Proof of Proposition 5

Proof. In both cases, $\bar{V} \leq V_{T/B}(\bar{\theta})$ ($\bar{V} \leq V_{T/B}(\theta)$, $\forall \theta \in [(1-h)\bar{\theta}; \bar{\theta}]$), therefore, no one uses traditional registration. The landowners resort to privacy when the value of their land is below $\frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}} \equiv V_{B/P}$.

□

Proof of Proposition 6

Proof. Consider the first assertion. First suppose that θ is lower than θ'. Under our assumption, $V_{T/B}(\theta)$ is greater than $\bar{V}$. Therefore, people only use the blockchain or resort to privacy: no one uses traditional registration. Second, suppose that $\theta > \theta'$. Then $V_{T/B}(\theta)$ is lower than $\bar{V}$. The owners of land whose value is in $[V_{T/B}(\theta), \bar{V}]$ prefer to rely on traditional registration. They are better-off than when the quality of traditional registration θ satisfies $\theta < \theta'$. Therefore, social welfare is greater when $\theta > \theta'$. Now, social welfare is a continuous function of θ. Therefore, by Weiestrass's Theorem, there exists a quality $\theta^* > \theta'$ that maximizes social welfare.

Let us now address the second assertion. For all $\theta < \theta'_1$ or $\theta'_2 < \theta$, landowners either use the blockchain or resort to privacy. Reasoning similar to that used above shows that the optimal quality of traditional registration is in θ'_1, θ'_2.

Let us now address the third assertion. For all $\theta < \theta'$, landowners either use the blockchain or resort to privacy. Then, using reasoning paralleling that used to prove the first assertion shows that the optimal value of traditional registration is greater than θ'.

□

Proof of Proposition 7

Proof. It can immediately be seen that whenever θ is in $[\hat{\theta}, (1-h)\bar{\theta}]$, $V_{T/B}(\theta) \leq 0$. Therefore, privacy will always be preferred to the traditional registration system. But people whose land value V is greater than $V_{B/P}$ will prefer the blockchain to privacy. □

Proof of Proposition 8

Proof. Since we have

$$V'_{T/B}(\theta) = \frac{\phi(\theta)}{\big(\theta - (1-h)\bar{\theta}\big)^2} \tag{21}$$

where

$$\phi(\theta) = -c'(\theta)\big((1-h)\bar{\theta} - \theta\big) + \frac{\tilde{a}}{(1-h)\bar{\theta} - \underline{\theta}} - c(\theta), \tag{22}$$

and $\phi'(\theta) = -c''(\theta)\big((1-h)\bar{\theta} - \theta\big) \leq 0$, then $V'_{T/B}(\underline{\theta}) \leq 0$ implies that $V_{T/B}(\theta)$ is non-increasing and thus $V_{T/B}(\theta) \leq \frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}}$ for all $\theta \leq \hat{\theta}$. Moreover, we have

$$\lim_{\theta \to \underline{\theta}} \frac{c(\theta)}{\theta - \underline{\theta}} = c'(\underline{\theta}) > \frac{\tilde{a}}{(1-h)\bar{\theta} - \underline{\theta}}, \tag{23}$$

and as the function $c(\theta)/(\theta - \underline{\theta})$ is increasing, no one ever chooses traditional registration.[16] People whose land value is greater than $\frac{\tilde{a}}{(1-h)\bar{\theta}-\underline{\theta}}$ choose the blockchain, and the others resort to privacy. □

Proof of Proposition 9

Proof. Consider two qualities θ_1 and θ_2 with $\theta_1 < \check{\theta} < \theta_2$. When $\theta = \theta_2$, no one uses traditional registration (people either use the blockchain or resort to privacy) and the social welfare function no longer depends on θ. More precisely, the owners of land whose values V are such that $V \leq V_{T/B}(\check{\theta}) = V_{B/P}$ resort to privacy, and those for whom $V > V_{T/B}(\check{\theta})$ register with the blockchain. Now, assume that $\theta = \theta_1$. How are the landowners' registration decisions affected by this change? Consider owners of land whose values are below $V_{T/B}(\check{\theta}) = V_{B/P}$.
If they continue to choose privacy when $\theta = \theta_1$, their welfare is unchanged. This is the case when $V \in [0, V_{T/P}(\theta_1)]$ (we have $V_{T/P}(\theta_1) < V_{T/B}(\theta_1)$ by definition of $\check{\theta}$). When $V \in [V_{T/P}(\theta_1), V_{T/B}(\theta_1)]$, landowners choose traditional registration. If they favor traditional registration instead of privacy, it is because they are better-off. Let us now concentrate on people whose land value is above $V_{T/B}(\check{\theta})$. If they choose the blockchain when $\theta = \theta_1$, their welfare is unchanged. If they favor traditional registration, they are better-off. So individuals lose when the quality of traditional registration is θ_2 instead of θ_1. Social welfare is thus greater with θ_1 than with θ_2 if the set of people who choose traditional registration is not negligible. But since the distribution function is strictly increasing, this means that the probability that land values belong to $[V_{T/P}(\theta_1), V_{T/B}(\theta_1)]$ is strictly positive. Thus the set of people who use traditional registration when its quality is θ_1 is not negligible and social welfare is strictly greater with $\theta = \theta_1$ than with $\theta = \theta_2$. □

[16] For instance, assume that: $c(\theta) = (\theta + k)^2 - (\underline{\theta} + k)^2$, with $k > 0$. Note that $c(\underline{\theta}) = 0$, and that $c'(\underline{\theta}) > 0$.

Proof of Proposition 10

Proof. The result follows directly from Propositions 5, 7, and 8. □

Proof of Proposition 11

Proof. This follows directly from Propositions 5, 7, and 9. □

Proof of Proposition 12

Proof. This follows directly from Propositions 6, 7, and 8. □

Proof of Proposition 13

Proof. In each of the cases listed in the Proposition, there is always a non-negligible set of lands whose owners choose to register traditionally, whether the quality of traditional registration is low or high. Therefore, the optimal quality is located either in $[\underline{\theta}, \check{\theta}[$ or in $](1-h)\bar{\theta}, \bar{\theta}]$ (or in both sets). □

Proof that $V_{T/P}(\theta_*) < V_{T/B}(\theta^*)$

Proof. From the definition $V_{T/B}(\theta) = \frac{c(\theta)-\tilde{a}}{\theta-(1-h)\bar{\theta}}$, we obtain that $V'_{T/B}(\theta) = 0$ implies $c(\theta) = \tilde{a} + c'(\theta)[\theta - (1-h)\bar{\theta}]$. Using this expression, observe that when $\theta < (1-h)\bar{\theta}$, $V_{T/B}(\theta)$ reaches its maximum value at a quality θ_1 such that $V_{T/B}(\theta_1) = c'(\theta_1)$. On the other hand, when $\theta > (1-h)\bar{\theta}$, $V_{T/B}(\theta)$ reaches its minimum value at a quality θ_2 such that $V_{T/B}(\theta_2) = c'(\theta_2)$. Since the function $c(.)$ is strictly convex it follows that $V_{T/B}(\theta_1) < V_{T/B}(\theta_2)$ and therefore that $V_{T/P}(\theta_*) < V_{T/B}(\theta^*)$. □

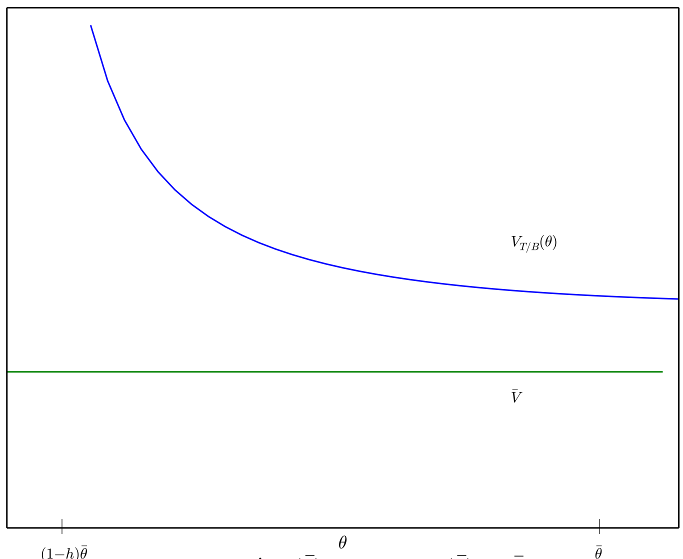

Figure 1: $V'_{T/B}(\bar{\theta}) < 0,\ V_{T/B}(\bar{\theta}) > \bar{V}$

Parameter values are: $\bar{V} = 1.5$, $\underline{\theta} = 1$, $\bar{\theta} = 2$, $h = 0.2$, $\tilde{a} = .02$, and $c(\theta) = 0.5(\theta - \underline{\theta})^2$.

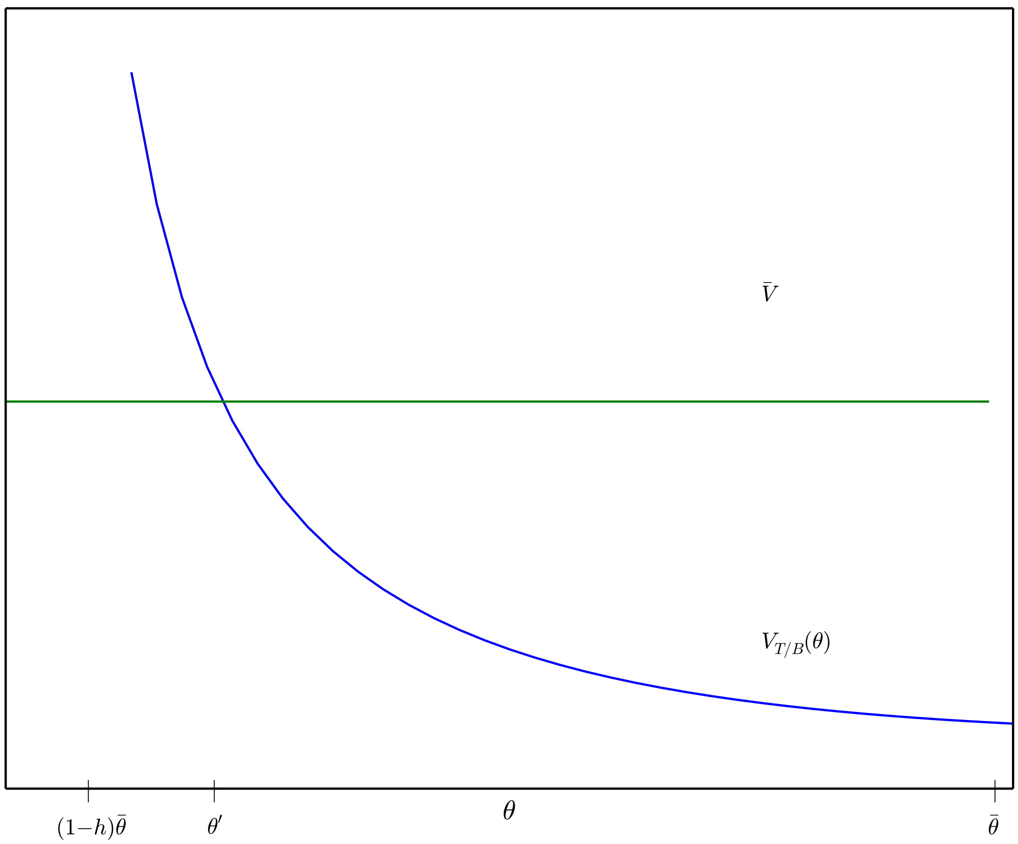

Figure 2: $V'_{T/B}(\bar{\theta}) < 0,\ V_{T/B}(\bar{\theta}) < \bar{V}$

Parameter values are: $\bar{V} = 1.5$, $\underline{\theta} = 1$, $\bar{\theta} = 2$, $h = 0.2$, $\tilde{a} = .02$, and $c(\theta) = 0.5(\theta - \underline{\theta})^2$.

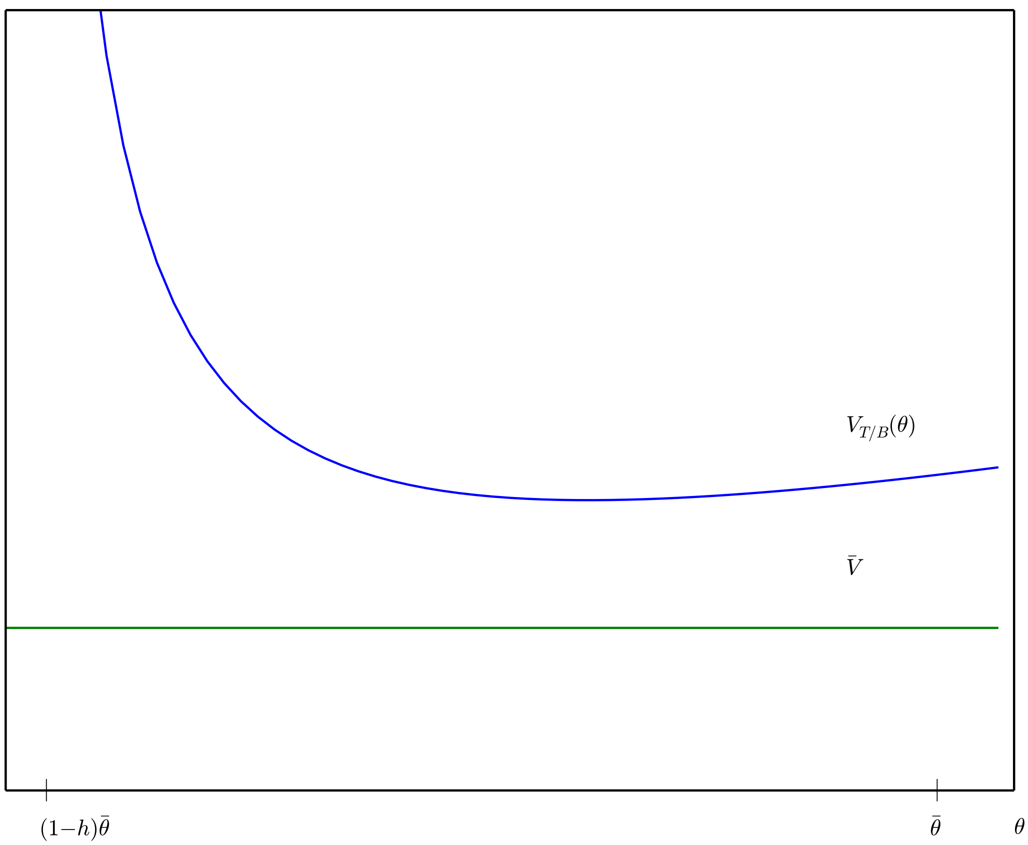

Figure 3: $V'_{T/B}(\bar{\theta}) > 0,\ \bar{V} < \min_\theta V_{T/B}(\theta)$

Parameter values are: $\bar{V} = .55$, $\underline{\theta} = 1$, $\bar{\theta} = 2$, $h = 0.2$, $\tilde{a} = .02$, and $c(\theta) = 0.5(\theta - \underline{\theta})^2$.

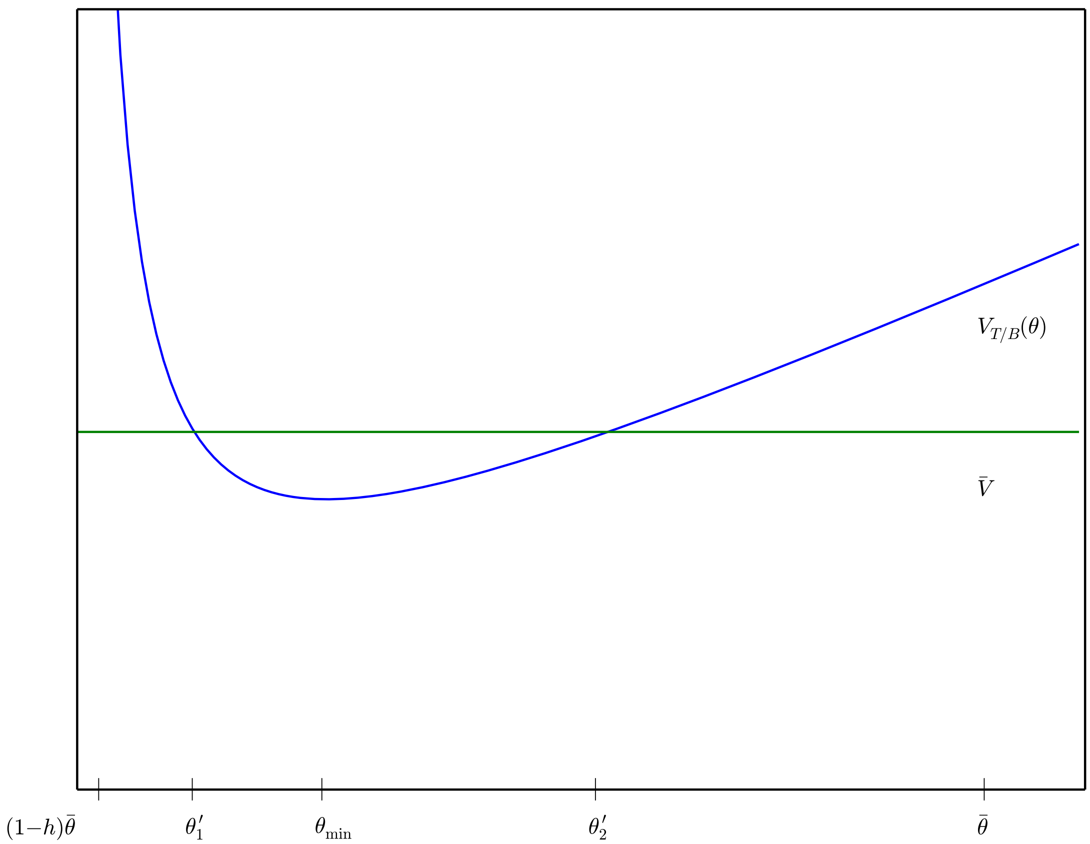

Figure 4: $V'_{T/B}(\bar{\theta}) > 0,\ \bar{V} > \min_\theta V_{T/B}(\theta),\ V_{T/B}(\bar{\theta}) > \bar{V}$

Parameter values are: $\bar{V} = .85$, $\underline{\theta} = 1$, $\bar{\theta} = 2$, $h = 0.3$, $\tilde{a} = .02$, and $c(\theta) = 0.5(\theta - \underline{\theta})^2$.

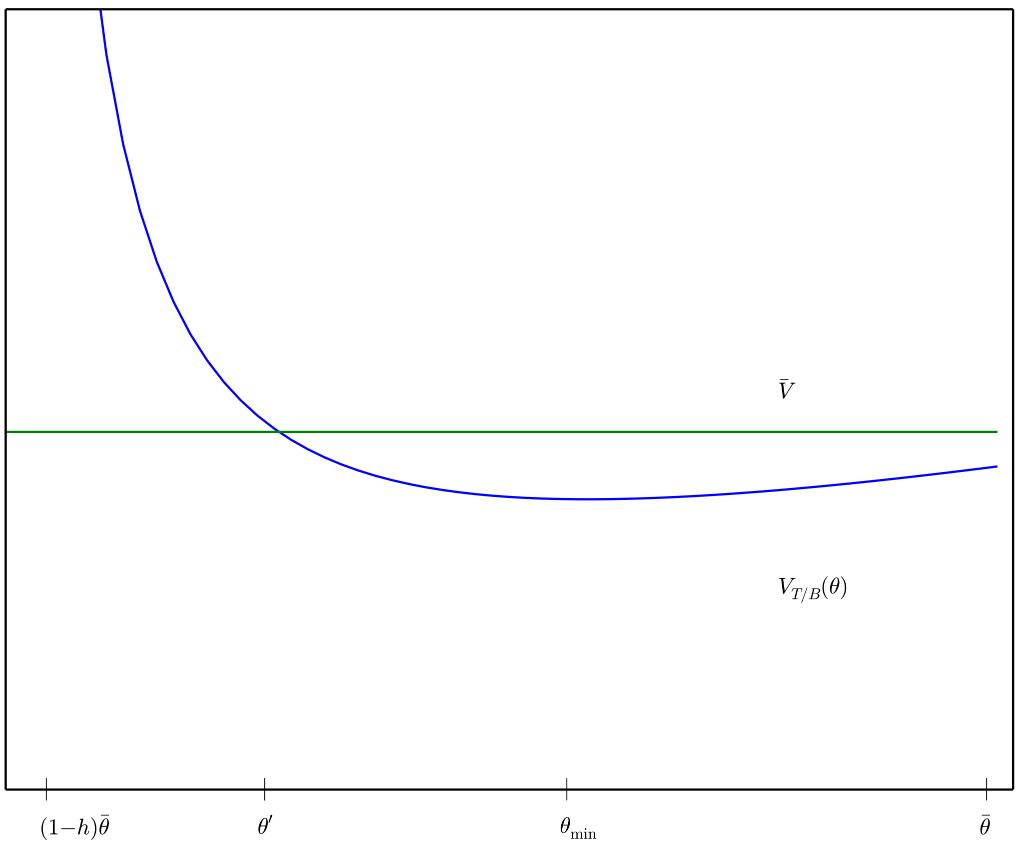

Figure 5: $V'_{T/B}(\bar{\theta}) > 0,\ V_{T/B}(\bar{\theta}) < \bar{V}$

Parameter values are: $\bar{V} = .85$, $\underline{\theta} = 1$, $\bar{\theta} = 2$, $h = 0.3$, $\tilde{a} = .02$, and $c(\theta) = 0.5(\theta - \underline{\theta})^2$.

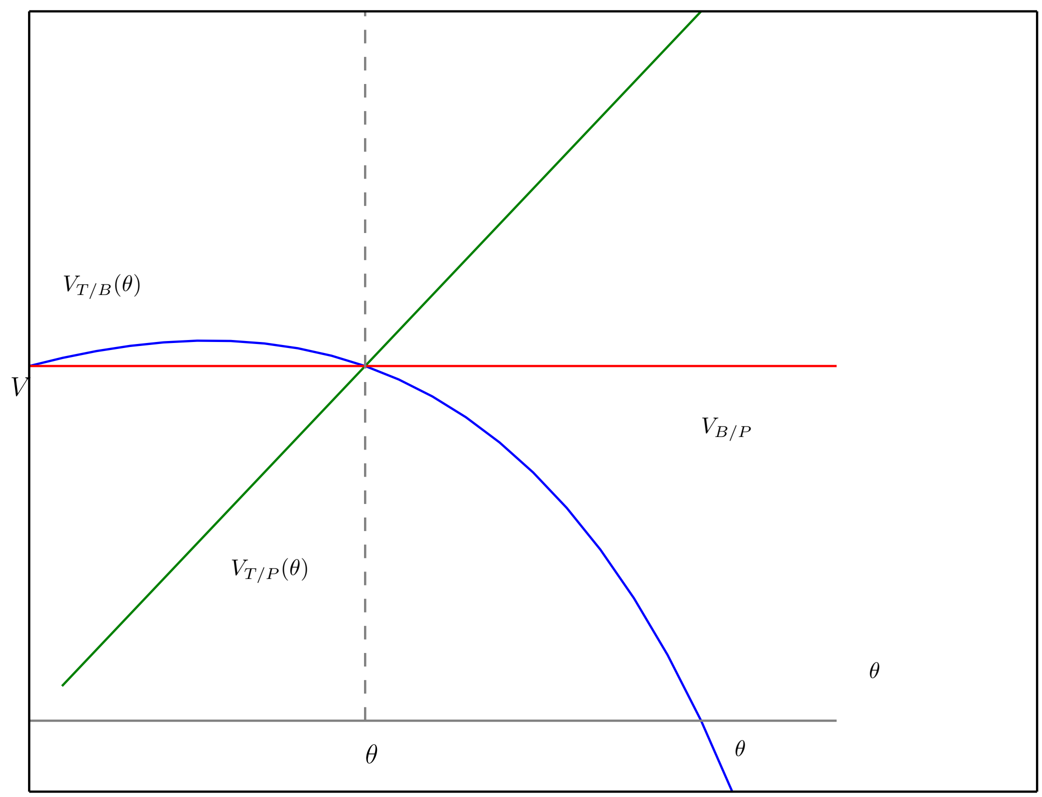

Figure 6: An example where $V_{T/P}(\check{\theta}) = V_{T/B}(\check{\theta}) = V_{B/P}$

Parameter values are: $\bar{V} = .85$, $\underline{\theta} = 1$, $\bar{\theta} = 2$, $h = 0.3$, $\tilde{a} = .02$, and $c(\theta) = 0.5(\theta - \underline{\theta})^2$.