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Optimal production of transplant care services

Bruno Deffains [a,b], Jean Mercier Ythier [c,d,e,*]

[a] Université de Paris Panthéon-Assas (Ermes), France
[b] European Business School, Wiesbaden, Germany
[c] Université de Metz, France
[d] BETA (CNRS), Strasbourg, France
[e] IDEP, Marseille, France

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ABSTRACT

Most organ transplants are from dead donors. National transplant organizations exhibit considerable differences in terms of their donor population rates. Spain's organization is by far the most efficient in this respect. We argue that much of the productivity advantage of Spain's transplant organization proceeds from an efficient organization of the production chain, from organ procurement to transplantation. Transplants from dead donors are analogous to a common resource for the transplant community. Their circulation through the national transplant organization creates public good externalities between the care units in charge of organ retrieval and those in charge of transplantation. A socially efficient production of transplant care services obtains through an optimal control, by the national transplant agency, of *both* the circulation *and the production* of transplants. In particular, transplant shortage makes the rotten kid theorem fail in this context. The analysis also produces a natural measure of public good externalities, evaluated from the standpoint of care units.

1. Introduction

The demand for life-saving transplant surgery is growing in most countries. However, the organ donor rates as well as the total number of transplants differ tremendously from one country to another. During the last ten years, Spain appears as the champion of the dead-donor league. Considering that 90% of organ transplants, in general, are coming from dead donors,[1] it is crucial to understand the reasons for such a success. In practice, the Spanish transplant system improves survival, it increases organ demand, and more and more people are taking advantage of transplantation. For many commentators, these achievements clearly show that organ donation is the limiting factor in treating certain pathologies. This is certainly partly true, but still insufficient to explain the differences, in terms of production efficiency, of transplant care systems around the world. We argue in this article that the problem is not only, and perhaps not mainly, with the lack of donors per se, but, rather, with the organization of the transplant system, and notably of its production side. A recent report of the Rand Corporation about organ donation and transplantation in the European Union thus states that «the Spanish model is an outstanding example of how organizational changes in the transplantation system can increase the number of organs available from deceased donors. Based on the premise that *the greatest barrier to organ transplantation was not a lack of suitable donors but the failure to identify and 'convert potential into real donors'*,[2] the Spanish government founded the National Transplant Organisation (ONT) in 1989 and began to set up a nationwide system to monitor potential organ donors.» (Tiessen et al., 2008, p. 38).

* Corresponding author. Université de Metz, France.
E-mail address: jean.mercier-ythier@wanadoo.fr (J. Mercier Ythier).

[1] IRODaT, 2006.

[2] Emphasis added.

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Table 1
Organs donor rates per 1 million population in 2006.

	2006
Australia	9
Canada	14.8
France	25.3
Greece	5.8
Israel	7.7
Italy	20.9
Spain	36.4
Sweden	14.5
UK	13
United States	26.6

The differences in donation rates between countries are commonly explained by the differences between the "opt-in" and the "opt-out" legal systems for organ donation. In the opt-out donation system, consent is presumed for deceased donors unless she/he registered on an appropriate refusal file when alive. In most opt-out systems, the next of kin's approval is also required. In the opt-in system of donation, on the contrary, those willing to give their organs upon death must sign up as donors. Countries with opt-out systems have higher deceased-donor rates on average. However, the evolution of most countries in the western world during the recent period is clearly opt-out oriented. Moreover, one observes considerable differences in the donation rates of countries that have the same legal regime (e.g., Spain and France) and similar donation rates in countries with opposite legal systems (e.g., France and the USA: see Table 1 of Section 2). The report of the Rand Corporation is categorical on this subject: "Only around 3% of all people dying in hospitals are potential donors. The conversion of this potential depends on the willingness of patients and their families to donate organs *and the participation of hospitals in organ retrieval activities*.[3] While public debate often centres on public awareness of organ donation and the organization of consent systems, recent research and experience from piloting new approaches point to the *organizational aspects of organ donation as one of the most important factors influencing organ procurement rates*.[4]" (Tiessen et al., 2008, p. 12).

A surprisingly small number of papers concentrate on organization aspects in the economic literature on organ transplants. Notable exceptions are the contributions of Roth et al. (2005a,b, 2006). They consider the case of live kidney donations, and design theoretical patterns of gift-exchange for efficient pairwise matching of kidney donors and recipients from a given set of pairs of incompatible donor and recipient. They present numerical simulations of the impact of such discrete optimization procedures on transplant provision, and consider the practical implementation of these procedures by means of specialized clearinghouses (Roth et al., 2005a).

The quasi-contractual arrangements between pairs of beneficiaries and living donors involved in these models of Pareto-efficient gift-exchange do not apply to cadaveric donation. Economic analysis misses a model of efficient allocation of cadaveric transplants, whereas the latter constitute the bulk of actual transplant resources as recalled above. Such a model should capture, notably, the following three interrelated features, common to (most) actual transplant economies: (i) organ retrieval is performed at some cost by care units; (ii) a non-profit national agency collects transplants, and redistributes them to care units for transplantation; (iii) transplant markets are banned, and, in particular, the compensation received by a care unit procuring an organ to the agency reduces to the payment of the cost of inputs consumed in organ retrieval. We develop a simple model of this type below, which, we argue, captures essential features of the Spanish experience and provides theoretical underpinnings for some of the main policy recommendations that it has inspired (e.g., Tiessen et al., 2008, notably points 1 and 2 of their list of causes of the Spanish success on p. 39).

The model of the transplant economy may be described informally as follows. The ban on markets of organs makes transplants a common resource, collected mainly by "exhortation," that is, notably, by public calls for donation (Thorne, 2000, 2006). The bulk of the "resource" is constituted by brain-dead patients randomly distributed in hospitals through the statistical variety of death circumstances, and physically non-transferable for a variety of reasons that notably include the stringent legal obligations relative to the body of the deceased. This initial distribution of the common resource is naturally mismatched, in general, with the statistical distribution of the needs in grafts for transplantation in care production units. Operating an appropriate match of resources and "needs" in transplant inputs is the basic reason for the existence of institutions in charge of circulating grafts, such as national transplant agencies, as substitutes for banned transplant markets. Grafts are produced by hospitals, and circulated by the transplant agency, to be used by other hospitals as inputs in their final production of transplant care services. A hospital's intermediary graft production thus induces *public good external effects* on others' final production of care services. The resulting public good issues are captured through principal–agent interactions, in subgame-perfect equilibria of two-stage games where hospitals are only concerned with *their own final production* of care services while the transplant agency maximizes a social utility function that aggregates hospitals' preferences (Bergstrom, 1989; Cornes and Silva, 1999). It is notably shown that an optimal control of the agency over *both the circulation and the production* of graft inputs achieves production optimum an optimal control by the agency of *circulation alone* generally implies suboptimal under-provision of transplant inputs and services. In particular, transplant shortage makes the rotten kid theorem fail in this context.

The analysis also yields, as a by-product, a useful theoretical measure of public good externalities in the transplant system, computed from the standpoint of each hospital. This measure can be used, notably, as an indicator of tension on a hospital's graft resources at the optimum and at the equilibrium of the transplant system.

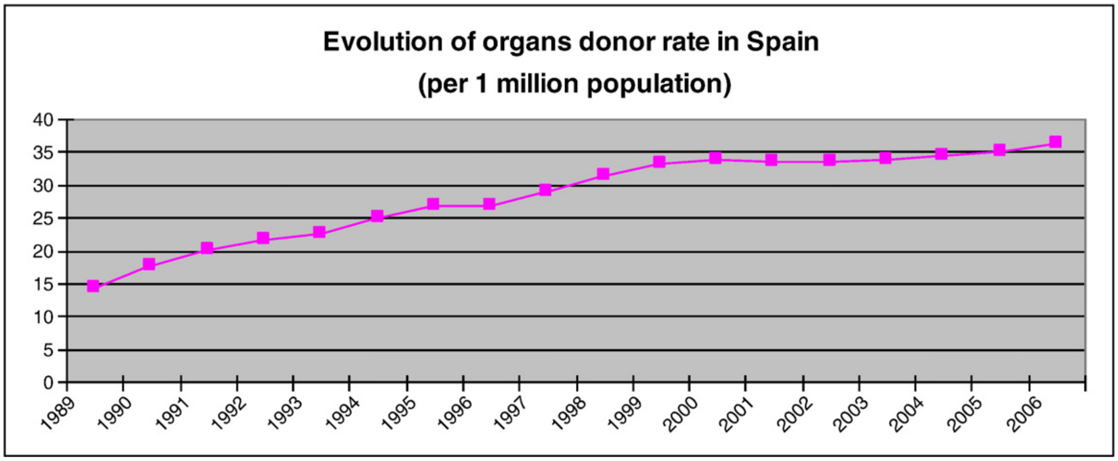
The paper develops as follows. Section 2 analyzes the Spanish transplant organization. Section 3 presents the model of the transplant care system. Section 4 sets and solves the public good problem of graft production and circulation. Section 5 concludes. An appendix collects the proofs.

2. Spain's transplant organization

The history of organ transplantation in Spain begins in 1965, with the first transplants in Madrid and Barcelona. In 1979, a law is adopted to favour the development of transplantation, but donations remain at a low level during the eighties. In 1989, the Organizacion Nacional de Trasplantes (ONT) is created to solve this problem. This institution is attached to the Ministry of Health and Consumption, and is in charge of developing the competencies relative to the provision and clinical utilization of organs and tissues. To carry out these tasks, it functions as a technical operative unit and fulfils its mission of coordinating the activities of donation, extraction, preservation, distribution, exchange, and transplantation of organs and tissues throughout the whole Spanish Health Care System. At the creation of the ONT, the main idea was that the problem was not with the number of donors but with their identification and the organization of the program.

After the creation of the ONT, Spain went from 14 donors per million population (pmp) in 1989 to 36.4 donors pmp in 2006. This evolution displayed in the graphic below made Spain evolve from donation rates ranked in intermediate-low positions in Europe to the highest rate not just in Europe, but also worldwide.

[3] Emphasis added.
[4] Emphasis added.



The origin of this spectacular change is internationally known as the "Spanish Model," a series of measures taken in this country to improve organ donation. This model, widely described in the scientific literature, has been recommended by the World Health Organization and is being applied in different regions of the world with outcomes very similar to those obtained in Spain.

The Spanish transplant law is similar to the corresponding laws in other Western countries. Although the law on transplant donation presumes the consent of deceased potential donors, according to a subsequent decree, relatives of a potential donor must be approached to determine the deceased's wishes regarding organ donation. In the absence of this knowledge, close relatives can sign the authorization, after internal discussion if required. At present, Spain's annual refusal rate for organ donation is around 15% of all donation interviews. Death is defined as the total and irreversible cessation of brain or cardio-respiratory functions. Clinical evaluation and complementary tests required are detailed within the legal text allowing organ retrieval either from brain-stem death donors or from non-heart-beating donors. Like other coordinating systems worldwide, the Spanish system has to monitor the management of waiting lists, organ allocation, and statistical analysis. Nevertheless, it was considered that a continuous monitoring system over the entire organ donation process was essential. A network of healthcare professionals responsible for the organ donation process as a whole has been set up at all levels (national, regional and hospital). This implies the need for training, organization and coordination of activities.

It was considered that these professionals working at the grass-roots level must feel involved and that they must be accountable for performance. Most of them are physicians, mainly intensive care unit (ICU) specialists, and they belong to the staff of the hospital. They generally continue in their medical role, but, as transplant coordinators, their main objective is to improve the organ donation rate. Currently, 155 hospitals are officially authorized to take care of organ donor programs. A quality control system has been developed for the organ donor process—the ICU mortality registry and the brain death registry—a common practice in most of them. By law (RD 2070, 30 December 1999), transplant coordinators are the professionals responsible for the whole donation and retrieval process.

National and regional offices are service agencies supporting the organ donation and transplantation programs. They deal with organ sharing and waiting list management. They arrange organ or team shifts. They are responsible for the official statistics and reports on organ donation and transplantation. They promote legal statements and binding consensus guidelines. They also promote public education and address any doubt or question about organ donation and transplantation. A 24-h hot line and E-mail system have been put in place to keep all interested groups or individuals informed. They are also concerned with and involved in training and research programs. Any activity that could improve donation or facilitate the transplant team activities can be promoted through this network.

Organ transplantation has been considered a hospital medical activity for which a specific budget and staff are allocated. This kind of activity does not induce any budgetary overload for hospitals. The annual general budget for transplantation procedures in Spain is around 180 million Euros. The annual budget for the organ procurement network is around 15 million Euros (less than 10% of the budget covering organ procurement activities). The general donation budget covers all extra-salary and extra-time activities of both coordinators and surgical retrieval teams, as well as any donor evaluation tests, the ICU bed daily costs, etc. This budget also covers coordinating offices, training courses and some of the educational programs. The type of payment for the extra work of coordination and organ retrieval for professionals in charge differs depending on the region. It can be a fixed amount, or it can be based on registered activity, or be determined according to a mixed system (it does not usually exceed 30% of total salary).

Table 1 shows that the Spanish organ donor rate per million population is the highest around the world. The British rate is only 37.5% of the Spanish rate and the French rate is 69.5% of the Spanish rate. We could compute in Table 2 a rough estimate of the number of patients waiting for kidney transplantation that would be obtained in several countries if they achieved the same donor rate as Spain.

Table 2
Projections of the 2006 Spanish rate on other countries.

	Cadaveric donors[a]	Kidney transplants[a]	Patients awaiting for a transplant in 2007[b]
Australia	202	330	1388
Australia	*565*	*1334*	*343*
Canada	468	712	4195
Canada	*1151*	*1751*	*1705*
France	1441	2352	6491
France	*2073*	*3383*	*4511*
Greece	74	144	903
Greece	*464*	*903*	*144*
Israel	68	87	540
Israel	*321*	*411*	*114*
Italy	1239	2932	7096
Italy	*2157*	*5106*	*4074*
UK	633	1240	6876
UK	*1772*	*3472*	*3472*
United States	8022	10,659	76,313
United States	*10,909*	*14,496*	*55,767*

Numbers in italic are calculated using the national rates of Table 1.
[a] Source: IRODaT 2006.
[b] Source: Council of Europe, Transplant Newsletter, September 2008.

To sum up, the Spanish model consists of a program designed to optimize every stages of the transplantation process from the identification of a potential donor. Many factors have contributed to the extraordinary increase of the Spanish dead-donor rate during the last 20 years. Of course, Spain was a pioneer of the opt-out system, but its success mainly proceeds from an excellent network of organ-transplant teams operating in hospitals, which routinely screen patients' records to identify donors, and which initiate and coordinate the multiple tasks following donors' identification.

3. A model of production of transplant care services

The simple medical care system that we consider here is made of care production units, named hospitals, and a transplant agency in charge of collecting transplants produced from cadavers by hospitals, and of distributing them to transplant care units. The use of grafts by hospitals is constrained by the following two complementary rules: they must transfer to the transplant agency any graft they produce; and they must use for transplant care services any graft they receive from the transplant agency. We suppose, for simplicity, undifferentiated resources and needs in transplant inputs (say, a single medical indication for transplantation, such as kidney pathology, for example), and hospitals identical in all respects except their potential resources in graft inputs (their brain-dead patients, principally).

3.1. Agents and commodities

There are n hospitals, $n \geq 2$, designated by an index i running in $N = \{1,\ldots, n\}$. The transplant agency is denoted by index $i = 0$.

We partition the set of care services provided by hospitals into two broad classes, namely: Care services requiring transplants of organs or tissues such as heart, kidney, liver, lung, skin, cornea, bone marrow, etc.; and all other care services. We assume that the transplant care services of hospital i, on the one hand, and its other care services, on the other hand, are measurable by homogeneous continuous variables, respectively denoted by x_i and y_i. Moreover, each hospital i produces grafts from cadavers in homogeneous continuous quantity z_i. The final output of the medical care system in transplant care services (resp. other care services) is vector $x = (x_1,\ldots, x_n)$ (resp. $y = (y_1,\ldots, y_n)$). Its intermediary production of transplants is vector $z = (z_1,\ldots, z_n)$. We denote by $z_{n/i}$ the vector obtained from z by deleting its ith-component z_i, and by $(z_{n/i}, z'_i)$ the vector obtained from z and z' by substituting z'_i for z_i in z.

Likewise, we bunch the inputs of the production of care services in two broad types, also viewed as homogeneous continuous quantities, that is, for any hospital i: Transplants, denoted by real variable t_i; and other inputs, labelled “general” inputs in the following, and denoted by real variables v_i^x if they are used in the production of transplant care services, v_i^y if they are used in the production of other (final) care services, and v_i^z if they are used in the production of grafts. We let $v_i = (v_i^x, v_i^y, v_i^z)$, $v^r = (v_1^r,\ldots, v_n^r)$ for any $r \in \{x, y, z\}$, and $v = (v_1,\ldots, v_n)$.

We use the following notations for vectors of R^n, $n \geq 1$: e_n is the diagonal vector $(1,\ldots, 1)$ of $\mathbb{R}^n$; for any pair (x, x') of vectors of $\mathbb{R}^n$, $x \geq x'$ if $x_i \geq x'_i$ for all i, $x > x'$ if $x \geq x'$ and $x \neq x'$, $x \gg x'$ if $x_i > x'_i$ for all i; $\mathbb{R}^n_+$ is the non-negative orthant of $\mathbb{R}^n$, that is, set $\{x \in \mathbb{R}^n\text{: } x \geq 0\}$, and $\mathbb{R}^n_{++}$ is its positive orthant $\{x \in \mathbb{R}^n\text{: } x \gg 0\}$.

3.2. Feasibility conditions

Hospitals' potential of graft production is mainly determined, in practice, by the random distribution of brain-dead patients in hospitals and by refusal rates in donation interviews. This essential feature of the reality of transplant activities, which may be appropriately construed as a set of operative rationing constraints on both graft production and transplant care services, is captured in the model notably through an exogenous endowment of potential graft production of the hospital, viewed as a non-negative homogeneous continuous quantity, and denoted by ω_i for hospital i.[5] This endowment operates as an upper bound on a hospital's graft production, that is, $z_i \leq \omega_i$. We let $\omega = (\omega_1,\ldots, \omega_n)$, and suppose that $\omega \gg 0$.

Hospitals can purchase any quantity of general inputs $v_i^x + v_i^y + v_i^z$ on perfectly competitive markets of inputs at fixed market price w. Graft provision is non-profit: It is billed at production cost to the transplant agency, which collects transplants and redistributes them to care units free of charge. Each hospital i finances its general inputs for care services from a *fixed* budget B, the same for all i, subject to budget constraint $w(v_i^x + v_i^y) \leq B$. The latter imposes an upper bound B/w on its aggregate consumption of general inputs for care services $v_i^x + v_i^y$. The market price of general inputs is normalized to 1 in the following, that is, we let $w = 1$, without loss of generality.

Budget B may be viewed as the (constant) short-run cost function of the hospital. An appropriate interpretation of this feature of the model is the following: Hospitals' production capacities are fixed, and rigidly determine the total amount of variable inputs $v_i^x + v_i^y$ (including capital consumption) required for final production.[6]

Technically efficient production of hospital i is depicted through a triple of production functions $f_i = (f_i^x, f_i^y, f_i^z)$ transforming non-negative combinations of inputs $(t_i, v_i) \in \mathbb{R}^4_+$ into technically efficient output combinations $(x_i, y_i, z_i) = (f_i^x(t_i, v_i), f_i^y(t_i, v_i), f_i^z(t_i, v_i))$. Assumption 1 below supposes, in addition to the standard working hypotheses of differentiability and concavity, the following main features of hospitals' identical production techniques. General inputs are *indispensable* for production of any type (Assumption 1(ii)), and are *productive* in each type of production taken separately (Assumption 1 (iv) and (v)) and also in the three types of production taken jointly (Assumption 1(vi)). Transplants are *indispensable* and *productive* in transplant care services (Assumption 1(iii)–(iv)), and in them only (Assumption 1(v)). Technology exhibits a *congestion externality* between the three types of activities of each hospital (transplant care services, other care services, and graft production), specified as follows: A ceteris paribus increase in the scale of production, measured by the total amount of variable inputs $v_i^x + v_i^y + v_i^z$ used in

[5] Refusal rates in donation interviews, in particular, are treated as exogenous in this model, the latter's object being the analysis of the efficiency of production organization, from organ retrieval to transplantation. Diminishing refusal rates and improving the organization of production are the two main channels for improving the global efficiency of transplant care systems as measured by their donor population rates (Tiessen et al., 2008). The first channel supposes appropriate exhortation policies, which may include an adequate management of donation interviews (see Thorne, 1996, 2006: 5.1 for an empirical estimation of the productivity of exhortation spending; see also elements 4 and 5 of the list of causes of the Spanish success in Tiessen et al., 2008, p. 39). Spain's low refusal rate accounts for a part of its high relative performance in terms of donor rate, but seemingly not for the main part of it. Comparing, for example, the refusal and donor rates of France and Spain, one can produce rough estimates of the relative contributions of exhortation policy (say, the “exhortation effect”) and production organization (say, the “organization of production effect”) to the productivity gap between these two countries quite simply as follows: Substituting the French refusal rate (27%) for the Spanish one (15%) in Spanish donation data yields a Spanish donor rate net of the difference in exhortation policies of $36.4 \times \frac{1-0.27}{1-0.15} = 31.26$ per million; the latter implies relative contributions of the exhortation effect and the organization of production effect to the productivity gap that are respectively of $\frac{36.4-31.26}{36.4-25.3} = 46.3\%$ and $\frac{31.26-25.3}{36.4-25.3} = 53.7\%$. Similar calculations conducted on UK data yield similar conclusions, namely, an exhortation effect and a production organization effect respectively accounting for 44% and 56% of the productivity gap between Spain and the UK.

[6] Transplantation represents only a marginal part of hospitals' total output. Variation in the share of transplantation activity in output is unlikely to induce any significant variation in a hospital's production costs, in realistic circumstances. The assumption of a fixed budget is a simple way of capturing this fact. It does not entail any substantial loss of realism for subsequent analysis, and allows us to concentrate on the main issues of transplant economics, which are basically non-monetary, namely, transplant under-provision and shortage (the conditions of determination of vector z, in the model). In terms of hospital's technology, this assumption means that the short-run cost function is invariant to changes in the feasible efficient combinations of final output (x_i, y_i) (that is, all efficient combinations compatible with the hospital's fixed production capacities have the same “content” in general inputs $v_i^x + v_i^y = B$). More concretely, we suppose that additional transplantations substitute for other types of surgery having the same average cost.

the hospital, diminishes the productivity of general inputs in all types of production of this hospital, due to the congestion of a number of fixed capital inputs implicit in the production function, such as wards, operating theatres, surgery teams, etc. (Assumption 1(iv) and (v)). Example 1 of Section 4.2.2 below provides an example of a Cobb–Douglas technology that verifies Assumption 1.

Assumption 1. (i) For all $r \in \{x, y, z\}$, f_i^r is of the type $(t_i, v_i) \to g_i^r(t_i, v_i^r, v_i^x + v_i^y + v_i^z)$, where g_i^r is continuous and concave in $\mathbb{R}_+^3$, C^2 and strictly concave in $\mathbb{R}_{++}^3$. (ii) $g_i^r(t_i, v_i^r, v_i^x + v_i^y + v_i^z) = 0$ whenever $v_i^r = 0$. (iii) $g_i^x(t_i, v_i^x, v_i^x + v_i^y + v_i^z) = 0$ whenever $t_i = 0$. (iv) g_i^x is >0, increasing in t_i, totally increasing in v_i^x, and is decreasing in total general input $v_i^x + v_i^y + v_i^z$ in $\mathbb{R}_{++}^3$ (that is, precisely: $g_i^x > 0$, $\partial_1 g_i^x > 0$, $\partial_2 g_i^x + \partial_3 g_i^x > 0$ and $\partial_3 g_i^x < 0$ in $\mathbb{R}_{++}^3$, where $\partial_k g_i^x$ denotes the partial derivative of g_i^x with respect to its k-th argument, $k \in \{1, 2, 3\}$). (v) For all $r \in \{y, z\}$, g_i^r is everywhere constant in t_i; it is >0, C^2, totally increasing in v_i^r, and decreasing in $v_i^x + v_i^y + v_i^z$ in $\mathbb{R}_+ \times \mathbb{R}_{++}^2$ (i.e., with the notations above: $\partial_1 g_i^r = 0$; $g_i^r > 0$, $\partial_2 g_i^r + \partial_3 g_i^r > 0$ and $\partial_3 g_i^r < 0$ in $\mathbb{R}_+ \times \mathbb{R}_{++}^2$). (vi) For all $(t_i, v_i) \in \mathbb{R}_+^3$ and all neighbourhood V of (t_i, v_i) in $\mathbb{R}_+^3$, there exists $\tilde{v}_i \in \mathbb{R}_+^2$ such that $(t_i, \tilde{v}_i) \in V$ and $g^r(t_i, \tilde{v}_i, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z) > g^r(t_i, v_i, v_i^x + v_i^y + v_i^z)$ for all $r \in \{x, y, z\}$. (vii) Hospitals' production constraints are identical, except for the upper bound on graft production, that is, there exists a triple of functions (g^x, g^y, g^z) such that, for all i: $(g_i^x, g_i^y, g_i^z) = (g^x, g^y, g^z)$.

Summing up, the physical constraints limiting a hospital's production are fourfold: fixed graft resources, imposing an upper bound on organ retrieval ($z_i \le \omega_i$); fixed capacity, imposing an upper bound on the total quantity of general inputs for final production ($v_i^x + v_i^y \le B$); the fixed amount t_i of transplants received from the agency; and input productivity, determined notably by the level of congestion of production capacities. They are summarized in the following set of feasible alternatives of hospital i: $A_i(t_i, \omega_i) = \{(x_i, y_i, z_i, v_i) \in \mathbb{R}_+^6 : z_i \le \omega_i,\ v_i^x + v_i^y \le B$, and $(x_i, y_i, z_i) \le g(t_i, v_i)\}$, where g is the mapping $(t_i, v_i) \to (g^x(t_i, v_i^x, v_i^x + v_i^y + v_i^z), g^y(t_i, v_i^y, v_i^x + v_i^y + v_i^z), g^z(t_i, v_i^z, v_i^x + v_i^y + v_i^z))$.

Finally, the transplant agency is endowed with fixed budget B_0, sufficient to cover the cost of graft production for any feasible z, that is, $B_0 \ge \sum_{i \in N} v_i^z$ for all v such that $g^z(t_i, v_i) \le \omega_i$ for all i. This notably implies, realistically enough we believe, that the rationing constraints on organ transplantation are entirely driven by technical and endowment limitations: They owe nothing in this model, and owe very little in practice, to the financial constraints of the medical care systems of developed economies. The set of feasible alternatives of the agency reads: $A_0(z) = (t = (t_1, \ldots, t_n) \in \mathbb{R}_+^n : \Sigma_{i \in N} t_i \le \Sigma_{i \in N} z_i)$.

3.3. Hospital's production possibility frontier

All relevant characteristics of a hospital's constraints can be conveniently summarized through the *production possibility frontier* of the hospital, describing the set of its accessible and technically efficient output combinations (x_i, y_i, z_i).

The details of its construction and properties are developed in the appendix (see Appendix A1). In particular, the quantity of general inputs required to produce z_i when the hospital works at full capacity ($v_i^x + v_i^y = B$) is well-defined, determined implicitly by $z_i = g^z(t_i, v_i^z, B + v_i^z)$. It is denoted by $(g_B^z)^{-1}(z_i)$. Likewise, the production of general care services y_i accessible at full capacity from any fixed accessible (x_i, z_i) and any fixed *positive* t_i is well-defined and positive. It is denoted by $F(x_i, z_i, t_i)$. The production possibility frontier is then defined as follows:

Definition 1. The *production possibility frontier* of hospital i is: set $\{(x_i, y_i, z_i) \in \mathbb{R}_+^3 : x_i = 0, y_i = g^y(0, B, B + (g_B^z)^{-1}(z_i))$ and $z_i \le \omega_i\}$ if $t_i = 0$; set $\{(x_i, y_i, z_i) \in \mathbb{R}_+^3 : x_i \le g^x(t_i, B, B + (g_B^z)^{-1}(z_i)), y_i = F(x_i, z_i, t_i)$ and $z_i \le \omega_i\}$ if $t_i > 0$.

Fig. 1A and 1B represents the canonical projection of some production possibility frontier on plane (x_i, y_i) for fixed pairs (z_i, t_i) such that t_i is respectively null (Fig. 1A) and positive (Fig. 1B).

The production and circulation of grafts through the transplant organization induce in-kind costs and benefits for hospitals, which notably imply public good externalities of the technological type. They are captured through hospitals' production possibility frontiers in the following way.

Any increase in graft production z_i ceteris paribus induces a contraction of the hospital's set of accessible final production (x_i, y_i), that is, a downward shift of the graph of $x_i \to F(x_i, z_i, t_i)$ in plane (x_i, y_i) (see Lemma 1 and Fig. 1C). This is due to the congestion effects of an increased use of general inputs in graft production, which lowers the productivity of general inputs in final production when the hospital works at full capacity. In other words, quite concretely, any additional organ retrieval delays some medical care by immobilizing fixed inputs (a surgical team and operating theatre, principally) for some period of time.

Symmetrically, any increase in transplant transfer t_i ceteris paribus induces an expansion of the hospital's set of accessible final production (x_i, y_i), that is, an upward shift of the graph of $x_i \to F(x_i, z_i, t_i)$ in plane (x_i, y_i) (Lemma 1 and Fig. 1D). This follows from the productivity of inputs (positive marginal productivities) and from the substitutability of transplant and general inputs in the production of transplant care services (as implied by the differentiability of g^x). It is interpreted, most appropriately, as follows: sending back home a newly transplanted patient after recovery saves more general inputs (housing, catering, working hours of medical staff, dialysis…,[7]) than are consumed in transplantation; the quantity of general inputs so released can then be freely reallocated between x_i and y_i production without inducing any additional congestion of production capacities.

These technological consequences of graft production and circulation are conveniently summarized in the *marginal rate of compensation* of graft contribution by graft transfer, defined as follows:

Definition 2. Let $t_i > 0$, and (x_i, y_i, z_i) belong to hospital i's associate production possibility frontier. The *marginal rate of compensation* (MRC) of transplant provision by transplant transfer at (x_i, y_i, z_i) is: $-\frac{\partial_2 F(x_i, z_i, t_i)}{\partial_3 F(x_i, z_i, t_i)}$.

The MRC synthesizes the *non-monetary costs and benefits, for hospitals, of the production and circulation of grafts*. It can be viewed as a measure of the (technological) *public good externalities* between hospitals in the transplant system, computed from the standpoint of each hospital. Precisely, $-\frac{\partial_2 F(x_i^*, z_i^*, t_i^*)}{\partial_3 F(x_i^*, z_i^*, t_i^*)}$ measures the marginal variation (increase) in the transplant transfer $t_i^*(>0)$ received by hospital i that is required for keeping the hospital's production constant ($= F(x_i^*, z_i^*, t_i^*)$), following a marginal increase in its graft production from z_i^*. It corresponds, geometrically, to the slope of the level curve through (z_i^*, t_i^*) of partial function $(z_i, t_i) \to F(x_i^*, z_i, t_i)$ in plane (z_i, t_i) (see Fig. 1E).[8] It is determined by the ratio of congestion costs to the marginal productivity of inputs, increasing in the former, as implied by the informal discussion above.[9]

A second essential characteristic of a hospital's technology that can be derived, more standardly, from its production possibility frontier is the *marginal rate of substitution* of general care services for transplant care services, defined as the marginal variation (decrease) in the provision of general care services that is required for maintaining the hospital's production combination on the production possibility frontier, following a marginal increase in the provision of transplant care services. It coincides with partial derivative $\partial_1 F(x_i, z_i, t_i)$ (see Lemma 1). It corresponds, geometrically, to the slope of the graph of

[7] Dialysis is more costly than kidney transplantation (e.g., Steiner, 2010, p. 258).

[8] More formally, it follows from Lemma 1 and the implicit function theorem that equation $F(x_i, z_i, t_i) - F(x_i^*, z_i^*, t_i^*) = 0$ implicitly defines t_i as a C^2 increasing function $[0, \omega_i] \to \mathbb{R}_{++}$ of z_i, the graph of which is the "level curve" of $(z_i, t_i) \to F(x_i^*, z_i, t_i)$through (z_i^*, t_i^*) in plane (z_i, t_i). The implicit function theorem moreover implies that the first derivative of this implicit function is $= -\frac{\partial_2 F(x_i^*, z_i, t_i)}{\partial_3 F(x_i^*, z_i, t_i)}$ at any point (z_i, t_i) of its graph.

[9] Calculations using the proof of Lemma 1 yield an $MRC = -\frac{1}{\partial_2 g^z + \partial_3 g^z}\left(\frac{\partial_3 g^x}{\partial_1 g^x} + \frac{\partial_2 g^x}{\partial_1 g^x} \cdot \frac{\partial_3 g^y}{\partial_2 g^y}\right)$, which is increasing in $|\partial_3 g^r|$ for all $r \in \{x, y, z\}$.

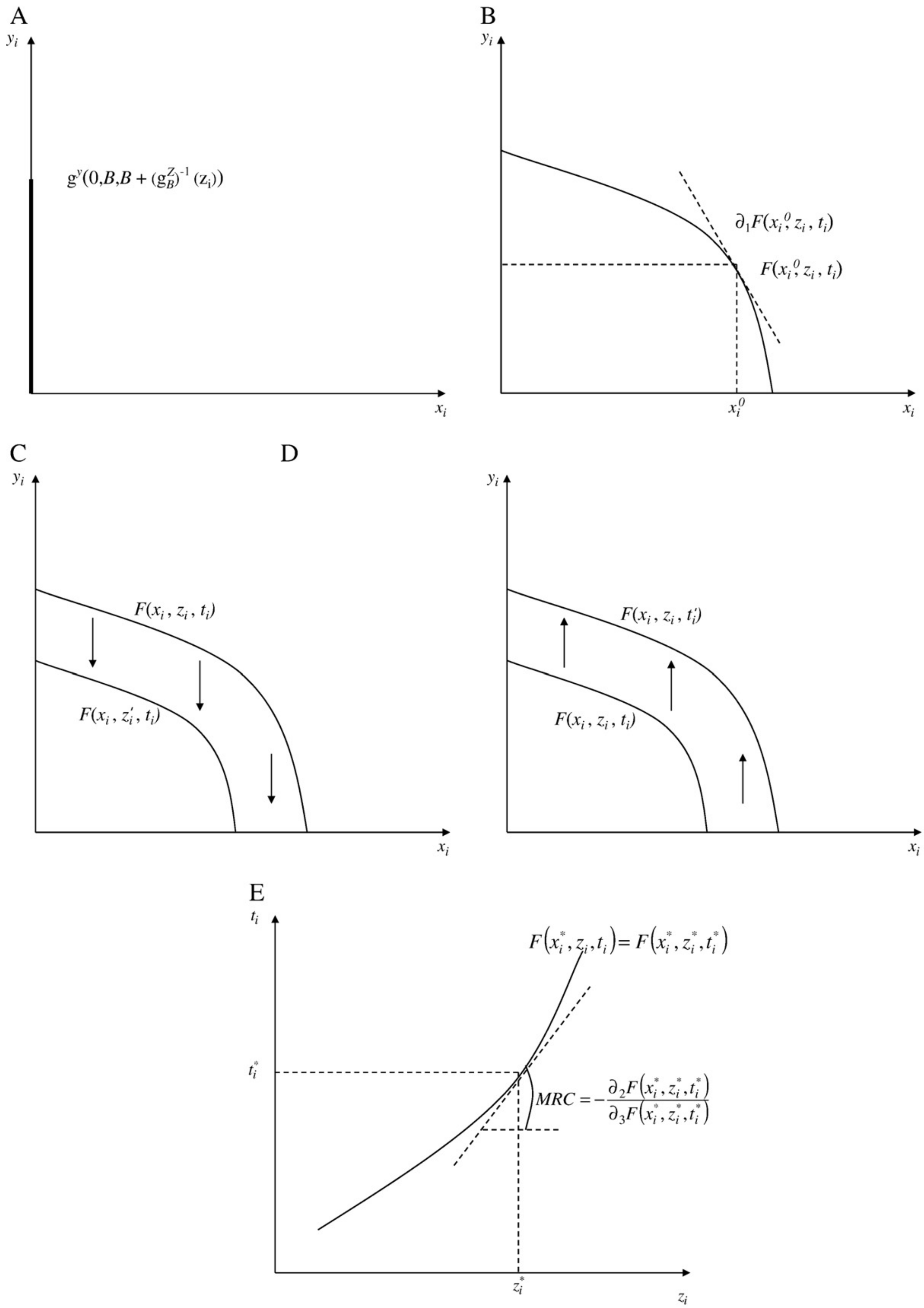


Fig. 1.

partial function $x_i \to F(x_i, z_i, t_i)$ in plane (x_i, y_i) (see Fig. 1B). We name it the *marginal rate of transformation* in the following, although it does not exactly coincide with the usual meaning of the latter notion, in order to distinguish it from the marginal rate of substitution defined from hospitals' utility function below.

4. Behavioural assumptions: preferences, interactions and equilibrium

In this section, we first return to one of the basic justifications for the existence of a transplant agency collecting and dispatching transplants (Section 4.1). We turn next to the modelling of the public good issues relative to transplants in the presence of the transplant agency (Sections 4.2.1 and 4.2.2), and finally design an optimal transplant care system (Section 4.3).

4.1. Transplant production optimum

Hospitals are viewed as rational agents, maximizing some objective function that depends on their provision of final care services. They can be non-profit or (partly) for-profit institutions.

Precisely, we suppose that hospital i maximizes utility function u, the same for all i, over the set of pairs (x_i, y_i) of combinations of transplant and other care services which it performs. We make the following set of standard assumptions on the utility function:

Assumption 2. Hospitals' utility function is a continuous, non-decreasing, quasi-concave function over $\mathbb{R}^2_+$, whose restriction to $\mathbb{R}^2_{++}$ is C^2 and strictly increasing. Moreover $u(x_i, y_i) > u(0)$ implies $(x_i, y_i) \gg 0$.

The convexity assumption on hospitals' preferences is compatible, in particular, with for-profit maximizing behaviour for *positive* production of final care services, that is, a hospital's choice of an optimal $(x_i, y_i) \gg 0$ determined by the relative price of the two types of care services (corresponding, geometrically, to a flat indifference curve through optimal $(x_i, y_i) \gg 0$, with slope equal to relative price).[10] The boundary condition of Assumption 2 may be interpreted as a social priority of the production of final care services, supported by implicit public policies. In other words, subsequent analysis applies to the special case of for-profit behaviour of care production units when equilibrium conditions imply an optimal production plan of hospitals that is sufficiently far from the axes.

In the absence of a market for transplant inputs, banned by law, and of any institutional substitute for the former such as a transplant agency, hospitals would be reduced to a situation of autarky, as far as transplant inputs are concerned, that is, to produce by themselves, from their own endowment ω_i and budget B_i, the grafts they use in their final production of transplant care services. Formally, each hospital would solve program max $\{u(x_i, y_i):(x_i, y_i, z_i, v_i, t_i) \geq 0, (x_i, y_i, z_i) \leq g(t_i, v_i), t_i \leq z_i \leq \omega_i$, and $v_i^x + v_i^y + v_i^z \leq B\}$, where the hospital's budget covers all expenses in general inputs, including the general inputs v_i^z used in intermediary graft production (the hospital's "autarkic" budget constraint). A production equilibrium of this autarkic transplant care system would then consist of an input–output combination of the care system (x, y, z, v, t) solving simultaneously the n independent programs of the hospitals. Common sense suggests that such unregulated equilibrium can very easily result in the waste of a part of total graft resources, that is, typically, in this highly aggregated model,[11] disposal, by best endowed hospitals, of the fraction of their endowment that exceeds the quantity of graft inputs they need for the provision of transplant care services that maximizes their utility in program above.

The disposal of a part of the resources of the hospitals that are best endowed in terms of their potential of graft production will very commonly appear as social waste if there exists a possibility of making a productive use of disposed resources in other hospitals, in terms of their final production of care services. Hospitals' endowments in the sense above (potential of graft production) being physically and legally non-transferable, the notion of production optimum implicit in this normative appreciation of "wasteful" disposal actually refers to implicit social preferences over pairs (x, y) of final production of the care system.[12] We now introduce such preferences, with the following basic normative priors, summarized in Assumption 3 below: The social preferences which aggregate hospitals' preferences are increasing in both types of final production of care services, express (like hospitals') a priority of production, and imply a preference for an "equal treatment of relevant equals," that is, a preference for an equal provision of final care services over hospitals whenever the latter is accessible.

Assumption 3. The social utility function is a continuous, non-decreasing, anonymous,[13] quasi-concave function $W: \mathbb{R}^n_+ \to \mathbb{R}_+$, whose restriction to $\mathbb{R}^n_{++}$ is a C^2, strictly increasing and strictly quasi-concave function $\mathbb{R}^n_{++} \to \mathbb{R}_{++}$. Moreover $(x, y) \to W(u(x_1, y_1),\ldots, u(x_n, y_n))$ is such that $W(u(x_1, y_1),\ldots, u(x_n, y_n)) > W(u(0),\ldots, u(0))$ implies $u(x_i, y_i) > u(0)$ for all i.

The utilitarian sum of hospitals' utility functions $\sum_{i \in N} u:(x, y) \to \sum_{i \in N} u(x_i, y_i)$ yields an example of a social utility function that verifies Assumption 3 if hospitals' utility function is strictly quasi-concave in $\mathbb{R}^2_{++}$.[14]

We can now introduce, as formal Definitions 3 and 4 below, two derived notions that will prove useful for the normative appreciation of production equilibrium, namely, the *socially efficient production* of final care services of the medical care system (in short, *production optimum*), and the *social scarcity of graft resources*.

Definition 3. A final production combination (x, y) of the medical care system, or associate input–output combination (x, y, z, v, t), is *socially efficient* if it maximizes the social utility function W in the set of socially accessible input–output combinations $\{(x,y,z,v,t) \in \mathbb{R}^{7n}_+ : \Sigma_{i \in N} t_i \leq \Sigma_{i \in N} z_i; z \leq \omega; \Sigma_{i \in N} v_i^x + v_i^y \leq nB$; and $(x_i,y_i,z_i) \leq g(t_i,v_i)$ for all $i\}$.[15]

[10] Letting p_x (resp. p_y) denote the price of x_i (resp. y_i), hospital i's profit from final production (x_i, y_i) reads $p_x x_i + p_y y_i - B$. Recall that we assumed a fixed budget B for simplicity, that is, essentially, a production cost independent of the scale of transplantation activity in accessible efficient combinations (x_i, y_i) (see the justification of this assumption in footnote 6 above).

[11] In a more accurate description of the medical care system, the problem under consideration here would be, realistically, formulated as mismatched vectors of potential graft resources (kidneys, corneas,…) and final transplant care services of the hospital at any moment in time. For an application of matching models and discrete optimization techniques to the health care system and the economics of transplants, see Roth et al. (2005a,b, 2006).

[12] Brain-dead patients cannot be physically transferred from one hospital to another mainly because of imperative legal constraints. In particular, lump-sum transfers of hospitals' graft endowments cannot be used as instruments of a public distribution policy in this context. Grafts are physically transferable between hospitals, subject to the legal constraints of the national transplant organization, but they must be retrieved on site, due to the reason above. Distribution ω cannot be an object of individual or social preference in our context (if preference underlies choice, as is assumed here, naturally). Graft production z is individually or socially valuable only as an intermediary for the final production of transplant care services x. Final production of care services seems, therefore, to be the most appropriate object of preferences, both at individual and at social level, in this model of the medical care system.

[13] The anonymity property states that any permutation in hospitals' names (and associate production (x_i, y_i)) leaves the social utility unchanged.

[14] This is not true anymore if hospitals are profit maximizers: social preferences, being strictly convex, cannot reduce to the sum of hospitals profits. Assumption 3 embodies two types of end-objectives of collective action, which, as suggested in the main text above, may be viewed as implied by some wider social preference relation aggregating the preferences of all concerned individuals (physicians, patients, citizens…). These objectives are the *utilization of the whole resource* in grafts (if the latter are scarce, which is generally assumed in the following) and *approximate equality in the distribution of transplants* between (relevantly equal) patients. We do not need a full-fledged social preference relation here, due to the narrowly defined object of the article, that is, the design of an appropriate *organization of the production* of transplant care services. We use, instead, a social preference relation exactly adjusted to the formulation of the specific organization issue that is addressed here, namely, coordination problems associated with the presence of public good externalities in production (Bergstrom, 1989; Cornes and Silva, 1999). These narrowly defined social preferences are so designed that their maximization entails the fulfilment of the two basic end-objectives outlined above.

[15] Note that the specification of the social opportunity set implied by this definition of the production optimum supposes, as already stated at the end of Section 3.2 above, that the constraints binding the final production of transplant care services, if any, are the rationing constraints on graft production, as opposed to the budget constraints limiting purchases of general inputs.

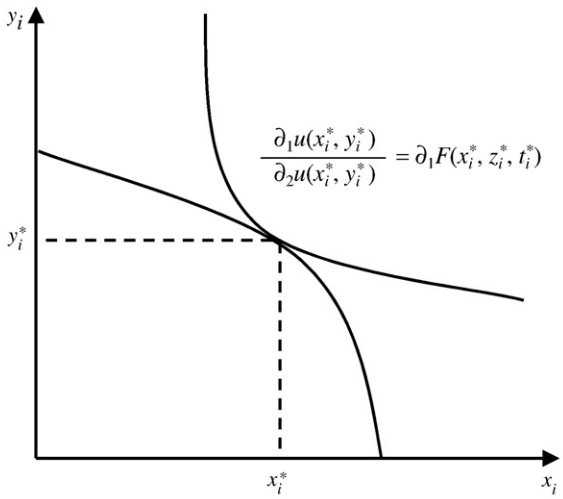


Fig. 2.

Definition 4. The graft resources of hospital i are *socially scarce* if a ceteris paribus increase in this hospital's endowment increases optimal social utility, that is, if max $\{W(u(x_1, y_1),\ldots, u(x_n, y_n)):(x, y, z, v, t)\geq 0;\ \sum_{i\in N}t_i \leq \sum_{i\in N}z_i;\ z\leq\omega';\ \sum_{i\in N}v_i^x+v_i^y\leq nB;$ and $(x_i, y_i, z_i)\leq g(t_i, v_i)\ \forall_i\}>$max $\{W(u(x_1, y_1),\ldots, u(x_n, y_n)):(x, y, z, v, t)\geq 0;\ \sum_{i\in N}t_i\leq\sum_{i\in N}z_i; z\leq\omega;\ \sum_{i\in N}v_i^x+v_i^y\leq nB;$ and $(x_i, y_i, z_i)\leq g(t_i, v_i)\ \forall_i\}$ whenever $\omega_i'>\omega_i$ and $\omega_{n/i}'=\omega_{n/i}$.

Theorem 1 in Appendix A2 characterizes production optimum and scarcity. It is conveniently reformulated in Corollary 1 below, using hospitals' production possibility frontiers. Notably, production optimum is such that: each hospital maximizes its utility subject to production possibilities (condition (ii) and Fig. 2); the marginal social utilities of transfers $\partial_i W\cdot\partial_2 u\cdot\partial_3 F$ are equal for all hospitals, implying that social utility is non-increasing relative to marginal variations in transfers (condition (iii)); retrieving one additional organ in hospital i and transferring it to any hospital (including i itself) does not increase social utility unless constraint $z_i\leq\omega_i$ is binding (complementary slackness $\delta_i\cdot(\omega_i-z_i^*)=0$ in condition (iii)).[16] Multipliers λ and δ_i are, respectively, the marginal social utility of transfers (and aggregate contribution $\sum_{i\in N}z_i$) and the marginal social utility of hospital i's contribution (and endowment ω_i) at social optimum. In particular, hospital i's graft resources are socially scarce if and only if $\text{MRC}_i<1$; and a lower MRC implies, ceteris paribus, more tension on the hospital's graft resources (a higher shadow value of the latter, both at the hospital and at the social level).[17]

Corollary 1. (x^*, y^*, z^*, t^*) is socially optimal if and only if: (i) $\sum_{i\in N}t_i^*=\sum_{i\in N}z_i^*$; (ii) for all i, $y_i^*=F(x_i^*, z_i^*, t_i^*)$ and $\frac{\partial_1 u(x_i^*,y_i^*)}{\partial_2 u(x_i^*,y_i^*)}=\partial_1 F(x_i^*,z_i^*,t_i^*)$; (iii) $z^*\leq\omega$ and there exists $(\lambda,\delta)\in\mathbb{R}_{++}\times\mathbb{R}_+^n$ such that, for all i, $\partial_i W\cdot\partial_2 u\cdot\partial_3 F=\lambda$, $\delta_i=\partial_i W\cdot\partial_2 u\cdot\partial_3 F\cdot\left(1+\frac{\partial_2 F}{\partial_3 F}\right)<\lambda$, and $\delta_i\cdot(\omega_i-z_i^*)=0$, where the partial derivatives are evaluated at the optimum. Hospital i's graft resources are scarce at production optimum if and only if this hospital's MRC is <1 at the optimum.

4.2. Regulated equilibrium with public good interactions

We suppose, from there on, that there exists a transplant agency of the type described at the beginning of this section, and moreover assume that this agency endorses the social preferences of Assumption 3. As noted above, the existence of a transplant agency induces public good externalities of the technological type between hospitals, as long as the latter control their production of grafts, that is: The graft production decided by any hospital has consequences on the production sets of all others through transplant redistribution by the agency (see Section 3.3, notably Fig. 1C to E).

Public good interactions between hospitals and the agency are modelled below through a device that has become standard in mechanism design theory, namely, subgame-perfect Nash equilibria of two-stage games (see notably, in the context of models of private contributions to a public good, Guttman, 1978, 1987; Bergstrom, 1989 or Cornes and Silva, 1999 and also the detailed references reviewed in Mercier Ythier, 2006: 6.3 and A.2.1). We successively consider three possible variants of the two-stage game, where hospitals and the agency alternate as first and second players in the game. The first two are defined below as the *myopic game* (Section 4.2.1) and the *clear-sighted game* (Section 4.2.2) respectively. The third notion of the two-stage game, labelled the *monitored game*, is defined and studied in Section 4.3.

4.2.1. Myopic equilibrium

In the first variant of the two-stage game, the transplant agency moves first, choosing transfers $(t_1,\ldots, t_n)$. This choice of the transplant agency is made knowing that, at the second stage of the game, each hospital i, having observed t_i, chooses a production combination (x_i, y_i, z_i, v_i) that maximizes its utility in its opportunity set $A_i(t_i, \omega_i)$. The agency anticipates, in other words, that hospitals' production plans, including contributions $(z_1,\ldots, z_n)$, depend on transfers, and take this dependence into account when choosing the transfers.

As is usual, subgame-perfect equilibrium is defined recursively, beginning with the second stage of the game. At second stage, each hospital i solves max $\{u(x_i, y_i):(x_i, y_i, z_i, v_i)\in A_i(t_i, \omega_i)\}$ with respect to (x_i, y_i, z_i, v_i) for any given t_i. We denote by φ_i hospital i's reaction correspondence at this stage, defined by $\varphi_i(t)=\arg\max\{u(x_i, y_i):(x_i, y_i, z_i, v_i)\in A_i(t_i, \omega_i)\}$, and let $\varphi=(\varphi_1,\ldots, \varphi_n)$. At first stage, the transplant agency solves max $\{W(u(x_1, y_1),\ldots, u(x_n, y_n)):(x, y, z, v)\in\varphi(t)$ and $t\in A_0(z)\}$ with respect to t. An *equilibrium* of the game is a state (x, y, z, v, t) that solves the latter program. We refer to this first notion of equilibrium as the *myopic equilibrium* in the following, due to the short-sighted free-riding behaviour of hospitals which it implies.

We have the following simple benchmark property for the myopic equilibrium:

Theorem 2. *The provision of transplant care services and grafts is null at myopic equilibrium.*

Proof. Graft production is costly for the hospital, due to its congestion effects on the hospital's production of final care services (x_i, y_i), and does not yield any advantage, ceteris paribus (that is, given agency's transfers), in terms of the hospital's utility. Therefore $z=0$, which implies $t=0$, which implies in turn $x=0$.[18] □

[16] The marginal variation in social utility from retrieving one additional organ in hospital i and transferring it to hospital j reads $\partial_i W\cdot\partial_2 u\cdot\partial_2 F+\partial_j W\cdot\partial_2 u\cdot\partial_3 F$. Since $\partial_i W\cdot\partial_2 u\cdot\partial_3 F=\partial_j W\cdot\partial_2 u\cdot\partial_3 F=\lambda$, this marginal variation is equal to $\partial_i W\cdot\partial_2 u\cdot\partial_3 F\cdot\left(1+\frac{\partial_2 F}{\partial_3 F}\right)=\delta_i$. Complementary slackness states that it is non-negative at production optimum, and positive only if $z_i^*=\omega_i$.

[17] In a similar fashion, i's marginal utility of its graft resources at the equilibrium of the transplant game studied in Section 4.2.2 below is $\partial_2 u\cdot\partial_3 F\cdot(\partial_i\varphi-\text{MRC}_i)$, where $\partial_i\varphi$ denotes i's marginal (graft) return on its graft contribution to the transplant agency (Lemma 3). In particular, $z_i\leq\omega_i$ binds at equilibrium if and only if $\text{MRC}_i<\partial_i\varphi$; and a lower MRC_i implies, ceteris paribus, more tension on ω_i. Therefore, the MRC can be viewed as an indicator of tension on the hospital's graft resources at optimum and at equilibrium as well.

[18] The set of myopic equilibria is $\{(x,y,z,v,t)\in\mathbb{R}_+^{7n}:x=z=t=0,\ v_i^z=0,\ v_i^x+v_i^y\leq B/w$ and $0\leq y_i\leq g^y(0, v_i^y, v_i^x+v_i^y)$ for all $i\}$, implying an equilibrium utility of hospitals and the agency everywhere equal to their minimal values in their respective domains, that is $u(0)$ and $W(u(0), \ldots, u(0))$ respectively.

The above result is interesting as a clear-cut, albeit extreme expression of the coordination problem of transplant activities. It is individually rational for myopic hospitals to free ride, or shirk, on graft production, that is, to attempt to shift onto others the congestion costs induced by graft production.[19] Myopia is interpreted as a lack of understanding, at individual level, of the collective damages that result from generalized free-riding, namely, the dramatic under-provision of transplants (no provision at all, in the case under consideration). The existence of a central agency collecting and redistributing transplants is not only insufficient, per se, for solving the public good problem; it also dramatically deteriorates production equilibrium, relative to the autarkic equilibrium, if public good interactions are of the myopic type.

4.2.2. Clear-sighted equilibrium

The consequence of Theorem 2 is too extreme to be accepted literally. Hospitals should be, and actually are, well aware of the damages of shirking (in the sense of footnote 19) to the medical care system as a whole, and for themselves as a part of it. Myopia does not appear a realistic assumption, in other words, both a priori and in view of its logical implication.

In the variant of the two-stage game that we introduce now, hospitals move first, each one choosing the production combination (x_i, y_i, z_i, v_i) that maximizes its utility in its opportunity set $A_i(t_i, \omega_i)$. A hospital's choice is made taking the contributions of others as given and knowing that at the second stage of the game, the transplant agency, having observed contributions $(z_1,\dots, z_n)$, chooses the transfers $(t_1,\dots, t_n)$ that maximize the social utility function subject to aggregate resource constraint $\sum_{i\in N} t_i \le \sum_{i\in N} z_i$ and hospitals' production possibilities. Hospitals anticipate, in other words, the dependence of the agency's transfers on their contributions, and take this dependence into account when making their production decisions.

Subgame-perfect equilibrium is defined recursively as follows. At the second stage of the game, the agency solves max $\{W(u(x_1, y_1),\dots, u(x_n, y_n)):(x_i, y_i, z_i, v_i)\in A_i(t_i, \omega_i)$ for all i, and $t\in A_0(z)\}$ with respect to (x, y, v, t) for any given $z\le\omega$. We denote by $\varphi^0=(\varphi_1^0,\dots, \varphi_n^0)$ the agency's transfer correspondence at this stage, where φ_i^0 yields the agency's optimal transfers to hospital i for any fixed z. Hospitals play first, each one solving max $\{u(x_i, y_i)$: $(x_i, y_i, z_i, v_i)\in A_i(t_i, \omega_i)$; $t_i\in\varphi_i^0(z)\}$ with respect to (x_i, y_i, z_i, v_i) for any given vector of graft production of other hospitals $z_{n/i}$. An equilibrium of the game is a Nash non-cooperative equilibrium of the first-stage game, that is, a state $(x^*, y^*, z^*, v^*, t^*)$ such that: $t^*\in\varphi^0(z^*)$; and for all i, $(x_i^*, y_i^*, z_i^*, v_i^*)$ solves max $\{u(x_i, y_i):(x_i, y_i, z_i, v_i)\in A_i(t_i, \omega_i)$; $t_i\in\varphi_i^0((z_{n/i}^*, z_i))\}$. We name it the *clear-sighted equilibrium* in the following, because it embodies hospitals' clear awareness of the public good externality associated with graft production, and individual damages from free-riding behaviour that it implies for them.

We establish below that clear-sightedness, if it actually improves the functioning of the transplant care system relative to the myopic game, by implying a positive production of grafts and transplant services (Theorem 3(i)), nevertheless does not suffice for solving the under-provision problem. Precisely, it is shown that a fraction of the system's resources for graft production remains unexploited, in general, at clear-sighted equilibrium when graft resources are socially scarce (Theorem 3(ii)).

We restrict attention, in this section, to the medical care systems that have clear-sighted equilibria. The existence property of clear-sighted equilibrium is analyzed in detail in Appendix A5. It is shown there (Lemma 5) that the critical feature which conditions existence is that hospitals' first-stage reaction correspondences be convex-valued. A minimal sufficient condition on preferences and technology for the latter is that the first-stage reduced form of hospitals' utility functions $(x_i, z_i)\to u(x_i, F(x_i, z_i, \varphi_i(z))$ be quasi-concave (Appendix A5: Lemma 4). Formally:

Definition 5. The medical care system (W, u, g, ω) is *convex* if, for all i, the first-stage reduced form of hospital i's utility function $(x_i, z_i)\to u(x_i, F(x_i, z_i, \varphi_i((z_{n/i}^*, z_i)))$ is quasi-concave over $\{(x_i,z_i)\in\mathbb{R}_+^2$: $0<z_i\le\omega_i\}$ for all $z^*\in\{z\in\mathbb{R}_+^n$: $z\le\omega\}$ and quasi-concave over $\{(x_i,z_i)\in\mathbb{R}_+^2$: $0\le z_i\le\omega_i\}$ for all $z^*\in\{z\in\mathbb{R}_+^n$: $z\le\omega$; $0<z_{n/i}\}$.

Theorem 3. *Let (W, u, g, ω) be convex. (i) Clear-sighted equilibria exist and are $\gg 0$. (ii) Transplant care services are underprovided, in general, at clear-sighted equilibrium (that is, equilibrium graft production is $<\omega$) when hospitals' graft resources are all scarce at production optimum. (iii) Clear-sighted equilibrium is a production optimum notably if programs max $\{u(x_i, F(x_i, z_i, z_i))$:$z_i\le\omega_i\}$ yield a same solution (x_i, z_i) for all hospitals.*

The details of the proof are given in Appendix A6. We concentrate here on the essence of the argument underlying the second and third parts of the theorem.

The first-order conditions characterizing production optimum (Corollary 1) and clear-sighted equilibrium (Lemmas 2 and 3) differ on a single main point, namely, hospitals' marginal utilities of graft resources, which read $\partial_2 u\cdot\partial_3 F\cdot(1-\mathrm{MRC}_i)$ at production optimum and $\partial_2 u\cdot\partial_3 F\cdot(\partial_i\varphi_i^0-\mathrm{MRC}_i)$ at social equilibrium.[20] The transfer policy φ^0 of the transplant agency will therefore completely solve the coordination problem of the care system, that is, make hospitals' equilibrium and optimum evaluation of graft resources coincide in all circumstances if, and, in general, only if $\partial_i\varphi_i^0(z)=1$ for all i and all z, that is, if φ^0 is the identity function $z\to z$ of $\mathbb{R}^n$. The latter consists of returning to each hospital its contribution in all circumstances (a "status quo" transfer policy).

Clearly enough, this complete solution to the coordination problem should, in most circumstances, conflict with the end-objectives of allocation efficiency and distribution equity implied by the social preference relation. The only notable exception corresponds to the case where hospitals spontaneously achieve production optimum because rationing constraints are either non-binding, as in Example 2 (Appendix A4), or identical for all of them. In realistic circumstances, where rationing constraints are binding and there is some diversity in hospitals' endowments, the status quo transfer policy is not optimal for the agency, that is, does not yield equal social marginal utilities of transfers for all hospitals. The agency's transfer policy then generally induces discrepancies between hospitals' marginal valuations at production optimum and at equilibrium, through non-unitary marginal returns on contributions $\partial_i\varphi_i^0$.

In Example 1 below, we present a family of calculable care systems in which the agency's second-stage optimal transfer policy is the equal sharing of aggregate contribution, that is, $\varphi_i^0(z)=\frac{1}{n}\sum_{i\in N} z_i$ for all i and z. Hospitals' marginal return on contribution $\partial\varphi_i^0(z)$ is

[19] Note that the formulation of the public good problem as a pure coordination problem here and below does not suppose imperfect or costly information. The reason for this is empirical: Accounts of the Spanish and other experiences of national transplant systems we are aware of put little emphasis, if any emphasis at all, on information issues in interactions (that is, strategic manipulations of information asymmetry by care units). The main difficulty, as far as production units are concerned, seems to be self-centredness, understood as the propensity of each hospital to concentrate on its own patients, and subsequent reluctance to consider costly actions that are not directly related to this priority. One of the main lessons of the Spanish experience, it seems to us, is that most problems are solved by simply discharging hospitals, in some appropriate way, from the concern of on-site organization of graft production (including identification of potential donors, and donation interviews).

[20] The other difference lies in the specification of budget constraints, namely, the aggregate budget constraint at production optimum versus individual budget constraints at equilibrium. We assume implicitly here and explicitly in the case of the monitored equilibrium studied in the next section that production optimum is always decentralizable, in the sense that if an input–output combination of the care system can be achieved from its aggregate budget, then it can also be achieved from the set of hospitals' individual budgets. This assumption is not very demanding in our setup, since hospitals are assumed identical in all respects except graft endowment, and production optimum verifies hospitals' rationing constraints by definition.

therefore equal to $1/n$ in the example, hence smaller than 1 if there is more than one hospital and decreasing to 0 as the number of hospitals grows to infinity. We show that clear-sighted equilibrium is reduced then to an example of the general class of symmetric Nash equilibrium with public goods of Chamberlin (1974)[21] when the number of hospitals is sufficiently large. In particular: hospitals' individual contribution is positive, decreasing in the number of hospitals, and tends asymptotically to 0 as the latter grows to infinity; hospitals' aggregate contribution increases in the hospitals' number, at a lower speed than the latter. This notably implies that no rationing constraint is binding in first-stage hospitals' programs, and that clear-sighted equilibrium is therefore independent of the initial distribution of graft resources, when the number of hospitals is sufficiently large. Moreover, we show that graft resources are scarce at production optimum, whatever the number of hospitals, for suitable equal distributions of graft resources.

Transfers actually practiced by transplant agencies certainly are much closer to equal sharing policy $z \to \frac{1}{n}\sum_{i\in N} z_i$ than to the status quo policy $z \to z$, so that hospitals' marginal returns on contribution $\partial\varphi_i^0(z)$ should be considered much closer to $1/n$ than to 1 in reality, hence much closer to 0 than to 1 in view of actual numbers of care production units in charge of providing the transplant care services (155 in the case of Spain, for example: see Section 2 above). Inefficient under-provision, therefore, seems the most plausible outcome of the clear-sighted game, for realistic assumptions on the agency's policy and the number of production units.

Theorem 3(iii), finally, is a rotten kid theorem (Becker, 1974, 1981). Precisely, it identifies configurations of principal–agent interactions where the optimal transfer policy of the (benevolent) principal drives the (non-cooperative, self-centred) agents to implement a production optimum which coincides with the principal's optimum. The most significant of them was already mentioned above, as the case of free graft resources (non-binding rationing constraints). This property of the model, and its relations to the analogous properties of Becker (1974, 1981), Bergstrom (1989) and Cornes and Silva (1999) are analyzed in Appendix A6.

Example 1. A calculated example of Olson–Chamberlin under-provision.

We study the following calculable medical care system (W, u, g, ω): production functions are the concave Cobb–Douglas $g(t_i, v_i) = (v_i^x + v_i^y + v_i^z)^{-\frac{1}{2}}\left((t_i v_i^x)^{\frac{1}{2}}, (v_i^y)^{\frac{1}{2}}, (v_i^z)^{\frac{1}{2}}\right)$; hospitals' utility function is the log linear $u(x_i, y_i) = \log x_i + \log y_i$; social utility function is the utilitarian sum $W(u_1(x_1, y_1), \ldots, u_n(x_n, y_n)) = \sum_{i\in N} u(x_i, y_i)$. It verifies Assumptions 1–3. Associate function F reads $F(x_i, z_i, t_i) = \left(1 - z_i^2 - \frac{x_i^2}{t_i}\right)^{\frac{1}{2}}$. The first-order conditions of Lemma 2 then yield $\varphi_i^0(z) = (1/n)\sum_{i\in N} z_i$ for all i, that is, the agency's optimal distribution policy is *equal sharing of aggregate hospitals' contribution*.[22] Substituting optimal transfer $\varphi_i^0(z)$ for t_i in F yields the following reduced form for hospital i's first-stage objective function: $u\left(x_i, F\left(x_i, z_i, \varphi_i^0(z)\right)\right) = \log x_i + (1/2)\log\left(1 - z_i^2 - n\frac{x_i^2}{\sum_{j\in N} z_j}\right)$, viewed as a function of (x_i, z_i) for fixed $z_{n/i}$. The path of hospital i's optimal final production conditional on z_i is $\left\{\left(\left(t_i\frac{1-z_i^2}{2}\right)^{\frac{1}{2}}, \left(\frac{1-z_i^2}{2}\right)^{\frac{1}{2}}\right) : 0 \le z \le \omega_i\right\}$. By further restricting to this path the objective function above, we get the following final reduced form for hospital i's first-stage program max $\{(1/2)\log\sum_{j\in N} z_j + \log(1 - z_i^2) - (1/2)\log n - \log 2 : 0 \le z_i \le \omega_i\}$, where the objective function is (differentiably) strictly concave. Let us provisionally ignore the rationing constraint in the latter program. The first-order necessary and sufficient condition for an unconstrained maximum reads $5z_i^2 + 4(\sum_{j\in N: j\neq i} z_j) z_i - 1 = 0$. Solving for z_i yields the unique >0 solution $z_i = -(2/5)\sum_{j\in N: j\neq i} z_j + (1/5)\sqrt{5 + 4(\sum_{j\in N: j\neq i} z_j)^2}$. Letting $z_i = z^*$ for all i in the solution and solving for z^* yields the symmetric individual contributions $z^* = \frac{1}{\sqrt{4n+1}}$. In particular, there exists n_0 such that $z^* \cdot e_n \ll \omega$ for all $n \ge n_0$, implying that z^* is a symmetric equilibrium contribution of the medical care system, with non-binding rationing constraints, when the number of hospitals is at least as large as n_0. This is then the unique equilibrium contribution, as a special case of Cornes and Hartley (2007).[23] Equilibrium individual contribution lies in]0, 1[for all $n \ge n_0$. It is decreasing, asymptotically equivalent to $\frac{1}{2\sqrt{n}}$, converging to 0 as the number of hospitals grows to infinity, while aggregate equilibrium contribution $\sum_{i\in N} z_i = \frac{n}{\sqrt{4n+1}}$ is increasing, growing to infinity with the number of hospitals but at a lower speed than the latter.[24] One verifies easily from the first-order conditions of Theorem 1 that these equilibria are socially inefficient. The marginal social utility of hospitals' aggregate contribution is $\lambda = \frac{n}{2\sum_{i\in N} z_i}$, and the marginal social utility of hospital i's graft resources is $\delta_i = \frac{n}{2\sum_{i\in N} z_i} - 2\frac{z_i}{1 - z_i^2}$ for all i in the f.o.c. Letting $z_i = \frac{1}{\sqrt{4n+1}}$ for all i in the latter yields positive values of δ_i for all $n \ge 2$, which are inconsistent with production optimality for $n \ge n_0$ (since $\frac{1}{\sqrt{4n+1}} < \omega_i$ for all i then). Suppose, finally, that initial endowments are equally distributed, that is, $\omega = \tilde{\omega} \cdot e_n$ for some $\tilde{\omega} \in \mathbb{R}_{++}$ for all n. The MRC at $(x_i, z_i, t_i) \gg 0$ is $2z_i(t_i/x_i)^2$. On the path of hospital i's optimal final production conditional on z_i, and for agency's optimal transfer associated with z, this yields: $4.\frac{\sum_{i\in N} z_i}{n}.\frac{1 - z_i^2}{z_i}$. Therefore, hospitals' graft resources are all scarce at production optimum if and only if $4\left(1 - \tilde{\omega}^2\right) \in]0, 1[$, that is, if and only if $\tilde{\omega} \in]0, \frac{1}{\sqrt{5}}[$.

4.3. Monitored graft production

A simple solution to the coordination problem raised in Section 4.2 is the monitoring of graft production by the transplant agency. This solution actually appears trivial in the setup above, from a logical point of view. It is interesting to develop because it captures, we believe, the organizational features of the Spanish transplant system that are at the origin of the latter's remarkable achievements analyzed in Section 2 above.

The model is amended as follows. The transplant agency hires physicians and delegates them in hospitals in order to supervise graft production in each of them, with an objective of maximization of the latter subject to the legal, technical and endowment constraints detailed above. Formally, this new organizational trait amounts to letting the agency decide (through its delegates in hospitals) on hospitals' levels of graft production $z = (z_1, \ldots, z_n)$. That is, the agency's *monitoring*

[21] See also the generalizations and extensions of Chamberlin's result by Andreoni (1988) and Fries et al. (1991), and the related literature reviewed in Mercier Ythier (2006: 6.2).

[22] Notably: f.o.c. $\frac{\partial_1 u(x_i, F(x_i, z_i, t_i))}{\partial_2 u(x_i, F(x_i, z_i, t_i))} = -\partial_1 F(x_i, z_i, t_i)$ yields $\frac{x_i^2}{t_i} = y_i^2$ for all i; substituting into f.o.c. $\partial_2 u(x_i, F(x_i, z_i, t_i)).\partial_3 F(x_i, z_i, t_i) = \lambda$ and adding up over i then yields both $\lambda = 2n/\sum_{j\in N} z_j$ and $t_i = (1/n)\sum_{j\in N} z_j$.

[23] The reader can check this by proceeding to the following change of variable: Let the utility function in the framework of Cornes et al. be $U(x_i, G) = \log(-x_i^2 + 2x_i) + (1/2)\log\sum_{j\in N} g_j + -(1/2)\log n - \log 2$, where x_i denotes their "private good" (not to be confused with our "provision of transplant care services"), $G = \sum_{j\in N} g_j$ is the public good, and g_j is j's individual contribution to G. Let their agent's endowment (not to be confused with our "potential of graft production") be $= 1$. Their reduced utility function, obtained by substituting budget constraint $x_i + g_i = 1$ in the former, is $U(1 - g_i, g_i + G_{-i}) = \log(1 - g_i^2) + (1/2)\log(g_i + G_{-i}) + -(1/2)\log n - \log 2$, where $G_{-i} = \sum_{j\in N: j\neq i} g_j$, which is identical to the reduced form of the utility function of our calculated example. A simple calculation shows that function $U(1 - g_i, g_i + G_{-i})$ verifies the normality condition of Chamberlin (1974), which implies, in turn, the condition for uniqueness of Cornes and Hartley (2007).

[24] $\frac{n}{\sqrt{4n+1}}$ is asymptotically equivalent to $\frac{1}{2}\sqrt{n}$. Its instantaneous growth rate is $\frac{1}{n} - \frac{1}{2n + \frac{1}{2}}$, which is positive and <1 for all $n \ge 1$, decreasing with n, asymptotically equivalent to $\frac{1}{2n}$, and, in particular, tending to 0 as n grows to infinity. The asymptotic behaviour of hospitals' contributions reproduces the qualitative features of the general property of Chamberlin (1974). We established above that the first-stage Nash equilibrium of this example reduces to a special case of Chamberlin's symmetric Nash equilibrium when the number of hospitals becomes large enough to make all rationing constraints slack at equilibrium.

opportunity set now reads $A_0^M(\omega)=\{\{(z,v^z,t)\in\mathbb{R}_+^{3n}\colon \Sigma_{i\in N}t_i\le\Sigma_{i\in N}z_i;\ z_i\le g^z(t_i,v_i^z,v_i^x+v_i^y+v_i^z)$ and $z_i\le\omega_i$ for all $i\}$, while its monitoring budget B_0^M now covers the wages of supervisors in addition to the other costs of transplant provision. Similarly, the hospitals' *monitored opportunity sets* are defined as: $A_i^M(v_i^z,t_i)=\{(x_i,y_i,v_i^x,v_i^y)\in\mathbb{R}_+^4\colon x_i\le g^x(t_i,v_i^x,v_i^x+v_i^y+v_i^z);\ y_i\le g^y(t_i,v_i^y,v_i^x+v_i^y+v_i^z)$, and $v_i^x+v_i^y\le B\}$.

The public good externality between hospitals vanishes in this new specification of the transplant system, since it followed from their individual choice of a level of graft production, which now is essentially endorsed by the agency. The transplant agency is the natural principal of the game in this setup. It moves first, choosing transfers and hospitals' graft production levels. This is done knowing that at the second stage of the game, each hospital i, having observed (z_i, v_i^z, t_i), chooses a final production (x_i, y_i) that maximizes its utility in its monitored opportunity set $A_i^M(v_i^z, t_i)$. Subgame-perfect equilibrium is specified, accordingly, as follows. Hospitals play second, each one solving max $\{u(x_i,y_i)\colon(x_i,y_i,v_i^x,v_i^y)\in A_i^M(v_i^z,t_i)\}$ with respect to (x_i, y_i, v_i^x, v_i^y) for any given (v_i^z, t_i). We denote by φ_i^M hospital i's *monitored* reaction correspondence at this stage (solving program above for any (v_i^z, t_i)), and let φ^M denote the associate product correspondence defined by $\varphi^M(v^z,t)=\{(x,y,v^x,v^y)\colon(x_i,y_i,v_i^x,v_i^y)\in\varphi_i^M(v_i^z,t_i)$ for all $i\}$. The transplant agency plays in the first stage, solving max $\{W(u(x_1,y_1),\ldots,u(x_n,y_n))\colon(x,y,v^x,v^y)\in\varphi^M(v^z,t)$ and $(z,v^z,t)\in A_0^M(\omega)\}$ with respect to (z, v^z, t). An *equilibrium* of the game is a state (x, y, z, v, t) that solves the latter program. We refer to this third notion of equilibrium as the *monitored equilibrium*.

We establish below that monitored equilibrium and production optimum coincide, provided that socially efficient production can be achieved by hospitals endowed with equal budgets B (Appendix A7). This optimality property implies, in particular, in view of Theorem 1, that a monitored equilibrium exists, and that the corresponding socially optimal production of final care services is unique.

Theorem 4. *Suppose that, for any production optimum (x, y, z, v, t), there exists a combination of general inputs $\tilde v$ such that $\tilde v_i^x+\tilde v_i^y=B$ and $(x_i, y_i, z_i)=g(t_i, \tilde v_i)$ for all i. Then, the monitored equilibrium is a production optimum.*

Theorem 4 implies a clear advantage of monitored equilibrium, relative to clear-sighted equilibrium, in terms of the production of final care services of the medical care system. Optimizing the distribution of transplants does not suffice, in other words, for achieving socially efficient production. The latter supposes that some control be exerted also on graft production. This implies in turn some additional monitoring costs, captured in the simple model above through the (positive) difference $B_0^M-B_0$ between the agency's budgets in the monitored and clear-sighted games. A complete comparative evaluation of the two modes of regulation of the transplant care system supposes that their differences in terms of socially efficient production be balanced against their differences in terms of budgetary costs. The data collected in Section 2 suggest that monitoring costs are actually low, relative to their remarkable impact on graft production. In other words, the Spanish experience displays a high productivity of monitoring expenses.

5. Conclusion

The economic organization of the transplant care system was characterized as a production economy of the non-market sector operating on the background of incomplete markets of inputs. The collection and circulation of transplants by the transplant agency induce public good interactions between hospitals. A socially optimal distribution policy of the agency cannot achieve alone the coordination of hospitals' production decisions at equilibrium and cannot in general attain alone the production optimum when graft resources are scarce, that is, equivalently, when the rationing constraints on the production of transplant inputs are binding at production optimum. Production optimum is attained by eliminating the public good interactions between hospitals through the optimal control of both the distribution and the production of transplant inputs by the agency. The data suggest that more than one half of Spain's donor rate differential with other countries proceeds from an adequate management of this public good problem by its national transplant organization. Improving the coordination of hospitals' production of transplants seems the principal and most efficient way for improving national donor rates. The other major way consists of lowering donation refusal rates through adequate exhortation policies and an adequate management of donation interviews.

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Appendix A

A1. Hospitals' production possibility frontier

Lemma 1. *There exists a function F such that, for any $(z_i,t_i)\in[0,\omega_i]\times\mathbb{R}_{++}$, the set of technically accessible output combinations of hospital i is: $\{(x_i,y_i,z_i)\in\mathbb{R}_+^3\colon x_i=g^x(t_i, B, B+(g_B^z)^{-1}(z_i)),\ y_i=F(x_i,z_i,t_i)$ and $z_i\le\omega_i\}$, where $(g_B^z)^{-1}$ denotes the inverse of increasing partial function $v_i^z\to g^z(t_i,v_i^z,B+v_i^z)$. Function F is defined over sets $\{(x_i,y_i,z_i)\in\mathbb{R}_+^2\times\mathbb{R}_{++}\colon x_i\le g^x(t_i, B, B+z_i^y);\ z_i=g^z(t_i,v_i^z,B+v_i^z);\ v_i^z\ge0\}$, and C^2 in the (non-empty) intersection of these convex domains with $\mathbb{R}_{++}^3$. It is decreasing and strictly concave in x_i, decreasing in z_i and increasing in t_i. Its partial derivatives read: $\partial_1F=-\frac{\partial_2g^y}{\partial_2g^x}$, $\partial_2F=\frac{1}{\partial_2g^z+\partial_3g^z}\left(\frac{\partial_3g^x}{\partial_2g^x}\partial_2g^y+\partial_3g^y\right)$, and $\partial_3F=\frac{\partial_1g^x}{\partial_2g^x}\partial_2g^y$ where the partial derivatives of F, g^x, g^y and g^z are respectively evaluated at (x_i, z_i, t_i), (t_i, v_i^x, s), $(t_i, B-v_i^x, s)$ and $(t_i, (g_B^z)^{-1}(z_i), s)$ such that $s=B+(g_B^z)^{-1}(z_i)$ and $x_i=g^x(t_i, v_i^x, s)$.*

Proof. Let $G^r\colon(t_i,v_i^r,z_i)\to g^r(t_i,v_i^r,B+(g_B^z)^{-1}(z_i))$, $r\in\{x,y\}$. Function G^x, being increasing in v_i^x in $\mathbb{R}_{++}^2\times\mathbb{R}_+$ (see Assumption 1), then admits a partial inverse relative to this variable, that is, there exists a function h^x such that $g^x(t_i,h^x(x_i,z_i,t_i),B+(g_B^z)^{-1}(z_i))=x_i$ for all $(x_i,z_i,t_i)\in\{(x_i,z_i,t_i)\in\mathbb{R}_{++}\times\mathbb{R}_+\times\mathbb{R}_+\colon x_i=g^x(t_i, B,B+z_i^y);\ z_i=g^z(t_i, v_i^z,B+z_i^y);\ z_i^y\ge0\}$. This domain of h^x is convex and has a non-empty intersection with $\mathbb{R}_{++}^3$ by Assumption 1. The implicit function theorem moreover implies that h^x is C^2 in the intersection of its domain with $\mathbb{R}_{++}^3$, with: $\partial_1h^x=1/\partial_2g^x$, $\partial_2h^x=-\partial_3g^x/(\partial_2g^x(\partial_2g^z+\partial_3g^z))$ and $\partial_3h^x=-\partial_1g^x/\partial_2g^x$, where the partial derivatives of h^x, g^x and g^z are respectively evaluated at (x_i, z_i, t_i), $(t_i, h^x(x_i, z_i, t_i), B+(g_B^z)^{-1}(z_i))$ and $(t_i, (g_B^z)^{-1}(z_i), B+(g_B^z)^{-1}(z_i))$. And h^x is: increasing and strictly convex in x_i as inverse of increasing strictly concave partial functions $v_i^x\to g^x(t_i,v_i^x,B+(g_B^z)^{-1}(z_i))$; increasing in z_i and decreasing in t_i by the derivatives calculated above and Assumption 1. We may let F be defined by: $F(x_i,z_i,t_i)=G^y(t_i,B-h^x(x_i,z_i,t_i),z_i)$ if $x_i>0$; $F(x_i,z_i,t_i)=G^y(t_i,B,z_i)$ if $x_i=0$. One verifies immediately that F is strictly concave in x_i, decreasing in x_i and in z_i, increasing in t_i. Its restriction to $\mathbb{R}_{++}^3$ is C^2, and $\partial_1F=-\frac{\partial_2g^y}{\partial_2g^x}$, $\partial_2F=\frac{1}{\partial_2g^z+\partial_3g^z}\left(\frac{\partial_3g^x}{\partial_2g^x}\partial_2g^y+\partial_3g^y\right)$, and $\partial_3F=\frac{\partial_1g^x}{\partial_2g^x}\partial_2g^y$ where the partial derivatives of F, g^x and g^y are respectively evaluated at (x_i, z_i, t_i), $(t_i, h^x(x_i, z_i, t_i), B+(g_B^z)^{-1}(z_i))$ and $(t_i, B-h^x(x_i, z_i, t_i), B+(g_B^z)^{-1}(z_i))$. □

A2. Production optimum

Theorem 1. *There exist production optima $(x^*, y^*, z^*, v^*, t^*)$, which are $\gg 0$, with a unique optimal production of final care services (x^*, y^*), and verify the following system of necessary and sufficient first-order conditions, where partial derivatives are evaluated at the optimum: (i) $\sum_{i\in N} t_i^* = \sum_{i\in N} z_i^*$; (ii) $\sum_{i\in N} v_i^{x*} + v_i^{y*} = nB$; (iii) for all i, $(x_i^*, y_i^*, z_i^*) = g(t_i^*, v_i^*)$ and $\frac{\partial_1 u}{\partial_2 u} = \frac{\partial_2 g^y}{\partial_2 g^x}$; (iv) $z^* \le \omega$ and there exists $(\lambda,\delta) \in \mathbb{R}_{++} \times \mathbb{R}_+^n$ such that $\partial_i W \cdot \partial_1 u \cdot \partial_1 g^x = \lambda$, $\delta_i = \lambda + \frac{\partial_i W}{\partial_2 g^z + \partial_3 g^z} \cdot (\partial_1 u \cdot \partial_3 g^x + \partial_2 u \cdot \partial_3 g^y) < \lambda$, and $\delta_i.(\omega_i = z_i^*) = 0$ for all i, where λ and δ_i are the marginal social utilities of aggregate hospitals' contribution and hospital i's graft resources respectively. The graft resources of hospital i are scarce at production optimum if and only if δ_i is >0.*

Proof. The social opportunity set $\{(x,y,z,v,t) \in \mathbb{R}_+^{7n}$: $\Sigma_{i\in N} t_i \le \Sigma_{i\in N} z_i$; $z \le \omega$; $\sum_{i\in N} v_i^x + v_i^y \le nB$; and $(x_i, y_i, z_i) \le g(t_i, v_i)$ for all $i\}$ is non-empty (it contains 0), compact (by continuity of g) and strictly convex (by the concavity assumptions on g). It has a non-empty intersection with $\mathbb{R}_{++}^{7n}$ by our assumptions relative to the productivity of hospital's technology (see Assumption 1, notably parts (iv), (v) and (vi)). The continuity of social utility function $(x, y) \to W(u(x_1, y_1),\ldots, u(x_n, y_n))$ therefore implies the existence of a social optimum$(x^*, y^*, z^*, v^*, t^*)$, which must be $\gg 0$ by Assumption 1 and the boundary conditions of Assumptions 2 and 3. The optimal production of final care services (x^*, y^*) is unique by the strict convexity of the social opportunity set. The Kuhn and Tucker first-order conditions are necessary and sufficient at a $\gg 0$ solution of convex program max $\{W(u(x_1, y_1),\ldots, u(x_n, y_n)):(x, y, z, v, t) \ge 0$; $\sum_{i\in N} t_i \le \sum_{i\in N} z_i$; $z \le \omega'$; $\sum_{i\in N} v_i^x + v_i^y \le nB$; and $(x_i, y_i, z_i) \le g(t_i, v_i)$ for all $i\}$ by Arrow and Enthoven (1961: Theorems 1 and 2). Strictly increasing utility and production functions in the positive orthant readily imply that constraints $\sum_{i\in N} t_i \le \sum_{i\in N} z_i$ and $(x_i, y_i, z_i) \le g(t_i, v_i)$ are binding, with positive associate multipliers, in the f.o.c. Strictly increasing utility and Assumption 1(vi) moreover imply that aggregate budget constraint $\sum_{i\in N} v_i^x + v_i^y \le nB$ is also binding, with a positive associate multiplier, at social optimum. These remarks and some calculations yield the following system of characterizing f.o.c., where partial derivatives are evaluated at the optimum: (i) $\sum_{i\in N} t_i^* = \sum_{i\in N} z_i^*$; (ii) $\sum_{i\in N} v_i^{x*} + v_i^{y*} = nB$; (iii) for all i, $(x_i^*, y_i^*, z_i^*) = g(t_i^*, v_i^*)$ and $\frac{\partial_1 u}{\partial_2 u} = \frac{\partial_2 g^y}{\partial_2 g^x}$; (iv) and there exists $(\lambda,\delta) \in \mathbb{R}_{++} \times \mathbb{R}_+^n$ such that $\partial_i W \cdot \partial_1 u \cdot \partial_1 g^x = \lambda$, $\delta_i = \lambda + \frac{\partial_i W}{\partial_2 g^z + \partial_3 g^z} \cdot (\partial_1 u \cdot \partial_3 g^x + \partial_2 u \cdot \partial_3 g^y) < \lambda$, and $\delta_i \cdot (\omega_i - z_i^*) = 0$ for all i, where λ is the multiplier associated with constraint $\sum_{i\in N} t_i \le \sum_{i\in N} z_i$. Finally, the characterization of scarcity in the last part of Theorem 1 is a simple consequence of definitions and the characterizing f.o.c. above. □

A3. Agents' behaviour at clear-sighted equilibrium

Lemma 2. Agency's transfer policy

The agency's transfer correspondence at the second stage of the clear-sighted game identifies with a continuous function φ^0: $\{z \in \mathbb{R}^n$: $0 \le z \le \omega\} \to \mathbb{R}_+^3$ such that $\varphi^0(0) = 0$ and $\varphi^0(z) \gg 0$ for all $z>0$. Its restriction to $\{z \in \mathbb{R}^n$: $0 \le z \le \omega\}$ solves, for any given z, the following system of first-order conditions in (x, t): (i) $\sum_{i\in N} t_i = \sum_{i\in N} z_i$; (ii) for all i, $\frac{\partial_1 u(x_i, F(x_i, z_i, t_i))}{\partial_2 u(x_i, F(x_i, z_i, t_i))} = -\partial_1 F(x_i, z_i, t_i)$; (iii) and there exists $\lambda \in \mathbb{R}_{++}$ such that, for all i, $\partial_i W(u(x_1, F(x_1, z_1, t_1)),\ldots, u(x_n, F(x_n, z_n, t_n))) \cdot \partial_2 u(x_i, F(x_i, z_i, t_i)) \cdot \partial_3 F(x_i, z_i, t_i) = \lambda$.

Proof. Sets of alternatives $A_i(t_i, \omega_i)$ and $A_0(z)$ being non-empty, compact and convex for all non-negative (t, z) such that $z \le \omega$, and the agency's utility function being continuous, program max $\{W(u(x_1, y_1),\ldots, u(x_n, y_n)):(x_i, y_i, z_i, v_i) \in A_i(t_i, \omega_i)$ for all i, and $t \in A_0(z)\}$ has one solution (x, y, t) at least, for any fixed non-negative $z \le \omega$. That is, correspondence φ^0: $\{z \in \mathbb{R}^n$: $0 \le z \le \omega\} \to \mathbb{R}_+^n$ is well-defined (i.e. has non-empty values over its domain). Its values are compact by continuity of u, and convex by convexity of $\{(x, y):(x_i, y_i, z_i, v_i) \in A_i(t_i, \omega_i)$ for all i, and $t \in A_0(z)\}$ and quasi-concavity of u.

$A_0(0) = \{0\}$ by definitions, and $A_i(0, \omega_i) = \{(0, y_i, z_i, v_i) \in \mathbb{R}_+^6$: $y_i \le g^y(0, v_i^y, v_i^x + v_i^y + v_i^z)$, $z_i \le g^z(0, v_i^z, v_i^x + v_i^y + v_i^z)$, $z_i \le \omega_i$ and $v_i^x + v_i^y \le B\}$ for all i by definitions and Assumption 1(iii). These facts and Assumptions 1 and 2 imply that the set of solutions of the agency's program when $z = 0$ coincides with the corresponding set of alternatives of the agency's program, that is, with set $\{(x, y, t):x = t = 0$ and $0 \le y_i \le g^y(0, B, B)$ for all $i\}$, the agency's and hospitals' utilities being then $= W(0)$ over this whole set. In particular: $\varphi^0(0) = \{0\}$.

Suppose from now on that $0<z \le \omega$.

$A_0(z) \cap \mathbb{R}_{++}^n$ is non-empty whenever $z>0$, that is, it is always possible for the agency to make positive transplant transfers to all hospitals whenever some >0 quantity of transplant is available. The agency's set of alternatives $\{(x,y,z,v,t) \in \mathbb{R}_+^{7n}$: $(x_i, y_i, z_i, v_i) \in A_i(t_i, \omega_i)$ for all i, and $t \in A_0(z)\}$ is convex for all z, by the concavity of production functions g^r, $r \in \{x, y, z\}$ (see Assumption 1). The boundary conditions of Assumptions 2 and 3 relative to utility functions and Assumption 1 then readily imply that the solutions of the agency's program are $\gg 0$ vectors (x, y, v, t), which moreover imply a unique optimal production of final care services (x, y), whenever $z>0$. From Lemma 1 and strictly increasing hospitals' utility, such interior solutions can be characterized, equivalently, as interior solutions to max $\{W(u(x_1, F(x_1, z_1, t_1)),\ldots, u(x_n, F(x_n, z_n, t_n))):0 \le x_i \le g^x(t_i, B, B + (g_B^z)^{-1}(z_i))$, and $t \in A_0(z)\}$, where "interior" now means either that $(x, y, t) \gg 0$ or, equivalently, that $t \gg 0$ and $0<x_i<g^x(t_i, B, B + (g_B^z)^{-1}(z_i))$ for all i. The necessary first-order conditions (f.o.c.) for the latter C^2 program read as follows (e.g., Mas-Colell, 1985: D.3.3): (i) $\sum_{i\in N} t_i \le \sum_{i\in N} z_i$; (ii) for all i, $\frac{\partial_1 u(x_i, F(x_i, z_i, t_i))}{\partial_2 u(x_i, F(x_i, z_i, t_i))} = -\partial_1 F(x_i, z_i, t_i)$; (iii) and there exists a ≥ 0 real number λ such that $\partial_i W(u(x_1, F(x_1, z_1, t_1)),\ldots, u(x_n, F(x_n, z_n, t_n))) \cdot \partial_2 u(x_i, F(x_i, z_i, t_i)) \cdot \partial_3 F(x_i, z_i, t_i) = \lambda$ and $\lambda(\sum_{i\in N} z_i - \sum_{i\in N} t_i) = 0$ for all i.

Utility functions being strictly increasing in the positive orthant (see Assumptions 2 and 3) and function F being strictly increasing relative to $t_i>0$ in $\mathbb{R}_{++}^3$ (see Lemma 1), the third part of the f.o.c. readily implies that $\lambda>0$ and $\sum_{i\in N} t_i = \sum_{i\in N} z_i$, that is, the agency's marginal utility of aggregate graft provision is >0 and aggregate graft production is entirely transferred to hospitals at the agency's optimum.

The non-convex program max $\{W(u(x_1, F(x_1, z_1, t_1)),\ldots, u(x_n, F(x_n, z_n, t_n))):0 \le x_i \le g^x(t_i, B, B + (g_B^z)^{-1}(z_i))$, and $t \in A_0(z)\}$ being equivalent to the convex program max $\{W(u(x_1, y_1),\ldots, u(x_n, y_n)):(x_i, y_i, z_i, v_i) \in A_i(t_i, \omega_i)$ for all i, and $t \in A_0(z)\}$, the necessary f.o.c. above are also necessary first-order conditions for the latter. And the f.o.c. of program max $\{W(u(x_1, y_1),\ldots, u(x_n, y_n)):(x_i, y_i, z_i, v_i) \in A_i(t_i, \omega_i)$ for all i, and $t \in A_0(z)\}$ are also sufficient conditions for an interior solution of the latter by Arrow and Enthoven (1961: Theorem 1). They characterize, therefore, the solutions whenever $z>0$.

Let us prove, to finish with, that φ^0 is single-valued and continuous over $\{z \in \mathbb{R}_+^n$: $z \le \omega\}$.

We already proved that $\varphi^0(0) = \{0\}$. Let $z>0$ and (x^*, y^*, t^*) solve max $\{W(u(x_1, F(x_1, z_1, t_1)),\ldots, u(x_n, F(x_n, z_n, t_n))):0 \le x_i \le g^x(t_i, B, B + (g_B^z)^{-1}(z_i))$; $t \in A_0(z)\}$. We established above that optimal (x, y) is unique, $= (x^*, y^*)$, and that $t^* \gg 0$. Function F being increasing in transfer, t_i^* is necessarily unique for all i, as unique solution of equation in t_i: $y_i^* = F(x_i^*, z_i, t_i)$. Therefore, φ^0 is single-valued over $\{z \in \mathbb{R}_+^n$: $z \le \omega\}$. It identifies, in other words, with a function $\{z \in \mathbb{R}_+^n$: $z \le \omega\} \to \mathbb{R}_+^3$ over this domain.

Let sequence $(z^q)_q \in \mathbb{N}$ of elements of $\{z \in \mathbb{R}_+^n$: $z \le \omega\}$ converge to z^*. Suppose first that $z^*>0$. Then $\varphi^0(z^*) \gg 0$ and there exists $q_0 \in \mathbb{N}$ such that $z^q>0$ and $\varphi^0(z^q) \gg 0$ for all $q \ge q_0$. Therefore, z^* and all z^q such that $q \ge q_0$ verify the system of C^1 f.o.c. above. $(\varphi^0(z^q))_{q \ge q_0}$, being a sequence of elements of compact set $\{t \in \mathbb{R}_+^n$: $\Sigma_{i\in N} t_i \le 1\}$, has at least one limit point t^* in that set. t^* verifies the f.o.c. at z^* by continuity of the latter. Therefore $t^* = \varphi^0(z^*)$, and continuity in $\{z \in \mathbb{R}_+^n$: $0 \le z \le \omega\}$ is established. Suppose, finally, that $z^* = 0$. By definition of φ^0, $\varphi^0(z^q)$ is ≥ 0 and verifies inequalities $0 \le \sum_{i\in N} \varphi_i^0(z^q) \le \sum_{i\in N} z_i^q$ for all q. Therefore $\lim_{z^q \to 0, z^q \ge 0} \varphi^0(z^q)$ is well-defined, $= 0 = \varphi^0(0)$, and continuity at 0 is established. □

Lemma 3. Hospital's behaviour

Hospital i's reaction correspondence at the first stage of the clear-sighted game is a well-defined, upper hemi-continuous correspondence φ_i^C: $z \in \{z \in \mathbb{R}_+^n: z \leq \omega\} \rightarrow \mathbb{R}_+^6$ *such that:* $\varphi_i^C(z) \subset \mathbb{R}_{++}^6$ *whenever* $z_{n/i} = 0$; $\varphi_i^C(z) \subset \mathbb{R}_{++}^2 \times \mathbb{R}_+ \times \mathbb{R}_+^2 \times \mathbb{R}_+$ *whenever* $z_{n/i} \neq 0$. *Let* $\tilde{z} \in \{z \in \mathbb{R}_+^{n-1}: z \leq \omega\}$ *be fixed,* $(x_i^*, y_i^*, z_i^v, v_i^*) \in \varphi_i^C(\tilde{z})$ *be such that* $z_i^* > 0$, *and suppose that* $z_i \rightarrow \varphi_i^0((\tilde{z}_{n/i}, z_i))$ *is* C^1 *in some interval open in* $(0, \omega_i]$ *containing* z_i^*. *Then,* $(x_i^*, y_i^*, z_i^*, v_i^*)$ *verifies the following system of first-order conditions: (i)* $y_i^* = F(x_i^*, z_i^*, \varphi_i^0((\tilde{z}_{n/i}, z_i^*)))$; *(ii)* $(x_i^*, y_i^*, z_i^*) = g(\varphi_i^0((\tilde{z}_{n/i}, z_i^*)), v_i^*)$; *(iii)* $\frac{\partial_1 u(x_i^*, y_i^*)}{\partial_2 u(x_i^*, y_i^*)} = -\partial_1 F(x_i^*, z_i^*, \varphi_i^0((\tilde{z}_{n/i}, z_i^*)))$; *(iv) and there exists* $\delta_i \in \mathbb{R}_+$ *such that* $\partial_2 u(x_i^*, y_i^*) \cdot (\partial_2 F(x_i^*, z_i^*, \varphi_i^0((\tilde{z}_{n/i}, z_i^*))) + \partial_3 F(x_i^*, z_i^*, \varphi_i^0((\tilde{z}_{n/i}, z_i^*))) \cdot \partial_i \varphi_i^0((\tilde{z}_{n/i}, z_i^*))) = \delta_i$ *and* $\delta_i(\omega_i - z_i^*) = 0$. *If function* $(x_i, z_i) \rightarrow u(x_i, F(x_i, z_i, \varphi_i^0((\tilde{z}_{n/i}, z_i))))$ *is, moreover, quasi-concave over* $\{(x_i, z_i) \in \mathbb{R}_+^2: 0 < z_i \leq \omega_i\}$, *then, the first-order conditions above characterize the* $\gg 0$ *elements of* $\varphi_i^C(\tilde{z})$, *that is,* $(x_i^*, y_i^*, z_i^*, v_i^*) \in \varphi_i^C(\tilde{z})$ *and is* $\gg 0$ *if and only if* $(x_i^*, y_i^*, z_i^*, v_i^*)$ *verifies the f.o.c. and is* $\gg 0$.

Proof. Note first that set $A_i(\varphi_i^0(z), \omega_i)$ being non-empty and compact for all $(z, \omega_i) \geq 0$ and utility function u being continuous, program max $\{u(x_i, y_i):(x_i, y_i, z_i, v_i) \in A_i(\varphi_i^0(z), \omega_i)\}$ has one solution at least for any fixed $(z, \omega_i) \geq 0$. Therefore, correspondence φ_i^C: $\mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+^6$ is well-defined.

Let $z_{n/i} = 0$. We established in Lemma 2 that $\varphi^0(0) = 0$ and $\varphi^0(z) \gg 0$ whenever $z > 0$. And we supposed in Assumption 1 that $g^x(0, v_i) = 0$ for all v_i. Therefore, hospital i's optimal graft production z_i is positive, for then and only then is a $> u(0)$ utility level accessible for hospital i by Assumption 2. In other words, if other hospitals contribute nothing, hospital i is willing to contribute something, in order to receive some positive transfer from the agency that allows for a $\gg 0$ final production (x_i, y_i) and $> u(0)$ utility.

Let $z_{n/i} \in \{z_{n/i} \in \mathbb{R}_+^{n-1}: z_{n/i} \leq \omega_{n/i}\}$ be fixed from there on. $\varphi^0((z_{n/i}, z_i^*)) \gg 0$ for any optimal graft production z_i^* of $\varphi_i^C(z)$ by the paragraph above, so that program max $\{u(x_i, y_i):(x_i, y_i, z_i, v_i) \in A_i(\varphi_i^0((z_{n/i}, z_i)), \omega_i)\}$ can be rewritten equivalently as max $\{u(x_i, F(x_i, z_i, \varphi_i^0((z_{n/i}, z_i)))):0 \leq x_i \leq g^x(\varphi_i^0((z_{n/i}, z_i)), B, B + (g_B^z)^{-1}(z_i))$, and $0 \leq z_i \leq \omega_i\}$ by Lemma 1. Solutions in (x_i, y_i) are "interior", that is, $(x_i, F(x_i, z_i^*, \varphi_i^0((z_{n/i}, z_i^*)))) \gg 0$, by the boundary condition of Assumption 2. But we may have a corner solution in z_i, that is, an optimal graft production z_i^* equal to either 0 or ω_i ($z_i^* = 0$ only if $z_{n/i} > 0$).

Suppose that optimal graft production z_i^* is positive, and that $z_i \rightarrow \varphi_i^0((z_{n/i}, z_i))$ is C^1 in an interval open in $(0, \omega_i]$ containing z_i^*. The necessary first-order conditions for solution (x_i, z_i^*) of the reduced program above then read as follows (e.g., Mas-Colell, 1985: D.1): (i) $\frac{\partial_1 u(x_i, F(x_i, z_i^*, \varphi_i^0((z_{n/i}, z_i^*))))}{\partial_2 u(x_i, F(x_i, z_i^*, \varphi_i^0((z_{n/i}, z_i^*))))} = -\partial_1 F(x_i, z_i^*, \varphi_i^0((z_{n/i}, z_i^*)))$; (ii) and there exists $\delta_i \in \mathbb{R}_+$ such that $\partial_2 u(x_i, y_i) \cdot (\partial_2 F(x_i, z_i^*, \varphi_i^0((z_{n/i}, z_i^*))) + \partial_3 F(x_i, z_i^*, \varphi_i^0((z_{n/i}, z_i^*))) \cdot \partial_i \varphi_i^0((z_{n/i}, z_i^*))) = \delta_i$ and $\delta_i(\omega_i - z_i^*) = 0$. Conversely, if $(x_i, z_i) \rightarrow u(x_i, F(x_i, z_i, \varphi_i^0((z_{n/i}, z_i))))$ is quasi-concave over $\{(x_i, z_i) \in \mathbb{R}_+^2: 0 < z_i \leq \omega_i\}$, if $z_i \rightarrow \varphi_i^0((z_{n/i}, z_i))$ is C^1 in an interval open in $(0, \omega_i]$ containing $z_i^* > 0$, and if $(x_i, z_i^*) \gg 0$ verifies the f.o.c. above, then (x_i^*, z_i^*) solves reduced program max $\{u(x_i, F(x_i, z_i, \varphi_i^0((z_{n/i}, z_i)))):z_i \leq \omega_i\}$, by Arrow and Enthoven (1961: Theorem 1). Let us establish, finally, that φ_i^C is upper hemi-continuous (u.h.c.) in $\{z \in \mathbb{R}_+^n: z \leq \omega\}$ for all i.

Let $(z^q)_q \in \mathbb{N}$ be a sequence of elements of $\{z \in \mathbb{R}_+^n: z \leq \omega\}$ converging to z^*, and sequence $((x_i^q, y_i^q, \bar{z}_i^q, v_i^q))_{q \in \mathbb{N}}$ be such that $(x_i^q, y_i^q, \bar{z}_i^q, v_i^q) \in \varphi_i^C(z^q)$ for all q and converge to $(x_i^*, y_i^*, \bar{z}_i^*, v_i^*)$. We want to prove that $(x_i^*, y_i^*, \bar{z}_i^*, v_i^*) \in \varphi_i^C(z^*)$. Note that $(x_i^*, y_i^*, \bar{z}_i^*, v_i^*) \in A_i(\varphi_i^0(z^*), \omega_i) = \{(x_i, y_i, z_i, v_i) \in \mathbb{R}_+^6: (x_i, y_i, z_i) \leq g(\varphi_i^0(z^*), v_i), z_i \leq \omega_i$, and $v_i^x + v_i^y \leq B\}$ by continuity of g and φ_i^0. Let $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i, \tilde{v}_i)$ be any element of $A_i(\varphi_i^0(z^*), \omega_i)$.

If $\tilde{x}_i$ or $\tilde{y}_i$ is $= 0$, then $u(x_i^*, y_i^*) \geq u(\tilde{x}_i, \tilde{y}_i) = u(0)$ by Assumption 2.

Suppose that $(x_i^*, y_i^*) \gg 0$. Note that, then, $\varphi_i^0(z^*) > 0$ and $x_i^* < g^x(\varphi_i^0(z^*), B, B + (g_B^z)^{-1}(\tilde{z}_i))$ by the definition of $A_i(\varphi_i^0(z^*), \omega_i)$ and Assumption 1. We construct a sequence $((\tilde{x}_i^q, \tilde{y}_i^q, \tilde{z}_i^q, \tilde{v}_i^q))_{q \in \mathbb{N}}$ that converges to $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i, \tilde{v}_i)$ and is such that $(\tilde{x}_i^q, \tilde{y}_i^q, \tilde{z}_i^q, \tilde{v}_i^q) \in A_i(\varphi_i^0(z^q), \omega_i)$ for all q. There exists $q_0 \in \mathbb{N}$ such that $\varphi_i^0(z^q) > 0$ and $\tilde{x}_i < g^x(\varphi_i^0(z^q), B, B + (g_B^z)^{-1}(\tilde{z}_i^q))$ for all $q \geq q_0$, by continuity of g^x, φ_i^0 and $(g_B^z)^{-1}$. If $\tilde{y}_i < F(\tilde{x}_i, \tilde{z}_i, \varphi_i^0(z^*))$, then, by construction of F (see the proof of Lemma 1), either $\tilde{v}_i^x + \tilde{v}_i^y = B$ and $\tilde{y}_i < g^y(\varphi_i^0(z^*), \tilde{v}_i^y, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z)$ or $\tilde{v}_i^x + \tilde{v}_i^y < B$; therefore, by continuity of F, φ_i^0 and g, there exists $q_1 \in \mathbb{N}$ such that, for all $q \geq q_1$: $\tilde{y}_i < F(\tilde{x}_i, \tilde{z}_i, \varphi_i^0(z^q))$, and either there exists $\bar{v}_i^{x,q} < \tilde{v}_i^x + \tilde{v}_i^y$ solving $g^x(\varphi_i^0(z^q), \bar{v}_i^{x,q}, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z) = g^x(\varphi_i^0(z^*), \tilde{v}_i^x, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z)$ such that $g^y(\varphi_i^0(z^q), \tilde{v}_i^x + \tilde{v}_i^y - \tilde{v}_i^{x,q}, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z) > \tilde{y}_i$, if $\tilde{v}_i^x + \tilde{v}_i^y = B$, or there exists $\bar{v}_i^{x,q}$ solving $g^x(\varphi_i^0(z^q), \bar{v}_i^{x,q}, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z) = g^x(\varphi_i^0(z^*), \tilde{v}_i^x, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z)$ and $\bar{v}_i^{y,q}$ solving $g^y(\varphi_i^0(z^q), \bar{v}_i^{y,q}, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z) = g^x(\varphi_i^0(z^*), \tilde{v}_i^y, \tilde{v}_i^x + \tilde{v}_i^y + \tilde{v}_i^z)$ such that $\tilde{v}_i^{x,q} + \tilde{v}_i^{y,q} < B$, if $\tilde{v}_i^x + \tilde{v}_i^y < B$. We let then: $(\tilde{x}_i^q, \tilde{y}_i^q, \tilde{z}_i^q) = (\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$, $\tilde{v}_i^{x,q} = \bar{v}_i^{x,q}$, $\tilde{v}_i^{y,q}$ be either $= \tilde{v}_i^x + \tilde{v}_i^y - \tilde{v}_i^{x,q}$ (if $\tilde{v}_i^x + \tilde{v}_i^y = B$) or $= \bar{v}_i^{y,q}$ (if $\tilde{v}_i^x + \tilde{v}_i^y < B$), and $\tilde{v}_i^{z,q} = \tilde{v}_i^z$, for all $q \geq$ max $\{q_0, q_1\}$; $(\tilde{x}_i^q, \tilde{y}_i^q, \tilde{z}_i^q, \tilde{v}_i^q)$ be an arbitrary element of $A_i(\varphi_i^0(z^q), \omega_i)$ for all $q <$ max $\{q_0, q_1\}$. If $\tilde{y}_i < F(\tilde{x}_i, \tilde{z}_i, \varphi_i^0(z^*))$, we have then $\tilde{v}_i^x + \tilde{v}_i^y = B$, $\tilde{v}_i^z = (g_B^z)^{-1}(\tilde{z}_i)$ and $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) = g(\varphi_i^0(z^*), \tilde{v}_i)$ by construction of F (see Lemma 1), and we let: $\tilde{v}_i^q = \tilde{v}_i$, $\tilde{z}_i^q = \tilde{z}_i$, $\tilde{y}_i^q = F(\tilde{x}_i, \tilde{z}_i, \varphi_i^0(z^q))$, $\tilde{v}_i^{x,q}$ solve $\tilde{x}_i = g^x(\varphi_i^0(z^q), \tilde{v}_i^{x,q}, B + (g_B^z)^{-1}(\tilde{z}_i))$, $\tilde{v}_i^{y,q}$ solve $\tilde{y}_i^q = g^y(\varphi_i^0(z^q), \tilde{v}_i^{y,q}, B + (g_B^z)^{-1}(\tilde{z}_i))$, and $\tilde{v}_i^{z,q} = (g_B^z)^{-1}(\tilde{z}_i) = \tilde{v}_i^z$ for all $q \geq q_0$; $(\tilde{x}_i^q, \tilde{y}_i^q, \tilde{z}_i^q, \tilde{v}_i^q)$ be an arbitrary element of $A_i(\varphi_i^0(z^q), \omega_i)$ for all $q < q_0$. One verifies immediately that the sequence converges to $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i, \tilde{v}_i)$. We have $u(x_i^q, y_i^q) \geq u(\tilde{x}_i^q, \tilde{y}_i^q)$ for all q by construction, so that $u(x_i^*, y_i^*) \geq u(\tilde{x}_i, \tilde{y}_i)$ by continuity of u. Therefore $(x_i^*, y_i^*, \bar{z}_i^*, v_i^*) \in \varphi_i^C(z^*)$, and the upper hemi-continuity of φ_i^C is established. □

A4. Example 2: Linear transferable transplant technology

In this example, we consider the case of convex medical care systems with constant unitary MRC. We label this special case the *transferable transplant* case, by analogy with transferable utility (Bergstrom and Cornes, 1983, Bergstrom and Varian, 1985a,b and Bergstrom, 1989).[25] We further restrict attention, for calculation purposes, to linear hospital technology. Linear technology being inconsistent with the boundary conditions of Assumption 1, we suppose, more precisely, that there exists a positive real number $\varepsilon \leq \inf\{\omega_i: i \in N\}$, which may be taken arbitrarily close to 0, such that $F(x_i, z_i, t_i) = -ax_i - bz_i + bt_i + c$, $(a,b,c) \in \mathbb{R}_{++}^3$, whenever $(x_i, z_i, t_i, F(x_i, z_i, t_i)) \geq (\varepsilon, \varepsilon, \varepsilon, \varepsilon)$ (see Fig. 3A and 3B). We also suppose that the hospital's utility function is strictly quasi-concave in $\mathbb{R}_{++}^2$.

Let (x_i^*, y_i^*) denote a local maximum of u in $\{(x_i, y_i) \in \mathbb{R}_+^2: y_i \leq -ax_i + c\}$. Note that such a point: necessarily exists by continuity of u; is $\gg 0$ by the boundary condition of Assumption 2, and therefore is the unique global maximum of u in $\{(x_i, y_i) \in \mathbb{R}_+^2: y_i \leq -ax_i + c\}$ by the strict quasi-concavity of utility in $\mathbb{R}_{++}^2$; is such that $y_i^* = -ax_i^* + c$ (u being strictly increasing in $\mathbb{R}_{++}^2$); and verifies first-order condition $\frac{\partial_1 u(x_i^*, y_i^*)}{\partial_2 u(x_i^*, y_i^*)} = a$. We suppose in the following that $(x_i^*, y_i^*) \gg (\varepsilon, \varepsilon)$ (see Fig. 3C).

The first-order conditions of Lemma 2 (Appendix A3) apply to this example. They readily imply that the agency's optimal distribution policy at the second stage of clear-sighted equilibrium is to *transfer to each hospital its own contribution* when all hospitals contribute at least ε, that is, formally: the restriction of φ^0 to $\{z \in \mathbb{R}_+^n: \varepsilon \cdot e_n \leq z \leq \omega\}$ is the identity $z \rightarrow z$.[26] The first-order conditions of Lemma 3 (Appendix A3) then imply, in turn, that all hospitals have essentially the same set of optimal production combinations at the first stage of clear-sighted equilibrium, precisely: all elements of set $\{(x_i^*, y_i^*, z_i) \in \mathbb{R}_+^3: \varepsilon \leq z_i \leq \omega_i\}$, where (x_i^*, y_i^*) is the same for all i (but where, of course, ω_i may vary with i), solve hospital i's first-stage program for all $z_{n/i} \geq \varepsilon \cdot e_{n-1}$. In

[25] These characteristics of F are obtained easily from Assumption 1 by letting functions g^r, $r \in \{x, y, z\}$, be linear whenever $(x_i, y_i, z_i) \geq (\varepsilon, \varepsilon, \varepsilon) \gg 0$. A suitable choice of coefficients in the linear representations of functions g^r yields a linear graph of F, with $F(x_i, z_i, t_i) \geq \varepsilon$ and unit MRC, for $(x_i, z_i, t_i) \geq (\varepsilon, \varepsilon, \varepsilon)$. As should be clear from footnote 28 below, transferable transplants neither implies, nor is implied by, transferable utility.

[26] Let $z_i = t_i$ and $(x_i, y_i) = (x_i^*, y_i^*) = (x^*, y^*)$ for all i in the f.o.c., and recall that the anonymity property of the social utility function implies that marginal social utilities of hospitals' utilities are equal whenever hospitals' utilities are equal.

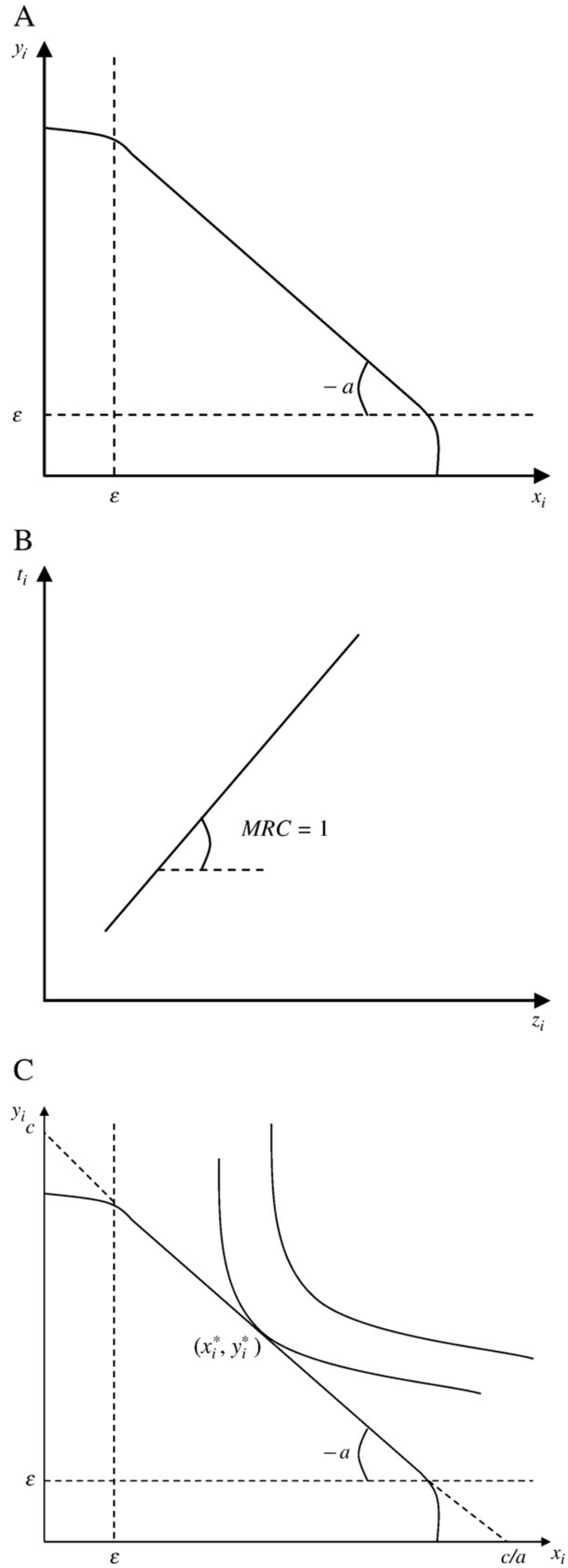

Fig. 3.

other words, if all hospitals contribute at least ε, the agency's transfer policy makes each hospital's decision independent of others' decisions, and also makes hospital's final production of transplant and other care services independent of its own intermediary production of grafts for the agency; so that all hospitals end up choosing the same output combination for their final care services, and also end up indifferent to their intermediary graft production over range $[\varepsilon, \omega_i]$.

Let $(x_i^*, y_i^*) = (x^*, y^*)$. It clearly follows from definitions and paragraph above that any $(x^* \cdot e_n, y^* \cdot e_n, z)$ such that $\varepsilon \cdot e_n \le z \le \omega$ is a clear-sighted equilibrium production of the medical care system, and that $t = z$ is the corresponding vector of equilibrium transfers. Note, finally, that: other equilibria might exist; $(x^* \cdot e_n, y^* \cdot e_n)$ is the unique socially efficient final production combination of the medical care system, as a simple consequence of the first-order conditions of Theorem 1 and Corollary 1; and $z < \omega$ for all equilibria above except $(x^* \cdot e_n, y^* \cdot e_n, \omega)$. In particular, the marginal social utility of the graft resources of all hospitals is null in this example (while the marginal social utility of hospitals' aggregate contribution $\sum_{i \in N} z_i$ is positive).[27] More precisely, some fraction of aggregate transplant resources does have a positive marginal social utility since hospitals and the agency have $u(0)$ and $W(u(0),\ldots, u(0))$ utility levels if $\sum_{i \in N} \omega_i = 0$, but any $\omega \ge \varepsilon \cdot e_n$ suffices for sustaining the equilibria above. That is: transferable transplants technology applies if aggregate resources exceed threshold $n\varepsilon$ (technology being linear only for $(x_i, y_i, z_i, t_i) \ge (\varepsilon, \varepsilon, \varepsilon, \varepsilon)$); and this particular technology then makes any amount of aggregate transplant resources in excess of this threshold (any positive difference $-n\varepsilon + \sum_{i \in N} \omega_i$) socially useless.

A5. Existence of clear-sighted equilibrium

The existence of a clear-sighted equilibrium is not warranted, in general, under Assumptions 1, 2 and 3. The appropriate tool for establishing existence is Debreu's (1952) social equilibrium existence theorem, applied to the Nash non-cooperative equilibrium of the first stage of the clear-sighted game. The general condition for existence, implied by this theorem, which may fail to hold in the case of clear-sighted equilibrium is convex-valued reaction correspondences of hospitals. We show below that an equilibrium exists in an acceptable subset of the wider class of medical care systems considered in this article.

Lemma 4. *If (W, u, g, ω) is convex, then φ_i^C is convex-valued for all i.*

Proof. We established in the proof of Lemma 3 that hospital i's first-stage program max $\{u(x_i, y_i):(x_i, y_i, z_i, v_i) \in A_i(\varphi_i^0((z_{n/i}, z_i)), \omega_i)\}$ was equivalent to program max $\{u(x_i, F(x_i, z_i, \varphi_i^0((z_{n/i}, z_i)))):0 \le z_i \le \omega_i\}$ for any fixed $z_{n/i} \in \{\tilde{z}_{n/i} \in \mathbb{R}_+^{n-1}: z_{n/i} \le \omega_{n/i}\}$, and yielded positive optimal graft production of hospital i whenever $z_{n/i} = 0$. The convexity assumption of Definition 5 is therefore exactly sufficient for the convexity of $\varphi_i^C(z)$ for all i and all $z \in \{z \in \mathbb{R}_+^n: z \le \omega\}$. □

Lemma 5. *Let the medical care system (W, u, g, ω) be such that φ_i^C is convex-valued for all i. Then there exists a clear-sighted equilibrium of (W, u, g, ω).*

Proof. φ^0 is a continuous function $\{z \in \mathbb{R}_+^n: z \le \omega\} \to \mathbb{R}_+^n$ by Lemma 2, and φ_i^C is an upper hemi-continuous correspondence $\{z \in \mathbb{R}_+^n: z \le \omega\} \to \mathbb{R}_+^6$ for all i by Lemma 3. Let the canonical projection $(x_i, y_i, z_i, v_i) \to z_i$ be denoted by pr_3. $\Phi^C: z \to (pr_3(\varphi_1^C(z)),\ldots, pr_3(\varphi_n^C(z)))$ is an upper hemi-continuous, convex-valued correspondence $\{z \in \mathbb{R}_+^n: z \le \omega\} \to \{z \in \mathbb{R}_+^n: z \le \omega\}$. Set $\{z \in \mathbb{R}_+^n: z \le \omega\}$ being non-empty compact and convex, Φ^C has a fixed point in $\{z \in \mathbb{R}_+^n: z \le \omega\}$ by Kakutani's fixed point theorem, that is, there exists $z^* \in \{z \in \mathbb{R}_+^n: z \le \omega\}$ such that $z^* \in \Phi^C(z^*)$. There exists, therefore, a state $(x^*, y^*, z^*, v^*, t^*)$ such that $t^* = \varphi_i^0(z^*)$ and $(x_i^*, y_i^*, z_i^*, v_i^*) \in \varphi_i^C(z^*)$ for all i. $(x^*, y^*, z^*, v^*, t^*)$ is an equilibrium of the clear-sighted game by construction. □

The medical care systems of Examples 1 and 2 are convex.

[27] From the corollary of Theorem 1: $\delta_i = \partial_i W \cdot \partial_2 u \cdot \partial_3 F \cdot \left(1 + \frac{\partial_2 F}{\partial_3 F}\right) = 0$, while $\lambda = \partial_i W \cdot \partial_2 u \cdot \partial_3 F = \partial_i W \cdot \partial_2 u \cdot c > 0$, for all i.

A6. Rotten kids and abundance

The public good externalities of first-stage equilibrium pre-exist to transfer policy in the game of Cornes and Silva. This and the neutrality property of transfers allow the principal to use transfer policy as a pure coordination device in their setup: in the absence of any trade off between allocation and distribution objectives (due to neutrality), the principal's optimal transfer policy achieves production optimum by equating individual marginal valuations of the public and private goods at first-stage equilibrium with their marginal valuations at social optimum. The public good externalities of the first-stage equilibrium of the transplant care game, if any, are, by contrast, generated by the principal's transfer policy (as in Becker's and Bergstrom's games); moreover, transfer policy induces public good externalities if and only if it is *not* of the status quo type, that is, if and only if it does not merely consist of returning each agent its contribution. Only if status quo is the agency's optimal transfer policy can the mechanism of Cornes and Silva be successfully replicated in the context of the transplant care game, that is, using transfer policy as a pure coordination device for achieving production optimum. Theorem 3(iii) gives the sufficient, and in general necessary, condition for status quo transfer policy to be the agency's second-stage optimal policy.

Bergstrom (1989) states that the rotten kid theorem applies if, and, in general, only if, agents' utilities are conditionally transferable. Bergstrom's general property, like Becker's original theorem, do not apply to the transplant care game if rationing constraints are binding at production optimum. Becker's theorem applies in the context of competitive market exchange, essentially because (perfect) competitive exchange automatically achieves allocation efficiency for any distribution of money income, thereby allowing the principal to optimize the sole distribution of income, by means of lump-sum endowment (or numeraire) transfers (see Hick's composite theorem in Bergstrom (1989), and also Example 2 of Mercier Ythier (2009)). The rotten kid property still obtains outside competitive market exchange if the allocation efficiency frontier is invariant to redistribution and if the principal can freely redistribute aggregate money income between self-centred agents (Bergstrom, 1989: Proposition 1). None of the latter conditions apply to the transplant care game, except in the special case where rationing constraints are non-binding at equilibrium (see the argument of footnote 28 and the proof of Theorem 3(iii) below[28]). Moreover, transplant shortage follows from the ban on markets of transplant inputs, which interprets as a case of market incompleteness caused by basic normative reasons. The virtuous rotten kids of Becker's theorem are, so to speak, daughters and sons of abundance.

Proof of Theorem 3. Part (i) of the Theorem is a simple consequence of Lemmas 4 and 5 (existence) and of Lemmas 2 and 3 (positivity). The qualitative aspects of parts (ii) and (iii) are supported by Examples 1 and 2 and the discussion above. Part (iii) is complemented by the following clear-cut statements, established below: *If all programs $\{u(x_i, F(x_i, z_i, z_i)):z_i \leq \omega_i\}$ have a same solution (x^*, z^*), then: rationing constraints are either all identical and binding or all non-binding at (x^*, z^*) in programs $\{u(x_i, F(x_i, z_i, z_i)):z_i \leq \omega_i\}$; $(x^* \cdot e_n, F(x^*, z^*, z^*) \cdot e_n, z^* \cdot e_n)$ is a clear-sighted equilibrium production combination, and the agency's corresponding equilibrium transfer is $\varphi^0(z^* \cdot e_n) = z^* \cdot e_n$; $\partial\varphi^0(z^*)$ is $= e_n$ if rationing constraints are all non-binding in programs $\{u(x_i, F(x_i, z_i, z_i)):z_i \leq \omega_i\}$, and $\ll e_n$ otherwise.*

Let (x^*, z^*) be a solution of $\{u(x_i, F(x_i, z_i, z_i)):z_i \leq \omega_i\}$, the same for all i, and let $F(x^*, z^*, z^*)$ be denoted by y^*. The boundary condition of Assumption 2 implies that production combination (x^*, y^*, z^*) is $\gg 0$. The characterizing first-order conditions for this maximum read: $\frac{\partial_1 u}{\partial_2 u} = -\partial_1 F$, $-\frac{\partial_2 F}{\partial_3 F} \leq 1$, and $\left(1 + \frac{\partial_2 F}{\partial_3 F}\right) \cdot (\omega_i - z^*) = 0$ for all i, where partial derivatives are evaluated at the optimum.

Identical (x^*, y^*) imply that marginal social utilities $\partial_i W(u(x^*, y^*), \ldots, u(x^*, y^*))$ are identical for all i by the anonymity property of Assumption 3. Identical (x^*, z^*) imply that hospitals have same $\partial F_3(x^*, z^*, z^*)$. The f.o.c. of Corollary 1 then imply that $(x^* \cdot e_n, y^* \cdot e_n, z^* \cdot e_n)$ is a socially optimal production combination of (W, u, F, ω).

If some rationing constraint is binding at (x^*, z^*) in programs above, that is, if $-\frac{\partial_2 F(x^*, z^*)}{\partial_3 F(x^*, z^*)} < 1$ and $z^* = \omega_i$ for some i, then, clearly, all rationing constraints are binding and identical, so that, in particular, all hospitals have the same endowment, $= z^*$. In other words, rationing constraints are either all identical and binding or all non-binding at (x^*, z^*) in programs $\{u(x_i, F(x_i, z_i, z_i)):z_i \leq \omega_i\}$.

Function F being C^2 wherever it is defined in $\mathbb{R}^3_{++}$, and then such that $\partial_3 F > 0$, the implicit function theorem implies the existence of open neighbourhoods U and V of z^* in $\mathbb{R}_{++}$ and of a C^1 function ψ: $U \to V$ such that $\psi(z^*) = z^*$, and, for all $s \in U$, $y^* = F(x^*, s, \psi(s))$ and $\partial\psi(s) = -(\partial F_2(x^*, s, \psi(s))/\partial F_3(x^*, s, \psi(s)))$. The f.o.c. of Lemma 2 then imply that the agency's second-stage optimal transfer policy identifies with function $(z_1, \ldots, z_n) \to (\psi(z_1), \ldots, \psi(z_n))$ over $\{z \in U^n$: $z \leq \omega$; $\exists \alpha \in \mathbb{R}_{++}$ such that $z = \alpha \cdot e_n\}$ (since all hospitals have same $\partial F_3(x_i^*, z_i, \psi(z_i))$ for all z in the latter set). In particular: $\varphi^0(z^*) = z^*$; and $\partial\varphi^0(z^*)$ is $= e_n$ if rationing constraints are all non-binding in programs $\{u(x_i, F(x_i, z_i, z_i)):z_i \leq \omega_i\}$, and $\ll e_n$ otherwise, that is, if rationing constraints are all binding and identical in these programs. The f.o.c. of Lemma 3 and the quasi-concavity properties of $(x_i, z_i) \to u(x_i, F(x_i, z_i, \varphi^0(z^*_{n/i}, z_i)))$ (implied by the convexity of (W, u, F, ω)) then imply that (x^*, z^*) solves $\max \{u(x_i, F(x_i, z_i, \varphi_i^0((z^*_{n/i}, z_i)))):z_i \leq \omega_i\}$ for all i (see Lemma 3), and therefore that $(x^* \cdot e_n, y^* \cdot e_n, z^* \cdot e_n)$ is a clear-sighted equilibrium production combination of (W, u, F, ω)), and that $z^* \cdot e_n$ is the corresponding optimal transfer of the agency. □

A7. Monitored equilibrium

Proof of Theorem 4. Let $(x^*, y^*, z^*, v^*, t^*)$ be a monitored equilibrium. Assumptions 1, 2 and 3 and the definition of monitored equilibrium clearly imply that $(x^*, y^*, z^*, v^*, t^*) \gg 0$. Monitored opportunity set $A_i^M(v_i^z, t_i^*) = \{(x_i, y_i, x_i^x, y_i^y) \in \mathbb{R}^4_+ : x_i \leq g^x(t_i^*, v_i^x, v_i^x + v_i^y + v_i^z), y_i \leq g^x(t_i^*, v_i^y, v_i^x + v_i^y + v_i^z), v_i^x + v_i^y \leq B\}$ is compact, convex, and has a non-empty intersection with $\mathbb{R}^4_{++}$. The Kuhn and Tucker first-order conditions are therefore necessary and sufficient for convex program $\max \{u(x_i, y_i):(x_i, y_i, v_i^x, v_i^y) \in A_i^M(v_i^z, t_i)\}$ at interior equilibrium solution $(x_i^*, y_i^*, v_i^{x*}, v_i^{y*})$ by Arrow and Enthoven (1961: Theorems 1 and 2).

[28] In this footnote, it is proved that *binding rationing constraints make the rotten kid theorem fail, even if hospitals' utility is transferable*. Transferable utility translates as follows into our framework: The preference relation underlying hospitals' (identical) reduced form utility functions $u(x_i, F(x_i, z_i, t_i))$ admits a utility representation of the type $A \cdot t_i + C(x_i, z_i)$, where A is a >0 real number and C is a real-valued function decreasing in z_i. Suppose for simplicity (without significant loss of generality by Mas-Colell, 1985: 2.3.11) that $u(x_i, F(x_i, z_i, t_i)) = A \cdot t_i + C(x_i, z_i)$ for all i. This implies $\partial_2 F / \partial_3 F = (\partial_2 u \cdot \partial_2 F)/(\partial_2 u \cdot \partial_3 F) = \partial_2 C / A$, where the points of evaluation of partial derivatives are omitted to alleviate notations. Denoting by u_i a utility level of hospital i, the utility possibility set conditional on the system of agents' actions (x, z) is the simplex $\{(u_1, \ldots, u_n) \geq (C(x_1, z_1), \ldots, C(x_n, z_n)): \sum_{i \in N} u_i \leq A \cdot \sum_{i \in N} z_i + \sum_{i \in N} C(x_i, z_i)\}$. The rotten kid theorem implies the maximization of "social income" $A \cdot \sum_{i \in N} z_i + \sum_{i \in N} C(x_i, z_i)$ relative to socially accessible agents' actions (x, z) at equilibrium. The f.o.c. for a maximum of $A \cdot \sum_{i \in N} z_i + \sum_{i \in N} C(x_i, z_i)$ such that $x \gg 0$ subject to rationing constraints $z \leq \omega$ read: $\partial_1 C(x_i, z_i) = 0$, $1 + \frac{\partial_2 C}{A} \geq 0$ and $\left(1 + \frac{\partial_2 C}{A}\right) \cdot (\omega_i - z_i) = 0$ for all i, and therefore coincide with the f.o.c. for the solutions of $\max \{u(x_i, F(x_i, z_i, z_i)):z_i \leq \omega_i\}$ with positive x_i. Supposing an anonymous utility function of the principal, this set of conditions characterizes a socially optimal clear-sighted equilibrium, with status quo second-stage optimal transfer policy $\varphi^0: z \to z$, if and only if programs $\max \{u(x_i, F(x_i, z_i, z_i)):z_i \leq \omega_i\}$ have a same solution, that is, *if and only if rationing constraints are either non-binding in all these programs or identical in all of them* (the latter implying identical hospitals' endowments). In particular, Proposition 1 of Bergstrom (1989) does not apply *if distinct rationing constraints are binding in at least two of these programs*. The assumption of Bergstrom's proposition that fails to hold in the latter case is that the principal can choose any vector of transfers in set $\{t \in \mathbb{R}^n_+ : \Sigma_{i \in N} t_i \leq \Sigma_{i \in N} z_i\}$, implying that the principal's transfers are not limited by rationing constraints in the cases covered by the proposition. □

They read: (i) $v_i^{x*} + v_i^{y*} = B$; (ii) $(x_i^*, y_i^*, z_i^*) = g(t_i^*, v_i^*)$; and (iii) $\frac{\partial_1 u(x_i^*, y_i^*)}{\partial_2 u(x_i^*, y_i^*)} = \frac{\partial_2 g^y(t_i^*, v_i^{y*}, v_i^{x*} + v_i^{y*} + v_i^{z*})}{\partial_2 g^x(t_i^*, v_i^{x*}, v_i^{x*} + v_i^{y*} + v_i^{z*})}$. Or equivalently, by Lemma 1: (i) $v_i^{x*} + v_i^{y*} = B$; (ii) $y_i^* = F(x_i^*, z_i^*, t_i^*)$; and (iii) $\frac{\partial_1 u(x_i^*, y_i^*)}{\partial_2 u(x_i^*, y_i^*)} = \partial_1 F\left(x_i^*, z_i^*, t_i^*\right)$.

Suppose that $(x^*, y^*, z^*, v^*, t^*)$ is not a production optimum and let us derive a contradiction. There exists then, by Theorem 1, a production optimum (x, y, z, v, t) such that $W(u(x_1, y_1),\ldots, u(x_n, y_n)) > W(u(x_1^*, y_1^*),\ldots, u(x_n^*, y_n^*))$. But then $(x, y, v^x, v^y) \in \varphi^M(v^z, t)$ if $v_i^x + v_i^y = B$ for all i, by the characterizing f.o.c. of Theorem 1 and of paragraph above. This may be supposed without loss of generality for (x, y, z, v, t) by the hypothesis of Theorem 4. But $(z, v^z, t) \in A_0^M(\omega)$, as an immediate consequence of the definition of a production optimum. Therefore $(x^*, y^*, z^*, v^*, t^*)$ is not a monitored equilibrium, the contradiction we were looking for. □

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