Regular Distributive Social Systems

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Abstract

I consider abstract social systems where individual owners make gifts according to their preferences on the distribution of wealth in the context of a noncooperative equilibrium. I define a condition of regularity and a condition of strong regularity of these social systems. I prove notably that: regularity is generic, and implies the local uniqueness of equilibrium and the uniqueness of status quo equilibrium; strong regularity is nongeneric, implies that an equilibrium exists for all initial distributions of wealth, whenever an equilibrium exists for one of them, and implies the connectedness of the range of the equilibrium correspondence. These properties have strong implications for distributive theory and policy, summarized in a general hypothesis of perfect substitutability of private and public transfers. The formulation and discussion of this hypothesis lead to a general assessment of the explanatory power of the theory.

1. Introduction

I consider pure distributive social systems, made of individual owners who consume and transfer wealth according to their preferences on the distribution of individual consumptions, in the context of a noncooperative equilibrium of gifts.

This type of construct is central in the economic theory of voluntary redistribution, born from the contributions of Kolm (1966), Hochman and Rodgers (1969), Barro (1974), and Becker (1974). It has been used to analyze the role of voluntary redistribution in a variety of contexts, including the microeconomic analysis of redistribution inside the family, charitable gift-giving and its interplay with public assistance policies, or the macroeconomic analysis of intergenerational transfers and public debt policies.

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In the landscape of the general theory that I consider here, the applications to family and charity of Becker's "theory of social interactions" (1974) possess, notably, the following distinctive features: a single agent (the "head" of the family) makes altruistic transfers to the other members of the family; and the distributive equilibrium coincides with the distributive optimum of the altruistic agent. Becker (1981) shows in a simple and intuitively appealing way how gift-giving can integrate individual behaviors within the family, by creating, then making them serve, the common interests of its members. It appears presently as the centerpiece of the microeconomic theory of redistribution inside the family (Laferrère 2000).

The application of Becker's theory to charitable gift-giving is less convincing than its application to family gift-giving. It matches situations where each charitable agent takes care, so to speak, of "his own poors." This corresponds to a type of practice often described in literary accounts of the best times of the "société bourgeoise" (say, of the period 1870–1914), but seems at variance with the social context of modern Welfare States, where the bulk of public or private assistance is provided by a large number of anonymous donors.

The identification of the distribution of wealth with a public good in the formal sense of economic theory was made by Kolm (1966) as a consequence of the assumption that individuals have preferences on the distribution of wealth. The same article and the article of Hochman and Rodgers (1969) define independently a distributive optimum as a Pareto optimum according to individual preferences on the distribution of wealth (implied, also, by the "maximum of utility for a collectivity" of Pareto 1913). Hochman and Rodgers consider Pareto-optimal redistribution from a wealthy to a poor. Musgrave (1970) raises the problem of noncooperative behavior when redistribution concerns a large number of individuals. Warr (1982) and Roberts (1984) analyze the noncooperative equilibrium of gifts when two wealthy individuals are altruistic to a single (egoistic) poor (and indifferent to each other), with the following conclusions: equilibrium is Pareto inefficient; and Pareto-efficient redistribution necessarily reduces private charity to 0. The full integration of distributive efficiency and noncooperative gift equilibrium for large social systems is realized in the concepts of distributive core (Mercier Ythier 1998b) and liberal distributive social contract (Kolm 1985 and Mercier Ythier 1998a). The former consists of the distributions of individual rights (i.e., the distributions of individual wealth endowments) that are strong equilibria with respect to the gifts of isolated individuals or of coalitions of individuals. The second consists of the elements of the distributive core that are preferred unanimously to the initial distribution of rights. The distributive core coincides with the set of distributive Pareto optima when individuals share the opinion that wealth transfers, if any, should flow downward, from the wealthier to the less wealthy (Mercier Ythier 1998a). The coincidence of the distributive core with the set of distributive Pareto optima is, essentially, necessary and sufficient for the existence of a liberal distributive social contract for all initial distributions of endowments (Mercier Ythier 2000b). It implies generally that, as in Warr (1982), or Roberts (1984), the achievement of distributive efficiency by means of public transfers reduces private charity to 0 (Mercier Ythier 2000a, p. 5).

Barro's proposition on the neutrality of the public debt (1974) outlines the consequences for distributive policy of a second fundamental property of this theory, namely, the perfect substitutability of public and private transfers (also noticed in Becker 1974). His analytical framework differs from the distributive systems that I consider here in two minor respects: it is a model of overlapping generations, with an infinite number of finitely lived agents (generations) and an infinite horizon; the utility of a generation depends on the wealth of the subsequent one through the indirect utility function of the latter. Warr (1982, 1983), Bergstrom, Blume, and Varian (1986), and Mercier Ythier (2000a) provide successive versions, of increasing generality, of the neutrality property of pure distributive social systems. Mercier Ythier (2000a) states, notably, that a public distributive policy operating small lump-sum transfers can redistribute wealth between the components of the graph of equilibrium transfers, but not inside them. In particular, small public transfers are neutral if (and only if) any pair of agents is connected by a chain of gifts at equilibrium.

The present paper studies regular distributive social systems.

A distributive social system is regular if, essentially, the linear system tangent to the set of equilibrium first-order conditions associated with the gifts which can take on a positive value at equilibrium, has full rank.

I establish below that regularity is a generic property of the set of distributive social systems, that is, a property verified in an open and dense subset of the latter. Informally, a slight linear perturbation of distributive utility functions will generally be sufficient to restore regularity when the latter happens, coincidentally, to be violated. This means, in other words, that one should view the regularity of distributive social systems as the rule, its violation as an exception.

This article elicits, and examines the implications of, three aspects of the determinacy of equilibrium in regular distributive social systems: equilibria are in finite number; status quo equilibrium is determinate, which means that if status quo (i.e., no gift) is an equilibrium, then status quo is the only equilibrium; the graphs of equilibrium gifts are forests, that is, have no circuit. These properties are generic, as consequences of regularity.

The generic local determinacy of equilibrium and generic global determinacy of status quo equilibrium are familiar implications of transversality theory (Mas-Colell 1985), verified by finite competitive economies, as well as by distributive social systems. Combined with the perfect substitutability of public and private transfers, the second implies that distributive policy can, generically, remove any potential indeterminacy of its outcome that would originate in private transfers, simply by crowding them out. This fact appears particularly interesting in the context of distributive theory (as opposed to competitive exchange theory) since, as recalled above, the achievement of

Pareto efficiency in the distribution of wealth precisely implies, in many relevant cases, the full crowding out of all private transfers.

The absence of circuits in the graphs of equilibrium gifts is another aspect of determinacy, equivalent to the fact that the linear map associating equilibrium distributions with equilibrium gifts is one-to-one. It has, notably, the interesting implication that gift exchange, which corresponds formally to the existence of directed circuits in the graph of equilibrium gifts, is bound to appear only coincidentally in this theory. In other words, the theory of gift-giving that I consider here cannot generally be used to explain reciprocity (see, nevertheless, the exceptions of Section 4).

A third implication of regularity adds precision to the informal statement that public lump-sum transfers are nonneutral if and only if they imply (net) redistributions between the connected components of the graph of equilibrium gifts. I establish that the set of equilibrium distributions that obtains from local public lump-sum transfers leaving the graph of equilibrium gifts unchanged (and, in particular, crowding out none of these gifts) is, generically, a differentiable manifold whose dimension equals the dimension of the set of feasible distributions minus the number of equilibrium gifts.

The regularity property holds only generically. A natural strengthening for a given vector of individual distributive utility functions, is to suppose that regularity holds for all initial distributions of wealth. Strong regularity has two substantial implications: the range of the equilibrium correspondence is connected; and an equilibrium exists for all initial distributions of wealth, provided that an equilibrium exists for at least one of them. I display an example showing that neither strong regularity nor connectedness nor existence are generic. Connectedness and the perfect substitutability of public and private transfers together imply that distributive policy can reach any equilibrium distribution by means of continuous public redistributions crowding out all individual gifts.

The paper is organized as follows. Section 2 defines and characterizes distributive equilibrium. Section 3 defines regularity. Section 4 establishes the determinacy and genericity properties. Section 5 defines strong regularity and examines its scope and consequences. The conclusion evaluates the whole theory in the light of its implications. An appendix gathers the proofs of theorems.

2. Distributive Social System and Equilibrium

This section applies the general notion of social equilibrium of Debreu (1952) to the context of a pure distributive social system.

2.1. Pure Distributive Social Systems

I consider pure distributive social systems, defined as abstract social systems where: (i) wealth is measured in money units and divisible; (ii) wealth is shared initially among individual owners; (iii) owners can, individually, consume, or transfer to others, any amount of their ownership, that is, of their initial endowment increased by the gifts received from others; (iv) owners make their consumption and transfer decisions according to their preferences on the final distribution of wealth, that is, on the vector of individual consumption levels; and (v) aggregate wealth is fixed, which implies notably that the latter does not depend on individual consumption and transfer decisions.

Formally, let individuals be designated by an index i running in $N = \{1, \dots, n\}$, and choose the money unit so that aggregate wealth is 1.

Individual i's *initial endowment* or *right*, that is, his share in total wealth prior consumption or transfer is denoted by $\omega_i \in [0, 1]$.

A *consumption* x_i of individual i is the money value of his consumptions of commodities. A *gift* t_{ij} *from individual i to individual* $j (j \neq i)$ is a nonnegative money transfer from individual i's estate (his initial ownership plus the gifts he received from others) to individual j's. A *gift-vector of individual i* is a vector[1] $t_i = (t_{ij})_{j \in N \setminus \{i\}}$ of $\mathbb{R}_+^{n-1}$.

I ignore alternative individual uses of wealth, like disposal or production, as well as potential costs associated with consumption and transfer activities, so that the following accounting identity is verified for all individual i, endowment ω_i, and decision (x_i, t_i):

$$x_i + \sum_{j:j \neq i} t_{ij} = \omega_i + \sum_{j:j \neq i} t_{ji}.$$

A distribution of initial rights $(\omega_1, \dots, \omega_n)$ is denoted by ω. It is an element of the unit simplex $S_n = \{x \in \mathbb{R}_+^n \mid \sum_i x_i = 1\}$ of $\mathbb{R}^n$. A distribution of individual consumption expenditures $(x_1, \dots, x_n)$ is denoted by x. It is *feasible* if it belongs to S_n. A *gift-vector* t is a vector $(t_1, \dots, t_n)$.

Individuals have well-defined preferences on the final distribution of wealth, that is, on the vectors of individual consumption expenditures: individual i is endowed with a *distributive utility function* $w_i : x \to w_i(x)$, defined on the space of consumption distributions $\mathbb{R}^n$. The vector $(w_1, \dots, w_n)$ of individual utility functions is denoted by w.

A distributive social system is a pair (w, ω).

I use the following notations. t^T is the transpose of row vector t. $t_{\setminus i}$ (resp. t_I, resp. $t_{\setminus I}$) is the vector of gifts obtained from t by deleting t_i (resp. t_{ij} for all $(i, j) \notin I$, resp. t_{ij} for all $(i, j) \in I$). $(t_{\setminus i}, t_i^*)$ (resp. $(t_{\setminus I}, t_I^*)$) is the gift-vector obtained from t and t^* by substituting t_i^* for t_i (resp. t_{ij}^* for t_{ij} for all $(i, j) \in I$) in t. $\Delta_i t$ is the net transfer $\sum_{j:j \neq i}(t_{ji} - t_{ij})$ accruing to individual i when t is the gift-vector. Δt is the vector of net transfers $(\Delta_1 t, \dots, \Delta_n t)$. $x(\omega, t)$ is the

[1] I will often have to use notations like $(t_{ij})_{(i,j) \in I}$, where I is a subset of $\{(i, j) \in N \times N : i \neq j\}$, to denote row vectors of $\mathbb{R}^{\#I}$. The entries t_{ij} are then ranked in increasing lexicographic order (that is, according to the ordering defined on $N \times N$ by: $(i, j) > (i', j')$ if either $i > i'$ or $i = i'$ and $j > j'$). *Unless an explicit mention of the contrary, notations t, t_I and t_{ij} will refer to nonnegative vectors and number.*

vector of individual consumption expenditures $\omega + \Delta t = (\omega_1 + \Delta_1 t, \ldots, \omega_n + \Delta_n t)$, that is, given the accounting identity above, the unique consumption distribution associated with the distribution of rights ω and the gift-vector t. $x_i(\omega, t)$ is the ith projection $pr_i x(\omega, t) = \omega_i + \Delta_i t$. $\partial_t x(\omega, t)$ (resp. $\partial_{t_i} x_i(\omega, t)$) is the Jacobian matrix of $t \to x(\omega, t)$ (resp. $t_i \to x_i(\omega, t)$).

2.2. Distributive Equilibrium

This subsection defines gift equilibrium and provides a characterization for differentiable social systems.

2.2.1. Definition

The general notion of social equilibrium of Debreu (1952), applied to the pure distributive social system, becomes the following. Every agent takes the transfers of others as fixed, and maximizes his utility with respect to his own gifts, subject to the constraint that his consumption must be nonnegative. An equilibrium is a gift vector that solves all individual programs simultaneously. Formally:

DEFINITION 1: *A distributive equilibrium of (w, ω) is a gift-vector t^* such that t_i^* is a maximum of $t_i \to w_i(x(\omega, (t^*_{\setminus i}, t_i)))$ in $\{t_i : x_i(\omega, (t^*_{\setminus i}, t_i)) \geq 0\}$ for all i.*

For a fixed w, we have the following equilibrium sets and correspondences. $T_w(\omega) = \{t : t$ is a distributive equilibrium of $(w, \omega)\}$ is the set of equilibrium gift-vectors of (w, ω); $X_w(\omega) = \{x : \exists t \in T_w(\omega)$ such that $x = x(\omega, t)\}$ is the corresponding set of equilibrium distributions; $\Omega_w(x) = \{\omega : \exists t \in T_w(\omega)$ such that $x = x(\omega, t)\}$ is the set of initial distributions ω supporting x as an equilibrium distribution of (w, ω). $T_w : \omega \to T_w(\omega)$ is then the *equilibrium correspondence* of w, $X_w : \omega \to X_w(\omega)$ is its *equilibrium distribution correspondence*, and $\Omega_w : x \to \Omega_w(x)$ is the inverse equilibrium distribution correspondence. The range $\cup_{\omega \in S_n} X_w(\omega)$ of X_w (and domain of Ω_w) will be denoted by M_w. The range $\cup_{x \in S_n} \Omega_w(x)$ of Ω_w (and domain of T_w and X_w) is denoted by Q_w. The subscript w will be omitted whenever this can create no confusion, that is, whenever we specify that w is fixed.

2.2.2. First-Order Conditions

In the remainder of this paper, I restrict myself to differentiable distributive social systems that verify the following standard assumptions:

ASSUMPTION 1: *(i) w_i is C^2 for all i (smooth preferences); (ii) w_i is quasi-concave for all i (convex preferences); (iii) w_i is strictly increasing in x_i for all i (utility increasing in own wealth); (iv) $M_w \subset \mathbb{R}^n_{++}$ (interior equilibrium distributions).*

Let $W = \{(w, \omega) : w$ verifies Assumption 1$\}$.

The first-order conditions characterizing equilibrium are given in the Theorem 1 below (proof in Appendix A). They can be restated as follows: t is an equilibrium of (w, ω) if and only if $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) \leq 0$ for all (i, j) and $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) = 0$ whenever $t_{ij} > 0$ ($\partial_{x_j} w_i$ denotes the partial derivative of w_i with respect to its jth argument, and $\partial w_i(x)$ the Jacobian matrix of w_i at x). Informally, these conditions say that, at equilibrium, a marginal incremental wealth transfer from i to j does not increase i's utility, and that a marginal incremental wealth transfer from j to i does not increase i's utility whenever the equilibrium transfer from i to j is positive.

THEOREM 1: *t is a distributive equilibrium of $(w, \omega) \in W$ if and only if for all i there exists a vector $\delta_i = (\delta_{ij})_{j \in N \setminus \{i\}} \in \mathbb{R}_+^{n-1}$ such that: (i) $\partial w_i(x(\omega, t)) \cdot \partial_{t_i} x(\omega, t) + \delta_i = 0$; (ii) and $\delta_i \cdot t_i^T = 0$.*

The Corollary 1 of Theorem 1 yields simple characterizations of M and Ω. Let: $g(t) = \{(i, j) \in N \times N : t_{ij} > 0\}$; $\gamma_w(x) = \{(i, j) \in N \times N : i \neq j$ and $-\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) = 0\}$. These sets will be viewed as directed graphs or digraphs. The *incidence matrix* $\Gamma_{w,i}(x)$ of digraph $\gamma_w(x)$ is the $(n, n-1)$-matrix defined in the following way: the rows of $\Gamma_{w,i}(x)$ are associated with the elements (vertices) of N, ranked in increasing order; the columns of $\Gamma_{w,i}(x)$ are associated with the elements (darts) of $\{(i, j) \in N \times N : j \neq i\}$, ranked in increasing lexicographic order; if $(i, j) \in \gamma_w(x)$, the entries of the corresponding column of $\Gamma_{w,i}(x)$ are -1 on row i, 1 on row j, 0 on the other rows; if $(i, j) \notin \gamma_w(x)$, the entries of the corresponding column of $\Gamma_{w,i}(x)$ are 0 on all rows. The incidence matrix $\Gamma_w(x)$ of $\gamma_w(x)$ is the $(n, n(n-1))$-matrix: $(\Gamma_{w,1}(x), \ldots, \Gamma_{w,n}(x))$. The subscript w will be omitted in notations of graphs and incidence matrices whenever w is fixed. We have then the following:

COROLLARY 1: *Let w verify Assumption 1. Then: (i) $M = \{x \in S_n \cap \mathbb{R}_{++}^n : -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) \leq 0$ for all $(i, j)\}$. (ii) For all $x \in M$, $\Omega(x)$ is the convex set $\{x - \Gamma(x) \cdot t^T \in S_n : t \in \mathbb{R}_+^{n(n-1)}\} = \{x - \Gamma(x) \cdot t^T \in S_n : g(t) \subset \gamma(x)\}$, of dimension rank $\Gamma(x)$.*

Theorem 2, finally, lists some useful consequences of Assumption 1 relative to the compactness and continuity properties of equilibrium sets and correspondences (proof in Appendix B).

THEOREM 2: *(i) If w verifies Assumption 1, then M_w is a compact subset of compact set Q_w. (ii) Correspondences $(w, \omega) \to M_w$, $(w, x) \to \Omega_w(x)$ and $(w, \omega) \to X_w(\omega)$ are compactvalued and upper hemicontinuous on their respective domains $D = \{(w, \omega) \in W : M_w \neq \emptyset\}$, $D_\Omega = \{(w, x) \in D : x \in M_w\}$ and $D_X = \{(w, \omega) \in D : \omega \in Q_w\}$.*

3. Regularity

This section defines the regularity of distributive social systems, and gives graphical examples of singular (i.e., nonregular) social systems.

3.1. Definition

The first-order conditions above can be viewed as a system of inequalities of the type $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) \leq 0$ and $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)))t_{ij} = 0$. The equilibrium gift t_{ij} associated with a strict inequality $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) < 0$ is locally determinate (=0) by complementary slackness and by the continuity of marginal utilities. The notion of regularity of a social system (w, ω) that I define below is designed to ensure that local determinacy holds also for the gifts which can take on a positive value at equilibrium. It states, essentially, that the linear system tangent to the sub-system of first-order conditions of the type $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) = 0$ has full rank at equilibrium.

Consider $t^* \in T_w(\omega)$, and let $x^* = x(\omega, t^*)$. Recall the definitions of the digraph $\gamma^* = \gamma_w(x^*)$ and of the incidence matrices $\Gamma_{w,i}(x^*)$, given above. Define $f_{w,\omega,t^*} : t \rightarrow (\partial w_1(x(\omega, (t^*_{\setminus\gamma^*}, t_{\gamma^*}))) \cdot \Gamma_{w,1}(x^*), \ldots, \partial w_n(x(\omega, (t^*_{\setminus\gamma^*}, t_{\gamma^*}))) \cdot \Gamma_{w,n}(x^*))$. And let the domain of f_{w,ω,t^*} be $\mathbb{R}^{n(n-1)}$. We say that:

Definition 2: *$(w, \omega) \in W$ is regular if rank $\partial f_{w,\omega,t}(t) = \#\gamma_w(x(\omega, t))$ for all $t \in T_w(\omega)$.*

3.2. Examples

The examples that I present here and below have the following common features. (w, ω) is in W. There are three agents $(n = 3)$. The set of feasible distributions of wealth S_3 is the triangle $O_1 O_2 O_3$, where O_i denotes the feasible distribution where agent i owns or consumes the total wealth. The feasible distribution x^i is the (supposed unique) distribution that maximizes w_i in S_3. $x^i m^{ij}$ $(i \neq j)$ is the locus of tangency points of the indifference map of agent i in S_3 with the segments $\{x \in S_3 : x_j = c\}$ such that $c \geq x^i_j$: in view of Assumption 1, this is, equivalently, the set $\{x \in S_3 : x_j \geq x^i_j$ and $-\partial w_i(x) + \partial w_j(x) = 0\}$. M_w is the area shaded grey.

Example 1: Coordination issue. In this example, represented in Figure 1, curves $x^1 m^{13}$ and $x^3 m^{31}$ overlap on ab. The set $X_w(\omega)$ of the equilibrium distributions of (w, ω) is ab, where $\gamma_w(x)$ is constant, equal to $\{(1, 2); (3, 2)\}$. For any x of ab, there is a single pair of equilibrium gifts, from agents 1 and 3 to agent 2. There is, thus, a continuum of possibilities of coordination of these individual transfers, between point a, where the burden is carried by agent 3 alone, and point b, where it is carried by agent 1 alone.

Example 2: Coincidental indifference. The second example is represented in Figure 2. $x^3 m^{31}$ contains here a "thick" region (the hatched area of Figure 2), due to flat sections of the indifference map of agent 3, parallel to segment $O_2 O_3$, between a and b. This means that, at any distribution

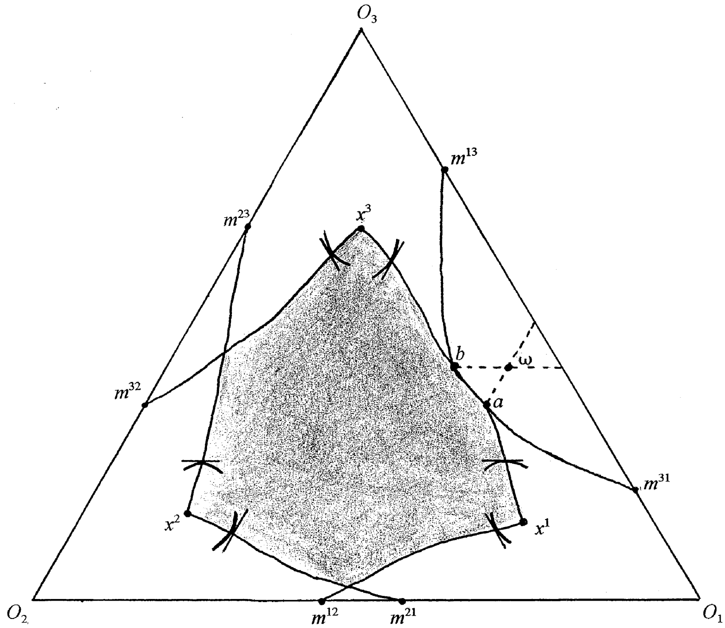

Figure 1: Coordination issue

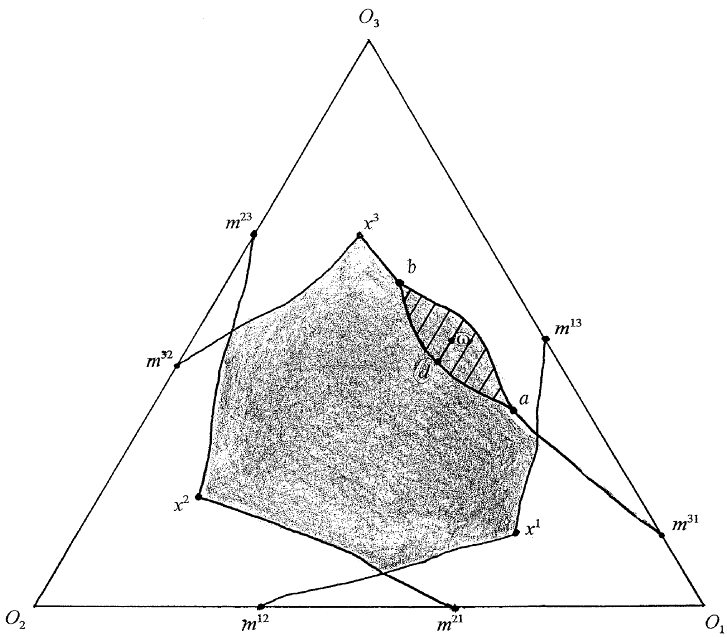

Figure 2: Coincidental indifference

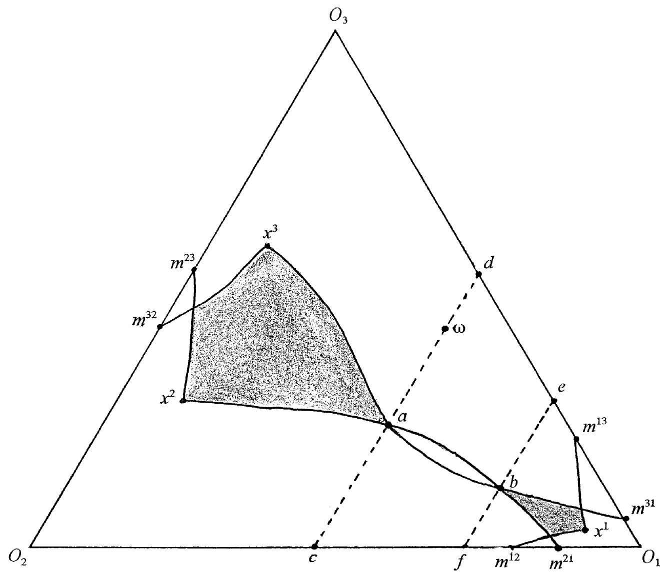

Figure 3: Reciprocal gifts

in the interior of the thick part of x^3m^{31}, agent 3 is locally indifferent to wealth transfers between himself and agent 1, in both directions. In particular, the set $X_w(\omega)$ of the equilibrium distributions of (w, ω) is the closed segment $d\omega$, where $\gamma_w(x)$ is constant, equal to $\{(3, 2)\}$. For any x of $d\omega$, there is a single equilibrium gift, from agent 3 to agent 2.

Example 3: Reciprocal gifts. In the third example, represented in Figure 3, curves x^2m^{21} and x^3m^{31} intersect at points a and b, where $\gamma_w(x) = \{(2, 3); (3, 2)\}$. The equilibrium distribution is determinate, but not the vector of equilibrium gifts, due to the presence of a circuit in $\gamma_w(a)$. Precisely: $X_w(\omega) = \{a\}$, which means that a is the sole equilibrium distribution of (w, ω); and $T_w(\omega) = \{t : t_{32} - t_{23} = a_2 - \omega_2;\ t_{ij} = 0\ \forall (i, j) \notin \gamma_w(a)\}$, which implies that any pair of reciprocal gifts between agents 2 and 3 inducing the positive net transfer $a_2 - \omega_2$ from agent 3 to agent 2 is an equilibrium.

4. Generic Properties

This section establishes the genericity of the regularity and determinacy of distributive equilibrium, and draws the consequences of these properties for distributive theory and policy (proofs in Appendix C).

4.1. Regularity

The next theorem applies transversality theory (e.g., Mas-Collel 1985, Chapter 1, I) to pure distributive social systems.

The conditions of the application of transversality theory to distributive equilibrium differ from the conditions of its application to economic equilibrium in two main respects.

First, in typical situations, most equilibrium gifts are equal to 0 at equilibrium: precisely, regular social systems have at most $n-1$ positive equilibrium gifts, hence at least $(n-1)^2$ equilibrium gifts equal 0, which means that at most one gift out of n is positive (cf. Theorem 4 below). Consequently, the set of equilibria of a social system (w, ω) will be described by a *family* of systems of equations of the type $(-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)))_{(i,j)\in\gamma} = 0$, where γ runs in the finite set $\{\gamma(x) : x \in X_w(\omega)\}$. The implicit function and transversality theorems will be applied, therefore, to such families of equations systems, instead of being applied to a single system of equations as this is ordinarily the case with economic equilibrium.

Second, it will generally not be sufficient to consider perturbations of the vector of individual endowments ω to establish genericity. The proof that regularity is a dense property of distributive social systems has to follow from the application of the transversality theorem to linear perturbations of *utility functions*, while perturbations of endowments (Debreu 1970) or utility functions (e.g. Mas-Collel 1985, Chap. 8) can be used indifferently to establish the same property for economic equilibrium.

THEOREM 3: *The set* $\{(w, \omega) \in W : (w, \omega)$ *is regular*$\}$ *is open and dense in* W.

4.2. Determinacy

This section elicits four aspects of the determinacy of regular distributive equilibrium. One is a straightforward general implication of regularity: the local determinacy of regular distributive equilibrium, established in Theorem 5, is the exact analogue of the local determinacy of regular market equilibrium, first established by Debreu 1970.

The other three are of more specific interest for the study of distributive social systems: their graphs of equilibrium gifts are forests, or, equivalently, equilibrium distributions are in one-to-one correspondence with equilibrium gift vectors (Theorem 4); M is locally, in the neighborhood of a regular equilibrium distribution x, a finite union of boundaryless C^1 manifolds of dimension $n - 1 - \#\gamma$, where $\#\gamma$ runs in $\{1, \ldots, \#\gamma(x)\}$ (Theorem 6 and Corollary 2); and a status quo equilibrium of a regular distributive social system, if any, is the unique equilibrium of this social system (Theorem 7 and Corollary 3).

These properties of determinacy, being verified by all regular social systems of W, are generic in the latter (Theorem 3).

THEOREM 4: *If* $(w, \omega) \in W$ *is regular, then it verifies the following three equivalent properties: (i) for all* $x \in X(\omega)$, $\gamma(x)$ *is a forest; (ii) for all* $x \in X(\omega)$, *dim* $\Omega(x) = \#\gamma(x) = v(\gamma(x)) - c(\gamma(x))$, *where* $v(\gamma(x))$ *and* $c(\gamma(x))$ *denote respectively the* *number of vertices and number of connected components of* $\gamma(x)$*; (iii) the restriction to* $T(\omega)$ *of function* $\mathbb{R}^{n(n-1)} \to \mathbb{R}^{n}: t \to x(\omega, t)$ *is one-to-one.*

THEOREM 5: *If* $(w, \omega) \in W$ *is regular, then* $T(\omega)$ *is finite and* $\#T(\omega) = \#X(\omega)$.

THEOREM 6: *If* $(w, \omega) \in W$ *is regular, and if* $\omega \in M$*, then there exists an open neighborhood* U *of* ω *in* $S_n \cap \mathbb{R}^n_{++}$ *such that, for all open neighborhood* $V \subset U$ *of* ω *in* $S_n \cap \mathbb{R}^n_{++}$ *and all* $\gamma \subset \gamma(\omega)$, $V \cap \{x \in M : \gamma(x) = \gamma\}$ *is a boundaryless* C^1 *manifold of dimension* $n - 1 - \#\gamma$.

COROLLARY 2: *Suppose that* $(w, \omega^0) \in W$ *is regular, and let:* $t^* \in T(\omega^0)$*; and* $A = \{\omega : \exists t \in T(\omega) \text{ such that } g(t) = g(t^*)\}$*. Then, there exists a neighborhood* U *of* $x(\omega^0, t^*)$ *in* $S_n \cap \mathbb{R}^n_{++}$ *such that* $\cup_{\omega \in A}(X(\omega) \cap U)$ *is a boundaryless* C^1 *manifold of dimension* $n - 1 - \#\gamma$.

THEOREM 7: *If* $(w, \omega) \in W$ *is regular, and if* $0 \in T(\omega)$*, then* $T(\omega) = \{0\}$.

COROLLARY 3: *If* $(w, \omega^0) \in W$ *is regular, then there exists a neighborhood* V *of* ω^0 *in* S_n *such that the restriction of* T_w (resp. X_w) *to* $V \cap M_w$ *is function* $\omega \to 0$ (resp. $\omega \to \omega$).

4.3. Implications for Distributive Theory

The fact that the graphs of equilibrium gifts are, generically, forest graphs, simple as it is, has far-reaching consequences for the domain of relevance of the theory of gift-giving which is considered in this article. This theory tends to view gift-giving as the mean of equalizing, hence essentially unilateral, benevolent redistributions of wealth. I argue below that: benevolent reciprocity appears in the theory as a limit, though interesting case, located on the boundary of its universe of logical possibilities; the theory implies a type of nonaltruistic reciprocity, through the Rotten Kid Theorem, but does not model it explicitly; and agonistic reciprocity appears definitely beyond its universe of logical possibilities.

4.3.1. Benevolent Reciprocity

Benevolent reciprocity corresponds formally to the presence of a directed circuit in the graph of equilibrium gifts, and we know from the second part of the proof of Theorem 3 and from Theorem 4(i) that linear perturbations of utility functions will generally disrupt any circuit, directed or not, of this graph.

The social system of Example 3 is a case of benevolent reciprocity between agents 2 and 3. The utility functions of this example generate the same graph of equilibrium gifts $\{(2, 3); (3, 2)\}$ for all $\omega \neq a, b$ taken in segments *cd*, with equilibrium distribution *a*, and *ef*, with equilibrium distribution *b*. Q_w is the union of surfaces $O_2 O_3 cd$ and efO_1: there exists no equilibrium when ω is in the relative interior of surface *cdef* in S_3, because of a "war of gifts" (Mercier Ythier 1989, 1992, 1993, and 2000a) between agents 2 and 3. Curves $x^i m^j$

and set Q_w move continuously in response to small enough continuous linear perturbations of w, with the following typical consequences on equilibrium of the perturbed w for the fixed ω of Figure 3: either there is a single equilibrium transfer, from agent 3 to agent 2, or there is a war of gifts between these two agents. The same typical consequences obtain when ω is perturbed and w is fixed.

The detailed analysis of this example helps to make precise the meaning and scope of genericity. The generic property under consideration here states essentially that circuits of equilibrium gifts are scarce relative to the universe of logical possibilities represented by the social systems of any open subset of W. Nevertheless, the theory views w and ω as fixed: the "perturbations" considered in the proof of genericity properties are but technical devices for eliciting the static diversity of initial conditions, to be clearly distinguished from systematic variations following well-defined and motivated stochastic or dynamic processes. The social system of Figure 3 illustrates this difference between exogenous and endogenous typicality. This social system is singular in several, interrelated senses: not only the mathematical sense above, but also an esthetic and an ethical sense, which bestow positive moral values on it, such as goodness or moral beauty. It "maximizes love subject to individual property," so to speak, in establishing a fragile balance of reciprocity and individuality, typically disrupted by the variation of w or ω, which either breaks the directed circuit of gifts or generates a war of gifts that undermines individual property. The social scientist confronting this social system can reasonably suppose that his consciousness of the logical, esthetic and ethical properties above is shared, to some extent, by the agents participating in its functioning, and find, in this speculation of a common consciousness of actors and spectators, a well motivated support, in the particular case under consideration, for his general contention that preferences and endowments should be the stable determinants of the theory. In other words, the social system of Figure 3, coincidental as it is, is a valuable, logically self-sustained theory.

4.3.2. The Rotten Kid Theorem

The Rotten Kid Theorem of Becker (1974), which is implied by the distributive theory considered in the present article, gives general conditions under which altruistic gift-giving induces reciprocity from nonaltruistic beneficiaries.

It captures the situations where the wealth, hence utility of an egoistic agent (the rotten kid) is an increasing function of the wealth of his altruistic benefactor (the family head) because the optimal gift of the benefactor is an increasing function of the benefactor's wealth. This happens when the wealth of the beneficiary is a normal good to the benefactor. The egoistic beneficiary, acting as the follower in a Stackelberg interaction with his benefactor, is then determined to undertake any action that increases the donor's wealth, when the cost of such an action does not exceed the induced increase in the transfer from the donor.

These acts of nonaltruistic reciprocity are not modeled explicitly by Becker. They cannot be in the context of the pure distributive social systems that I consider here: if individuals respect the right of private property, the nonaltruistic gift from the rotten kid to the family head is bound to operate through exchange or production.

The interaction of benevolent wealth transfers with competitive exchange and production, as analyzed in Mercier Ythier (2000a), leave little room, moreover, for nonaltruistic acts of reciprocity. The Rotten Kid Theorem, and, more generally, any nonaltruistic act of gift-giving, can appear there, in principle, only as an aspect of the transfer paradox, when an egoistic agent realizes that he can influence equilibrium market prices by giving up some of his own endowment, and is able to increase, that way, the value of the pre-transfer endowment of his altruistic benefactor, the value of the latter's transfer to him, and/or the value of his own post-transfer endowment. While logically admissible, this possibility appears largely irrelevant for two reasons: competitive exchange supposes implicitly that individual wealth is "small" relative to aggregate wealth, in the sense that variations of individual endowments can have only negligible consequences on equilibrium market prices; and individual participants in a competitive, hence large economy, are not supposed to know nor to be able to learn the equilibrium correspondence, hence to compute the consequences of the variation of their individual endowment on equilibrium prices. In short, the individual participants in a competitive economy will safely consider prices as given, and, therefore, gifts as "dollar-for-dollar" wealth transfers from the donors to the beneficiaries. This lets us back, essentially, to the distributive theory of the present article.

The acts of nonaltruistic reciprocity implied by the Rotten Kid Theorem will consist, thus, of exchanges and productions at nonmarket prices, or of violations of property rights. The Rotten Kid Theorem means, in other words, that the individual relation created by gift-giving is liable to induce nonaltruistic acts of reciprocity, that can be analyzed as a type of individual nonmarket exchange when agents respect property rights.

4.3.3. Agonistic Reciprocity

Agonistic reciprocity or *potlatch* has been much studied by Anthropology since the classical work of Mauss (1924) (see the recent references of Godelier 1996 and 2000).

The benevolent equalization of wealth, which lies at the heart of the present theory of gift-giving, and potlatch gift-giving, appear in many respects as polar cases, intimately associated with their respective, systematically contrasted anthropological backgrounds.

The comparison of benevolent equalization with the potlatch of the Trobriand Islands, the celebrated *kula* (Godelier 1996, 2000), will allow us to characterize and situate more precisely the domain of relevance of the former. It yields four articulated differences on: the nature of transferable wealth; the type of individual donors and beneficiaries of transfers; the definition of property rights on transferable wealth; and the motives of transfers. Transferable wealth consists of: abstract money wealth for individual consumption in the first case; specific types of symbolic objects (armbands and necklaces) intended for kula gifts in the second case. Individual donors and beneficiaries consist of: private owners unequally endowed, the donor being much wealthier than the beneficiary, in the first case; nobles (chiefs and "Big Men") representing their clans in the second case. Individual property rights on transferable wealth consist of: a full right of *usus* and *abusus*, that is, of selling, giving, consuming and disposing of owned wealth in the first case; in the second case, a partial right of *usus*, limited to a right of enjoying the possession of potlatch objects (that is, having them at home) during a period of time precisely determined by custom. And the motives of transfers consist, eventually, of: benevolent equalization of wealth in the first case; competition for rank or fame in the second case.

It is interesting, given this systematic opposition, to point out, first, a fundamental common feature between the two: the social links materialized by transfers are operated by individuals who feel and/or understand their participation as the expression of their individual right of property on transferable wealth. Both systems build up social relations on individual participation, realizing hence a delicate balance of individual existence and the existence of society, through the articulation of: on the one hand, the social orientation of individual motives, reflecting by definition an individual sense of responsibility in the existence of society, and perhaps, though indirectly, an individual sense of the reliance of individual existence on the existence of society; and on the other hand, the social enforcement of individual property rights on transferable wealth.

From this common foundation, benevolent equalization and potlatch draw an opposition of two types of societies.

Benevolent equalization concerns large democratic societies, which consist, ideally, of large populations of individual owners operating on the background of competitive market economies and an impersonal representative State.[2] The population and economy are large notably in the sense that individual endowments and consumptions are negligible relative to aggregate wealth. Transferable wealth consists, there, of the exchange value of the market commodities destinated to individual consumption. It is entirely appropriated by individuals in the two interrelated senses of being: entirely consumed

[2]Mercier Ythier (2004) analyzes the determination of the distribution of wealth in such a large democratic social system: the process of determination tends to the realization of a liberal social contract; and the social system is large in the sense, analogous to the notion of a large economy defined in Debreu and Scarf (1963), that the number of types of individuals is negligible relative to the total number of individuals.

by them; and the object of a full right of *usus* and *abusus* of individual owners. The individual members of this ideal democratic society all have the same status of owner–consumer, and their ownership and consumption are all negligible relative to the sum of individual ownerships, so they can be compared only on the basis of their individual differences in owned or consumed wealth. The individual sense of commonality finds then a natural expression in (mildly) equalizing preferences on the distribution of individual consumption expenditures.

Potlatch concerns small aristocratic societies (though a limited number of important aspects of it might possibly concern also the distinguished subgroups of large democratic societies, such as the so called "meritocracies" of modern developed countries). Transferable wealth is a dignity, that distinguishes, simultaneously: the initiator of a chain of potlatch exchanges, whose name remains attached to the potlatch object and who keeps his ownership of it till the closure of potlatch exchanges; and the temporary owner, who received the potlatch object, with the obligation of accepting the gift, and the obligation of giving it himself in precisely defined customary conditions. The dignity is positional, conveying a representation of the aristocratic status of the participants in potlatch, and relational, establishing, through the specificities of the property right on potlatch objects, a temporal sequence of relations between the initiator of the chain of potlatch exchanges and each of the subsequent participants in it. The distinction between potlatch objects and consumable wealth, hence, is a manifestation of the more fundamental distinction of nobles with ordinary people that characterizes aristocratic societies. Potlatch concerns aristocratic societies that are small notably in the sense that the ownership of consumables by nobles, and particularly by chiefs, is a sizeable fraction of the consumable wealth of the community. This makes a significant difference with distinguished sub-groups of large democratic societies, integrated in the impersonal system of representations of society constituting the State, which establishes, in particular, a clear-cut distinction between common wealth, conceived ideally as the sum or proceeds of the sum of individual wealths, and the individual, proportionately negligible wealth of its distinguished members. The individual sense of commonality of nobles then finds a natural expression in their eagerness to participate in the circulation of dignities in potlatch exchanges, which appears individually as a competition for reknown, but ends, in a paradox analogous in some respects to Adam Smith's "invisible hand" of competitive market exchange, in an intertemporal equalization of individual positions, through the permanent renewal of their permutations.

4.4. Implications for Distributive Policy

I consider now the implications of generic determinacy for public distributive policies operating by means of lump-sum transfers, that is, by manipulating the vector of individual endowments ω.

These implications are twofold.

One concerns "local" distributive policies (Mercier Ythier 2000a), where public redistributions are confined in an arbitrarily small neighborhood of 0. We know already that such policies can operate redistributions between the connected components of the graph of equilibrium transfers, but not inside them unless public transfers are large enough to crowd out some of the existing equilibrium transfers (Mercier Ythier 2000a, Theorem 4). The analysis above brings the additional precisions that, generically: the number of components of the graph of equilibrium transfers is the number of agents connected by gifts minus the number of gifts (Theorems 3 and 4(ii)); and the set of equilibrium distributions that obtains from local public lump-sum transfers leaving the graph of equilibrium gifts unchanged (and therefore crowding out none of these gifts) is a differentiable manifold whose dimension equals the dimension of the set of feasible distributions minus the number of equilibrium gifts (Theorems 3 and 6 and Corollary 2). In particular, generically, local distributive policy is neutral if and only if the graph of equilibrium transfers is a tree (i.e., connected forest) connecting all the agents of the social system.

Note that the scope of these results is limited, nevertheless, in the case of local distributive policy, by the theoretical possibility of the existence of multiple equilibria when equilibrium is not a status quo. The issues of distributive policy raised by equilibrium multiplicity are illustrated in Example 4.

Example 4: Discontinuity and/or indeterminacy of public redistribution. The social system (w, ω) of Figure 4 is regular. It has two equilibrium distributions a and b, such that $\gamma_w(a) = \{(1,2); (3,2)\} = \gamma_w(b)$. There is only one positive equilibrium gift at a (resp. b), from agent 3 (resp. 1) to agent 2.

Suppose now that the actual equilibrium is a, and that the state, considering that the burden of redistribution must be shared more equally between agents 3 and 1, decides to make a small lump-sum transfer from agent 1 to agent 2, hence moving the vector of endowments from ω to ω'. The unique equilibrium distribution of (w, ω') is b. The public policy misses its objective: it induces a jump of the equilibrium distribution, from distribution a where the burden is carried by agent 3 alone, to distribution b, where it is carried by agent 1 alone. Moreover, the same jump will occur for all $\omega' \in \,]\omega, b]$, because of the discontinuity of the equilibrium correspondence at ω (X_w is upper hemicontinuous, by Theorem 8(ii), but generally not lower hemicontinuous, when w verifies Assumption 1).

Suppose, conversely, that the original social system is (w, ω'), whose unique equilibrium distribution is b, and that the state, for the same type of considerations as above, decides to move the vector of endowments from ω' to ω. The public policy misses its objective again because: either agents 1 and 3 coordinate on b, and we have a case of neutrality of distributive policy;

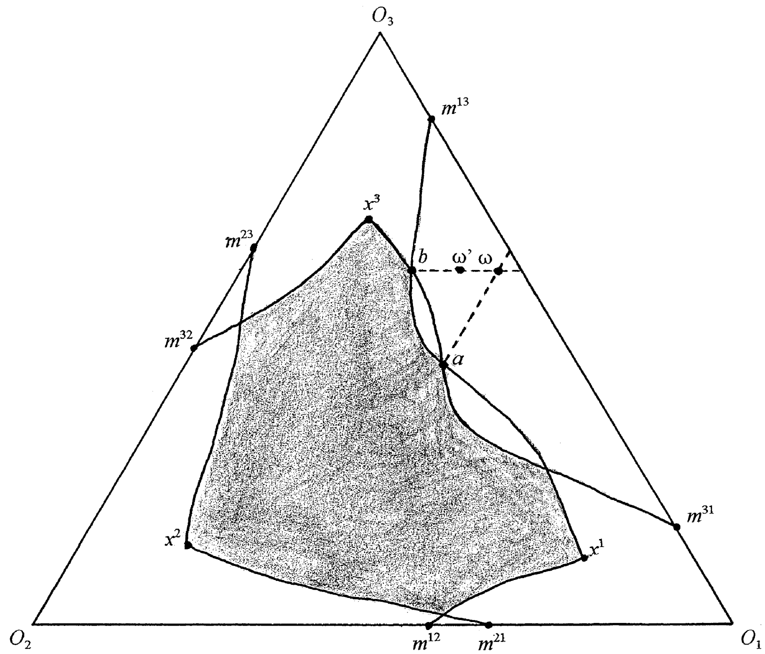

Figure 4: Discontinuity and/or indeterminacy of public redistribution

or they coordinate on a, and we end up with the same type of nondesired discontinuous jump as above. Moreover, distributive policy is neutral for all vector of public transfers $z \in \,]0, \omega - \omega'[$.

This example does not mean, of course, that distributive policy becomes ineffective in the presence of multiple equilibria. It illustrates, rather, the fact that the ways of public action cannot always be deduced in a simple, univocal manner from its objectives, because of the complexity that stems from individual interactions: the local distributive policy will reach its aim, in Example 4, by means of a suitable combination of public redistribution in favour of agent 2 with public redistribution between agents 1 and 3.

The second implication of generic determinacy for distributive policy, drawn from the generic uniqueness of status quo equilibrium (Theorem 7 and Corollary 3), states that such complexities are particular to local distributive policy: if distributive policy is allowed to fully crowd out all private transfers, then it can reach any, generically determinate, equilibrium distribution (that is, any, generically determinate, element of M). Since the full crowding out of all private transfers is accessible at any equilibrium distribution by simple substitution, for all equilibrium gifts, of identical public transfers (Corollary 1), distributive policy can always reach any, generically uniquely defined, equilibrium distribution.

5. Nongeneric Properties

It is tempting to skip from the *property*, established in the proof of Theorem 3, that any $(w, \omega) \in W$ is regular up to an arbitrary linear perturbation of w, to the *assumption* that $(w, \omega) \in W$ is regular for all $\omega \in S_n$. This defines strong regularity, which is a substantial property: it is nongeneric (Section 5.1); and it implies the following two nongeneric consequences that Q_w is either empty or equal to S_n (Section 5.2), and that M_w is connected and a finite union of pairwise disjoint boundaryless C^1 manifolds whose dimensions run over $\{0, 1, \ldots, n\}$ (Section 5.3).

5.1. Strong Regularity

Strong regularity is defined formally as follows.

DEFINITION 3: *w is strongly regular if $(w, \omega) \in W$ is regular for all $\omega \in S_n$.*

Example 3 yields a counterexample to the genericity of strong regularity: curves x^2m^{21} and x^3m^{31} react continuously to continuous perturbations of w, so their intersection remains a nonempty subset of $S_n \cap \mathbb{R}^n_{++}$ if the vector of utility functions is picked in a small enough neighborhood of w.

5.2. Existence

THEOREM 8: *If w is strongly regular, then either Q_w is empty or $Q_w = S_n$.*

A strongly regular social system, therefore, either has an equilibrium for all initial distributions, or has an equilibrium for none. Example 5, drawn from Mercier Ythier 1998a, illustrates the latter case.

Example 5: Generalized war of gifts. The system w of Figure 5 has an empty set M_w, and therefore is strongly regular. It is the place of generalized, bilateral and trilateral wars of gifts.[3] At equal distribution $e = (1/3,1/3,1/3)$, for instance, a marginal wealth transfer from agent 1 (resp. 2, resp. 3) to agent 2 (resp. 3, resp. 1) increases the utility of the former.

Strong regularity, hence regularity, are not sufficient for the existence of a distributive equilibrium. A simple corollary of Theorem 8 is, nevertheless, that the strong regularity of w implies the existence of a distributive equilibrium for all ω provided that M_w is nonempty. Mercier Ythier 1998b,

[3]The wars of gifts of examples 3 and 5 express an incompatibility of individual plans concerning the distribution of wealth, in a context where individuals enjoy unrestricted property rights on their wealth. They differ from the potlatch studied by anthropologists in essential respects relative to the definition of wealth, property rights and individual motives (cf. 4.3.3 above and Mercier Ythier 2000b, 2.3). Obviously, they call for the institution, in the corresponding social systems, of a collective ownership of wealth and a collective decision process for its distribution.

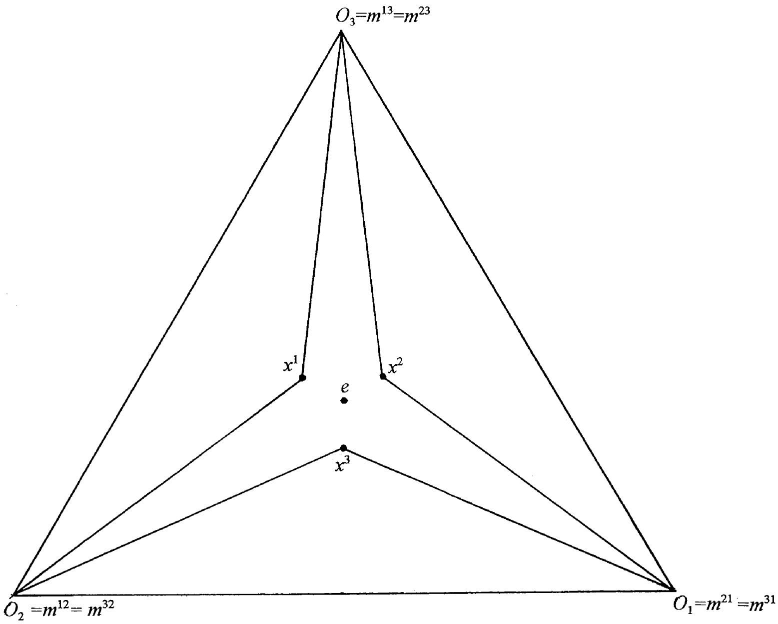

Figure 5: Generalized war of gifts

Theorem 1, gives a weak sufficient condition on individual utility functions for the nonemptiness of M_w. And Mercier Ythier 1993, Theorem 2 and 2000a, Theorem 3, provide a natural condition on these utility functions implying existence without implying regularity nor being implied by strong regularity.

Examples 3 and 5 yield counterexamples to the genericity of the existence of an equilibrium. In Figure 3, surface *cdef* changes continuously in response to continuous perturbations of w, and is therefore nonempty for all vector of utility functions taken in a small enough neighborhood of w. In Figure 5, curves $x^i m^{ij}$ react continuously to continuous perturbations of w, so $M_{w'}$ remains empty for all w' taken in a small enough neighborhood of w.

5.3. Connectedness

THEOREM 9: *If w is strongly regular, then M_w is connected.*

THEOREM 10: *Suppose that $w \in W$ is strongly regular and that M is nonempty. Then M is a finite union of pairwise disjoint boundaryless* C^1 *manifolds* $\{x \in M : \gamma(x) = \gamma\}$ *of dimension* $n - 1 - \#\gamma$, *where* $\#\gamma$ *runs over* $\{0, 1, \ldots, n-1\}$ (*that is, where* $\{\#\gamma(x) : x \in M\} = \{0, \ldots, n-1\}$).

These properties of strongly regular social systems have interesting implications for distributive policy. Combined with the determinacy of status quo equilibria (Theorem 7 and Corollary 3), they imply that a distributive policy operating by lump-sum transfers from an initial distribution ω lying in M will be able to reach any objective in M by redistributing endowments along a *continuous* path contained in M (a C^1 path if the target lies in the same connected component of $\{x \in M : \gamma(x) = \gamma(\omega)\}$ as ω). The same remains essentially true when the distribution of rights ω does not lie in M, with the minor qualification that the distributive policy must crowd out first all equilibrium private transfers, before being able to proceed by continuous adjustments inside M.

Example 3 yields, again, a counterexample to the genericity of the connectedness of M_w: the latter is modified in a continuous way by continuous perturbations of w, and remains therefore disconnected if the vector of utility functions is picked in a small enough neighborhood of w.

6. Conclusion: An Evaluation of the Theory

The basic properties of this theory can be summarized in, or deduced from, an assumption of perfect substitutability of public and private transfers encompassing, notably, the following important features: (a) the perfect divisibility of private wealth; (b) complete and enforced property rights on private wealth; (c) lump-sum public and private transfers; (d) public and private transfers aiming at equalizing the distribution of private wealth; (d) the complete and perfect information of distributive agents on all variables relevant for public and private distributive decisions; and (e) the Nash conjecture for private transfers.

A fundamental property of this explanation of the distribution of wealth is the so-called "public good problem," namely that, in the presence of common distributive concerns, the distributive equilibrium is generally not Pareto efficient with respect to distributive preferences.

Becker's theory of social interactions is a remarkable exception to this general property: the sole donor of his gift equilibrium manages to reach his optimum, that is, his best distribution subject to the sole aggregate feasibility constraint, which corresponds to a strong Nash equilibrium of the game, and is therefore Pareto efficient.

This exception is essentially unique (Mercier Ythier 2000a,[10] 2000b, Section 2.2, 2002, Section 5.1.). The conciliation of distributive equilibrium with distributive efficiency supposes, in general, a social contract between the individuals who share the same distributive concerns. Particularly important is the case where distributive concerns are public, that is, in the exact acceptation of this term, shared by *all* individual members of the society, though possibly to variable degrees for each of them ("local public goods" should rather be termed social or community or club goods, depending on the context). This corresponds, practically, to poverty relief, where the simultaneous achievement of gift equilibrium and distributive efficiency obtains through the public transfers of the liberal social contract. I noticed in the introduction that such public transfers implied, generally, the full crowding out of all private transfers.

Perfect substitutability, confronted with the requirement of distributive efficiency (that can be viewed, in the context of this theory, as a basic requirement of democratic rationality), is therefore a knife edge, eliminating all equilibrium configurations but two: Becker's microsocial equilibrium, associated with individual distributive concerns limited to close relatives, where redistribution is fully private; and the distributive liberal social contract, associated with general distributive concerns relative to poverty, where redistribution is fully public.

I argued above (Section 4.3.3.) that this distributive theory was suitable for the explanation of the distribution of wealth in a large democratic society, combining three features: a large number of individual owners–consumers; a competitive economy; and an impersonal representative State.

First, the representation of wealth implied by points (a), (b), and (c) of the assumption of perfect substitutability above supposes perfectly competitive markets, hence a large economy.

Second, in the pure or ideal type of this large democratic society, individuals are differentiated objectively by their ownership and consumption of market wealth only, and each individual wealth is negligible relative to aggregate market wealth. The individual sense of commonality must find, therefore, in such a context, its expression in individual preferences for averaging, mildly equalizing redistributions of wealth, notably from the most to the least wealthy. And the large size of population and negligible relative wealth of its individual members, combined with the public (in the sense recalled above) character of these distributive concerns, imply the Nash conjecture for individual transfer decisions.

Third, the State of this ideal democratic society is a mere expression of the common concerns of the individual members of this society, which implies notably, in view of the first and second points above, that: common wealth is nothing other than the sum of individual market wealths; public policy reduces to equalizing redistributions of wealth that are unanimously required by its individuals members (or, at least, required by some of them and vetoed by none of the others).

The large democratic society, so defined, is an abstract type, synthesizing the abstract notions of (perfect) competitive economy and (putative) liberal social contract. This abstract concept can serve the explanation of actual wealth distribution in two distinct ways.

The large democratic society can be viewed, first, as a hypothetical limit to which real societies tend in the course of their economic development. The principle of explanation is, here, implicitly dynamic, and emphasizes the development of market exchange as the fundamental driving force of the evolution of society. It provides interesting intuitions for the interpretation and evaluation of the theory but exposes to the danger of substituting a prophecy for a causal explanation of reality (Karl Marx and Alexis de Tocqueville produced some of the most famous of such "analytic prophecies," whose contrasted successes illustrate the difficulties inherent to prediction in the social sciences; one is tempted to think, nevertheless, that the astonishing perceptiveness of the predictions of the second was correlated, at least partly, to the accuracy of his analysis of modern politics). The proper use of the notion, understood as an abstract representation of an anthropological structure associated with a certain stage or type of economic development, supposes, therefore, its systematic confrontation with the observation of anthropological evidence on developed and other societies. Section 4.3.3. illustrates the analytical fruitfulness of such confrontations.

The notion can be used, second, as an atemporal system of reference, a collection of analytical concepts endowed with sense and inner consistency, relative to which both the social reality and its explanatory models can be situated. A large part of modern economic theory proceeds from the systematic application of such heuristics from the abstract notion of perfect competition: the notion is used to identify a host of potential sources of "imperfections" in the actual functioning of economies such as nonconvexities, incomplete markets or information etc., which, properly formulated, are then incorporated in suitable models, whose logical properties are deduced, and confronted both with the theoretical "ideal" and with observed reality. The same task has now been largely undertaken, though in a less conscious and systematic way, from the notion of perfect substitutability outlined above. Let us review briefly some of these works.

Much empirical work has been done to test the most salient implication of perfect substitutability, namely, the "one-for-one" crowding out effect, stating that individuals will adjust their transfers so as to offset variations in the transfers from another, public or private source, as long as the nonnegativity conditions on their transfers are not binding. The evidence for charitable donations (e.g., Kingma 1989, Schokkaert and Van Ootegem 2000) and intrafamily transfers (Altonji, Hayashi, and Kotlifoff 1997) yields mixed conclusions: on the one hand, it elicits significant crowding-out effects; but, on the other hand, these crowding-out effects are only partial, significantly smaller than the "one-for-one" substitution predicted by the theory.

A flourishing stream of literature develops, accordingly, that challenges the various aspects of the assumption of perfect substitutability outlined at the beginning of this section, for instance by: deducing the nonneutrality of the public debt from *capital market imperfections* (Altig and Davis 1993); finding a raison d'être to charitable institutions in the Pareto improvements that they are able to induce in the private provision of an *indivisible* public good (Andreoni 1998) and in the case of *imperfect information* of donors (Andreoni 2000); deducing an "exchange motive" for transfers (Bernheim, Shleifer, and Summers 1985) or a "Samaritan's dilemma" (Lindbeck and Weibull 1988, Coate 1995) from *strategic interactions* between donors and beneficiaries; or assuming that individual transfers serve *more complex motives* than flat redistribution of wealth, such as "warm glow" (Andreoni 1989), or various forms of reciprocity (Akerlof 1982, Falk and Stark 2000, Kolm 1984, and 2000).

The challenge raised by the last class of models appears more fundamental than the other ones, for it paves the way to the recognition and analysis of the role of gift-giving in the production and reproduction of society, the dynamics of social links so to speak.

I argued in Section 4.3.3. that gift-giving was the expression of a delicate balance between socially enforced individual property rights and socially perceptive individual motives. The paradox of this symmetric reliance of social life on individual senses of commonality on the one hand, and of individual lives on the social enforcement of individual rights on the other hand, making room both to autonomous individual actions that express complex intellectual and sensitive representations of their human environment, and to complex institutions that permit these individual expressions and contribute to shape their interaction, this paradox, thus, is at the heart of what Godelier names the "enigma of the gift."

This tension between individual rights and individual social motives is particularly sensitive in the contexts where gift-giving is related to individual life and death, namely: support to the least favored members of society; and individual acts of intergenerational transmission of such valuable "assets" as life, wealth, education, culture or political power.

Support to the least favored is part of the *definition* of society, for the social category of the least favored individuals is a fundamental part of the inner structure of society, and the level of support contributes to the determination of the borderline separating those who, among the least favored, are still able and willing to maintain their participation in social life, and those who are not and suffer consequently some kind of social death. I argued above that the distributive theory considered in this article succeeds in accounting for this type of redistribution in the context of its abstract representation of the large democratic society.

Individual acts of intergenerational transmission are direct consequences of the finiteness of individual biological lives, of course. But their overall content conveys an essential part of the "assets" of a society, from genes to capital goods, financial assets or education, hence makes an essential contribution to the reproduction and, more generally, the *dynamics* of society. The controversy on the determinants of aggregate savings and capital accumulation which began in the early eighties with Kotlikoff and Summers 1981, is a nice illustration of this point: Franco Modigliani was right to assert that the bulk of aggregate savings and capital accumulation of contemporary developed economies originated in the human wealth of the middle classes; but he was wrong to infer from this fact that most (80–90%) of the stock of accumulated wealth originated in the life cycle motives for saving, as opposed to the transfer motives, for the simple reason that human wealth is inherited also, though admittedly in a different and somewhat looser sense than what is ordinarily meant by bequests and donations of nonhuman wealth (see notably Modigliani 1986 and the synthesis of Gale and Scholtz 1994). It is debatable whether, and to what extent, the distributive theory of the present article can be applied to the analysis of intergenerational transmission. The central issue lies, here, in the conditions of transmission of human wealth, notably education, which seem at variance with the assumption of perfect substitutability above, in at least three (possibly interrelated) respects: the specificities ("imperfections") of the markets for education, health and social insurance; the specific limits of individual property rights on human wealth; and the existence of "merit wants" in the preferences and common concerns of individuals relative to education, health and social insurance, which point, notably, to specific transfer motives, driven by more complex ends than the redistribution or transmission of market wealth, such as, for instance, intentions relative to the social status of own descendants or to the social behavior of future generations. The proper identification and formulation of these conditions of transmission of human wealth seems to be a major challenge for future research in the theory of economic and social equilibrium.

Appendix A: First-Order Conditions for Equilibrium

Proof of Theorem 1: Let t^* be a distributive equilibrium of $(w, \omega) \in W$. Equalities (i) and (ii) of Theorem 1 are the first-order conditions for a maximum of $t_i \to w_i(x(\omega,(t^*_{\setminus i}, t_i)))$ in $\{t_i : x_i(\omega,(t^*_{\setminus i}, t_i)) > 0\}$. These conditions are necessary by Assumption 1-(i), and Assumption 1-(iv) (e.g. Mas-Colell 1985, D.1). They are sufficient by Assumption 1-(ii), Assumption 1-(iii), and Assumption 1-(iv) (Arrow and Enthoven 1961, Theorem 1-(b) or (c)). ■

Proof of Corollary 1: (i) If $t \in T(\omega)$, then $x(\omega, t) \in \{x \in S_n : -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) \leq 0$ for all $(i, j)\}$ by Theorem 1. Conversely, if $\omega \in \{x \in S_n : -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) \leq 0$ for all $(i, j)\}$, then $0 \in T(\omega)$ by Theorem 1.

(ii) Notice that: $\Gamma(x) \cdot t^T = \Gamma(x) \cdot (0_{\setminus \gamma(x)}, t_{\gamma(x)})^T$ for all t. Suppose therefore without loss of generality that $g(t) \subset \gamma(x)$. Notice then that $\Gamma(x) \cdot t^T = (\Delta_1 t, \ldots, \Delta_n t)$ and apply Corollary 1-(i), Theorem 1 and the definition of Ω. ■

Appendix B: Topological Properties of Equilibrium Correspondences and Sets

In the proof of Theorem 2, two lemmas are needed. These are formulated and proved now first.

LEMMA 1[4]: *For all t, there is a t' such that $g(t')$ is a forest and $\Delta_i t' = \Delta_i t$ for all i.*

[4]Established in Mercier Ythier (1992) as first step of the proof of Proposition 3.

Proof: Consider a circuit $C = ((i_k, j_k))_{1 \leq k \leq m}$ of $g(t)$.

Suppose without loss of generality that $t_{i_1 j_1} = \min_k t_{i_k j_k}$, and define recursively the following two orientation classes of the darts of C: (i_1, j_1) has positive orientation; (i_{k+1}, j_{k+1}) has positive (resp. negative) orientation if either (i_k, j_k) has positive orientation and $j_k = i_{k+1}$ (resp. $j_k = j_{k+1}$), or (i_k, j_k) has negative orientation and $i_k = i_{k+1}$ (resp. $i_k = j_{k+1}$) (with the convention that $(i_{m+1}, j_{m+1}) = (i_1, j_1)$). This orientation is well defined, for if some dart had simultaneously a positive and negative orientation, then this should be the case of all darts by the recursive definition above, and this would imply in turn that C has a single vertex i and a single dart (i, i), which contradicts the definition of $g(t)$.

Define now gift vector t^1 such that: $t^1_{i_k j_k} = t_{i_k j_k} - t_{i_1 j_1}$ whenever (i_k, j_k) has positive orientation in C; $t^1_{i_k j_k} = t_{i_k j_k} + t_{i_1 j_1}$ whenever (i_k, j_k) has negative orientation in C; $t^1_{i_k j_k} = t_{i_k j_k}$ whenever i or j is not a vertex of C. One verifies readily that $g(t^1)$ does not contain circuit C (dart (i_1, j_1) has been deleted) and that $\Delta_i t^1 = \Delta_i t$ for all i. The conclusion follows then from a recursive application of the algorithm above to all circuits of $g(t)$ (in finite number since $g(t)$ is finite). ■

LEMMA 2: *$\{t : g(t) \subset \gamma$ and $\exists\, \omega \in S_n$ such that $x(\omega, t) \in S_n\}$ is compact whenever γ is a forest.*

Proof: Let $T = \{t : g(t) \subset \gamma \text{ and } \exists\, \omega \in S_n \text{ such that } x(\omega, t) \in S_n\}$. I establish first that T is closed, and then that it is bounded whenever γ is a forest.

Consider a converging sequence $(t^q)_{q \in \mathbb{N}}$ of elements of T and denote t^* its limit. By continuity, t^* is a gift vector such that $g(t^*) \subset \gamma$. For all q, let $\omega^q \in S_n$ be such that $x(\omega^q, t^q) \in S_n$. The compactness of S_n implies that sequence $(\omega^q)_{q \in \mathbb{N}}$ has at least one limit point $\omega^* \in S_n$. The continuity of function $(\omega, t) \to x(\omega, t)$ and closedness of S_n imply then that $x(\omega^*, t^*) \in S_n$. Therefore $t \in T$ and T is closed.

Suppose that γ is a forest and let us prove that, then, $T \subset \{t : t_{ij} \leq 1$ for all $(i, j)\}$. This will conclude the proof since T is bounded below by definition of a gift vector. For all i, let $I(i)$ be the set of vertices j such that there is a directed path contained in γ, originating in j and ending in i. Since γ contains no circuit, we have: $i \notin I(i)$; $I(j) \subset I(i)$ for all $j \in I(i)$, the inclusion being strict since $j \notin I(j)$; $I(j) \cap I(k) = \emptyset$ whenever $(j, k) \in I(i) \times I(i)$ and $k \notin I(j)$. The definition of T implies moreover that there is an $\omega \in S_n$ such that $t_{ij} \leq \omega_i + \sum_{k \in N} t_{ki}$ for all (i, j). We have $\sum_{k \in N} t_{ki} = \sum_{k \in I(i)} t_{ki}$. The fact that $I(k)$ is strictly included in $I(k')$ whenever $k \in I(k')$, that $I(k') \cap I(k'') = \emptyset$ whenever $(k', k'') \in I(k) \times I(k)$ and $k'' \notin I(k')$, and that N is finite, combined with the inequalities above, imply then recursively that $t_{ij} \leq \sum_{k \in I(i)} \omega_k$ for all (i, j). ■

Proof of Theorem 2: $\{w : w$ verifies Assumption 1$\}$ is endowed with the topology of the C^2 uniform convergence on compacta (which is metrizable), S_n

with any usual metric topology, and W with the corresponding product topology.

Let w verify Assumption 1. $0 \in T_w(\omega)$ for all $\omega \in M_w$ by Theorem 1, so that $M_w \subset Q_w$. M_w is compact since it is contained in compact set S_n by definition and closed by Corollary 1-(i), and Assumption 1-(i).

If $x \to \Omega_w(x)$ is upper hemicontinuous (u.h.c.), then Q_w is compact, since $Q_w = \cup_{x \in M_w} \Omega_w(x)$: if $x^q \to x$ in Q_w, there exists a sequence (z^q) such that $x^q \in \Omega_w(z^q)$ for all q; z^q has a limit point z by compactness of M_w; the upper hemicontinuity of $x \to \Omega_w(x)$ implies then $x \in \Omega_w(z)$; therefore Q_w is closed, hence compact as a closed subset of the compact set S_n.

The domains of correspondences $(w, \omega) \to M_w$, $(w, x) \to \Omega_w(x)$ and $(w, \omega) \to X_w(\omega)$ are metric spaces and their values are subsets of the metric space S_n. I establish below that they are compactvalued and use the following sufficient condition for u.h.c. at y^0 for a compactvalued correspondence $\varphi: Y \to Z$ such that Y and Z are metric spaces: for every pair of sequences (y^q), (z^q) such that $y^q \to y^0$ and $z^q \in \varphi(y^q)$, there is a converging subsequence (z^q) whose limit belongs to $\varphi(y^0)$.

I established the compactness of M_w above. I prove now that the values $\Omega_w(x)$ and $X_w(\omega)$ are closed, hence compact subsets of S_n. Let $(x^q, \omega^q) \to (x, \omega)$ be such that $\omega^q \in \Omega_{w^q}(x^q)$ or equivalently $x^q \in X_{w^q}(\omega^q)$ for all q. Then, for all q, there is a $t^q \in T_{w^q}(x^q)$ such that $g(t^q)$ is a forest and $x^q = x(\omega^q, t^q)$ (Lemma 1). The sequence (t^q) has a limit point t by Lemma 2. We have $t \in T_w(\omega)$ by Theorem 1 and Assumption 1-(i), and therefore $\omega \in \Omega_w(x)$ and $x \in X_w(\omega)$.

Next, let $((w^q, \omega^q, x^q))_{q \in \mathbb{N}}$ be such that $(w^q, \omega^q) \to (w, \omega)$ in D and $x^q \in M_{w^q}$ for all $q \cdot (x^q)$ is then a sequence of elements of S_n. The compactness of the latter implies the existence of a subsequence of (x^q) that converges to a limit in S_n. Let x denote this limit. We have clearly $x \in M_w$ by Assumption 1-(i) and Corollary 1-(i). $(w, \omega) \to M_w$, being compact valued, is therefore u.h.c..

Consider then $((w^q, x^q, \omega^q))_{q \in \mathbb{N}}$ such that $(w^q, x^q) \to (w, x)$ in D_Ω and $\omega^q \in \Omega_{w^q}(x^q)$ for all $q \cdot (\omega^q)$ is then a sequence of elements of S_n, and has therefore a limit point ω in $S_n \cdot (x, \omega) \in M_w \times S_n$ by u.h.c. of $(w, \omega) \to M_w$ and closedness of S_n. For all q, there is a $t^q \in T_{w^q}(x^q)$ such that $g(t^q)$ is a forest and $x^q = x(\omega^q, t^q)$ (Lemma 1). The sequence (t^q) has at a limit point t by Lemma 2. We have $t \in T_w(\omega)$ by Theorem 1 and Assumption 1-(i), and therefore $\omega \in \Omega_w(x)$. This establishes the u.h.c. of Ω (and compactness of Q).

Consider finally $((w^q, \omega^q, x^q))_{q \in \mathbb{N}}$ such that $(w^q, \omega^q) \to (w, \omega)$ in D_X and $x^q \in X_{w^q}(\omega^q)$ for all $q \cdot (x^q)$ has a limit point x as a sequence of elements of $S_n \cdot (\omega, x) \in Q \times S_n$ by closedness of Q and S_n. As above: for all q, there is a $t^q \in T_{w^q}(x^q)$ such that $g(t^q)$ is a forest and $x^q = x(\omega^q, t^q)$; the sequence (t^q) has a limit point $t \in T_w(\omega)$. And therefore $x \in X_w(\omega)$. This establishes the u.h.c. of X. ■

Appendix C: Generic Properties

Proof of Theorem 3: Let $W' = \{(w, \omega) \in W : (w, \omega) \text{ is regular}\}$. $\{w : w$ verifies Assumption 1$\}$ is endowed with the topology of the C^2 uniform convergence on compacta, S_n with the usual metric topology, and W with the corresponding product topology.

The proof proceeds in two steps. I establish first that W' is open, by means of the implicit function theorem. I prove next that W' is dense in W by applying transversality theory to linear perturbations of w.

(i) Let us prove that W' is open in W.

Let (w^*, ω^*) be a fixed element of W'. For all element t^* of its set of equilibrium gift-vectors $T(\omega^*)$, let: $\gamma(x(\omega^*, t^*))$ be denoted by γ; $F_{t^*}: W \times \mathbb{R}^{\#\gamma} \to \mathbb{R}^{\#\gamma}$ be defined by $F_{t^*}(w, \omega, t_\gamma) = (-\partial_{x_i} w_i(x(\omega, (t^*_{\setminus\gamma}, t_\gamma))) + \partial_{x_j} w_i(x(\omega, (t^*_{\setminus\gamma}, t_\gamma))))_{(i,j)\in\gamma}$.

From Theorem 1: $-\partial_{x_i} w_i(x(\omega^*, t^*)) + \partial_{x_j} w_i(x(\omega^*, t^*)) < 0$ for all $(i, j) \notin \gamma$ such that $i \neq j$; and $F_{t^*}(w^*, \omega^*, t^*_\gamma) = 0$.

Function $\partial_{t_\gamma} F_{t^*}(w, \omega, t_\gamma)$ is well defined and continuous on $W \times \mathbb{R}^{\#\gamma}$, and the regularity of (w^*, ω^*) implies that $\partial_{t_\gamma} F_{t^*}(w^*, \omega^*, t^*_\gamma)$ is nonsingular. Therefore, from Mas-Colell 1985, C.3.3, for all $t^* \in T(\omega^*)$, there exist open sets $U(t^*) \subset W$ and $V(t^*) \subset \mathbb{R}^{\#\gamma}$ and a continuous function $h_{t^*}: U(t^*) \to V(t^*)$ such that $h_{t^*}(w^*, \omega^*) = t^*_\gamma$ and $F_{t^*}(w, \omega, t_\gamma) = 0$ hold for $(w, \omega, t_\gamma) \in U(t^*) \times V(t^*)$ if and only if $t_\gamma = h_{t^*}(w, \omega)$. From the continuity of the derivatives of utility functions and of function h_{t^*}, we can choose $U(t^*)$ so that $-\partial_{x_i} w_i(x(\omega, (t^*_{\setminus\gamma}, h_{t^*}(w)))) + \partial_{x_j} w_i(x(\omega, (t^*_{\setminus\gamma}, h_{t^*}(w)))) < 0$ for all $(i, j) \notin \gamma$ such that $i \neq j$ and all $(w, \omega) \in U(t^*)$. We have then $T_w(\omega) = \cup_{t^* \in T(\omega^*)} \{t : t_{\setminus\gamma} = 0, \text{and } t_\gamma = h_{t^*}(w) \geq 0\}$ for all $(w, \omega) \in U(t^*)$. From the continuity of the determinant and of functions $\partial_{t_\gamma} F_{t^*}$ and h_{t^*}, we can choose, moreover, $U(t^*)$ so that $|\partial_{t_\gamma} F_{t^*}(w, \omega, h_{t^*}(w, \omega))| \neq 0$ for all $(w, \omega) \in U(t^*)$. Then $U(t^*) \subset W'$. Finally, $\cap_{t^* \in T(\omega)} U(t^*)$ is an open neighborhood of (w^*, ω^*) in W', since the equilibria of (w^*, ω^*) are in finite number (Theorem 5). ■

(ii) I prove now that W' is dense in W.

Consider a fixed element (w^*, ω^*) of W'.

For all $b_i \in \mathbb{R}^n$ and $b = (b_1, \ldots, b_n) \in \mathbb{R}^{n^2}$, let $w^b : \mathbb{R}^n \to \mathbb{R}^n$ be defined by $w^b(x) = (w^*_1(x) + b_1 \cdot x^T, \ldots, w^*_n(x) + b_n \cdot x^T)$. For all element t^* of the set of equilibrium gift-vectors $T(\omega^*)$, let $\gamma(x(\omega^*, t^*))$ be denoted by γ^*. And for all nonempty $\gamma \subset \gamma^*$, let $F_\gamma: \mathbb{R}^{\#\gamma} \times \mathbb{R}^{\#\gamma} \to \mathbb{R}^{\#\gamma}$ be defined by $F_\gamma(b_\gamma, t_\gamma) = (-\partial_{x_i} w^*_i(x(\omega^*, (t^*_{\setminus\gamma}, t_\gamma))) - b_{ii} + \partial_{x_j} w^*_i(x(\omega^*, (t^*_{\setminus\gamma}, t_\gamma))) + b_{ij})_{(i,j)\in\gamma}$.

From Theorem 1: $-\partial_{x_i} w_i(x(\omega^*, t^*)) + \partial_{x_j} w_i(x(\omega^*, t^*)) < 0$ for all $(i, j) \notin \gamma^*$ such that $i \neq j$; and $F_{\gamma^*}(0, t^*_\gamma) = 0$.

F_γ is C^1; and rank $\partial F_\gamma(b_\gamma, t_\gamma) = \#\gamma$ for all (b_γ, t_γ), since $\partial_{b_\gamma} F_\gamma(b_\gamma, t_\gamma)$ is the identity $\mathbb{R}^{\#\gamma} \to \mathbb{R}^{\#\gamma}$. Therefore, from Mas-Colell 1985, 8.3.1, except for a set of $b \in \mathbb{R}^{n^2}$ of Lebesgue measure 0, $H_{\gamma,b}: \mathbb{R}^{\#\gamma} \to \mathbb{R}^{\#\gamma}$ defined by

$H_{\gamma,b}(t_\gamma) = F_\gamma(b_\gamma, t_\gamma)$ has 0 as a regular value. Hence $B_\gamma = \{b \in \mathbb{R}^{n^2}$: 0 is a regular value of $H_{\gamma,b}\}$ is dense in $\mathbb{R}^{n^2}$.

Let $B = \cap_\gamma B_\gamma$. Of course, B is dense in $\mathbb{R}^{n^2}$. From the upper hemicontinuity of correspondence $w \to M_w$ (Theorem 2-(ii)), there is a neighborhood U of 0 in $\mathbb{R}^{n^2}$ such that $M_{w^b} \subset \mathbb{R}^n_{++}$ for all $b \in U$. Therefore, the set $\{(w^b, \omega^*): b_{ii} > 0$ for all i and $M_{w^b} \subset \mathbb{R}^n_{++}\}$ has a nonempty intersection with W. Hence there exists a sequence $(w^{b^q}, \omega^*) \to (w^*, \omega^*)$ in W, with $b^q \in B$ for all q. From the upper hemicontinuity of correspondence $w \to X_w(\omega^*)$ (Theorem 2-(ii)) and continuity of marginal utilities, there exists q_0 such that, for all $q \geq q_0$: $-\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) < 0$ for all $x \in X_{w^{b^q}}(\omega^*)$ and all $(i, j) \notin \gamma^*$ such that $i \neq j$. Since 0 is a regular value of H_{γ,b^q} for all $\gamma \subset \gamma^*$ and all q, the social systems (w^{b^q}, ω^*) are regular for all $q \geq q_0$ (Theorem 1). ■

Proof of Theorem 4: Let w be fixed.

Berge 1970, Theorem 1, implies the equivalence of the following three propositions: $\gamma(x)$ is a forest; $\text{rank}\Gamma(x) = \#\gamma(x)$; $\#\gamma(x) = v(\gamma(x)) - c(\gamma(x))$. In view of Corollary 1-(ii), the following two propositions, therefore, are equivalent: for all $x \in X(\omega)$, $\gamma(x)$ is a forest; for all $x \in X(\omega)$, $\dim\Omega(x) = \#\gamma(x) = v(\gamma(x)) - c(\gamma(x))$.

It will be sufficient, hence, to establish that: the restriction to $T(\omega)$ of function $\mathbb{R}^{n(n-1)} \to \mathbb{R}^n$: $t \to x(\omega, t)$ is one-to-one if and only if $\text{rank}\Gamma(x) = \#\gamma(x)$ for all $x \in X(\omega)$; and if $(w, \omega) \in W$ is regular, then $\text{rank}\Gamma(x) = \#\gamma(x)$ for all $x \in X(\omega)$.

For all $x \in X(\omega)$: the dimension of the linear space $\{t \in \mathbb{R}^{n(n-1)}$: $g(t) \subset \gamma(x)\}$ is $\#\gamma(x)$; and therefore, the restriction of linear function $\mathbb{R}^{n(n-1)} \to \mathbb{R}^n: t \to \Gamma(x) \cdot t^T$ to $\{t \in \mathbb{R}^{n(n-1)}: g(t) \subset \gamma(x)\}$ is one-to-one if and only if $\text{rank}\Gamma(x) = \#\gamma(x)$. By Corollary 1-(ii): $T(\omega) \subset \{t \in \mathbb{R}^{n(n-1)}: g(t) \subset \gamma(x)\}$; and $x(\omega, t) = \omega + \Gamma(x(\omega, t)) \cdot t^T$ for all $t \in \mathrm{T}(\omega)$. Therefore, the restriction of function $\mathbb{R}^{n(n-1)} \to \mathbb{R}^n$: $t \to x(\omega, t)$ to $T(\omega)$ is one-to-one if and only if $\text{rank}\Gamma(x) = \#\gamma(x)$ for all $x \in X(\omega)$.

Suppose now that $(w, \omega) \in W$ is regular. Let $t^* \in T(\omega)$ and $x = x(\omega, t^*)$. From the definition of $\Gamma(x)$, we have $\text{rank}\Gamma(x) \leq \#\gamma(x)$. Let us prove that $\text{rank}\Gamma(x) \geq \#\gamma(x)$. This is clear if $\gamma(x)$ is empty. Suppose that $\gamma(x)$ is nonempty. We have $\partial f_{\omega,t^*}(t^*) = (\partial^2 w_1(x) \cdot \partial_t x(\omega, t^*) \cdot \Gamma_1(x), \ldots, \partial^2 w_n(x) \cdot \partial_t x(\omega, t^*) \cdot \Gamma_n(x))$, so that $\text{rank}\Gamma(x) \geq \text{rank}\ \partial f_{\omega,t^*}(t^*)$, and therefore $\text{rank}\Gamma(x) \geq \#\gamma(x)$ by the regularity assumption. ■

Proof of Theorem 5: Let $(w, \omega) \in W$ be fixed and regular, $t^* \in T(\omega)$, $g = g(t^*)$, $x^* = x(\omega, t^*)$ and $\gamma = \gamma(x^*)$. Let F_{ω,t^*}: $\mathbb{R}^{\#\gamma} \to \mathbb{R}^{\#\gamma}$ be defined by: $F_{\omega,t^*}(t_\gamma) = (-\partial_{x_i} w_i(x(\omega, (t^*_{\setminus\gamma}, t_\gamma))) + \partial_{x_j} w_i(x(\omega, (t^*_{\setminus\gamma}, t_\gamma))))_{(i,j)\in\gamma}$. Regularity readily implies that F_{ω,t^*} is a local homeomorphism at t^*_γ, which implies in turn that t^* is isolated. $T(\omega)$ is therefore a discrete set. From

Theorem 4-(iii), $X(\omega)$ is thus a discrete set and $\#\mathrm{T}(\omega) = \#X(\omega)$. Since $X(\omega)$ is compact (Theorem 2), it is a finite set. ■

Proof of Theorem 6: Let w be fixed.

Theorem 6 is a straightforward consequence of Theorem 1 and Assumption 1-(i) if $\omega \in \{x \in M : \gamma(x) = \emptyset\}$.

Suppose from there on that $\omega \in \{x \in M : \gamma(x) \neq \emptyset\}$, notice that $0 \in T(\omega)$, and define $F_{\omega,0}: \mathbb{R}^{\#\gamma(\omega)} \to \mathbb{R}^{\#\gamma(\omega)}$ by $F_{\omega,0}(t_{\gamma(\omega)}) = (-\partial_{x_i} w_i(x(\omega, (0_{\setminus\gamma(\omega)}, t_{\gamma(\omega)}))) + \partial_{x_j} w_i(x(\omega, (0_{\setminus\gamma(\omega)}, t_{\gamma(\omega)}))))_{(i,j)\in\gamma(\omega)}$. Let $H_\omega:\{x \in \mathbb{R}^n: \sum_{i\in N} x_i = 1\} \to \mathbb{R}^{\#\gamma(\omega)}$ be such that $H_\omega(x) = (-\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x))_{(i,j)\in\gamma(\omega)}$. For all $t_{\gamma(\omega)} \in \mathbb{R}^{\#\gamma(\omega)}$ we have $F_{\omega,0}(t_{\gamma(\omega)}) = H_\omega(x(\omega, (0_{\setminus\gamma(\omega)}, t_{\gamma(\omega)}))$, so that rank $\partial F_{\omega,0}(0) \leq$ rank $\partial H_\omega(\omega)$. The regularity condition implies therefore that rank $\partial H_\omega(\omega) = \#\gamma(\omega)$. Hence there is an open neighborhood U of ω in $\{x \in \mathbb{R}^n: \sum_{i\in N} x_i = 1\}$ such that $0\ (=H_\omega(\omega))$ is a regular value of the restriction of H_ω to U. We can let $U \subset S_n \cap \mathbb{R}^n_{++}$ by Assumption 1-(iv).The implicit function theorem (Mas-Colell 1985, H.2.2.) implies then that $U \cap \{x \in M : \gamma(x) = \gamma(\omega)\}$ is a boundaryless C^1 manifold of dimension $n - 1 - \#\gamma(\omega)$.

I established, at this stage, that if $(w, \omega) \in W$ is regular, and if $\omega \in M$, there exists an open neighborhood U of ω in $S_n \cap \mathbb{R}^n_{++}$ such that $U \cap \{x \in M : \gamma(x) = \gamma(\omega)\}$ is a boundaryless C^1 manifold of dimension $n - 1 - \#\gamma(\omega)$. Regularity being an open property in W by Theorem 1, we can suppose that (w, x) is regular for all $x \in U$. And we can let $U \subset \{x \in S_n \cap \mathbb{R}^n_{++}: \gamma(x) \subset \gamma(\omega)\}$ by Assumption 1-(i). Therefore, for all $\gamma \subset \gamma(\omega)$ and all $\omega' \in \{x \in M : \gamma(x) = \gamma)\}$, there exists an open neighborhood $U_\gamma \subset U$ of ω' in $S_n \cap \mathbb{R}^n_{++}$ such that $U_\gamma \cap \{x \in M : \gamma(x) = \gamma\}$ is a boundaryless C^1 manifold of dimension $n - 1 - \#\gamma$. Hence, $U \cap \{x \in M : \gamma(x) = \gamma\}$ is either empty, or a boundaryless C^1 manifold of dimension $n - 1 - \#\gamma$, for all $\gamma \subset \gamma(\omega)$.

It will suffice, to finish with, to establish that $V \cap \{x \in M : \gamma(x) = \gamma\}$ is nonempty for all neighborhood $V \subset U$ of ω in $S_n \cap \mathbb{R}^n_{++}$. Notice that $F_{\omega,0}$ is a local homeomorphism at 0 by the regularity condition and the inverse function theorem, and recall that $F_{\omega,0}(0) = 0$. Therefore, for any given $\gamma \subset \gamma(\omega)$, there is a sequence $t^q_{\gamma(\omega)} \to 0$ in $\mathbb{R}^{\#\gamma(\omega)}$ such that: for all q, the entry $-\partial_{x_i} w_i(x(\omega, (0_{\setminus\gamma(\omega)}, t^q_{\gamma(\omega)}))) + \partial_{x_j} w_i(x(\omega, (0_{\setminus\gamma(\omega)}, t^q_{\gamma(\omega)})))$ of $F_{\omega,0}(t^q_{\gamma(\omega)})$ is 0 if $(i, j) \in \gamma$, negative otherwise; and $F_{\omega,0}(t^q_{\gamma(\omega)}) \to 0$. By continuity $x(\omega, (0_{\setminus\gamma(\omega)}, t^q_{\gamma(\omega)}) \in U$ for any large enough q. Therefore $V \cap \{x \in M : \gamma(x) = \gamma\}$ is nonempty for all neighborhood $V \subset U$ of ω in $S_n \cap \mathbb{R}^n_{++}$. ■

Proof of Corollary 2: Let $(w, \omega^0) \in W$ be fixed and regular, $t^* \in T(\omega^0)$, $g(t^*) = \gamma$ and $A = \{\omega: \exists t \in T(\omega)$ such that $g(t) = \gamma\}$. From Theorem 1 and Corollary 1, $x(\omega^0, t^*) \in \{x \in M : \gamma(x) = \gamma\}$, while, from Theorem 6, there exists an open neighborhood U of $x(\omega^0, t^*)$ in $S_n \cap \mathbb{R}^n_{++}$ such that $U \cap \{x \in M : \gamma(x) = \gamma\}$ is a boundaryless C^1 manifold of dimension $n - 1 - \#\gamma$.

And $U \cap \{x \in M : \gamma(x) = \gamma\}$ is identical to $\cup_{\omega \in A}(X(\omega) \cap U)$ by Theorem 1 and Corollary 1. ■

Proof of Theorem 7: Let w be fixed, and $(w, \omega^*) \in W$ be regular and such that $0 \in T(\omega^*)$.

From Theorem 1, we have $\omega^* \in M$.

Let $t^* \in T(\omega^*)$, $x^* = x(\omega^*, t^*)$ and $\gamma^* = \gamma(x^*)$. I want to prove that $t^* = 0$.

From regularity and Theorem 6, there exists a neighborhood U of x^* in S_n such that, for all $\gamma \subset \gamma^*$, $M_\gamma = U \cap \{x \in M : \gamma(x) = \gamma\}$ is a boundaryless C^1 manifold of dimension $n - 1 - \#\gamma$. If $\gamma^* = \emptyset$, then $t^* = 0$ by Theorem 1.

Suppose now that $\gamma^* \neq \emptyset$. For all $\gamma \subset \gamma^*$, define $T_\gamma = \{t : g(t) \subset \gamma\}$. T_γ is a C^∞ manifold (with corner) of dimension $\#\gamma$. $M_\gamma \times T_\gamma$ is therefore a boundaryless C^1 manifold of dimension $n-1$. And $A = \cup_{\gamma \subset \gamma^*}(M_\gamma \times T_\gamma)$ is a boundaryless C^1 manifold of dimension $n - 1$, as finite union of pairwise disjoint boundaryless C^1 manifolds of dimension $n - 1$. Consider now the linear function $h : \{x \in \mathbb{R}^n : \sum_{i \in N} x_i = 1\} \times \mathbb{R}^{n(n-1)} \to \{x \in \mathbb{R}^n : \sum_{i \in N} x_i = 1\}$ defined by $h(x, t) = (x_1 - \Delta_1 t, \ldots, x_n - \Delta_n t)$. The rank of h is $n - 1$. Since $\partial A = \emptyset$, the restriction $h_{|A}$ of h to A is a linear diffeomorphism $A \to h(A)$ (inverse function theorem: Mas-Colell 1985, H.2.1.(i)), and $h(A)$ is a neighborhood of $\omega^* = h(x^*, t^*)$ in $\{x \in \mathbb{R}^n : \sum_{i \in N} x_i = 1\}$. Let $V = h(A) \cap S_n$ be the induced neihborhood of ω^* in S_n.

Since $h_{|A}$ is a diffeomorphism $A \to h(A)$, $h(A)$ is the union $\cup_{\gamma \subset \gamma^*} h(M_\gamma \times T_\gamma)$ of the family of pairwise disjoint C^1 manifolds of dimension $n - 1$ $(h(M_\gamma \times T_\gamma))_{\gamma \subset \gamma^*}$. Since, moreover, $\omega^* \in M$ and (w, ω^*) is regular, we can choose U so that $V \cap M$ is the union of the family $(M'_\gamma)_{\gamma \subset \gamma(\omega^*)}$ of the pairwise disjoint C^1 manifolds $M'_\gamma = V \cap \{x \in M : \gamma(x) = \gamma\}$ (Theorem 6). $(x, t) \in A$ being by construction an equilibrium of $(w, h(x, t))$, we must have $M'_\emptyset = h(M_\emptyset \times T_\emptyset) = M_\emptyset$. Noticing finally that $M'_\emptyset = V \cap \operatorname{Int} M$ (Theorem 6), we get $x^* = \omega^*$ and $t^* = 0$ by continuity of h. ■

Proof of Corollary 3: Let w be fixed and suppose that $(w, \omega^0) \in W$ is regular. Regularity being an open property (Theorem 3), there exists a neighborhood V of ω^0 in S_n such that (w, ω) is a regular element of W for all $\omega \in V$. If $\omega \in V \cap M$, then $0 \in T(\omega)$ by Theorem 1, and therefore $T(\omega) = \{0\}$ by Theorem 6, which establishes the corollary. ■

Appendix D: Strong Regularity

Proof of Theorem 8: The proof makes no explicit or implicit use of the fixed point theorems of Brouwer-Kakutani. The inverse function theorem suffices. It is used to establish that Q_w is open in S_n. Since Q_w is closed by

Theorem 2, it must either be empty or coincide with S_n, by connectedness of the latter.

Let w be fixed and strongly regular. I prove that, then, Q is open in S_n. This is clearly true if Q is empty. Suppose therefore that Q is nonempty and let $\omega^* \in Q$, $t^* \in T(\omega^*)$, $x^* = x(\omega^*, t^*)$, $\gamma^* = \gamma(x^*)$. I prove that there exists a neighborhood of ω^* in S_n which is contained in Q. From regularity and Theorem 6, there exists a neighborhood U of x^* in S_n such that, for all $\gamma \subset \gamma^*$, $M_\gamma = U \cap \{x \in M : \gamma(x) = \gamma\}$ is a boundaryless C^1 manifold of dimension $n - 1 - \#\gamma$. If $\gamma^* = \emptyset$, then $x^* = \omega^*$ (Theorem 1) and $M_\emptyset = U \cap \mathrm{Int}M$ (Theorem 6), so $M_\emptyset$ is contained in Q (Theorem 2) and is a neighborhood of ω^* in S_n. Suppose now that $\gamma^* \neq \emptyset$. For all $\gamma \subset \gamma^*$, let $T_\gamma = \{t : g(t) \subset \gamma\}$. I established above (proof of Theorem 6) that $A = \cup_{\gamma \subset \gamma^*}(M_\gamma \times T_\gamma)$ is a boundaryless C^1 manifold of dimension $n - 1$, and that the linear function h defined on A by $h(x, t) = (x_1 - \Delta_1 t, \ldots, x_n - \Delta_n t)$ is a linear diffeomorphism $A \to h(A)$. Therefore $h(A) \cap S_n$ is a neighborhood of ω^* in S_n, and is contained in Q by construction. ■

Proof of Theorem 9: Suppose that M is not connected, and let us derive a contradiction. Let C_1 and C_2 be two distinct connected components of M. The compactness of M implies that its components are compact. Denote $A_1 = \cup_{x \in C_1}\Omega(x)$ and $A_2 = \cup_{x \in C_2}\Omega(x)$. The reasoning developed in the proof of Theorem 8 can be adapted straightforwardly to establish that $A_1 = S_n$. There exist therefore $(\omega^1, \omega^2) \in C_1 \times C_2$ and a nonzero $t \in T(\omega^2)$ such that $\omega^1 = x(\omega^2, t)$. But this contradicts Corollary 3. ■

Proof of Theorem 10: Let $M_\gamma = \{x \in M : \gamma(x) = \gamma\}$. Strong regularity implies that this set is either empty or a boundaryless C^1 manifold of dimension $n - 1 - \#\gamma$ (Theorem 6). It is sufficient, therefore, to prove that $\{\#\gamma(x) : x \in M\} = \{0, \ldots, n - 1\}$. The closedness of M (Theorem 2) implies that the boundary of $\mathrm{cl}M_\gamma$ is a nonempty subset of M whenever M_γ is nonempty. $x \in \mathrm{cl}M_\gamma$ implies that $\gamma \subset \gamma(x)$ by Assumption 1-(i), and the emptiness of ∂M_γ implies then that the inclusion is strict. We know from Theorem 6, that if $x \in M$ is such that $\gamma(x)$ strictly contains γ, then there exists an $x' \in M$ such that $\gamma(x')$ contains γ and $\#\gamma(x') = \#\gamma + 1$. Theorem 6 implies that $\{x \in M : \#\gamma(x) = 0\}$ is nonempty whenever M is nonempty. The result follows then recursively from the assumption that M is nonempty and from the fact that, by Theorem 4-(i), $\#\gamma(x) \leq n - 1$ for all $x \in M$. ■

References

AKERLOF, G. A. (1982) Labor contract as partial gift exchange, *Quaterly Journal of Economics* **97**, 543–569.

ALTIG, D., and S. J. DAVIS (1993) Borrowing constraints and two-sided altruism with an application to social security, *Journal of Economic Dynamics and Control* **17**, 467–494.

ALTONJI, J., F. HAYASHI, and L. KOTLIFOFF (1997) Parental altruism and inter vivos transfers: Theory and evidence, *Journal of Political Economy* **105**, 1121–1166.

ANDREONI, J. (1989) Giving with impure altruism: applications to charity and Ricardian equivalence, *Journal of Political Economy* **97**, 1447–1458.

ANDREONI, J. (1998) Toward a theory of charitable fundraising, *Journal of Political Economy* **106**, 1186–1213.

ANDREONI, J. (2000) Signalling the quality of a public good: a model of fund raising campaigns. *Contribution to the International Conference of the Association for Public Economic Theory*, Warwick, July 2000.

ARROW, K. J., and A. C. ENTHOVEN (1961) Quasi-concave programming, *Econometrica* **29**, 779–800.

BARRO, R. J. (1974) Are government bonds net wealth? *Journal of Political Economy* **82**, 1095–1117.

BECKER, G. S. (1974) A theory of social interactions, *Journal of Political Economy* **82**, 1063–1093.

BECKER, G. S. (1981) Altruism in the family and selfishness in the market place, *Economica* **48**, 1–15.

BERGE, C. (1970) *Graphes et Hypergraphes*. Paris: Dunod.

BERGSTROM, T. C., L. E. BLUME, and H. R. VARIAN (1986) On the private provision of public goods, *Journal of Public Economics* **29**, 25–49.

BERNHEIM, B. D., A. SHLEIFER, and L. H. SUMMERS (1985) The strategic bequest motive, *Journal of Political Economy* **93**, 1045–1076.

COATE, S. (1995) Altruism, the Samaritan's dilemma, and government transfer policy, *The American Economic Review* **85**, 46–57.

DEBREU, G. (1952) A social equilibrium existence theorem, *Proceedings of the National Academy of Sciences* **38**, 886–893. (Reprinted in Debreu, G. (1983) *Mathematical Economics*. Cambridge: Cambridge University Press, pp. 50–58.)

DEBREU, G., and H. SCARF (1963) A limit theorem on the core of an economy, *International Economic Review* **4**, 235–246. (Reprinted in Debreu, G. (1983) *Mathematical Economics*. Cambridge: Cambridge University Press, pp. 151–162.)

DEBREU, G. (1970) Economies with a finite set of equilibria, *Econometrica* **38**, 387–392. (Reprinted in Debreu, G. (1983) *Mathematical Economics*. Cambridge: Cambridge University Press, pp.179–185.)

FALK, I., and O. STARK (2000) Transfers, empathy formation and reverse transfers, Chap. 8, in *The Economics of Reciprocity, Giving and Altruism*, L.-A. Gérard-Varet, S.-C. Kolm, and J. Mercier Ythier, eds. London: Macmillan.

GALE, W. G., and J. K. SCHOLTZ (1994) Intergenerational transfers and the accumulation of wealth, *Journal of Economic Perspectives* **8**, 145–160.

GODELIER, M. (1996) *L'Enigme du Don*. Paris: Fayard. (Translated as *The Enigma of the Gift*. Cambridge: Polity Press, 1999.)

GODELIER, M. (2000) Things you don't give or sell but which you keep: Valuable and social objects, Chap. 9, in *The Economics of Reciprocity, Giving and Altruism*, L.-A. Gérard-Varet, S.-C. Kolm, and J. Mercier Ythier, eds. London: Macmillan.

HOCHMAN, A. M., and J. D. RODGERS (1969) Pareto optimal redistribution, *The American Economic Review* **59**, 542–557.

KINGMA, B. R. (1989) An accurate measurement of the crowd-out effect, income effect, and price effect for charitable contributions, *Journal of Political Economy* **97**, 1197–1207.

KOLM, S.-C. (1966) The optimal production of social justice. In International Economic Association on Public Economics, Biarritz. Proceedings ed. by H. Guitton, and J. Margolis. *Economie Publique,* Paris: CNRS, 1968, pp. 109–177. *Public Economics,* London: Macmillan, 1969, pp. 145–200.

KOLM, S.-C. (1984) *La Bonne Economie: La Réciprocité Générale*. Paris: Presses Universitaires de France.

KOLM, S.-C. (1985) *Le Contrat Social Libéral*. Paris: Presses Universitaires de France.

KOLM, S.-C. (2000) The theory of reciprocity, Chap. 5, in *The Economics of Reciprocity, Giving and Altruism,* L.-A. Gérard-Varet, S.-C. Kolm, and J. Mercier Ythier, eds. London: Macmillan.

KOTLIKOFF, L. J., and L. H. SUMMERS (1981) The role of intergenerational transfers in aggregate capital accumulation, *Journal of Political Economy* **89**, 706–732.

LAFERRÈRE, A. (2000) Intergenerational transmission models: a survey, Chap. 11; in *The Economics of Reciprocity, Giving and Altruism,* L.-A. Gérard-Varet, S.-C. Kolm, and J. Mercier Ythier, eds. London: Macmillan.

LINDBECK, A., and J. W. WEIBULL (1988) Altruism and time consistency: The economics of fait accompli, *Journal of Political Economy* **96**, 1165–1182.

MAS-COLELL, A. (1985) *The Theory of General Economic Equilibrium: A Differentiable Approach.* Cambridge: Cambridge University Press.

MAUSS, M. (1924) Essai sur le don. Forme et raison de l'échange dans les sociétés archaïques. *L'Année Sociologique* **1**. Translated as *The Gift: The Form and Reason for Exchange in Archaic Societies.* New York and London: W. W. Norton, 1990.

MERCIER YTHIER, J. (1989) *Equilibre Général et Don.* Thèse de Doctorat en Sciences Economiques, Institut d'Etudes Politiques de Paris.

MERCIER YTHIER, J. (1992) Existence of a General Equilibrium of Individual Gifts. Working Paper, Université de Paris-Panthéon-Sorbonne.

MERCIER YTHIER, J. (1993) Equilibre général de dons individuels, *Revue Economique* **44**, 925–950.

MERCIER YTHIER, J. (1998a) The distribution of wealth in the liberal social contract, *European Economic Review* **42**, 329–347.

MERCIER YTHIER, J. (1998b) Existence of an equilibrium of property rights, *Mathematical Social Sciences* **35**, 261–172.

MERCIER YTHIER, J. (2000a) The effectiveness of distributive policy in a competitive economy, *Journal of Public Economic Theory* **2**, 43–69.

MERCIER YTHIER, J. (2000b) Gift equilibrium and the liberal distributive social contract: a tentative synthesis, Chap. 4, in *The Economics of Reciprocity, Giving and Altruism*, L.-A. Gérard-Varet, S.-C. Kolm, and J. Mercier Ythier, eds. London: Macmillan.

MERCIER YTHIER, J. (2002) The economic theory of gift-giving: Perfect substitutability and the redistribution of wealth. Contribution to the conference "The Economics of Giving, Reciprocity and Altruism," Marseilles, January 2002. Forthcoming as Chap. 2 of the *Handbook on the Economics of Giving, Reciprocity and Altruism*, ed. L.-A. Gérard-Varet, S.-C. Kolm, and J. Mercier Ythier. Amsterdam: North-Holland/Elsevier.

MERCIER YTHIER, J. (2004) A limit theorem on the dual core of a distributive social system. Contribution to the conference "Nonwelfaristic Issues in Normative Economic Theory" of the Society for Social Choice and Welfare at Caen, June 2000. Forthcoming in the special issue "Nonwelfaristic Issues in Normative Economic Theory" of *Social Choice and Welfare*, forthcoming.

MODIGLIANI, F. (1986) Life cycle, individual thrift, and the wealth of nations, *The American Economic Review* **76**, 297–313.

MUSGRAVE, R. A. (1970) Pareto optimal redistribution: Comment, *The American Economic Review* **60**, 991–993.

PARETO, V. (1913) Il massimo di utilità per una colletività, *Giornale degli economisti* **3**, 337–341.

ROBERTS, R. D. (1984) A positive model of private charity and public transfers, *Journal of Political Economy* **92**, 136–148.

SCHOKKAERT, E., and L. VAN OOTEGEM (2000) Preference variation and private donations, Chap. 3, in *The Economics of Reciprocity, Giving and Altruism*, L.-A. Gérard-Varet, S.-C. Kolm, and J. Mercier Ythier, eds. London: Macmillan.

WARR, P. G. (1982) Pareto optimal redistribution and private charity, *Journal of Public Economics* **19**, 131–138.

WARR, P. G. (1983) The private provision of a public good is independent of the distribution of income, *Economic Letters* **13**, 207–211.