Chapter 5

THE ECONOMIC THEORY OF GIFT-GIVING: PERFECT SUBSTITUTABILITY OF TRANSFERS AND REDISTRIBUTION OF WEALTH

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Abstract

This chapter reviews the theory of the voluntary public and private redistribution of wealth elaborated by economic analysis in the last forty years or so. The central object of the theory is altruistic gift-giving, construed as benevolent voluntary redistribution of income or wealth. The theory concentrates on lump-sum voluntary transfers, individual or collective, which aim at equalizing the distribution of wealth from altruistic reasons or sentiments (perfectly substitutable altruistic transfers). It implies: (i) the Pareto-inefficiency of the non-cooperative interaction of individual altruistic transfers; (ii) the neutralization of public transfers by individual altruistic transfers; (iii) and the crowding out of private altruistic transfers by Pareto-efficient public redistribution. The chapter is organized as follows. Section 2 presents an informal overview of the general intent and content of the theory. Section 3 gives a first formal version of the theory in a one-commodity setup (pure distributive social system). Non-cooperative distributive equilibrium is characterized, and its fundamental properties of existence and determinacy are analyzed. Section 4 extends the definitions and fundamental properties of pure distributive social systems to general social systems that combine competitive market exchange with the non-cooperative altruistic transfers of individuals endowed with non-paternalistic interdependent preferences. Section 5 states the neutrality property in two versions of the theory successively: the general social systems of Section 4; and the important special case of the pure distributive social systems of Section 3, where the set of agents is partitioned in two subsets, namely, a subset of "poor" individuals with zero endowments and egoistic preferences, and a subset of "rich" individuals altruistic to the poor and indifferent to each other. Section 6 reviews the theory of Pareto-efficient redistribution in pure distributive social systems. Section 7 returns to the fundamental assumption of perfect substitutability of transfers through a selective review of theoretical models of imperfectly substitutable transfers and empirical tests of perfect substitutability.

Keywords

gift-giving, altruism, neutrality, distributive efficiency, liberal social contract

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1. Introduction

This chapter reviews the theory of the voluntary public and private redistribution of wealth elaborated by economic analysis in the last forty years or so.

The main feature of the theory, captured in the subtitle of the chapter under the notion of perfectly substitutable transfers, is the existence of a fundamental identity of all voluntary transfers, whether public or private, both in terms of their means (endowment redistribution) and in terms of their ends (making the distribution of wealth more equal).

In formal theory, perfect substitutability translates into a complex set of assumptions which combine, in their most elaborate form, elements from three key constructs: (i) competitive markets; (ii) individual preferences relative to the distribution of wealth; (iii) and Cournot–Nash interactions in transfer activities.

The first two constructs have a long history in economics, whether considered separately or combined.

The idea that human behavior in society can be fruitfully analyzed in a number of autonomous components (such as the "Market", the "Family" and the "State") corresponding to well-characterized differences in the motives of individual action, can be traced at least as far back to the work of Adam Smith, whose psychological explanation of the construction of the (pro-social and altruistic) moral self of individuals in the *Theory of Moral Sentiments* (1759) sharply contrasts with his representation of self-interested market behavior in the *Wealth of Nations* (1776). The first two constructs outlined above can be viewed, in many respects, as workable formal representations of Smith's theories: market exchange as interaction of self-interested individuals resulting in a socially efficient outcome; and the construction of the moral self of individuals as the outcome of acts of imaginative sympathy by which individuals imagine themselves in the position of others and experience, to some limited extent, the feelings associated with these positions.

The contrast between the representations of human behavior conveyed by Smith's two major works was sometimes viewed as a contradiction in nineteenth century controversies on the Economic Man (the "Adam Smith problem"). Two analytical contributions at the beginning of the twentieth century proved particularly useful to overcome this difficulty and to bridge (some of) the gaps between Smith's original insights and a formulation compatible with the stringent methodological demands of modern economic theory. The first one is Wicksteed's characterization of market behavior as "non-tuistic" (1910), that is, a behavior which is neither egoistic nor altruistic but proceeds, rather, from a type of instrumental rationality narrowly adapted to the context of market exchange, and compatible a priori with the large variety of goals that human beings pursue in other contexts. The second analytical contribution, which has proved very useful to the modern economic theory of gift-giving reviewed in this chapter, is Pareto's concept of a "maximum of utility for a collectivity" (1916), that is, in modern terms, a Pareto optimum defined according to (non-paternalistic) interdependent individual utilities. This notion places the Economic Man in the Social Man and, accordingly, market equilibrium in social equilibrium, in such a way that the (social) Pareto-efficiency of the latter entails the (market) Pareto-efficiency of the former.

The third feature of the perfect substitutability of transfers is much more recent than the former two [despite the reference to Cournot's (1838) early contribution]. It proceeds from the systematization of the use of game theory concepts in economics, from the 1950s, and notably from the contributions of Nash (1950) and Debreu (1952).

Combined with individual preferences on the distribution of wealth (distributive preferences, in short), Cournot–Nash interactions result in a public good problem, as: firstly, distributive preferences make the distribution of wealth an object of common concern, that is, a pure (non-rival, non-excludable) public good in the formal sense of modern economic theory [Kolm (1968)]; and secondly, non-cooperative individual contributions to a public good generally result in a socially inefficient (in the sense of Pareto) outcome, which in turn can be analyzed as a coordination deficiency of collective action [Samuelson (1954), Olson (1965)].

The joint assumption of distributive preferences and Cournot–Nash interactions has a second characteristic consequence, the so-called neutrality property, which essentially states that exogenous lump-sum wealth redistribution between agents connected directly or indirectly by operative (that is, positive) equilibrium gifts leaves the equilibrium distribution of wealth unchanged [Becker (1974) and, in an intertemporal setup, Barro (1974)]. Although noticed relatively late, the neutrality property is certainly the most obvious and salient aspect of the theory of redistribution reviewed in this chapter, as the direct translation, in general equilibrium terms, of the perfect substitutability of transfers.

The combination of the public good problem with the neutrality property results in a simple but powerful consequence, which can be viewed as the main prediction of the theory and can serve as a basis for an evaluation of its relevance and scope: the social efficiency of distribution requires the full crowding-out of all equilibrium transfers [Warr (1982)], unless a single agent is willing and able to make gifts to all others [Becker (1974)]. Put another way, the theory implies that if distributive concerns are widespread enough, they result in the socialization of a fraction of aggregate wealth (redistributed wealth) – in other words a redistributive welfare state – as the outcome of a Pareto-efficient social contract on the distribution of wealth [a distributive "liberal social contract": Kolm (1985)].

The chapter is organized as follows.

Section 2 presents a preliminary overview of the general intent and content of the theory.

Section 3 gives a first formal version of the theory in a one-commodity setup (pure distributive social systems). Non-cooperative distributive equilibrium is characterized, and its fundamental properties of existence and determinacy are analyzed.

Section 4 extends the definitions and fundamental properties of pure distributive social systems to general social systems, combining competitive market exchange with the non-cooperative benevolent transfers of individuals acting according to their non-paternalistic interdependent preferences (Pareto social systems).

Section 5 states the neutrality property in two versions of the theory successively: the general Pareto social systems of Section 4; and the important special case of the pure distributive social systems of Section 3, where the set of agents is partitioned in two subsets, namely, a subset of "poor" individuals with zero endowments and egoistic preferences, and a subset of "rich" individuals benevolent to the poor and indifferent to each other [Cornes and Sandler (1985a); Bergstrom, Blume and Varian (referred to as BBV in the sequel) (1986)].

Section 6 reviews the theory of Pareto-efficient redistribution in pure distributive social systems of the general type and of the BBV type.

Section 7 returns to the fundamental assumption of perfect substitutability through a selective review of theoretical models of imperfectly substitutable transfers and empirical tests of perfect substitutability. Special attention is given here to the meaning and degree of relevance of Cournot–Nash interactions and to the basic prediction of the full crowding-out of private redistributive transfers by public transfers.

2. Gift-giving in social equilibrium theory: A preliminary overview

This section briefly reviews, in an informal way, the object of the theory, the elements or determinants it mobilizes, the main solution concepts and results, and their interpretation and confrontation with facts.

The object of the theory is altruistic gift-giving, construed as benevolent voluntary redistribution of income or wealth.[1]

2.1. Preferences and rights

The benevolence of redistribution is understood as an expression of both the individual rationality of the donor(s) and his (their) favorable intentions relative to the beneficiary of his (their) gifts.

Individual rationality (in the usual sense of economic theory) translates into the maximization of well-behaved (that is, reflexive, complete and transitive) ordinal preferences of individuals on the distribution of wealth within a social group. Gift-giving appears, in other words, in the theory, as a "logical action" in the sense of Pareto (1916), designed to maximize donors' preferences on the distribution of wealth within the group; see Chapter 1 of this Handbook and the introduction to Section 4 below. Such preferences are defined directly on the distribution of income or wealth in the one-commodity setup of Section 3, or indirectly, via non-paternalistic interdependent preferences on the allocation of resources (see Footnote 2[2] for a precise formulation) in the setup of Section 4 with multiple market commodities exchanged on competitive markets.

[1] Wealth is understood as monetary wealth throughout Section 2.

[2] Following Pareto (1916) we name *ophelimity* the utility that an individual derives from his own consumption of market commodities. The assumption of non-paternalistic utility interdependence supposes that every

The favorable intentions of donors relative to beneficiaries translate into a positive valuation of the wealth of the latter in the preferences of the former, that is, donors' preferences increasing in the wealth of the beneficiaries of their gifts. This positive valuation of the beneficiary's wealth by the donor usually is named altruism in economic theory, notably since Becker (1976), following a long-running tradition in sociology initiated by Auguste Comte.[3] Formal altruism, in the sense just defined, is susceptible to cover various psychological contents in terms of donors' motives, depending on the context of redistribution, and notably on the size of the social group within which redistribution takes place. Three types of social contexts are considered in applications: families, or, by extension, small groups of close relatives; charity networks, where donors and beneficiaries may or may not be (and most frequently are not) in direct individual relation with each other; and general redistribution within large social groups, possibly whole political communities such as states or nations. Formal altruism usually is interpreted, accordingly: in family contexts, as feelings of individual sympathy, such as liking or love; in charity contexts, as philanthropy (sympathy towards mankind[4]), frequently associated with feelings or emotions of pity or compassion; in socio-political contexts, as feelings of solidarity (sympathy towards community members) or fraternity (sympathy towards equals).

Individual rational altruistic preferences on wealth distribution, as outlined above, make a first fundamental class of determinants in the social equilibrium theory of gift-giving. A second fundamental class of determinants, which refers to the voluntary character of gift-giving, consists of the property rights of individuals on income and wealth.

Gift-giving as voluntary wealth redistribution is an expression of the property right of donors. Property right is defined as the possibility, legitimated and protected by society, for the individual owner of a scarce resource, to freely decide on its use or abuse,

individual i has well-defined preferences on both: his own consumption of market commodities; and the vectors of individual ophelimities. In the sequel, the former is represented by an individual *ophelimity function*, denoted by u_i, and the latter by an individual *social utility function* of the type $w_i(\hat{u}_1, \dots, \hat{u}_n)$, where $\hat{u}_j$ denotes an ophelimity level of individual j for all j. In the one-commodity setup, ophelimity is identified with wealth and interpreted as individual consumption expenditure.

[3] Auguste Comte was one of the founders of scientific sociology. He probably coined the neologism "altruism" (or perhaps Andrieux), which, to the best of my knowledge, first appeared in print in his *Cours de Philosophie Positive* (1830). Former authors usually employed such terms as "benevolence", "beneficence", "love" or "sympathy". This notably was the case of Adam Smith, whose *Theory of Moral Sentiments* is deduced from sympathy [defined as "fellow-feeling with any passion whatever", a notion more commonly designated under the name of "empathy" in modern vocabulary (1759, Part First, Chapter I of Section I)] and characterizes benevolence as one of the three fundamental virtues in his classification of the systems of moral philosophy [the other two being the virtues of "propriety" and "prudence" (1759, Part Seventh, introduction to Section II)]. Vilfredo Pareto who, besides his fundamental contributions to economic theory, is also considered, like Auguste Comte, as a founder of scientific sociology, uses the term "humanitarianism" in the *Traité de Sociologie Générale* (1916, Chapter XII, notably Footnote 1 of §2131).

[4] The reader can find an exquisite literary illustration of philanthropic psychology (and so modern!) in the character of Mrs Birdseye in *The Bostonians* of Henry James (1886, notably Chapter IV).

within a conventional list of alternatives which typically consist of own consumption, consumption as input in a production process, disposal, selling and giving. Gift-giving is construed, consequently, as a free act of the donor(s), that is, notably, as a choice within a range of several accessible gifts, including the possibility of giving nothing.

An important extension of the freedom of agents so postulated by the theory lies in the representation of resulting social interactions as, firstly, non-strategic, and as, secondly, open to cooperation by means of explicit or implicit contracts between donors.

Non-strategic interactions, on the one hand, suppose that every individual agent or, in the case of cooperation, every group of cooperating individuals, makes the instrumental choice of taking the actions of others (for example, their gifts) as independent of its own. This corresponds to the so-called Cournot–Nash behavioral assumption, and opposes to strategic interactions, where agents, or at least some of them (called “leaders”, “principals”, ..., depending on the game under consideration), base their decisions on an accurate prediction of the reactions of others to their own actions. Contractual cooperation, on the other hand, supposes that individuals pool their resources in order to make collective decisions on the actions of the members of the resulting cooperating group (notably their gifts inside and outside this group) when such association is individually beneficial to all members. In the sequel, for the sake of brevity, we name *non-strategic cooperation* the combination of contractual cooperation (if any) with non-strategic interactions of non-cooperating agents or groups. This corresponds, in the formal definitions of Section 6.1.1, to the strong Nash distributive equilibrium.

The social equilibrium theory of gift-giving develops the view that the individual freedom of agents should result in non-strategic cooperation in the context of rational altruism. This view stems, in part, from a priori considerations reviewed in the sequel of this section, and also finds a posteriori justifications in the third and fourth characteristic properties of the theory considered in the next subsection (see notably the last paragraph of Section 2.2).

It seems to be a basic natural presumption that the independent acts of a genuinely free individual cannot be predicted with an objective certainty by the individuals who interact with him. While partly a postulate, as a “natural” consequence of the abstract notion of liberty in action, this proposition nevertheless can be given practical content in a variety of contexts relevant for us. Let us briefly enumerate three such realistic interpretations.

A first interpretation, expressed in terms of the cognitive abilities of any individual non-cooperatively interacting with a free individual agent (that is, more concretely, with a right-holder, such as an individual owner susceptible, for example, to make gifts, or consume them), is that the former knows the past and present acts of the latter or, still more realistically, is able to get such knowledge at sufficiently low cost and with sufficient accuracy to make it useful for his own practical purposes, but is unable to predict the latter’s acts at similar practical conditions (that is, to perform by himself sufficiently accurate predictions at sufficiently low cost). In short, the act is known (or can be), the agent is not. This interpretation clearly appears suitable for individual interactions in large social groups.

The second interpretation corresponds to the relativistic variant of the first, where the costs and hazards of non-strategic cooperation (essentially, transaction and enforcement costs and associate uncertainties) appear significantly lower than the costs and hazards of the individual prediction of others' reactions. This might, conjecturally, apply to social groups of any size, although more easily perhaps to groups of intermediate size, as the practical prevision of individual reactions can be presumed less difficult in (stable) micro-social units, and practical impediments to cooperation obviously increase with the size of the cooperation pool.

The third interpretation, finally, applies to situations where the ability of an agent to predict the reactions of another largely follows from his ability to relevantly constrain the latter, that is, relevantly restrain the set of alternatives accessible to his opponent by various means such as credible threats of retaliation in case of "bad conduct", "fait accompli", reliance on social norms etc. This is the interpretation which is most commonly retained, although implicitly, in the theoretical literature on strategic gift-giving reviewed in Section 7.1.3 below, notably models of strategic bequests and Samaritan's dilemma. It seems to apply, most relevantly, to long-lasting interactions, notably (but not only) in social contexts which imply close individual relations between agents. It must somehow contradict, by definition, the full liberty of action of private owners, and also formal altruism (in the sense of the former paragraph) when the use of the means of constraint is due to the donor, because of the characteristics that the donor's psychology then usually takes on. Let us briefly illustrate these points with an informal discussion of strategic bequests and Samaritan's dilemma, to finish with (see Section 7.1.3 for more detailed accounts).

Models of strategic bequest illustrate game situations where a testator, wishing to receive attention and care from "egoistic" heirs, obtains satisfaction, and moreover manages to reap the whole surplus from corresponding interactions, when he is in position to credibly threaten recalcitrants with disinheritance [Bernheim, Shleifer and Summers (1985); and also, putting specific emphasis on the importance of credibility, Hirshleifer (1977)]. Both articles find a nice literary illustration in the misfortunes met by Shakespeare's King Lear (1608) with his heirs. Balzac's *Eugénie Grandet* (1833) develops a similar vein in another historical and sociological context.[5] Both provide us with a lively illustration of a psychological process by which the interests induced by massive wealth transmission shape, and finally determine individual psychologies, expelling or deeply altering the ties of "natural" affection. Such literary archetypes, magnified by talent, and the abstract models above, yield pictures of individual interactions relative to inheritance, from which feelings of affection are not absent, but where they appear dominated by other features of the transmission relation (the various other "interests" of participants, which include material interests, although they do not necessarily reduce to them), and where gift-giving (bequest) notably appears as a powerful mean of constraint over beneficiaries.

[5] The first blooming of French bourgeois society, which followed the end of Napoleonic Wars.

Samaritan's dilemma [Buchanan (1975), Lindbeck and Weibull (1988)] refer to game situations where the generosity of the donor is negatively related to some characteristic over which the beneficiary has control, typically the beneficiary's current income or wealth from work and past savings. If the loss of one unit of the beneficiary's income, induced by a decrease in his labor or saving effort, is more than compensated, in terms of the beneficiary's welfare, by the corresponding increase in aid and decrease in disutility of effort, the beneficiary has an incentive to "exploit" the donor by choosing the low levels of effort (lower than in the absence of aid) that maximizes his welfare. Moreover, the resulting equilibrium then generally is Pareto-inefficient, implying the possibility of rearranging individual actions in such a way that all agents end up better off, including aid recipients. These models are contemporary echoes, with considerable attenuations in strength and tone though, of the traditional suspicion of parasitism and general misconduct of the beneficiaries of charity and public aid such as reflected, for example, in the debates which surrounded the British Poor Laws of the seventieth and nineteenth centuries. The essence of the argument in these past and present discussions on the political economy of poverty relief refers to aspects of the reality of aid practices which involve paternalistic motives on behalf of donors (notably the state) and their translation into various forms of control over beneficiaries, with a gradual evolution, over three centuries of economic development, from initial coercion and repression to contemporary policies of education and prevention. Note that, characteristically, in the contemporary models above, the agents who suffer de facto restrictions of their liberty of action are not the beneficiaries of aid as suggested above, but donors, who confront constraining "fait accompli" from beneficiaries (in addition to the obligation of respecting the property of others, sole liberty limitation implied, in principle, by property right). As if increasing economic affluence finally had shifted the burden from aid recipients to donors (this actually is the thesis (and regret) of Buchanan in the quoted reference).

The discussion above emphasizes types of interactions which do not fit in the joint assumption of (non-paternalistic) rational altruism and (full) freedom of action of individual owners outlined in the beginning. This raises in turn the question of the specific relevance, and adequate utilization of the latter in social equilibrium analysis. We return to this question in the third part of the section.

2.2. *Four characteristic properties*

Let us now turn to the characteristic properties of the theory.

For the sake of clarity, we name, from now on: *market optimum* a Pareto optimum relative to individual ophelimities;[6] *distributive optimum* a Pareto optimum relative to

[6] This terminology implicitly supposes a market economy that verifies the assumptions of the first and second fundamental theorems of welfare economics. These assumptions will be explicitly made in Sections 4 and 5, where we develop the study of the social systems that combine, on the one hand, non-tuistic (see Section 1 above, and Section 4.1.2 below) exchange of consumption goods and services on complete and perfect competitive markets, with, on the other hand, non-paternalistic altruistic individual transfers.

individual interdependent utilities (for the definitions of individual ophelimities and interdependent utilities, see Footnote 2 above). The associate notions of Pareto-efficiency are named, accordingly, *market* and *distributive efficiency* respectively (the latter sometimes also *social efficiency* in the sequel).

Summarizing, we consider social systems of rational altruists, endowed with property rights, who non-strategically interact, and possibly cooperate, in making voluntary wealth transfers.

The main characteristic properties of such systems, presented below in a logical order, are fourfold: the separability of redistribution by non-paternalistic altruistic transfers, from resource allocation by complete systems of competitive markets [Mercier Ythier (1989)]; neutrality [Barro (1974), and Becker (1974)]; a social aggregation property, which entails the Rotten Kid Theorem as a corollary [Becker (1974)]; and the full crowding out of altruistic transfers at distributive optimum [Warr (1982) and Mercier Ythier (1998a, 1998b)]. They are briefly summarized below.

The first characteristic property specifically applies to the multi-commodity setup, with non-paternalistic interdependent utilities and a complete system of competitive markets for consumption goods and services. The social system then involves the interaction of a subsystem of market exchange and production of the Walrasian type (a Walrasian economy) with voluntary wealth transfers from rational altruists. The property states that the *social equilibrium allocation is a market optimum*, under the usual conditions for the Pareto-efficiency of Walrasian economies. Note that the property notably supposes that altruistic donors non-strategically interact with the market, that is, are price takers, whether they act individually, or collectively by contractual cooperation. As a consequence, the type of non-altruistic gifts involved in the so-called transfer problem or paradoxes are excluded from the field of rational transfers considered by the theory (see Section 4.3).

Neutrality states, notably, that *exogenous lump-sum transfers between any pair of individual agents* (say, agents i and j) *leave equilibrium distribution unchanged if individual agents are linked by a gift at social equilibrium, provided that the gift be at least as large as the exogenous transfer when both transfers* (that is, the exogenous transfer and the gift) *have the same direction* (say, from i to j). Exogenous lump-sum transfers are determined outside the social system under consideration, that is, outside the set of interacting individual agents. They most frequently correspond to public decisions (or their consequences) in applications. The property holds under the basic assumptions of the theory, essentially: rational altruism and non-strategic interactions, complemented, in the multi-commodity setup, with non-paternalism and competitive markets. These assumptions are not only sufficient for neutrality (see Section 5) but also, in general, necessary for it as established by the numerous theoretical cases of non-neutralities reviewed in Section 7.1 below and in other chapters of this Handbook (see notably Chapters 18, 13 and 15 by Andreoni, by Laferrère and Wolff, and by Michel, Thibault and Vidal, respectively). Neutrality merely translates in general equilibrium terms the fact that with the assumptions above, and in general with them only, exogenous lump-sum transfers and equilibrium gifts are perfect substitutes: any variation in the exogenous transfer is exactly ("dollar-for-dollar") compensated by an opposite variation in the corresponding equilibrium gift, decided by the donor in order to keep the distribution of wealth unchanged (the best distribution from his point of view, among those he can attain given his budget constraint and the non-negativity of gifts). A simple and important consequence of the property is that gifts and exogenous (say, public) bilateral wealth transfers between the same pair of individual agents (say agents i and j) should not coexist at social equilibrium, as: if gift and exogenous transfer have opposite directions, then an exogenous transfer motivated by an intention to redistribute wealth between i and j is pointless, since the distributive objective of the donor must prevail on the exogenous distributive objective in such circumstances; and if the gift and the exogenous transfer have the same direction, then the exogenous transfer must crowd out the gift (that is, cancel it) in order to modify the equilibrium wealth distribution between i and j.

The third characteristic property states that *if social equilibrium is such that a single agent* (named family or community "head") *gives to all others, then the equilibrium distribution of wealth maximizes this agent's utility relative to the whole set of socially accessible distributions of wealth* (that is, relative to the set of distributions that verify the aggregate resource constraint of the community). The corresponding specific type of social equilibrium, where a single agent gives to all others, is named Becker's social equilibrium in the sequel. Two notable consequences follow, as simple corollaries, from this third characteristic property. Firstly, the social equilibrium trivially is a Pareto optimum relative to individual preferences on the distribution of wealth, since any socially accessible deviation from equilibrium distribution makes the family head worse off. Secondly, the characteristic property above implies Becker's Rotten Kid Theorem when social interactions are embedded in a two-stage sequential game where: the beneficiaries of the head's gifts play first, by (possibly) undertaking individual actions that increase social wealth at some cost for them (that is, increase the aggregate wealth of others by an amount larger than the individual cost they incur); the family head plays next, by making utility-maximizing gifts to all others, given the distribution of wealth which obtains at the first stage. The Rotten Kid Theorem states that, in such sequential games: if the individual wealth of every community member is a normal good for the head, that is, if the individual wealth which maximizes the head's utility is increasing in social wealth, then the egoistic beneficiaries of the head's gifts (the "rotten kids", who feel concerned only with their own wealth and welfare) seize all opportunities to maximize social wealth, because they know that, due to the altruistic behavior of the family head, this maximizes their individual wealth.

The fourth characteristic property states that *non-strategic cooperation in altruistic gift-giving results in status quo* (that is, in a social equilibrium without any individual or collective gift) *if and only if the initial distribution of wealth endowments is a distributive optimum*. The latter property requires, in general, additional assumptions on preferences. A natural sufficient condition states that: progressive wealth transfers[7] between any pair of agents (say, between individuals i and j) are weakly preferred[8] by all others, that is, are not vetoed by any agent distinct from i and j (this part of the condition is named *non-jealousy* in the sequel, for reasons detailed in Section 6.1.1); and individuals object to any bilateral wealth transfers from themselves to individuals wealthier than themselves (*self-centredness* in the sequel). The condition implies that altruistic gifts, if any, should be progressive (by self-centredness) and therefore weakly preferred to the status quo by non-contributors (by self-centredness and non-jealousy), that is: Pareto-efficient initial distribution implies the status quo. Since, moreover, cooperation implies Pareto-efficiency by definition, the characteristic property above follows from the assumption. A variant of the same reasoning obtains, with identical consequences, when the social system is made of "egoistic poor", only interested in their individual wealth, and "altruistic rich", who feel concerned about the aggregate wealth of the poor and are indifferent to the other rich: charitable gifts, if any, flow from rich to poor, and are preferred to the status quo by non-contributors. Note that the latter social system is formally identical to the standard public good model with additive technology,[9] so that the property above and the social contract solution below apply to the latter as well.

The fourth property provides a firm logical basis for a liberal social contract solution to the public good problem of redistribution (and also, by extension, as just noted, for a social contract solution to the financing of public spending on any set of pure public goods).

A liberal social contract consists of a Pareto-efficient arrangement of individual rights, which is unanimously preferred to an (historical) initial arrangement. Such collective agreements find their raison d'être in the inefficiency of individual or collective interactions, non-strategic or otherwise, notably in the presence of public goods or externalities. The contract necessarily remains implicit in many practical circumstances, due to various sources of contract failure in corresponding contexts, such as informational issues (notably preference revelation problems), transaction costs (for example when the number of concerned individuals is large), and so on. The implementation of corresponding Pareto-improving transfers generally supposes public interventions, therefore, with two main variants for the latter in practice: implementation by the state when the efficiency problem under consideration involves universal common concerns, as in the case of the national provision of a general public good; and, when the efficiency problem concerns a large part of society but can be separately solved in each component of

[7] A bilateral wealth transfer between individuals i and j is said to be *progressive* if it reduces, without reverting, the difference in wealth between them.

[8] That is, preferred or indifferent.

[9] With notations which have become standard in the literature, the utility of "rich" i reads $u_i(x_i, G_1, \ldots, G_n)$, where x_i denotes his consumption in the private good (his ophelimity), and G_j the aggregate provision of public good j (for example, the aggregate wealth of the poor of type j). Letting t_{ij} denote i's lump-sum contribution to public good j, which can be voluntary or forced, additive technology reads as: $G_j = \sum_i t_{ij}$ for all j, and most conveniently interprets as the financing of public expenditure by lump-sum transfers from private money wealth.

a partition of it, implementation by a set of public or private collective actions such that each action applies to the relevant social subset and the complete set of actions covers the whole society (as in the case of the national provision of a type of local public good by the autonomous actions of all concerned local authorities). With, in the latter variant, a role of the state which then mainly consists in providing an institutional framework that favors the expression and coordination of decentralized initiatives in favor of the public good.

The specific relevance of the notion in the context of the theory of altruistic gift-giving developed here stems from the public good problem of redistribution, which combines the following interrelated aspects: (i) the common distributive concerns of individuals, embodied in their distributive preferences, make the distribution of wealth a pure (that is, non-excludable and non-rival) public good in the formal sense; (ii) non-cooperative gifts generally yield socially inefficient distribution, notably in the presence of multiple donors (more formally, Nash non-cooperative gift equilibrium with multiple donors generally is Pareto-inefficient: see Section 6.1 and the examples in Section 3.3); (iii) as a first consequence, non-strategic cooperation generally fails to produce any equilibrium with gifts (formally, non-zero strong Nash equilibrium of gifts generally does not exist), with Becker's social equilibrium as sole notable exception; (iv) and, as a joint consequence of the public good problem and neutrality, the achievement of distributive optimum by means of exogenous lump-sum redistributions of initial endowments generally supposes the full crowding-out of private transfers. Except in Becker's equilibrium configuration, the achievement of a distributive optimum therefore supposes a re-arrangement of initial endowments, which, under the additional requirement of unanimous preference, precisely corresponds to a (distributive) liberal social contract. The fourth characteristic property above implies that such liberal social contracts are (status quo) social equilibria relative to the non-strategic interactions and contractual cooperation of rational altruists, when all individuals agree (in the sense of weak preference) that wealth transfers, if any, should flow downwards, from the wealthier to the less wealthy. Several important features of the theory follow from this basic fact, such as the existence and indeterminacy of these Pareto-efficient solutions to the public good problem of redistribution (see Section 6.1.2), and the uniqueness of corresponding social equilibria (see Section 3.4.2).

Note, finally, that the third and fourth characteristic properties provide ex post justifications to the Cournot–Nash behavioral assumption relative to altruistic gift-giving at corresponding social equilibria. In Becker's equilibrium, first, the head has no incentive whatsoever to behave strategically; and the strategic gifts of rotten kids, if any, are strictly self-interested. The status quo equilibrium of distributive liberal social contracts, second, exhaust, by construction, the opportunities of social exchange on the public good (that is, on the distribution of wealth), again leaving no room for strategic deviations of individuals or coalitions in the form of altruistic gifts. Note, nevertheless, that the distributive liberal social contract, like Becker's equilibrium, is potentially compatible with strategic non-altruistic gift-giving, notably through the transfer paradox (see Section 4.3), that is, interactions of voluntary redistribution with market exchange such that: endowment redistribution is substantial enough to significantly alter the system of equilibrium market prices (this presumably supposes large collective gifts); and this change in the terms of trade reveals so beneficial to donors that they end up better off in terms of their own ophelimities (that is, the utility they derive from their individual consumption of market commodities is increased).

2.3. Theory and facts

Let us now address, to finish with, the question of the relations between the theory just outlined and facts.

A characteristic structural feature of the theory is the representation of redistribution by altruistic transfers, and of allocation by the market, as autonomous processes, both operated by the non-strategic or cooperative actions of free rational individuals, and resulting in mutually compatible and Pareto-efficient outcomes, that is, respectively, distributive optimum and market optimum.

The main axiomatic constituents underlying this structural feature are: Walrasian economy; non-paternalistic utility interdependence; lump-sum transfers; Cournot–Nash interactions; and free contracting.

The first four elements of this list of constituents form a general hypothesis of perfect substitutability of transfers, as they are sufficient, and in general necessary for the separability and neutrality properties of social equilibrium (first and second characteristic properties).

When transfers are motivated by universal distributive concerns, and there is a unanimous (weak) preference for redistributing, and also sufficient conformity of individual preferences on redistribution (e.g. self-centredness and non-jealousy), the perfect substitutability of transfers generally implies the full crowding out of all private individual transfers at distributive liberal social contract (fourth characteristic property). In that sense, the theory predicts a redistributive welfare state.

Social contracting also provides partial (that is, ceteris paribus) solutions to social efficiency problems when common concerns are restricted to social sub-groups, as in the case of local public goods or club goods. In such contexts, the perfect substitutability of transfers might permit, possibly in association with other assumptions, to consistently combine the partial social contracts into a universal liberal social contract, by allowing for a separate treatment of all partial efficiency problems at an adequate sub-society level. This trivially is the case, for example, in the one-commodity setup, when distributive concerns partition society in a set of "families" in the sense of Becker (that is, of small groups of closely related individuals, who benefit from the altruistic gifts of a family head, and whose altruistic sentiments, if any, are reserved to group members; note that, in this very simple case, the liberal social contract is implemented without any public intervention, by the altruistic gifts of family heads). Interesting issues concerning such decentralized variants of the liberal social contract relate to the dynamics of public good provision in a context of competition of local public and/or private initiatives for the public good, and, in particular, to the corresponding variants of the Coase conjecture as statements relative to the shape and evolution of social equilibrium in the long run (see notably Sections 7.1.3 and 7.2 below).

The possible sources of gaps between theory and facts are transparent from the list of constituents above. They may consist of: market failures; distortionary transfers; individual motives of non-market transfers distinct from altruistic redistributive motives (that is, from motives of maximization of altruistic non-paternalistic interdependent utilities); and the various conceivable impediments to social contracting on public goods, such as costs of information, transaction or enforcement, and possibilities of strategic manipulations. They can be grouped into two large categories.

One consists of the imperfect substitutability of transfers (see Section 7). This refers to forms of complex interdependency between non-market transfers and market allocation, or between non-market transfers themselves. Imperfect substitutability may notably result in violations of the neutrality property, and also in the non-separability of non-market redistribution from market allocation. Non-separability may stem in particular from imperfections in transfer techniques (distortionary taxes, essentially). It may derive, alternatively, from imperfections in the functioning of markets, which are susceptible, notably, to inefficiently bind altruistic redistributive transfers, by superimposing market exchange motives (that is, ophelimity-maximizing motives) upon their original altruistic motives, in situations where the two types of motives cannot be simultaneously fulfilled (a type of second-best problem). Non-neutralities may follow from the two sources above and also, in addition, from the existence of alternative transfer motives, distinct both from market exchange motives and altruistic redistributive motives, such as: tutelary motives, which imply the use of transfers as a means of control on beneficiaries' behavior or conduct; and the various motives which imply that transfers matter per se, independently of their influence on wealth distribution or market allocation (joy of giving, "warm glow", demonstration effects, reciprocity motives etc). They may also derive, finally, from the existence of strategic transfers, notably when they stem from non-altruistic motives (for example strategic bequests and transfer paradoxes). In all such cases, social interactions generally involve some degree of complementarity between public and private redistributive transfers, which can contribute to explain the lasting coexistence of both types of transfer at social equilibrium in the long run (see Sections 7.1 and 7.2.2).

The second category of potential gaps between theory and facts consists of the practical limits to social contracting on public goods, essentially transaction costs and issues of imperfect information, enforceability and manipulability. These problems remain largely unexplored for redistribution as a public good (see nevertheless the remarks and references of Sections 6.3 and A.2 concerning the design of incentive compatible mechanisms in public good economies). They explain why the distributive liberal social contract is bound to remain partly implicit in many practical circumstances, and generally requires public interventions for its implementation. The efficacy of public action and its limitations in terms of the various administrative costs and other disadvantages associated with it contribute, in turn, to determine the practical size and shape of the transfers of the actual social contract.[10]

3. Perfectly substitutable transfers in a pure distributive social system

This section considers the simplest version of the present theory, where individuals interact non-strategically and non-cooperatively by means of altruistic individual gifts of a single commodity ("money wealth"). That is, we concentrate on the (generalized) Nash non-cooperative equilibrium of individual gifts (Section 3.1.2) of pure distributive social systems (Section 3.1.1).

This simple setup is illustrated in Section 3.3, through three classical applications to family gift-giving [Becker (1974)], Pareto-efficient redistribution [Arrow (1981)], and the private provision of public goods [Bergstrom, Blume and Varian (1986)]. These three studies retain the same non-strategic, non-cooperative scheme of social interactions for altruistic gift-giving. They differ in the nature and scope of the altruistic concerns they consider: *microsocial* family altruism with Becker; "*mesosocial*" charitable altruism from rich to poor with Bergstrom et al.; and altruistic concerns for distributive justice (nevertheless biased by some degree of self-centredness) at *macrosocial* level in Arrow's study. Special attention is devoted, in the presentation of these studies, to the *public good problem of redistribution*. We notably provide several graphical examples, using the geometric device introduced in Section 3.2, which substantiate the contention (formulated in Section 2 above as consequence of the third and fourth characteristic properties, and established in Section 6.1 below) that, except in the specific type of equilibrium configuration considered by Becker, non-trivial (that is, non-zero) gift equilibrium generally is Pareto-inefficient relative to individual distributive preferences.

The last Section 3.4 reviews known results on the existence and determinacy of the non-cooperative equilibrium of individual gifts of pure distributive social systems.

[10] Social contract theory traditionally defines the social contract relative to some hypothetical initial position (a hypothetical "state of nature", "original position" etc.), where the contingent obstacles to social contracting are consistently assumed away [the nature of the contingent obstacles so removed depending on the nature of the social contract considered; e.g. Kolm (2003)]. Rawl's theory of justice, for example, uses this type of hypothetical device for abstracting from individual characteristics, as contingent obstacles to the impartiality of individual judgments of justice: individuals are thus placed, by hypothesis, under a "veil of ignorance" relative to their actual position in society. The liberal social contract, likewise, is defined relative to an ideal state of society, where the contingent obstacles which are abstractly assumed away are the impediments to (generalized) exchange per se (mainly, transaction costs and enforcement issues). These *abstract social contracts* define ideal norms for public action. Their implementation by public policies is subject, in turn, to the actual limitations of public action. *Actual social contract* policies then consist of the set of public actions, rational and democratic by construction, which implement the ideal norm of the abstract social contract within the practical limits of actual public action.

3.1. Pure distributive social system and equilibrium

3.1.1. Pure distributive social systems

Pure distributive social systems are defined as abstract social systems where: (i) wealth is measured in money units and divisible; (ii) wealth is shared initially among individual owners; (iii) owners can, individually, consume or transfer to others any amount of their ownership, that is, of their initial endowment increased by the gifts received from others; (iv) owners make their consumption and transfer decisions according to their preferences on the final distribution of wealth, that is, on the vector of individual consumption expenditures; (v) aggregate wealth is fixed, which implies notably that the latter is independent of individual consumption and transfer decisions.

Formally, let individuals be designated by an index i running in $N = \{1, \ldots, n\}$, and choose the money unit so that aggregate wealth is 1.

Individual i's *initial endowment* or *right*, that is, his share in total wealth prior consumption or transfer is denoted by $\omega_i \in [0, 1]$.

A *consumption* x_i of individual i is the money value of his consumption of commodities. A *gift* t_{ij} *from individual* i *to individual* j $(j \neq i)$ is a non-negative money transfer from individual i's property (his initial endowment plus the gifts he received from others) to individual j's. A *gift-vector of individual* i is a vector[11] $t_i = (t_{ij})_{j \in N\setminus\{i\}}$ of $\mathbb{R}^{n-1}_+$.

We ignore alternative individual uses of wealth, like disposal or production, as well as "transaction" costs (including taxes) associated with consumption and transfer activities. The property rights (jus utendi et abutendi) of individuals translate then into the following budget identity, which holds for all individual i, endowment ω_i, and decision (x_i, t_i):

$$x_i + \sum_{j:\ j \neq i} t_{ij} = \omega_i + \sum_{j:\ j \neq i} t_{ji}.$$

A distribution of initial rights $(\omega_1, \ldots, \omega_n)$ is denoted by ω. This is an element of the unit simplex $S_n = \{x \in \mathbb{R}^n_+ \colon \sum_{i \in N} x_i = 1\}$ of $\mathbb{R}^n$. A distribution of individual consumption expenditures $(x_1, \ldots, x_n)$ is denoted by x. It is *feasible* if it belongs to S_n. A *gift vector* t is a vector $(t_1, \ldots, t_n)$. Individuals have ordinal preferences on the final distribution of wealth, that is, on the vectors of individual consumption expenditures, represented by their *distributive utility functions* $w_i : x \to w_i(x)$, defined on the space of consumption distributions $\mathbb{R}^n$. These preferences may express individual moral sentiments such as benevolence, malevolence or indifference to others, but also individual opinions of distributive justice relative, for instance, to the equity or fairness of the distribution of wealth. I will say notably that an individual is benevolent or altruistic (resp.

[11] Notations like $t, t_i, (t_{ij})_{j \in I}$ or t_I (where I is a subset of N), will refer to row vectors. The entries t_{ij} of these vectors are ranked in increasing lexicographic order (that is, according to the ordering defined on $N \times N$ by: $(i, j) > (i', j')$ if either $i > i'$ or $i = i'$ and $j > j'$).

malevolent, resp. indifferent or egoistic) to another individual in the neighbourhood of a distribution x if the former's utility is locally increasing (resp. decreasing, resp. constant) in the latter's wealth.

The vector $(w_1, \dots, w_n)$ of individual utility functions is denoted by w.

A *distributive social system* is a pair (w, ω).

We use the following notations. t^{T} is the transpose of row vector t. $t_{\setminus i}$ (resp. t_I, resp. $t_{\setminus I}$) is the vector of gifts obtained from t by deleting t_i (resp. t_i for all $i \notin I$, resp. t_i for all $i \in I$). $(t_{\setminus i}, t_i^*)$ (resp. $(t_{\setminus I}, t_I^*)$) is the gift-vector obtained from t and t^* by substituting t_i^* for t_i (resp. t_i^* for t_i for all $i \in I$) in t. $\Delta_i t$ is the net transfer $\sum_{j:\ j \neq i} (t_{ji} - t_{ij})$ accruing to individual i when t is the gift-vector. Δt is the vector of net transfers $(\Delta_1 t, \dots, \Delta_n t)$. $x(\omega, t)$ is the vector of individual consumption expenditures $\omega + \Delta t = (\omega_1 + \Delta_1 t, \dots, \omega_n + \Delta_n t)$, that is, given the accounting identity above, the unique consumption distribution associated with the distribution of rights ω and the gift-vector t. $x_i(\omega, t)$ is the ith projection $\mathrm{pr}_i\, x(\omega, t) = \omega_i + \Delta_i t$. $\partial_t x(\omega, t)$ (resp. $\partial_{t_i} x_i(\omega, t)$) is the Jacobian matrix of $t \to x(\omega, t)$ (resp. $t_i \to x_i(\omega, t)$) at (ω, t). Finally, for any pair $(z, z') = ((z_1, \dots, z_n), (z'_1, \dots, z'_n))$ of vectors of $\mathbb{R}^n$, we write: $z \geqslant z'$ if $z_i \geqslant z'_i$ for all i; $z > z'$ if $z \geqslant z'$ and $z \neq z'$; $z \gg z'$ if $z_i > z'_i$ for all i.

3.1.2. Distributive equilibrium

This subsection defines the gift equilibrium as a Nash non-cooperative equilibrium of individual gifts, and provides a characterization of gift equilibrium for differentiable social systems.

3.1.2.1. Definition The general notion of social equilibrium according to Debreu (1952),[12] applied to the pure distributive social system, becomes the following: every individual agent takes the transfers of others as fixed, and maximizes his utility with respect to his own gifts, subject to the constraint that his consumption be non-negative. An equilibrium is a gift vector that solves all individual maximization problems simultaneously. Formally:

DEFINITION 1. A *distributive equilibrium* of (w, ω) is a gift-vector t^* such that t_i^* is a maximum of $t_i \to w_i(x(\omega, (t^*_{\setminus i}, t_i)))$ in $\{t_i\colon x_i(\omega, (t^*_{\setminus i}, t_i)) \geqslant 0\}$ for all i.

[12] This notion is often labeled "Cournot–Nash" equilibrium, by reference to its early definitions by Auguste Cournot (1838), and John R. Nash (1950) [see for instance Cornes and Sandler (1986, Chapter 5)]. Modern game theory often refers to it as "generalized Nash equilibrium". I will stick to the vocabulary of Gérard Debreu in the sequel, because it fits well the substantive object of the theory reviewed in this chapter, and moreover corresponds to the words (if not the precise notion) of Vilfredo Pareto in the pioneering Chapter 12 of his *Traité de Sociologie Générale* (1916, §2067 to 2078, pp. 1308–1315). Note that Debreu's main application of his general notion was the proof of existence of a competitive economic equilibrium [Arrow and Debreu (1954)].

For a fixed w, I define the following equilibrium sets and correspondences. $T_w(\omega) = \{t\colon t$ is a distributive equilibrium of $(w, \omega)\}$ is the set of equilibrium gift-vectors of (w, ω); $X_w(\omega) = \{x\colon \exists t \in T_w(\omega)$ such that $x = x(\omega, t)\}$ is the corresponding set of equilibrium distributions; $\Omega_w(x) = \{\omega\colon \exists t \in T_w(\omega)$ such that $x = x(\omega, t)\}$ is the set of initial distributions ω supporting x as an equilibrium distribution of (w, ω). $T_w : \omega \to T_w(\omega)$ is then the *equilibrium correspondence* of w, $X_w : \omega \to X_w(\omega)$ is its *equilibrium distribution correspondence*, and $\Omega_w : x \to \Omega_w(x)$ is the *inverse equilibrium distribution correspondence*. The *range* of X_w (and domain of Ω_w) will be denoted by M_w. The *range* of Ω_w (and domain of T_w and X_w) is denoted by Q_w. The subscript w will be omitted in the sequel.

3.1.2.2. First-order conditions The remainder of the chapter is restricted to differentiable distributive social systems[13] that verify the following standard assumptions:

ASSUMPTION 1. For all i: (i) w_i is differentiable (smooth preferences); (ii) w_i is quasi-concave (convex preferences: $w_i(x) \geqslant w_i(x')$ implies $w_i(\lambda x + (1-\lambda)x') \geqslant w_i(x')$ for all real number $\lambda \in [0, 1]$); (iii) w_i is strictly increasing in x_i (utility increasing in own wealth); (iv) and $\omega_i > 0$.

Let $W = \{(w, \omega)\colon (w, \omega)$ verifies Assumption 1$\}$.

The first-order conditions characterizing equilibrium are given in Theorem 1 below.[14] Informally, these conditions state that, at equilibrium, a marginal incremental wealth transfer from i to j is either impossible or does not increase i's utility, and that a marginal incremental wealth transfer from j to i does not increase i's utility whenever the equilibrium transfer from i to j is positive.

THEOREM 1. *Let $(w, \omega) \in W$. Then, t is a distributive equilibrium of $(w, \omega) \in W \times S_n$ if and only if for all (i, j):* (i) *Either $x_i(\omega, t) = 0$ or $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) \leqslant 0$;* (ii) *and $(-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)))t_{ij} = 0$.*

PROOF. Let t^* be a distributive equilibrium of $(w, \omega) \in W \times S_n$. Inequalities (i) and (ii) of Theorem 1 are the first-order conditions for a maximum of $t_i \to w_i(x(\omega, (t^*_{\setminus i}, t_i)))$ in $\{t_i \in \mathbb{R}^{n-1}_+\colon x_i(\omega, (t^*_{\setminus i}, t_i)) \geqslant 0\}$. These conditions are necessary by Assumption 1(iv) and Arrow and Enthoven (1961: Theorem 2). They are sufficient by Assumption 1 and Arrow and Enthoven (1961: Theorem 1(b)). $\square$

[13] A natural strategy for the study of continuous social systems (that is, social systems with continuous individual preference preorderings) consists of "smoothing" them by means of appropriate approximation techniques, and examining then whether, as is often the case, the properties of smooth social systems extend by continuity to continuous ones. This is done, for instance, in Mercier Ythier (1992), for the existence of a social equilibrium in pure distributive social systems.

[14] Where $\partial_{x_j} w_i(x)$ denotes the partial derivative of w_i with respect to its jth argument, and $\partial w_i(x)$ the Jacobian matrix of w_i at x.

The characterization of M and Ω below is a simple consequence of Theorem 1.

Let:

$$g(t) = \{(i, j) \in N \times N \colon t_{ij} > 0\};$$
$$\gamma_w(x) = \{(i, j) \in N \times N \colon -\partial_{x_i} w_i\big(x(\omega, t)\big) + \partial_{x_j} w_i\big(x(\omega, t)\big) = 0\}.$$

These sets will be viewed as directed graphs or digraphs. The *incidence matrix* $\Gamma_{w,i}(x)$ is the $(n, n-1)$-matrix defined in the following way: the rows of $\Gamma_{w,i}(x)$ are associated with the elements (vertices) of N, ranked in increasing order; the columns of $\Gamma_{w,i}(x)$ are associated with the elements (darts) of $\{(i, j) \in N \times N \colon j \neq i\}$, ranked in increasing lexicographic order; if $(i, j) \in \gamma_w(x)$ is such that $i \neq j$, the entries of the corresponding column of $\Gamma_{w,i}(x)$ are -1 on row i, 1 on row j, 0 on the other rows; if $(i, j) \notin \gamma_w(x)$, the entries of the corresponding column of $\Gamma_{w,i}(x)$ are 0 on all rows. The incidence matrix $\Gamma_w(x)$ of $\gamma_w(x)$ is the $(n, n(n-1))$-matrix: $(\Gamma_{w,1}(x), \ldots, \Gamma_{w,n}(x))$. The subscript w will be omitted in subsequent notations of graphs and incidence matrices.

We have then the following corollary:

COROLLARY 1. *Let w verify Assumption 1, and suppose moreover that $M \subset \mathbb{R}^n_{++}$. Then:* (i) $M = \{x \in S_n \cap \mathbb{R}^n_{++} \colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) \leqslant 0$ *for all* $(i, j)\}$. (ii) *For all $x \in M$, $\Omega(x)$ is the convex set* $\{x - \Gamma(x).t^{\mathrm{T}} \in S_n \colon t \in \mathbb{R}^{n(n-1)}_{+}\} = \{x - \Gamma(x).t^{\mathrm{T}} \in S_n \colon g(t) \subset \gamma(x)\}$, *of dimension* rank $\Gamma(x)$.

PROOF. (i) If $t \in T(\omega)$, then $x(\omega, t) \in \{x \in S_n \colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) \leqslant 0$ for all $(i, j)\}$ by Theorem 1. Conversely, if $\omega \in \{x \in S_n \colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) \leqslant 0$ for all $(i, j)\}$, then $0 \in T(\omega)$ by Theorem 1.

(ii) Notice that: $\Gamma(x).t^{\mathrm{T}} = \Gamma(x).(0_{\setminus\gamma(x)}, t_{\gamma(x)})^{\mathrm{T}}$ for all t. Suppose therefore without loss of generality that $g(t) \subset \gamma(x)$. Notice then that $\Gamma(x).t^{\mathrm{T}} = (\Delta_1 t, \ldots, \Delta_n t)$ and apply Corollary 1(i), Theorem 1 and the definition of Ω. □

3.2. Diagrammatic representation

Individual preferences relative to the distribution of wealth make each individual's wealth a public good, at least potentially.

More formally, the consumption x_i of individual i is a public good (or bad) at some distribution x if there exists at least another agent j whose utility is either increasing (public good) or decreasing (public bad) in i's consumption at x, that is, if $\partial_{x_i} w_j(x) \neq 0$ for some $j \neq i$. Individual i's consumption is then a common concern for both i (due to the natural assumption of utility increasing in own wealth: Assumption 1(iii) above) and j. This is a *pure* public good in this setting: its "consumption" by individual j consists of his observation of x_i, which has the two classic properties of non-rivalry (observation by j induces no restriction on observation by k) and non-excludability (x_i is correctly observed by all concerned agents).[15] The public good is local if $\{j\colon\ \partial_{x_i} w_j(x) \neq 0\}$ is some relevant ("small") subset of N, general otherwise and notably when $\{j\colon\ \partial_{x_i} w_j(x) \neq 0\} = N$ (universal common concern).

The diagrammatic representation of distributive social systems presented below is adapted from a geometric device first used by Kolm (1969, Chapter 9), and since often referred to as Kolm's triangle [Thomson (1999)]. Ley (1996) gives a good account of the use of Kolm's technique in models of private provision of public goods, as well as a presentation of frequently used alternative techniques such as the Dolbear triangle (1967) and the diagrams of Cornes and Sandler (notably 1985a, Figure 6, p. 112, and the Cornes–Sandler box, 1986, Figure 5.3, p. 77).[16] The present application of the diagram to the analysis of voluntary redistribution was developed by Mercier Ythier (1989, 1993).

The choice of this geometric device is essentially related to the fact that the elicitation of the so-called public good problem, that is, in this context, the elicitation of the Pareto-inefficiency of distributive equilibrium, requires the existence of at least three agents. We recall and establish in Section 6.1.1.2 below the simple fact that the distributive equilibrium must be Pareto-efficient when the number of agents is $n = 2$ [Nakayama (1980)].

The set $S_3 = \{x \in \mathbb{R}^3_+\colon\ \sum_{i=1}^{3} x_i = 1\}$ of feasible distributions of wealth of a three-agent social system is represented, in the canonical system of Euclidean coordinates of $\mathbb{R}^3$, by the equilateral triangle $O_1O_2O_3$ (Figure 1), where O_i denotes the element of $\mathbb{R}^3$ whose ith coordinate is $= 1$ and jth coordinate is $= 0$ for all $j \neq i$. Any point of the triangle reads therefore as a vector of individual shares in the unit of aggregate wealth available for individual ownership or consumption. We abstract from the axes but maintain the Euclidean coordinates in the subsequent representations of S_3, which means that the plane of physical representation is implicitly identified with the Euclidean plane $\{x \in \mathbb{R}^3\colon\ \sum_{i=1}^{3} x_i = 1\}$.[17]

The loci of the feasible distributions with a constant $x_i \in [0, 1[$, or isowealth lines of individual i, are the straight lines parallel to O_jO_k, where $j \neq i$ and $k \neq i, j$, that is, the straight lines parallel to the side of the triangle opposite to O_i (cf. Figure 2, with $i = 1$; x_1 increases south east, from 0 at segment O_2O_3 to 1 at point O_1).

Figure 3 represents the indifference map of an individual (say, agent 1) whose preferences are convex and benevolent. Distribution x^1 is the best feasible distribution for

[15] Non-rivalry is clearly an innocuous feature of the setup. Non-excludability, on the contrary, appears much more demanding, in that it does not take into account interesting situations of the real world, where individuals feel concerned about the wealth of others that they do not observe correctly. In other words, this analytical framework recognizes only two types of agents: those who feel concerned about the wealth of some other agent and observe it correctly; and those who are indifferent to the latter.

[16] See also, among others, Chamberlin (1974) and Danziger (1976).

[17] This makes several differences with the usual definition of Kolm's triangle, notably: there are three agents and, at least potentially, three public goods (and, potentially again, no private good), instead of the two agents, the two private goods and the single public good of the usual versions of Kolm's triangle.

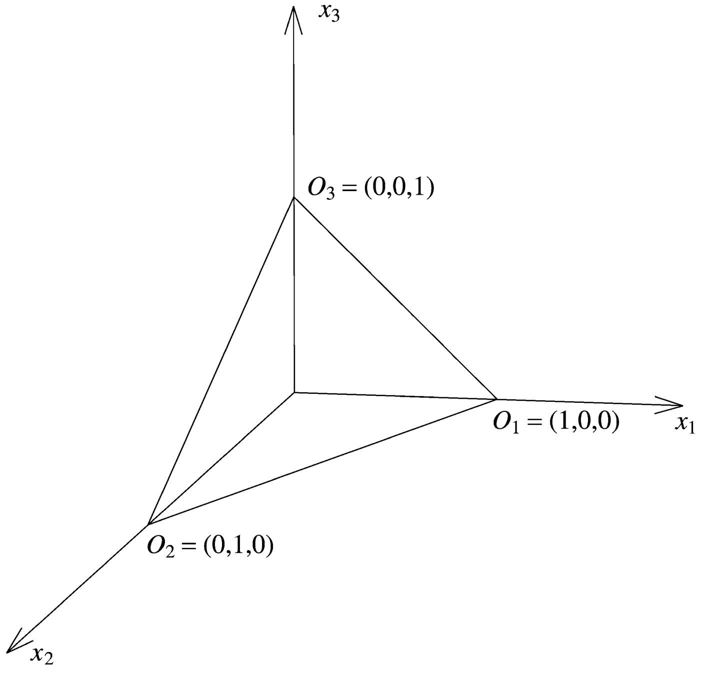

Figure 1. Feasible wealth distributions.

agent 1, that is, the distribution that maximizes $w_1(x)$ in S_3. More generally, the best feasible distribution of agent i will be denoted by x^i in the sequel.

Figure 4 provides the geometric device for the determination of the sign of $-\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x)$ for relevant feasible distributions x, that is, of the sign of the consequence on i's utility of a marginal wealth transfer from i to j at x. We let $i = 1$ and suppose for simplicity that w_1 is strictly quasi-concave. The curve $x^1 m^{1j}$ $(j = 2, 3)$ is the locus of tangent points of the indifference map of agent 1 in S_3 with the isowealth lines $\{x \in S_3\colon x_k = c, k \neq 1, j\}$ such that $c \geqslant x_k^1$. In view of Assumption 1, $x^1 m^{1j}$ is equivalently the set $\{x \in S_3\colon -\partial_{x_1} w_1(x) + \partial_{x_j} w_1(x) = 0$; and $x_k \geqslant x_k^1, k \neq 1, j\}$, that is, the subset of $\{x \in S_3\colon x_k \geqslant x_k^1, k \neq 1, j\}$ where agent 1's utility is stationary with respect to marginal wealth transfers between individual j and himself. The strict quasi-concavity of w_1 readily implies then that $-\partial_{x_1} w_1(x) + \partial_{x_j} w_1(x) < 0$ (resp. > 0) when x is obtained from some distribution of $x^1 m^{1j}$ by means of a wealth transfer from 1 to j (resp. from j to 1), that is, when x is a distribution of the isowealth line $\{x \in S_3\colon x_k = x_k^*, k \neq 1, j\}$ such that $x_1 < x_1^*$ (resp. $x_1 > x_1^*$) for some x^* of $x^1 m^{1j}$.

Figure 5 replicates the construct of Figure 4 for all three agents. The range $M = \{x \in S_n \cap \mathbb{R}^n_{++}\colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) \leqslant 0$ for all $(i, j)\}$ (Corollary 1(i)) of the correspondence of equilibrium distributions is the area shaded gray. The values of the inverse equilibrium correspondence $\Omega(x)$ are easily represented from the values of the

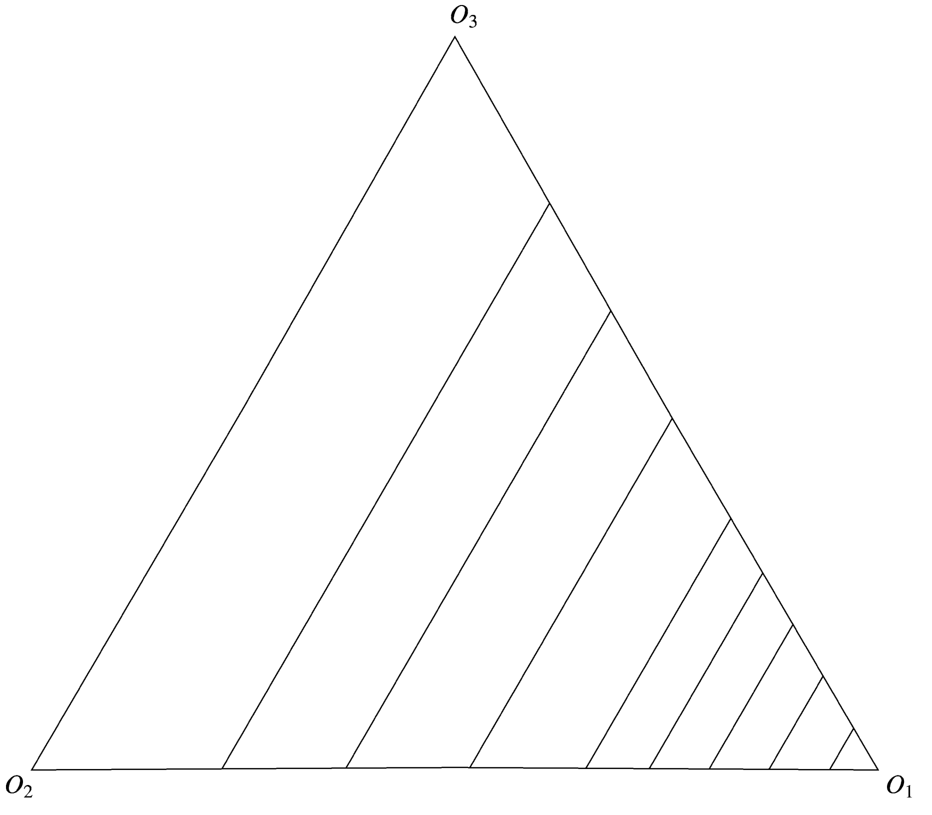

Figure 2. Agent 1's isowealth lines.

digraph $\gamma(x)$ at equilibrium distributions $x \in M$. Recall that $\gamma(x)$ is defined as $\{(i, j) \in N \times N\colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) = 0\}$. The subdigraph $\{(i, j) \in \gamma(x)\colon i \neq j\}$ corresponds therefore to the digraph of potential equilibrium gifts at x (potential because t_{ij} and $-\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x)$ can be simultaneously $= 0$ at equilibrium). One verifies then in Figure 5 that: $\gamma(x^i) = \{(i, j)\colon j \in N\}$; $\{(i, j) \in \gamma(z^k)\colon i \neq j\} = \{(i, k)\colon i \neq k\}$; $\{(j, m) \in \gamma(x)\colon j \neq m\} = \{(i, k)\}$ if x is an element of the topological boundary ∂M of M between x^i and z^k; $\gamma(x) = \varnothing$ if x is in the topological interior $\operatorname{Int} M$ of M. $\Omega(x)$ is then: $\{x\}$ if $x \in \operatorname{Int} M$; the line segment $\{\omega \in S_3\colon \omega_i \geqslant x_i$ and $\omega_k = x_k$ for $k \neq i, j\}$ if $x \in \partial M$ is between x^i and z^j; the triangle $\{\omega \in S_3\colon \omega_j \geqslant x_j$ for all $j \neq i\}$ if $x = z^i$; the parallelogram $\{\omega \in S_3\colon \omega_j \leqslant x_j$ for all $j \neq i\}$ if $x = x^i$. $\Omega(x)$ is, therefore, geometrically, at any $x \in \partial M$, the intersection with S_n of the convex cone generated by the set of half-tangents, outward pointing relative to M, to the indifference curves of the potential donors at x, that is, to the indifference curves of agents i such that $(i, j) \in \gamma(x)$ for some $j \neq i$.

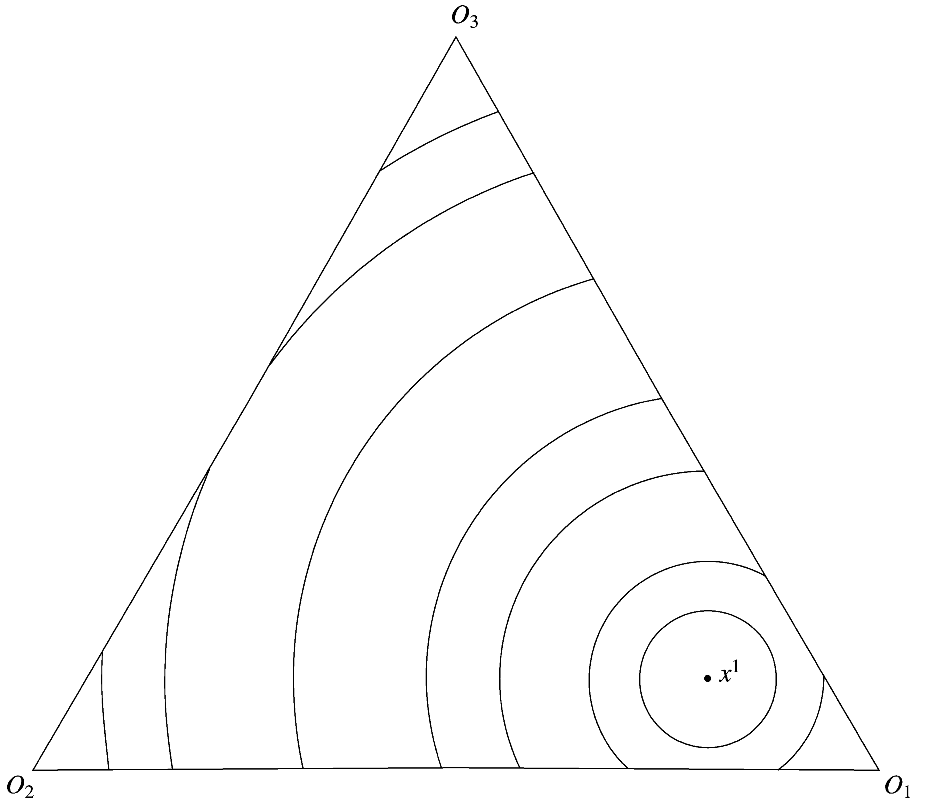

Figure 3. Agent 1's indifference map.

If w_i is the Cobb–Douglas $(x_1, x_2, x_3) \to \beta_{i1} \ln x_1 + \beta_{i2} \ln x_2 + \beta_{i3} \ln x_3$, with $\beta^i = (\beta_{i1}, \beta_{i2}, \beta_{i3}) \in S_3$, then $x^i = \beta^i$ and $x^i m^{ij}$ is the line segment $\beta^i O_k$ such that $k \neq i, j$ (Figure 6).

3.3. Three studies of pure distributive equilibrium

We now examine three classic studies of the distributive equilibrium that were decisive for the elaboration and subsequent popularization of the concept in economic analyses of voluntary redistribution, namely, Becker's "Theory of social interactions" (1974), Arrow's "Optimal and voluntary redistribution" (1981) and Bergstrom, Blume and Varian's "On the private provision of public goods" (1986).

We will show how the three models relate to the general setup of Section 3.1, elicit their particular assumptions with respect to the latter, and recall the salient properties of their respective equilibria, regarding notably their efficiency.

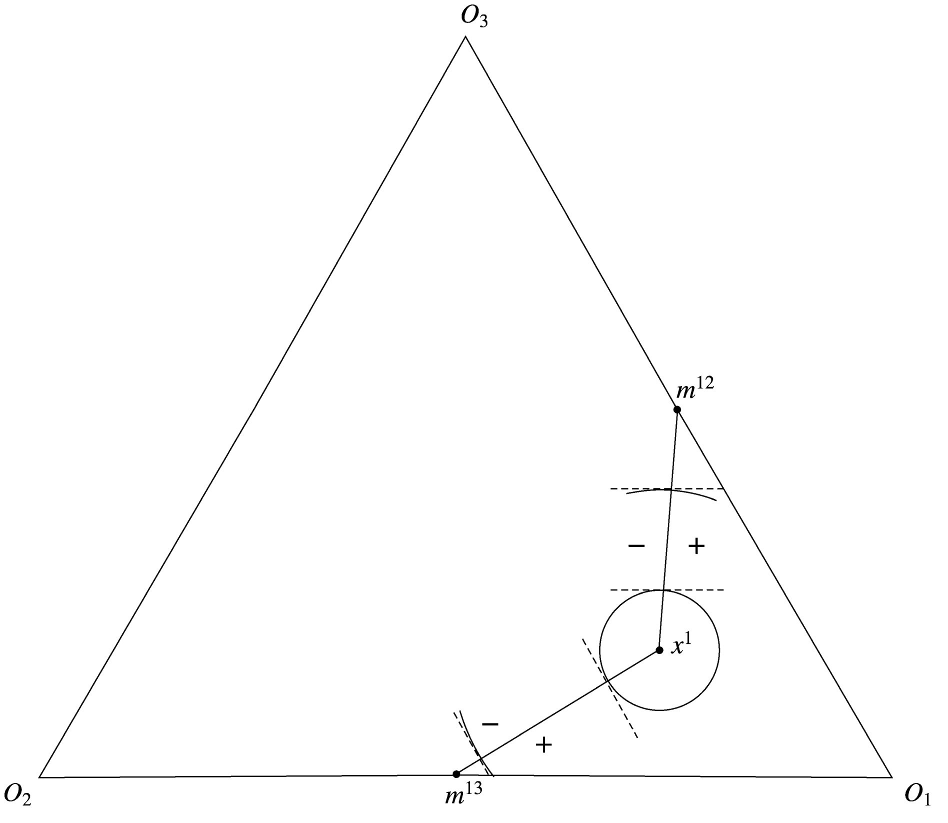

Figure 4. Sign of $-\partial_{x_1} w_1(x) + \partial_{x_j} w_1(x)$.

3.3.1. Becker (1974): The theory of social interactions

Becker's theory concentrates typically on social interactions in small groups, essentially the family. Interactions consist mainly of altruistic wealth transfers, although extensions to merit wants and malevolence are also considered and discussed at some length. The theory concentrates specifically on equilibria where a single agent (the "head") makes altruistic transfers to all other members of the group.[18]

The theory is presented initially in the framework of the household production model, where individual utility depends on a list of basic commodities that are produced from market goods and services, own time, education and the characteristics of others. But this general framework is immediately specialized by assuming a single commodity,

[18] This can be viewed as a definition of the social group in Becker's theory. In other words, Becker studies the social groups shaped by the altruistic equilibrium transfers of heads.

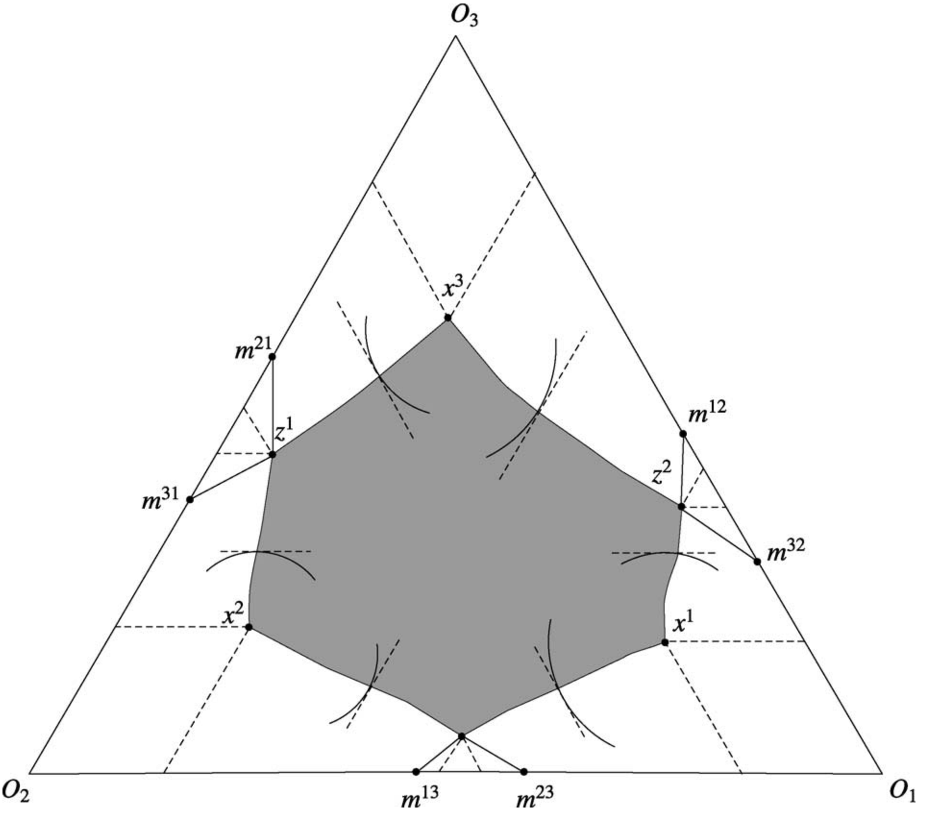

Figure 5. Inverse equilibrium correspondence.

a single market good and a single characteristic of others, so that utility derives from the individual consumption of a single market good and from a single characteristic of others.

The latter is identified in applications (family and charity) to the consumption of market good of the beneficiary of transfers. Let our set $N = \{1, \ldots, n\}$ designate a family in the sense of Becker, that is, the small group made of a head (say, $i = 1$) and the beneficiaries of his altruistic transfers. The utility functions of family members are then of the type $w_i(x_1, \ldots, x_n)$. The head is altruistic to the other members of the family. This translates formally into the strict monotonicity of w_1 (that is, $w_1(x) > w_1(x')$ whenever $x > x'$). Characteristically, Becker does not make any explicit assumption on the distributive preferences of non-heads (but the usual requirements of convexity and utility increasing in own wealth). He only assumes, implicitly, that their altruism, if any, is not strong enough to determine them to make gifts at equilibrium. His "Rotten Kid Theorem" (1974, 3.A, p. 1080; and 1976, p. 820) explicitly assumes selfish beneficiaries

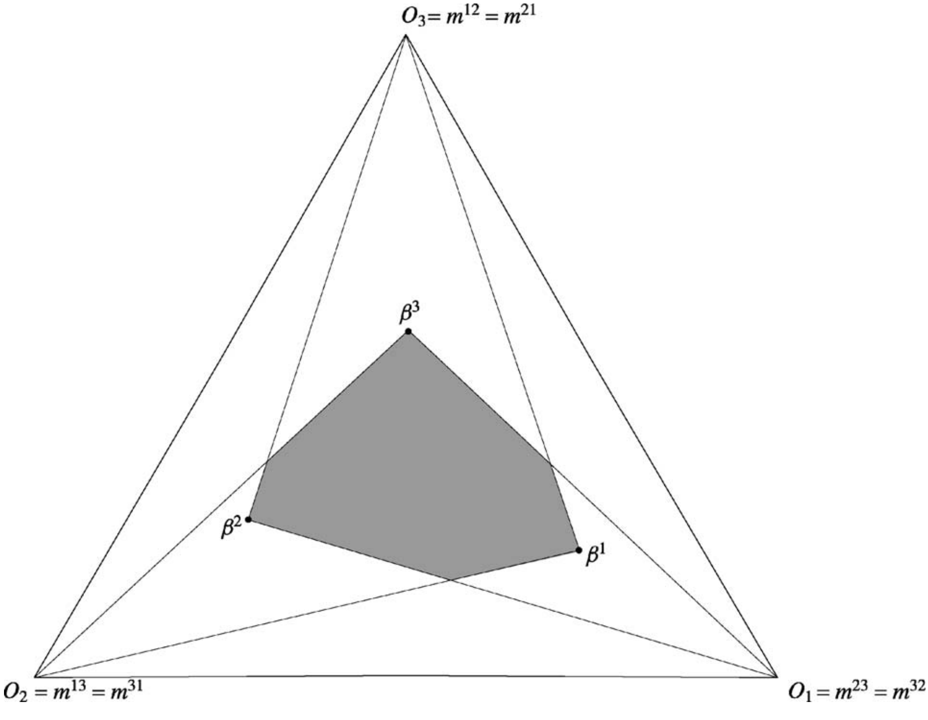

Figure 6. Cobb–Douglas social system.

(that is, $w_i(x) = x_i$ for all x and all $i \neq 1$). And a version of this theorem, in Becker (1981a, corollary, p. 183) or (1981b, p. 7), applies to malevolent ("envious", in Becker's terminology) beneficiaries.

The head chooses his consumption and levels of effort enhancing the characteristics of others so as to maximize the utility function above, subject to the budget constraint for money income: $x_1 + \sum_{j:\ j \neq 1} t_{1j} = \omega_1$.[19] Since the head is the sole donor in Becker's construct, the budget constraint of any other member of the family reads: $x_i = \omega_i + t_{1i}$, or equivalently $x_i - \omega_i = t_{1i}$. Substituting into the head's constraint, one gets the following equivalent formulation of the latter: $\sum_{i \in N} x_i = \sum_{i \in N} \omega_i$, where the right-hand side corresponds to the head's "social income" [Becker (1974, p. 1067)]. The

[19] Becker adopts the following more general formulation: $p_x x_i + p_t \sum_{j:\ j \neq i} t_{ij}$, for the left-hand side of the head's budget constraint in Sections 1 and 2, where p_x is the price of own consumption and p_t the price to the head of a unit of wealth of others. Discrepancies between p_x and p_t can stem from transaction costs of transfers (that may include the taxes paid on some types of transfers such as bequests or donations), or fiscal incentives such as the deductibility of charitable transfers from taxable income. They are assumed away in applications, nevertheless, and can be easily accommodated in the general framework presented in Section 3.1.1 above.

positive transfers of the head translate equivalently into the inequalities $x_i > \omega_i$ for the other members of the family.

The equilibrium distribution of Becker's microsocial system, therefore: is the (supposed unique) solution x^1 to $\max\{w_1(x)\colon x \in S_n\}$; and is such that $x_i^1 > \omega_i$ for all $i > 1$. In other words, the equilibrium of his version of the distributive social system defined in Section 3.1.1 above is essentially characterized by the following specific features: redistribution is achieved by a single donor, who gives to all family members, and manages to reach his most favored distribution in the whole set of feasible distributions of the family. These peculiarities of the distributive equilibrium of Becker are illustrated in Figures 7 and 8. Figure 7 represents a social system ("family") of three altruistic Cobb–Douglas agents: Becker's equilibrium obtains, with individual 1 (resp. 2, resp. 3) as family head, if and only if the initial distribution ω lies in the parallelogram $\{\omega \in S_3\colon \omega_j > x_j$ for all $j \neq 1$ (resp. 2, resp. 3)$\}$, that is, geometrically, in the relative interior of $\beta^1 a O_1 b$ (resp. $\beta^2 c O_2 d$, resp. $\beta^3 e O_3 f$) in S_3. Figure 8 represents the social system of the Rotten Kid Theorem (see the third basic property of Section 2), with three Cobb–Douglas agents: one altruistic head (individual 1), and two egoistic kids ($w_i(x) = x_i$, $i = 2, 3$). As in Figure 7, Becker's equilibrium obtains if and only if $\omega \in \{\omega \in S_3\colon \omega_j > x_j^1$ for all $j \neq 1\} = \beta^1 a O_1 b$.

In summary, Becker's configuration of distributive equilibrium is a perfect illustration of what might be called, paraphrasing Boulding (1973, notably p. 27), the "integrative" virtue of gift-giving: the gifts of the head "make" the family, whose equilibrium happens to coincide in turn with the rational choice of its individual head (individual utility-maximizing behavior).

3.3.2. Arrow (1981): Optimal and voluntary income distribution

Arrow's article, and particularly his charity game (pp. 217–223), is formulated directly in the general framework of Section 3.1.1. Its originality or specificity with respect to the latter lies in the assumptions on distributive preferences. Formally, Arrow supposes that there exist $n + 1$ strictly concave, differentiable and increasing functions $\mathbb{R} \to \mathbb{R}$ $\varphi, \varphi_1, \dots, \varphi_n$, such that, for all i, all $x \in \mathbb{R}^n$ and all $z \in \mathbb{R}$, $w_i(x) = \varphi_i(x_i) + \sum_{j\colon j \neq i} \varphi(x_j)$ and $\partial\varphi_i(z) > \partial\varphi(z)$. This means that, besides their familiar though non-trivial properties of additive separability and strict convexity, the preferences of Arrow's distributive agents exhibit: benevolence (w_i is monotonic strictly increasing); a (strong) variant of self-centredness,[20] stating that a wealth transfer from i to j makes the former worse off whenever their pre-transfer wealth are identical; and identical impartial-utilitarian views on redistribution affecting others.

[20] Arrow uses the word "selfishness" instead of self-centredness. I prefer the latter in order to avoid confusions with common formal definitions of selfishness as indifference to others, i.e. the constancy of utility with respect to the wealth of others [see the account of Becker (1974) above]. The relation between this assumption of Arrow and the similar notions of self-centredness discussed in this chapter (Sections 2.2, 3.4.1.2 and 6.1.1.3) is briefly examined, notably, in Footnotes 23 and 59.

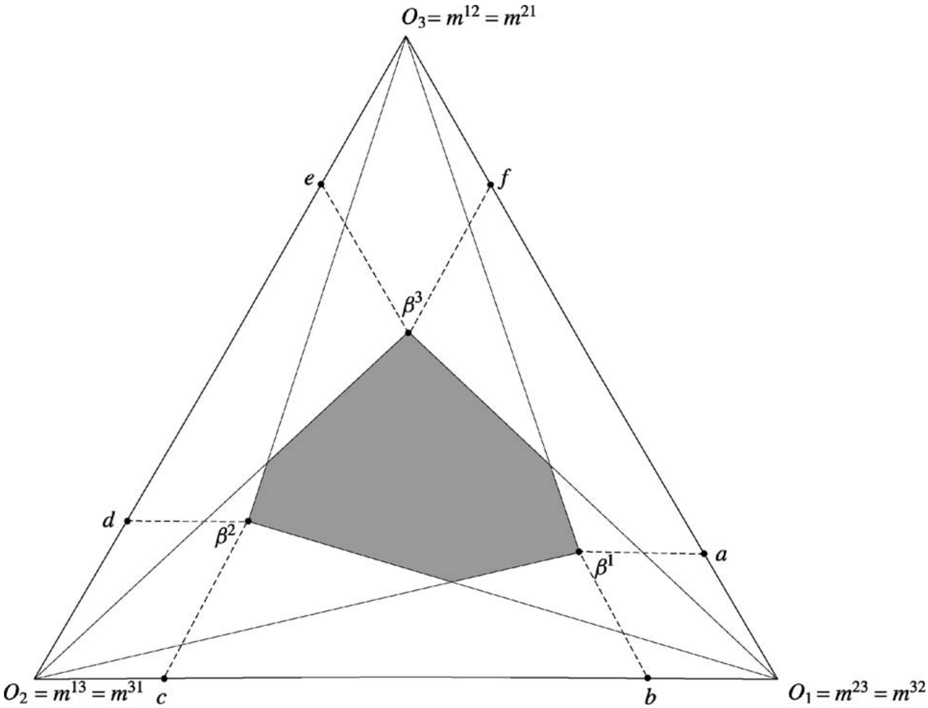

Figure 7. Becker's equilibrium with altruistic family members.

There is a unique equilibrium, which is characterized in the Theorem 5 (p. 221). The characterization relies on the functions $\xi_i : \mathbb{R} \to \mathbb{R}$ defined implicitly by $-\partial\varphi_i(\xi_i(z)) + \partial\varphi(z) = 0$, $i = 1, \ldots, n$. The set of donors is $\{i\colon\ \omega_i > \xi_i(\mathrm{Min}_j\, \omega_j)\}$, that is, the set of individuals whose utility is increasing in wealth transfers to the least favored at the endowment point ω. Letting x^* denote the equilibrium distribution, the set of receivers is $\{i\colon\ x_i^* > \omega_i\}$, that is, the set of individuals whose equilibrium wealth is larger than their initial wealth. The two sets have an empty intersection, which means that individuals cannot be simultaneously donors and receivers at equilibrium. All receivers have the same equilibrium wealth, which corresponds to the minimum equilibrium wealth $\mathrm{Min}_j\, x_j^*$, denoted by $x_{\min}$ (that is: $x_i^* = \mathrm{Min}_j\, x_j^* = x_{\min}$ for every receiver i). These properties are illustrated in the Cobb–Douglas social system of Figure 9, with log linear utility functions (recall that preferences are ordinal) such that $\beta_{ii} > \beta_{ij} = \beta_{ik}$ for all i, $j \neq i$, $k \neq i, j$: the minimum endowment is individual 2's; the unique donor is individual 1; the unique receiver is individual 2.

Theorem 6 (p. 222) yields an interesting characterization of Pareto-efficient equilibrium when the latter is not a status quo, that is, when the equilibrium distribution differs from the initial distribution ("non-trivial" equilibrium, in Arrow's terminology). It states that a non-trivial equilibrium is Pareto-efficient if and only if the equilibrium

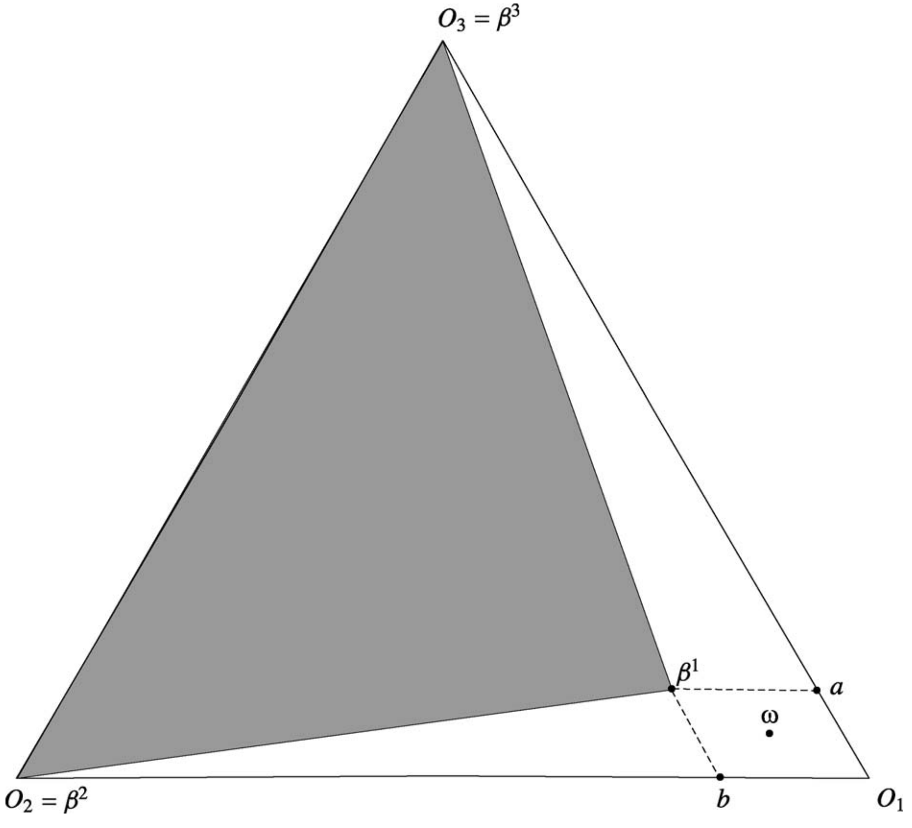

Figure 8. Becker's equilibrium with altruistic head and egoistic kids.

distribution x^* has exactly one individual (say, individual i) above the minimum equilibrium wealth $x_{\min}$. One deduces from the former paragraph that individual i then is the sole donor at equilibrium. And one easily verifies that the equilibrium then is an equilibrium of Becker.[21] In other words, a non-trivial equilibrium of Arrow is Pareto-efficient if and only if it is a social equilibrium of Becker. Figure 10 reproduces the social system of Figures 6 and 7, with the symmetries of distributive preferences that follow from Arrow's assumptions (namely: $\beta_{ij} = \beta_{ik}$ for all $i, j \neq i, k \neq i, j$): the set of Pareto-efficient distributions is the triangle $\beta^1\beta^2\beta^3$; its intersection with the set ∂M (the topological boundary of M) of non-trivial equilibrium distributions is the set $\{\beta^1, \beta^2, \beta^3\}$ of Beckerian equilibria; x^i obtains as an equilibrium if and only if the initial distribution is in the parallelogram $\{\omega \in S_3\colon \omega_j \geqslant x^i_j \text{ for all } j \neq i\}$.

[21] With one minor qualification: the donor needs not make gifts to *all* others, because we might have the coincidental situation where the endowment of an individual is equal to the minimum equilibrium wealth.

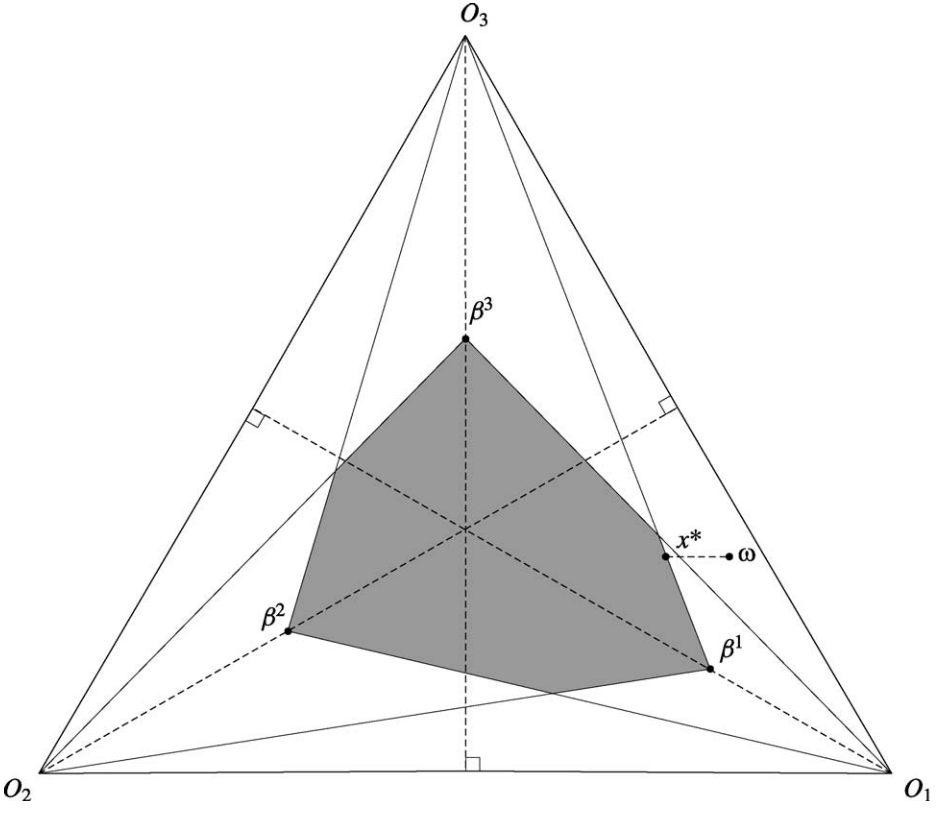

Figure 9. Arrow's distributive equilibrium.

3.3.3. Bergstrom, Blume and Varian (1986): On the private provision of public goods

The model of private provision of public goods of Bergstrom–Blume–Varian (BBV), although formulated in a slightly different setting, can be easily embedded in the framework of Section 3.1.1.

Let the set $N = \{1, \ldots, n\}$ of agents be partitioned in two subsets: the non-poor $\{1, \ldots, m\}$, and the poor $\{m+1, \ldots, n\}$, where $1 \leqslant m < n$. I suppose, accordingly, that $\omega_i > \omega_j$ for all $(i, j) \in \{1, \ldots, m\} \times \{m+1, \ldots, n\}$. The poor have selfish distributive preferences: $w_i(x) = \mathrm{pr}_i\, x = x_i$ for all x for all $i \in \{m+1, \ldots, n\}$, where pr_i denotes the ith canonical projection $x \to x_i$ of $\mathbb{R}^n$. The non-poor are indifferent to the other non-poor and benevolent to the poor: $w_i(x) = \mu_i(x_i, x_{m+1}, \ldots, x_n)$, with μ_i monotonic strictly increasing, for all $i \in \{1, \ldots, m\}$. This implies that the wealth of the poor is a pure (non-rival, non-excludable) public good for the non-poor, while the wealth of the non-poor is a pure private good. An important special case for the shape of non-poor utility functions is $w_i(x) = \nu_i(x_i, x_{m+1} + \cdots + x_n)$ with ν_i monotonic strictly

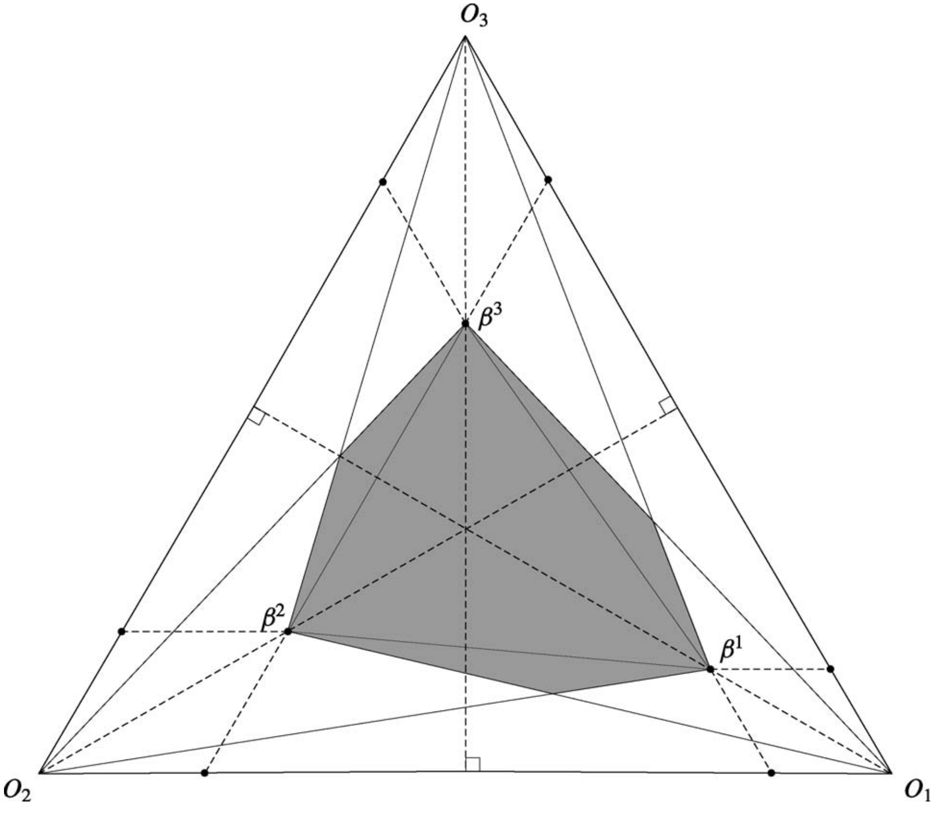

Figure 10. Pareto-efficient redistribution in Arrow's distributive social system.

increasing $\mathbb{R}^2 \to \mathbb{R}$. This is then the aggregate wealth of the poor which appears as the public good.[22]

This setup relates to BBV in the following way. The poor do not contribute to the public good: they choose $t_i = 0$ and consume $x_i = \omega_i + \sum_{j>m} t_{ji}$ for all vector $(t_1, \ldots, t_m)$ of contributions of the non-poor. The contributions of the non-poor, if any, benefit to the poor: non-poor i maximizes $\mu_i(\omega_i - \sum_{j>m} t_{ij}, \omega_{m+1} + \sum_{j\leqslant m} t_{jm+1}, \ldots, \omega_n + \sum_{j\leqslant m} t_{jn})$ with respect to $(t_{im+1}, \ldots, t_{in})$ given $t_{\setminus i}$. A BBV equilibrium of the set

[22] The model can be viewed as a crude stylization of traditional charitable redistribution from rich to poor. The assumed preferences of the "rich" and "poor" interpret, most conveniently, as revealed preferences, narrowly conditioned by the specific context of charitable redistribution: agents are endowed with the distributive preferences corresponding to their individual position in this context, as either "donor and rich" or "beneficiary and poor". A more refined and more satisfactory version of the model would assume individual preferences such that the (charitable) altruism of any individual i towards any individual j depend on x_i and x_j (in a natural way: non-decreasing in the former and non-increasing in the latter).

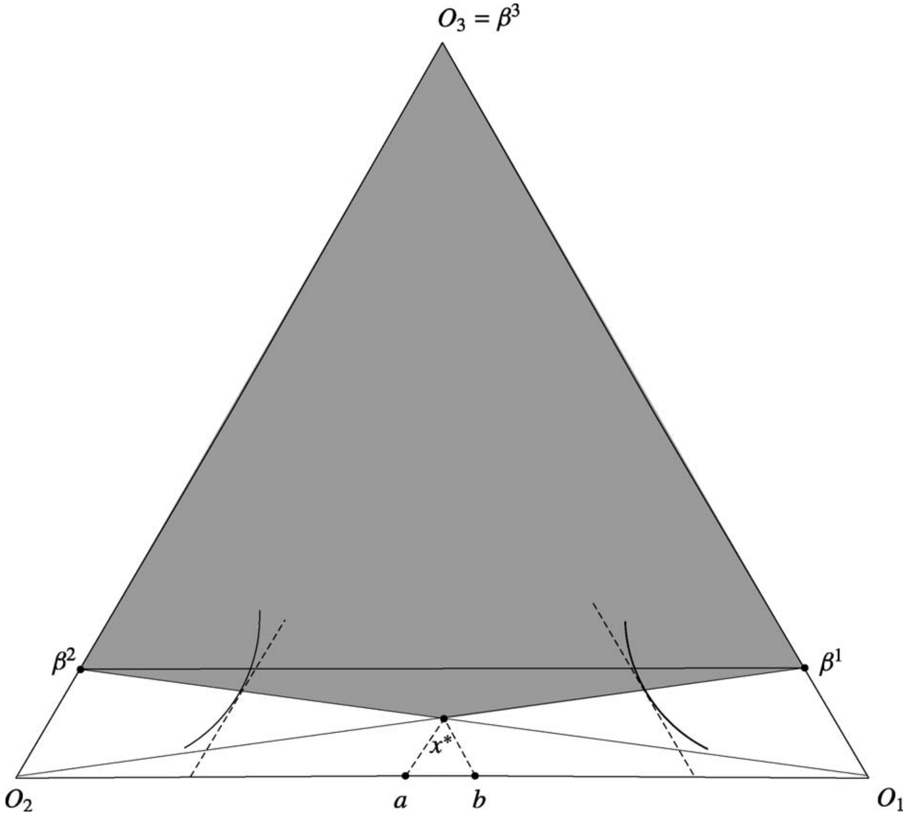

Figure 11. BBV distributive social system.

of non-poor agents $\{1, \ldots, m\}$ [also called Cornes–Sandler equilibrium in the literature, by reference to Cornes and Sandler (1985a), or sometimes also subscription equilibrium] is then a vector of gifts $(t_1, \ldots, t_m)$ such that t_i solves the maximization problem above for each non-poor i. One verifies immediately from the definitions that $(t_1, \ldots, t_m)$ is a BBV equilibrium of $\{1, \ldots, m\}$ if and only if the gift-vector $t = (t_1, \ldots, t_m, 0, \ldots, 0)$ is a distributive equilibrium of $\{1, \ldots, n\}$.

When w is of the type $(\mu_1, \ldots, \mu_m, \mathrm{pr}_{m+1}, \ldots, \mathrm{pr}_n)$, the social system is named a *BBV social system*. When it is of the type $(\nu_1, \ldots, \nu_m, \mathrm{pr}_{m+1}, \ldots, \mathrm{pr}_n)$, it is named a *strong* BBV social system.

The positive and normative properties of the BBV distributive equilibrium will be reviewed later in the chapter. It is sufficient at this stage to illustrate the model by an example. Figure 11 represents a BBV social system. There is a single poor, agent 3, whose utility function is $w_3(x) = x_3$. The non-poor are agents 1 and 2, with Cobb–Douglas utility functions $w_i(x_i, x_3) = \beta_{ii} \ln x_i + \beta_{i3} \ln x_3$. And ω is in the open line segment $]O_1, O_2[$ (that is: $\omega_3 = 0$ and ω_1 and ω_2 are > 0). One verifies easily, using the technique developed in Section 3.2, that: if $\omega \in]O_1, a]$, then agent 1 is the sole donor, and the equilibrium distribution is the projection of ω on the line segment $\beta^1 O_2$ parallel to $O_1 O_3$; if $\omega \in]a, b[$, then both non-poor give to the poor, and the equilibrium distribution is the intersection x^* of $\beta^1 O_2$ and $\beta^2 O_1$; finally, if $\omega \in [b, O_2[$, then agent 2 is the sole donor, and the equilibrium distribution is the projection of ω on the line segment $\beta^2 O_1$ parallel to $O_2 O_3$. The set of Pareto-efficient distributions is the triangle $\beta^1 O_3 \beta^2$, so that none of these BBV equilibria are efficient: it is possible to increase the utilities of the three agents by properly increasing the income support of agents 1 and 2 to the poor. In other words, charity is under-provided by voluntary contributions.

3.4. Existence, determinacy

This section reviews the most fundamental properties of distributive equilibrium. The latter explains the voluntary redistribution of wealth through essentially three types of determinants: individual preferences; initial endowments; and a mode of interaction, namely, the assumption that distributive agents take the transfers of others as fixed when making their own transfer decisions (the conjecture of Cournot–Nash, sometimes called also "zero conjecture" in the literature, and referred to as the "Cournot–Nash behavioral assumption" in this chapter). The study of the existence of equilibrium explores the general conditions under which these determinants are able to generate some equilibrium distribution of wealth from *any* initial distribution. While the study of its determinacy examines the general conditions under which the number of equilibria is finite (local determinacy), or, ideally, equal to 1 (full determinacy). In other words, these studies test the internal consistency and the precision of the determination of the distribution of wealth by the distributive preferences of individuals, their wealth endowments, and the Cournot–Nash behavioral assumption relative to wealth transfers.

The distributive equilibrium shares essentially the same existence and determinacy properties as competitive equilibrium, with only one significant exception: the possibility of logically robust (i.e. generic) non-existence in situations characterized below as "wars of gifts" (Section 3.4.1). Distributive equilibrium is generically locally determinate, and status quo distributive equilibrium is generically unique, as are competitive equilibrium and status quo competitive equilibrium respectively (Section 3.4.2).

3.4.1. Existence

Although the distributive equilibrium is a special case of Debreu's social equilibrium, the corresponding existence theorem [Debreu (1952, pp. 52–53)] does not apply, because the set $\{t\colon\ x(\omega, t) \geqslant 0\}$ of gift-vectors $t = (t_1, \ldots, t_n)$ jointly accessible to the set of all individuals is unbounded above.

We review below examples of non-existence of a distributive equilibrium. The existence problem is characterized as a "war of gifts". We give then a general existence theorem.

3.4.1.1. Non-existence of a distributive equilibrium We consider here examples of distributive social systems which have no equilibrium despite the upper hemicontinuity of individual reaction correspondences. The existence failure stems from the non-compactness of their domain. It is generally robust to small perturbations of utility functions or endowments, that is: existence is *not* a *generic* property of the distributive social systems of W [Mercier Ythier (2004b, 5.2)], and this contrasts with the general existence of market equilibrium in competitive economies with similar characteristics.

The non-existence of a distributive equilibrium implies the presence of a "war of gifts" between two agents or more, that is, more formally, the existence of some distribution and circuit of agents such that the utility of each agent is locally strictly increasing in bilateral wealth transfers from himself to the subsequent other in the circuit (Theorem 2 below). Bilateral wars of gifts are occasionally discussed in the literature as cases of logical inconsistency of models of two-sided altruism [e.g. Abel (1987, Equation (9), p. 1041); or Stark (1993, Footnote 1, p. 1416)]. In the context of two-agent distributive social systems, there is a war of gifts if and only if $x_i^j > x_i^i$ whenever $i \neq j$, when the best feasible distributions for i and j, x^i and x^j, are unique [e.g. Mercier Ythier (1989, P.3.11, p. 103)]. Mercier Ythier (1993) gives an example of a bilateral war of gifts in a three-agent distributive social system (pp. 939–940). And the Cobb–Douglas social system of Figure 12, drawn from Mercier Ythier (1998a, Counterexample 1, p. 340), is the place of generalized, bilateral and trilateral wars of gifts: for instance, agent 1's (resp. 2's, resp. 3's) utility is locally increasing in wealth transfers from himself to agent 2 (resp. 3, resp. 1) in the neighborhood of equal distribution $e = (1/3, 1/3, 1/3)$.

These examples display interesting analogies with the phenomenon known as *potlatch* in anthropology, and conceptualized notably by Mauss in his celebrated *Essai sur le Don* (1924), from the ethnographical works of Boas (1897) and Malinowski (1922), under a comprehensive notion of competitive gift-exchange [see Godelier (1996), for a well documented account of this stream of anthropological literature]. The analogy is formal, not substantial, but it can serve as a starting point for an anthropological interpretation of the abstract social system of Section 3.1. The characteristic features of such abstract systems and of social practices of competitive gift-exchange such as the *potlatch* and the *kula* [e.g. Godelier (1996, 2000)] differ on three articulated aspects. The nature of transferable wealth first: market money wealth for individual consumption, versus symbolic objects for circulation in competitive gift-exchange. The extension of individual property rights on transferable wealth, second: unrestricted *jus utendi et abutendi*, versus the three obligations of giving, accepting, and returning gifts. The motives of gift-giving, third: benevolent correction of wealth inequality, versus competition for rank or fame. In short, the abstract social system conveys a representation of gift-giving as benevolent individual equalization of private wealth, which stands in sharp contrast to the competition for rank or fame that characterizes competitive gift-exchange [Mercier Ythier (2000b, 2.3, pp. 100–101, 2004b, 4.3.3)].

3.4.1.2. Existence theorem The existence theorem presented below is drawn from Mercier Ythier (1993, Theorem 2, p. 941). It states essentially that non-existence im-

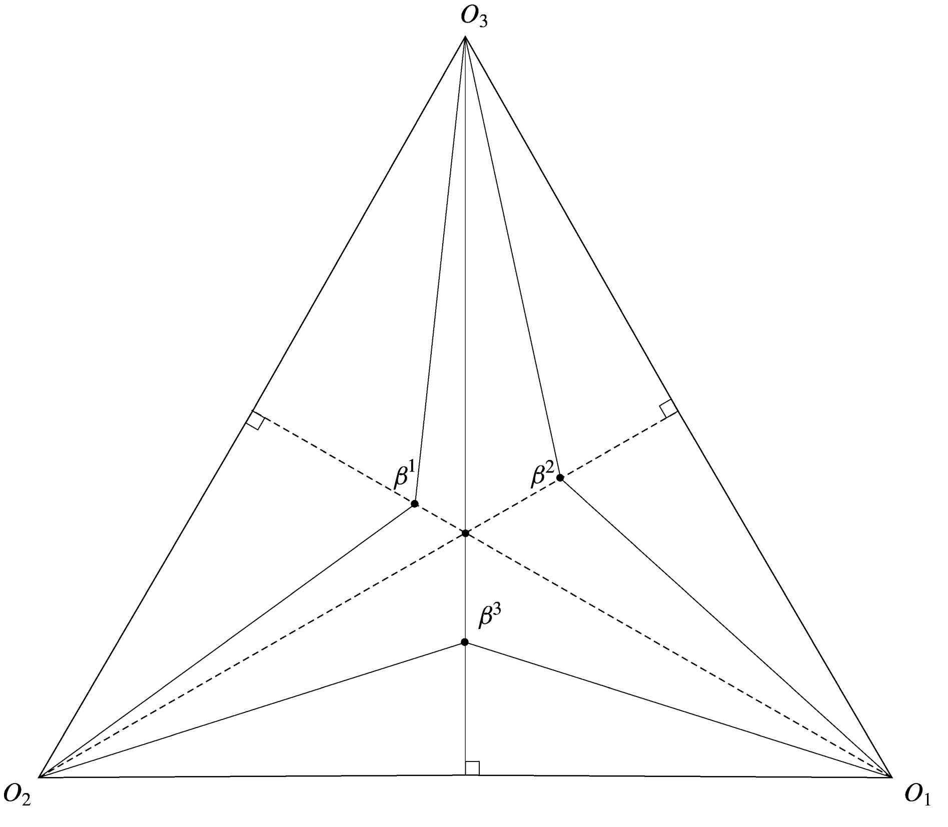

Figure 12. War of gifts.

plies the presence of a war of gifts. It implies (Corollary 2) the existence results of Arrow (1981, Theorem 5, p. 221) or Bergstrom, Blume and Varian (1986, Theorem 2, p. 33); see also Cornes, Hartley and Sandler (1999, Theorem, p. 505).

THEOREM 2. *Let w be twice differentiable and verify Assumptions 1(ii) and 1(iii). Then: if w has no equilibrium for some $\omega \in S_n$, there exists $x \in S_n$ such that the digraph $\{(i, j)\colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) > 0\}$ has a directed circuit (that is, contains a sequence $((i_k, j_k))_{1 \leqslant k \leqslant m}$ such that $m \geqslant 2$, $j_k = i_{k+1}$ for all $k = 1, \ldots, m-1$ and $j_m = i_1$).*

PROOF. Suppose that for all $x \in S_n$ the digraph $\{(i, j)\colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) > 0\}$ has no directed circuit. I want to prove that, then, (w, ω) has an equilibrium for all $\omega \in S_n$.

Let Φ_ω denote the correspondence $S_n \to S_n$ defined by: $\Phi_\omega(x) = \{x(\omega, t) \in \mathbb{R}^n_+\colon t_{ij} = 0$ whenever $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) < 0$; and $x_i(\omega, t) = 0$ whenever there exists j such that $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) > 0\}$. I first establish that a fixed point of Φ_ω is an equilibrium distribution of (w, ω), and next that Φ_ω has a fixed point.

Let x^* be a fixed point of Φ_ω. Then, by definition of Φ_ω, there exists t^* such that $x^* = x(\omega, t^*)$ and: for all i, either $x_i(\omega, t^*) = 0$ or $-\partial_{x_i} w_i(x(\omega, t^*)) + \partial_{x_j} w_i(x(\omega, t^*)) \leqslant 0$ for all j; for all (i, j), $(-\partial_{x_i} w_i(x(\omega, t^*)) + \partial_{x_j} w_i(x(\omega, t^*)))t^*_{ij} = 0$. But then t^*_i maximizes $w_i(x(\omega, (t^*_{\setminus i}, t_i)))$ in $\{t_i\colon x_i(\omega, (t^*_{\setminus i}, t_i)) \geqslant 0\}$ for all i by the assumptions on w and Arrow and Enthoven (1961: Theorem 1(c)), that is, t^* is a distributive equilibrium of (w, ω).

Correspondence Φ_ω is clearly compact- and convex-valued. By Kakutani's fixed point theorem, it is sufficient to prove that Φ_ω is: well-defined (that is, its values are non-empty) everywhere in S_n; and upper hemicontinuous. The first point is a simple consequence of the definition of Φ_ω and the assumption that the digraphs $\{(i, j)\colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) > 0\}$ have no directed circuit for all $x \in S_n$. And the second point follows straightforwardly from definitions and the continuity of the partial derivatives of utility functions. □

COROLLARY 2. *Let (w, ω) verify Assumption 1, with w twice differentiable, and suppose that: either w is a BBV social system; or w verifies the assumption of* weak self-centredness, *meaning that $-\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) \leqslant 0$ whenever $x_j \geqslant x_i$ (i's utility is non-increasing in wealth transfers from himself to j whenever j's consumption is at least as large as i's).*[23] *Then (w, ω) has an equilibrium.*

PROOF. Both assumptions of Corollary 2 readily imply that $\{(i, j)\colon -\partial_{x_i} w_i(x) + \partial_{x_j} w_i(x) > 0\}$ has no directed circuit for all $x \in S_n$. One applies then Theorem 2. □

3.4.2. Determinacy

A detailed formal discussion of the determinacy property of distributive equilibrium is beyond the scope of this chapter. We will provide instead a literary account of the main results of the analysis developed in Mercier Ythier (2004b) and recall the well-known property of uniqueness of the BBV equilibrium.

3.4.2.1. Generic determinacy of distributive equilibrium *Generic determinacy* is a property of regular distributive social systems. A distributive social system (w, ω) is *regular* if, essentially, the linear system tangent to the subsystem of first-order conditions of the type $-\partial_{x_i} w_i(x(\omega, t)) + \partial_{x_j} w_i(x(\omega, t)) = 0$ (with $i \neq j$) has full rank at equilibrium. Regularity is *generic* in $\{(w, \omega) \in W \times S_n\colon M_w \subset \mathbb{R}^n_{++}\}$, that is,

[23] Arrow's notion of self-centredness reads: $-\partial\varphi_i(x_i) + \partial\varphi(x_j) < 0$ whenever $x_i = x_j$. Combined with the concavity of φ_i and φ, it implies that $-\partial\varphi_i(x_i) + \partial\varphi(x_j) < 0$ whenever $x_j \geqslant x_j$, which is clearly stronger than the corresponding assumption of Corollary 2.

verified in an open and dense subset of the latter (Mercier Ythier, 2004b, Theorem 3). In other words, singularity (i.e. non-regularity) is *coincidental*: any linear perturbation of the preferences of a singular distributive social system will almost certainly restore regularity.

I establish the following three consequences of regularity: (i) there is a finite number of equilibria (op. cit: Theorem 5); (ii) status quo equilibrium is unique (op. cit.: Theorem 7); (iii) and the digraph of equilibrium gifts is a forest, that is, has no circuit[24] (op. cit.: Theorem 4).

The finiteness of the equilibrium set is certainly the most familiar, almost trivial implication of regularity. It is the exact analogue of the finiteness of the equilibrium set of finite regular competitive economies established in Debreu (1970).

The second point might appear more intriguing, although the analogous property of uniqueness of autarkic equilibrium is verified by finite regular competitive economies also. This fact usually receives only little attention in the theory of competitive exchange and production, for the simple reason that autarkic equilibrium presents little theoretical and practical interest as a situation of market equilibrium. Distributive equilibrium appears very different from market equilibrium in this respect, because of the public good problem, and particularly the type of inefficiency of equilibrium, encountered in many interesting theoretical cases, that is characterized by insufficient redistribution (see the account of the models of Arrow and BBV above, or Section 6 below). In other words, in many situations of theoretical interest, notably from the viewpoints of normative analysis and policy design, efficient distributive equilibria are status quo equilibria.

The third aspect of determinacy has no equivalent in the theory of competitive market equilibrium. It means, equivalently, that equilibrium gift-vectors and equilibrium wealth distributions are in one-to-one correspondence in regular distributive systems. And it implies that reciprocity, corresponding formally to the presence of a *directed* circuit in the digraph of equilibrium transfers, can appear only by coincidence in the distributive social systems of Section 3.1.

3.4.2.2. Uniqueness of BBV equilibrium We have mentioned already the uniqueness of Arrow's distributive equilibrium. We now recall below a similar property of BBV equilibrium when the aggregate wealth of the poor is the public good.

Let $w_i(x) = \nu_i(x_i, x_{m+1} + \cdots + x_n)$ be the utility function of non-poor i, $i = 1, \ldots, m$, as in Section 3.3.3 above. For any rich individual i, let $g_i = \sum_{k>m} t_{ik}$ denote the sum of his charitable contributions to the poor, and $G_{-i} = \sum_{j \leqslant m:\ j \neq i} g_j$ the aggregate charitable contribution of the other rich. Let $G = \sum_{i \leqslant m} g_i$ be the aggregate charitable contributions of the rich. If the utility functions ν_i of the rich (see

[24] A *circuit* of $N \times N$ is a sequence $((i_k, j_k))_{1 \leqslant k \leqslant m}$ of pairs of agents (darts) such that, $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ is non-empty for all k (that is, darts (i_k, j_k) and (i_{k+1}, j_{k+1}) have at least one common vertex for all k), with $(i_{m+1}, j_{m+1}) = (i_1, j_1)$ by convention. The circuit $((i_k, j_k))_{1 \leqslant k \leqslant m}$ is *directed* if $j_k = i_{k+1}$ for all k, that is, if the head-vertex j_k of dart (i_k, j_k) coincides with the tail-vertex i_{k+1} of dart (i_{k+1}, j_{k+1}) for all k. See also Footnote 45.

Section 3.3.3) are strictly quasi-concave, there exists, for all $i \leqslant m$, a function f_i that solves $\max\{v_i(x_i, \omega_{m+1} + \cdots + \omega_n + G)\colon x_i + G = \omega_i + G_{-i}\}$ with respect to G for any positive value of i's social income $r = \omega_i + G_{-i}$. $f_i(r)$ is i's demand for the public good when his endowment and the contributions of others to the public good add up to r, ignoring the non-negativity constraint on his own contributions. The gift-giving behavior of rich i is then described by the reaction function ρ_i such that $\rho_i(G_{-i}) = \max\{0, f_i(\omega_i + G_{-i}) - G_{-i}\}$. And we have the following property, that synthesizes two independent results of Cornes, Hartley and Sandler [(1999, Theorem, p. 505); see also the former, less general versions of Bergstrom, Blume and Varian (1986, Theorem 3, p. 33), Fraser (1992), and Bergstrom, Blume and Varian (1992)] and Shitovitz and Spiegel (2001, p. 221, §3):

THEOREM 3. *Suppose that v_i is monotonic strictly increasing, and strictly quasi-concave for all $i \leqslant m$, and that the social system verifies one of the following two normality conditions:* (i) *either, for all $i \leqslant m$, there exists a real number $\alpha_i < 1$ such that, for all G'_{-I} and G''_{-I} satisfying $0 \leqslant G''_{-I} < G'_{-I} \leqslant \sum_{j \leqslant m\colon j \neq i} \omega_j$, $0 \leqslant \rho_i(G''_{-I}) - \rho_i(G'_{-I}) \leqslant \alpha_i(G'_{-I} - G''_{-I})$ (normality);* (ii) *or, for all $i \leqslant m$, v_i is C^2, and $\partial_{x_i}(\partial_{x_i} v_i(x_i, y)/\partial_y v_i(x_i, y)) < 0$ and $\partial_y(\partial_{x_i} v_i(x_i, y)/\partial_y v_i(x_i, y)) > 0$ (ordinal normality). Then, there is a unique equilibrium vector $(g_1, \ldots, g_m)$.*

The first normality condition of Theorem 3, due to Cornes et al., is satisfied, in particular, whenever own wealth and the aggregate wealth of the poor are both normal goods for the rich, that is, equivalently, supposing the differentiability of f_i, whenever $0 < \partial f_i(r) < 1$ for all $r > 0$ and all $i \leqslant m$.

The second normality condition (ordinal normality), due to Shitovitz and Spiegel, is essentially equivalent to the (strict) gross substitutability of private goods and the public good at Lindahl prices [see the reference to Gaube (2001) in Section 6.2.3 below]. It states that an individual's marginal rate of substitution between his consumption of the private good and his consumption of the public good, that is, his relative (shadow Lindahl) price of private versus public consumption, is decreasing (resp. increasing) in is his private (resp. public) consumption.

We will omit proofs, and comment instead on the empirical relevance of the first normality assumption, which is by far the most commonly made in the literature. This question is addressed, notably, by Becker (1974, 1981a), whose Rotten Kid Theorem supposes that the wealth of the beneficiary (that is, the public good) is a normal good to the family head, and who argues, on theoretical grounds, that the income elasticity of gift-giving is likely to be positive (1981a, pp. 178–179), but also that it could be larger than 1 in this context of microsocial altruistic redistribution (1974, p. 1072), an empirical conjecture that is at variance with the normality assumption above.

Empirical findings on donors' income elasticities of *inter vivos* transfers were invariably found to be positive, and generally found to be below unity. Altonji, Hayashi and Kotlikoff (1997), for instance, find a 0.05 income elasticity of *inter vivos parental* transfers to children. And most of the 16 estimates of income elasticities of *charitable* giving reviewed in Foster et al. (2000, 6.2, pp. 125–129, notably Table 6.1) and in Schokkaert's Chapter 2 in this volume (Table 1), though much larger than the latter, are below unity also, with a typical value of 0.8 [notable exceptions are Taussig (1967) and Reece and Zieschang (1985)]. The estimates of elasticities of *bequests* relative to parental life resources reviewed in Chapter 14 of Arrondel and Masson in this Handbook, on the contrary, although fairly scattered (ranging from 0.5 to 2.9), are in the majority larger than 1, and in fact much larger than unity for the top quintile of permanent incomes [Menchik and David (1983), Arrondel and Laferrère (1991), and Arrondel and Masson (1991)].The idea that gift-giving is a normal good for donors is therefore supported by the data without ambiguity. The evidence relative to the nature of luxury good of gift-giving, on the contrary, is mixed: the assumption is clearly rejected in the context of inter vivos family gift-giving on the one hand; but, on the other hand, the average income elasticities obtained for charitable gift-giving are often close to 1, usually somewhat lower but sometimes significantly higher, suggesting that this type of transfer could be in fact a luxury good for a significant subsample (top quintile?) of the set of donors; and bequest definitely appears as a luxury good for the top quintile of permanent incomes.

4. Perfectly substitutable transfers in a competitive market economy

We now turn to abstract social systems that involve the simultaneous, non-strategic and non-cooperative interaction of altruistic gift and egoistic or "non-tuistic" [Wicksteed (1910)] competitive market exchange.

The analytical distinction between the motives of human action in market exchange and in other dimensions of social life, at least its conscious and systematic elaboration by Adam Smith in the two major works that span his intellectual life, the *Theory of Moral Sentiments* (1759) and the *Wealth of Nations* (1776), can be viewed as the point of departure for the development of economics as an autonomous social science. Since Smith's work, the difference and potential or actual contradiction between the narrowly self-regarding intentions driving individual market behavior, and other-regarding motives driving individual action in many other circumstances of life (beginning with family life) has been often noted, questioned and criticized as hypothesis and fact, inside economic theory as well as from outside [see for instance the famous "conclusions of morals" of Mauss (1924, Chapter IV), where he expresses his regret of the absence, in modern market exchanges, of the warmth and generosity of potlatch exchanges].

The theoretical constructs reviewed in this section build on the solutions to these questions elaborated by economic theory, and notably those formulated by Edgeworth

(1881), Pareto (1913, 1916) and Wicksteed (1910).[25] Let us examine them briefly, with some of their modern extensions.

Edgeworth emphasizes the abstractness of the representation of human behavior in economic science. Economic theory, at least the hardcore of it, retains from actual human behavior only what is strictly necessary for the understanding of its object, namely, of the determination of market prices and exchanges. This abstract representation of man is characterized as "unsympathetic isolation" (1881, p. 12) rather than substantive egoism. The existence of moral sentiments is actually recognized as a pervasive social fact ["the concrete nineteenth century man is for the most part an impure egoist, a mixed utilitarian": (1881, p. 104)]. To define economic man by abstracting away moral sentiments simply means that the corresponding theory of market exchange *can* dispense with these aspects of human reality (an observation that by no means implies the prescription that it *must* dispense with them).

Pareto (1913, 1916, Chapter 12, notably §2111-38) fits the economic man into the social man, the economic equilibrium into the social equilibrium, and the economic optimum into the social optimum, in the manner of Russian dolls. He distinguishes two types of actions: logical actions, characterized as those actions which involve, both, the adequacy of means to ends, and the coincidence of the objective (that is, real, effective) ends of action with the subjective ends of the agent (his conscious intention when performing his action); and non-logical actions. The economic equilibrium is construed, in the main, as the outcome of a subclass of logical actions, namely, those individual actions that tend to the maximization of individual "ophelimity" defined as the satisfaction derived from individual consumption of market goods and services. The social equilibrium, as an outcome of individual and collective actions, is far from being determined only by logical actions: non-logical actions make up an essential part (e.g. 1916, Chapter 12, §2079). Pareto considers, nevertheless, a broader class of logical actions which is directly interesting for our purposes: the individual actions that tend to the maximization of "utility" defined as the individual satisfaction derived from own ophelimity, the ophelimities (1913) or utilities (1916, Chapter 12, §2115) of others, and other external effects from its membership of a social group. This second class of logical actions makes up a part of the general social equilibrium, that contains economic equilibrium, but is significantly larger than the latter notably because moral sentiments such as altruistic feelings can take place into it.

Wicksteed (1910) notices that the "unsympathetic isolation abstractly assumed in economics" (Edgeworth, op. cit.) can be attenuated considerably without altering the explanation of market exchange provided by economic theory. He observes that all that is required by the latter is "non-tuism", defined as the absence of concern of exchangers for the purposes of their partners in exchange. This minimal notion of self-centredness

[25] See also Alfred Marshall's thoughts about the characteristics of individual behavior in modern industrial life, in his *Principles of Economics* (1890), notably §4 in the first chapter of Book I: "It is deliberateness, and not selfishness, that is the characteristic of the modern age".

of traders, strictly limited to the way they conduct their market operations, is compatible with virtually any type of individual behavior outside market exchange, logical or not, selfish, altruistic or otherwise.

Wicksteed's flexible, close to tautological conception of economic man certainly remains the most perfect expression of the abstract representation of human behavior implied by economic theory. Related contributions, such as those of von Mises (1936) and Robbins (1932), have emphasized the individual and social efficiency of non-tuistic market behavior as an explanation of its pervasiveness as individual market behavior and as an explanation of the development of market exchange itself. Becker (1981b) is quite representative of this line of reasoning: he shows (III, p. 11) that a company altruistic to its consumers can generate a greater social surplus, greater profits for itself and greater utilities for its consumers by charging the market price and giving them cash gifts, than by pricing its products below the market price.[26] Kirzner (1990) develops the same type of argument from the perspective of market exchange viewed as a continuous process of learning, with a particular insistence on the role of purposeful non-tuistic behavior in promoting continuous improvements of the mutual awareness of traders. Kolm (1983, 1984, Part III) emphasizes the limits of these arguments: non-tuistic market behavior, as well as market exchange, are not necessarily efficient when information is imperfect or when it proves impossible to constrain agents to respect the rights of others; and the efficiency criterion does not take into account the societal preferences of individuals, and notably their preferences relating to the relative shares of market (non-tuistic) and non-market (e.g. altruistic) behavior in social equilibrium.

The sequel to this review briefly examines the literature on utility interdependence, the subsequent extensions of the fundamental theorems of welfare economics, and the extension of the distributive social system of Section 3 in order to include competitive market economies. It concludes with a brief examination of the transfer paradox.

4.1. Interdependent preferences

Pareto (1913, 1916) suggests two alternative notions of interdependent preferences: in one of them, individual utility depends on the utilities of others (1916, §2115), while in the other one it depends on the ophelimities of others (1913, 1916, §2128[1] and 2131[1]).

The two notions are conceptually distinct. Utility appears, at least a priori, as a primitive notion in the first type of approach, preferences being defined there on mixed objects that combine objective characteristics such as consumption of goods and services with subjective ones, the psychological states of others, reflected by their utility levels. In the second type of approach, on the contrary, interdependent utilities consist solely of preferences on the allocation of resources, that is, on the vector of individual consumption of goods and services. Nevertheless, the first approach reduces to the second one when suitable assumptions are made [see for instance, among many: Becker (1974, Foot-

[26] This also is a favorite topic of Maurice Allais.

note 30, pp. 1080–1081); Bergstrom (1970, 1989a, 1999); and Kolm (1968, 2(A)–(F), 1984, Footnote 2, pp. 316–317, or 2000, Chapter 1, 5.2.4)].

4.1.1. Interdependence of primitive utilities

We will assume throughout Section 4 that there are l consumption goods and services, denoted by an index h running in $L = \{1, \ldots, l\}$. The consumption x_i of individual i is reinterpreted as a vector $(x_{i1}, \ldots, x_{il})$ of quantities of his consumption of these goods. x denotes accordingly the allocation $(x_1, \ldots, x_n)$.

Let $\widehat{U}_i$ denote a utility level of individual i, $\widehat{U}$ a utility vector $(\widehat{U}_1, \ldots, \widehat{U}_n)$. A system of interdependent preferences, with utility levels as primitive objects of preferences, consists then of n utility functions of the type $U_i(x_i, \widehat{U})$ that verify the consistency requirement that $\widehat{U} = (U_1(x_1, \widehat{U}), \ldots, U_n(x_n, \widehat{U}))$ for all $(x, \widehat{U})$ of the domain of the product function $U = (U_1, \ldots, U_n)$.

A straightforward application of the implicit function theorem to the functional equation $\widehat{U} = U(x, \widehat{U})$ yields the local existence and uniqueness of a function $\varphi : x \to \widehat{U}$, solving the latter in the neighborhood of any $(x^0, \widehat{U}^0)$ such that $\widehat{U}^0 = U(x^0, \widehat{U}^0)$, provided that U is continuously differentiable on an open domain containing $(x^0, \widehat{U}^0)$ and $\mathrm{I} - \partial_{\widehat{U}} U(x^0, \widehat{U}^0)$ has full rank (where I denotes the identity function of $\mathbb{R}^n$). In other words, a system of smooth utility functions "usually" (that is, generically) induces local systems of individual preferences defined solely on allocations (the local functions $x \to \varphi(x)$).

A special case of singularity of $\mathrm{I} - \partial_{\widehat{U}} U(x, \widehat{U})$, and a special case of pathology of function φ have received some attention in the literature. They describe situations where individuals are so benevolent to each other that any reasonable connection between utility vectors and allocations is lost, either because there is no function φ (singular $\mathrm{I} - \partial_{\widehat{U}} U(x, \widehat{U})$) or because φ is decreasing in all of its arguments: such individuals live, literally, of love and fresh water. Bergstrom (1989a) gives a nice humorous exposition of these paradoxes through the puzzles of Romeo and Juliet grappling with arbitrages between love and (individual consumption of) spaghetti, and in particular: difficulties disentangling love from spaghetti (non-existence of a function φ); and the conclusion that "true lovers hate spaghetti" (a decreasing function φ). Note that these problems are conceptually distinct from the wars of gifts discussed in Section 3.4.1.1 above: the former raise the question of the existence of non-pathological systems of individual preferences on the allocation of resources, while the latter refer to mutually incompatible acts of redistribution derived from well-defined and well-behaved (increasing and convex, notably) individual preferences on the distribution of wealth.

Utility levels and functions, therefore, are not only primitive notions in this version of the interdependence of preferences. They turn out also to be irreducible to preferences on allocations in the presence of singularities of $\mathrm{I} - \partial_{\widehat{U}} U(x, \widehat{U})$. Most applications, nevertheless, introduce assumptions that rule out this special case as well as monotonic decreasing preferences on allocations. Bergstrom (1999) provides an extensive discussion of the case where individual utility is weakly separable in own consumption (there is an individual "ophelimity"), increasing in own ophelimity, and non-decreasing in the utilities of others (non-malevolence). Formally, individual i's utility function is of the type $U_i^*(u_i(x_i), \widehat{U})$, increasing in its first argument and non-decreasing in the other ones. He shows (op. cit.: Proposition 3) that such systems of interdependent preferences are reducible to non-malevolent preferences of the type $\varphi_i^*(u_1(x_1), \ldots, u_n(x_n))$, defined on the ophelimity vectors associated with allocations, whenever $\mathrm{I} - \partial_{\widehat{U}} U^*(u(x), \widehat{U})$ is dominant diagonal for all $(x, \widehat{U})$ such that $\widehat{U} = \varphi^*(x)$ (with the following notations: $u(x) = (u_1(x_1), \ldots, u_n(x_n))$; $U^*(u(x), \widehat{U}) = (U_1^*(u_1(x_1), \widehat{U}), \ldots, U_n^*(u_n(x_n), \widehat{U}))$, and $\varphi^*(x) = (\varphi_1^*(x), \ldots, \varphi_n^*(x)))$. The condition that $\mathrm{I} - \partial_{\widehat{U}} U^*(u(x), \widehat{U})$ is dominant diagonal is logically equivalent to the non-singularity of the matrix if, as follows from non-malevolence, $\partial_{\widehat{U}} U^*(u(x), \widehat{U})$ is $\geqslant 0$; its inverse is then the non-negative sum of a geometric series $\sum_{t=0}^{\infty} (\partial_{\widehat{U}} U^*(u(x), \widehat{U}))^t$ (op. cit.: Lemma 1). These results extend to denumerable sets of agents, and apply therefore to the systems of interdependent preferences considered in the literature on intergenerational altruism initiated by Barro (1974) [see notably Kimball (1987), Hori and Kanaya (1989), and Hori (1992)]. In short, benevolent preferences weakly separable in own consumption reduce to well-defined and well-behaved preferences on ophelimity vectors provided that mutual benevolence is not so intense that it implies the divergence of $\sum_{t=0}^{\infty} (\partial_{\widehat{U}} U^*(u(x), \widehat{U}))^t$.

Kolm (1968, 2(F)) states the same type of condition in the language of marginal surplus theory. Let v_{ij} denote the money value to individual i of an additional dollar to individual j ($v_{ii} = 1$). The social value to individual i of an additional dollar to individual j is the sum of its direct individual valuation by i and indirect valuation through i's social valuations of others' individual valuations of the additional dollar to j, that is: $s_{ij} = \delta_{ij} + \sum_{k \neq i} v_{ik} s_{kj}$, where δ_{ij} is the number of Kronecker ($= 1$ if $i = j$, $= 0$ otherwise). Letting S and V denote respectively the n-dimensional matrices (s_{ij}) and (v_{ij}), we have therefore by definition $S = \mathrm{I} + VS$. This system of interdependent individual social values is well-defined if and only if $\mathrm{I} - V$ is non-singular, and we have then $S = (\mathrm{I} - V)^{-1} = \sum_{t=0}^{\infty} V^t$. In other words, individual social valuations are well-defined, and then reducible to combinations of direct individual valuations, provided, again, that mutual benevolence is not so intense that it implies the divergence of $\sum_{t=0}^{\infty} V^t$.

4.1.2. *Interdependent preferences on allocations and the fundamental theorems of welfare economics*

The alternative approach to the interdependence of preferences considers individual preferences defined directly on allocations. It can be traced back to Pareto (1913, 1916), and was maintained in a French tradition of economists notably by Divisia, and reintroduced in contemporary normative economic theory by Kolm (1968) extending the tradition above, and, independently, by Winter (1969).

This approach distinguishes two types or "levels" of individual preferences: "private" preferences, defined on the private consumption of market goods and services of the individual (Pareto's "ophelimities"); and "social" preferences, defined on allocations

(Pareto's "utilities"). It is frequently assumed, moreover, as a condition of individual integrity, that individual social preferences are weakly separable in own consumption and that the unique preference preordering that they induce on individual consumption coincides with his private preferences.

One defines accordingly, following Pareto (1913, 1916), two notions of allocative efficiency: market efficiency, which is Pareto-efficiency relative to the private preferences of individuals [Pareto's "maximum of ophelimity for a collectivity" (1916, Chapter 12, §2128, p. 1338)]; and distributive efficiency, which is Pareto-efficiency relative to the social preferences of individuals (Pareto's "maximum of utility for a collectivity": ibid, §2131, pp. 1341–1342).[27]

The simplest framework for a precise general formulation of essential ideas is the competitive exchange economy with free disposal. We make therefore, and maintain in the remainder of Section 4, the following assumptions: (i) the total quantity of each good available for individual consumption is given once and for all (exchange economy) and equal to 1 (this is a simple choice of unit of measurement of physical quantities); (ii) an allocation x is feasible if x_i is in the consumption set X_i of consumer i for all i and $\sum_{i \in N} x_{ih} \leqslant 1$ for all h (this definition of feasibility implies free disposal and the perfect divisibility of physical quantities of goods and services). Note that the definitions and properties below (Sections 4.1.2 and 4.2) extend in a straightforward way to full-fledged Walrasian economies with profit-maximizing firms and standard (notably convex) technology.

Denote by: X the Cartesian product $\prod_{i \in N} X_i$; F the set $\{x \in X\colon \sum_{i \in N} x_{ih} \leqslant 1$ for all $h\}$ of feasible allocations of the economy; $u_i : \mathbb{R}^l \to \mathbb{R}$ the ophelimity function of individual i; u the function $\mathbb{R}^{ln} \to \mathbb{R}^n$ defined by $u(x) = (u_1(x_1), \ldots, u_n(x_n))$; X^i the Cartesian product $\mathbb{R}^l \times \cdots \times \mathbb{R}^l \times u_i(\mathbb{R}^l) \times \mathbb{R}^l \times \cdots \times \mathbb{R}^l$; $W_i : X^i \to \mathbb{R}$ the utility function of individual i, supposed strictly increasing in its ith argument (that is, in i's own ophelimity). A social system is then a list $((W_1, u_1), \ldots, (W_n, u_n))$, and the notions of market and distributive efficiency receive the following precise definitions:

DEFINITION 2. An allocation x is a strong market optimum (resp. strong distributive optimum) of the social system $((W_1, u_1), \ldots, (W_n, u_n))$ if it is feasible and if there exists no feasible allocation x' such that $u_i(x'_i) \geqslant u_i(x_i)$ (resp. $W_i(x'_1, \ldots, x'_{i-1}, u_i(x'_i), x'_{i+1}, \ldots, x'_n) \geqslant W_i(x_1, \ldots, x_{i-1}, u_i(x_i), x_{i+1}, \ldots, x_n)$) for all i, with a strict inequality for at least one i.

This formulation of utility interdependence leads in a natural way to questions on the possibility of extending the first and second fundamental theorems of welfare economics from market to distributive optima, that is, more precisely, to questions of the distributive efficiency of competitive market equilibrium on the one hand, and of the existence of systems of market prices supporting distributive optima on the other hand.

[27] For a justification of this terminology, see Section 2.2 (notably Footnote 6).

In view of the first and second theorems of welfare economics themselves [e.g. Debreu, (1954, Theorems 1 and 2)], the issue reduces essentially to the identification of properties of utility functions implying that any market optimum is a distributive optimum (extension of the first theorem) and that any distributive optimum is a market optimum (extension of the second theorem).

The second question received a positive answer for a broad class of systems of interdependent utilities combining two features: non-paternalism, first, which appears by far as the main condition for the extension of the second welfare theorem to distributive optima, and is construed as the respect or endorsement by all individual social preferences of the preferences of others on their own consumption of market goods; and a restriction on malevolence, ensuring that there is always some way of reallocating resources that is preferred to disposal (social non-satiation relative to individual consumption of market goods).

Such properties were first introduced by Winter (1969) with an assumption of non-paternalistic non-malevolence of individual social preferences [Assumption b.3, p. 100; see also Bergstrom (1970, II-A, pp. 385–386)]. Expressed in terms of utility representations, Winter's assumption combines: the existence of functions $w_i : u(X) \to \mathbb{R}$ such that $w_i(u(x)) = W_i(x_1, \ldots, x_{i-1}, u_i(x_i), x_{i+1}, \ldots, x_n)$ for all x; and w_i non-decreasing in j's ophelimity for all i and all $j \neq i$. It generalizes a similar assumption of Edgeworth (1881) and Pareto (1913), where individual utilities are additively separable and strictly increasing in ophelimities, that is, with present notations, where w_i is of the type $\sum_{j \in N} a_{ij} u_j$ with $a_{ij} > 0$ for all i and all j. Non-paternalistic non-malevolence straightforwardly implies that any distributive optimum is a market optimum, and therefore, under classical conditions, that it is attainable as a competitive market equilibrium.

Archibald and Donaldson (1976) and Rader (1980) relax the original assumption of Winter by allowing for malevolence. They simply suppose the existence of the functions w_i above,[28] and prove, essentially, that Winter's result extends to their more general systems of non-paternalistic interdependent utilities, provided that mutual malevolence is not so intense that it induces the disposal of a part of aggregate resources of society at some distributive optima. Rader's main result (1980, Theorem 2, p. 423), which is slightly more general than Archibald and Donaldson's, states precisely that: if the so-

[28] Their assumption of utility interdependence is still more general in fact, for they do not assume, as we did above, that utility is increasing in own ophelimity. In other words, their assumption is compatible with an indifference or aversion of an individual to his own satisfaction as a consumer. This introduction of the possibility of a contradiction between individual views on own consumption as a consumer and as a member of society is interesting on logical grounds, notably as a progress in generality, but it does not appear very appealing on more substantive grounds, partly because of the systematic character of this opposition of views, suggesting a severe problem of personal integrity, and partly because of the object of this opposition, own consumption of market goods, which cannot plausibly give rise, as a whole at least, to such an internal debate. For an alternative critique of the same assumption, see Lemche (1986, Remark 1, pp. 272–274).

cial system verifies free disposal,[29] quasi-transferability[30] and local non-satiation of the distributive Pareto preordering,[31] then any distributive optimum is a market optimum.

Lemche (1986) further relaxes the assumptions of Archibald and Donaldson by giving up the weak separability of individual social preferences in own wealth.[32] His notion of non-paternalistic preferences is defined from the conditional preferences on own and on others' consumption sets induced by individual social preferences. It states that whenever an individual is indifferent between any pair of his consumption vectors conditional on some given vector of consumption of others, then all individuals are indifferent between the same pair conditional on the same vector. In order to facilitate the comparison with Rader's result, I will slightly rephrase Lemche's theorem (op. cit.: Theorem 1, p. 278), using the notion of conditional Pareto optimum of Arrow and Hahn[33] (1971, Chapters 6, 2, pp. 132–136) in place of Lemche's essentially equivalent notion of conditional competitive equilibrium. The theorem states that: if individual social preferences are non-paternalistic in the sense above, if the social system verifies Archibald and Donaldson's version of the local non-satiation of the social Pareto preordering,[34] and if conditional individual preferences on own consumption are strictly increasing, quasi-concave and differentiable, then any distributive optimum is a conditional Pareto optimum (op. cit.).

A casual examination of the converse problem relative to the possibility of extending the first theorem of welfare to distributive optima shows that this supposes both non-paternalism and non-benevolence. Even mild benevolence, in particular, will often suffice to exclude from the set of distributive optima the market equilibria that imply situations of extreme poverty for some [see for instance Winter (1969, 5, p. 102)]. Parks (1991) shows that the first theorem extends to the case of non-paternalistic non-benevolence, provided again that malevolence remains limited, although in a different sense than the local non-satiation of the social Pareto preordering. He assumes utility functions of the type $w_i : u(X) \to \mathbb{R}$, strictly increasing in own ophelimity and non-increasing in the ophelimities of others, and shows, essentially, that any market optimum is a distributive optimum whenever the Jacobian matrices $\partial w(\hat{u})$ of the product function $w = (w_1, \dots, w_n)$ have non-negative inverses (which supposes that the off-diagonal elements of $\partial w(\hat{u})$, that is, the marginal utilities of others' ophelimities, are not "too" negative).

[29] That is: $(\{\hat{u}\} - \mathbb{R}^n_+) \cap u(X) \subset u(F)$ for all $\hat{u} \in u(X)$. This notion of free disposal is equivalent to the notion implicit in the definition of the set of feasible allocations as $\{x \in X\colon \sum_{i\in N} x_{ih} \leqslant 1 \text{ for all } h\}$ whenever ophelimity functions are continuous monotonic increasing and consumption sets are equal to $\mathbb{R}^l_+$.

[30] That is: for all $\hat{u}$ and $\hat{u}'$ in $u(X)$ such that $\hat{u} > \hat{u}'$, there exists $\hat{u}'' \in u(X)$ such that $\hat{u}'' \gg \hat{u}'$.

[31] That is: for all $\hat{u} \in u(X)$ and all neighborhood V of $\hat{u}$, there exists $\hat{u}' \in V$ such that $w(\hat{u}') > w(\hat{u})$.

[32] More precisely, Lemche's notion of non-paternalism implies, and is not implied by, Archibald and Donaldson's notion complemented with the assumption that an individual's utility is strictly increasing in his own ophelimity.

[33] A conditional Pareto optimum is, in our context, a Pareto optimum relative to the conditional preferences of individuals on their own consumption.

[34] Stronger than Rader's (cf. Footnote 31 above) in general, but equivalent to it when ophelimity functions are monotonic strictly increasing.

4.2. General equilibrium with benevolent gift-giving and competitive market exchange

The social system and social equilibrium of Section 3.1 are now extended in order to include competitive market exchange (Section 4.2.1). The corresponding functioning involves the non-cooperative and non-strategic interaction of utility-maximizing individual gifts and ophelimity-maximizing exchanges on competitive markets, of individuals endowed with non-paternalistic interdependent preferences. We name Pareto social system this extension of the pure distributive social system, by reference to Pareto (1916).

It is shown (Section 4.2.2) that the market sub-equilibria of Pareto social systems are competitive equilibria. The first fundamental theorem of welfare economics extends, consequently, to social equilibrium, that is, the equilibrium allocation is Pareto-efficient relative to individual ophelimities (market efficiency). And the characterization and the existence property of the social equilibrium of pure distributive social systems then extend in a natural way to the social equilibrium of Pareto social systems (Sections 4.2.2 and 4.2.4 respectively).

We also establish (Section 4.2.3) the equivalence of in-kind and cash transfers, as a joint consequence of non-paternalism, perfect competitive market exchange and free disposal.

4.2.1. Social equilibrium

The setup of Section 3.1 is amended along the lines of Section 4.1.2 above and Mercier Ythier (1989, 2000a).

Agent i's initial endowment ω_i is now a non-negative element of the space of goods $\mathbb{R}^l$. We consider social systems of private property, where by definition the total endowment of society in all consumption goods is shared initially between its individual members, that is: $\sum_{i \in N} \omega_i = (1, \dots, 1)$. The vector $(\omega_1, \dots, \omega_n)$ of individual endowments is denoted by ω.

The agents can use commodities in three different ways: private consumption and individual gift-giving as in Section 3.1; and exchange on competitive markets.

A gift t_{ij} from i to j ($j \neq i$) is a non-negative element of $\mathbb{R}^l$, whose hth coordinate t_{ijh} is a non-negative quantity of consumption good h transferred from i to j. In other words, individuals are allowed to make both "cash" (numéraire) and in-kind transfers. The gift set of individual i is set $T_i = \mathbb{R}_+^{l(n-1)}$. The other notations of Section 3.1 relative to transfers are extended to the multi-commodity setting in the obvious way.

A *net trade* of agent i is a vector z_i of the space of commodities. Its hth coordinate z_{ih} is the net trade of agent i in good h, that is, the difference between his physical purchases and sales of commodity h. We denote by z a vector $(z_1, \dots, z_i, \dots, z_n)$ of individual net trades.

A *social state* is then a vector (x, t, z). Since individual uses of commodities are restricted to private consumption, gift-giving, and market exchange, a state (x, t, z) must verify the following physical accounting identities for all i: $x_i = z_i + \omega_i + \Delta_i t$, equating consumption to net physical inflows from trade, gift-giving and initial endowment, for all individuals and commodities.

An *action* of individual i, denoted by a_i, is a pair (z_i, t_i). An *action vector* is then a vector $a = (a_1, \dots, a_i, \dots, a_n)$ of individual actions. For all action vector a and all individual action a_i^*, we denote, as above, by: $a_{\setminus i}$ the vector of individual actions obtained from a by deleting its ith component a_i; $(a_{\setminus i}, a_i^*)$ the action vector obtained from a by replacing its ith component a_i by a_i^*. We suppose that every agent considers the actions of others as independent of his own (Cournot–Nash behavioral assumption). It follows from this and the accounting identities above that, given some $a_{\setminus i}$, the choice by agent i of some action $a_i^* = (z_i^*, t_i^*)$ determines the realization of one and only one allocation, namely allocation $x((a_{\setminus i}, a_i^*))$ whose jth component is $z_j + \omega_j + \Delta_j(t_{\setminus i}, t_i^*)$ for all j. We also suppose that every agent perceives market prices as independent from his individual actions (competitive markets). The vector of market prices is denoted by p. The unique social state determined by action vector a is denoted by $(x(a), t(a), z(a))$. The unique action vector associated with (x, t) is $((x_1 - \omega_1 - \Delta_1 t, t_1), \dots, (x_n - \omega_n - \Delta_n t_n, t_n))$, denoted by $a(x, t)$.

Individuals have interdependent preferences on the allocation of resources that are non-paternalistic in the sense of Archibald and Donaldson (1976), cf. Section 4.1.2 above. We suppose moreover that an individual's utility is strictly increasing in his own ophelimity (cf. Footnote 28). We let, without loss of generality, $u_i(0) = 0$ for all i.

The picture concerning individual behavior is, at this point, the following: each agent chooses his gifts and net trades in order to achieve some allocation of resources according to his non-paternalistic preferences.

We can now complete this description of individual behavior with a specification of the constraints binding individual choices. Consider some price-action vector (p^*, a^*), defining an environment for individual decisions. Individual i will choose his action in the *budget set* $B_i(p^*, a^*) = \{a_i = (z_i, t_i) \in \mathbb{R}^l \times T_i \colon x_i((a^*_{\setminus i}, a_i)) \in \mathbb{R}^l_+$ and $p^* z_i \leqslant 0\}$, in order to maximize his utility according to the program: $\max\{w_i(u(x((a^*_{\setminus i}, a_i)))) \colon a_i \in B_i(p^*, a^*)\}$.

An *extended distributive social system* is a pair of n-tuples of utility and ophelimity functions $((w_1, \dots, w_n), (u_1, \dots, u_n))$, denoted by (w, u). We name an abstract social system of this type a *social system of Pareto*, by reference to the Chapter 12 of his *Traité de Sociologie Générale* (1916). A *Pareto social system of private property* is a triple (w, u, ω). A *Pareto–BBV social system*, likewise, is a pair (w, u) such that society is partitioned into a subset of "egoistic" poor $\{m + 1, \dots, n\}$ with utility functions of the type $w_i(\hat{u}) = \hat{u}_i$ and a complementary subset of non-poor $\{1, \dots, m\}$ whose preferences are of the type $w_i(\hat{u}) = \mu_i(\hat{u}_i, \hat{u}_{m+1}, \dots, \hat{u}_n)$, where μ_i is monotonic strictly increasing (non-paternalistic benevolence to the poor).

DEFINITION 3. A social equilibrium of (w, u, ω) is a price-action vector (p^*, a^*) such that: (i) $\sum_{i \in N} z_i^* \leqslant 0$ and $p^* \sum_{i \in N} z_i^* = 0$ (market equilibrium with free disposal); (ii) and a_i^* solves $\max\{w_i(u(x((a^*_{\setminus i}, a_i)))) \colon a_i \in B_i(p^*, a^*)\}$ for all i (everyone is satisfied with his own choice, given prices and the actions of others).

One verifies easily that this definition implies the definition of a pure distributive equilibrium (Section 3.1.2.1, Definition 1) when there is a single commodity (just let, then: $p = 1$, $z = 0$ and u_i be the identity map $\mathbb{R} \to \mathbb{R}$ for all i).

4.2.2. *First-order conditions*

We will consider differentiable social systems, as in Section 3. The following assumptions on preferences and endowments will be maintained throughout the remainder of Section 4:

ASSUMPTION 2.[35] (i) For all i, $X_i = \mathbb{R}^l_+$ and u_i is: (a) continuous in $\mathbb{R}^l_+$, and differentiable in $\mathbb{R}^l_{++}$ (the interior of $\mathbb{R}^l_+$); (b) monotonic strictly increasing in $\mathbb{R}^l_{++}$ (i.e. $u_i(x_i) > u_i(x'_i)$ for all $(x_i, x'_i) \in \mathbb{R}^l_{++} \times \mathbb{R}^l_{++}$ such that $x_i > x'_i$); (c) and such that $x_i \gg 0$ whenever $u_i(x_i) > 0 (= u_i(0))$. (ii) For all i, w_i is: (a) continuous in $\mathbb{R}^n_+$, and differentiable with respect to its jth argument in $\{\hat{u} \in \mathbb{R}^n_+ \colon \hat{u}_j > 0\}$ for all j; (b) strictly increasing in its ith argument. (iii) For all i, $w_i \circ u$ is: (a) quasi-concave; (b) and such that $w_i(u(x)) = 0$ whenever $u_i(x_i) = 0$. (iv) For all i, $\omega_i > 0$.

The following theorem extends Theorem 1 to the equilibria of Pareto social systems. It provides an analogous system of necessary and sufficient conditions for equilibrium.

Its conditions (ii) and (iii) state that *the price system and the allocation of resources induced by a social equilibrium* (p, a) *of* (w, u, ω) *make a competitive market equilibrium of the induced exchange economy of private property* $(u, (\omega_i + \Delta_i t(a))_{i \in N})$ (see also Footnote 39 below).

In particular, the multipliers λ_i correspond to the marginal ophelimities of wealth of the consumers. Its condition (iv) therefore means the following: *at equilibrium, a marginal incremental wealth transfer from i to j does not increase i's utility* ((iv)(a)), *and a marginal incremental wealth transfer from j to i does not increase i's utility whenever the equilibrium transfer from i to j is positive* ((iv)(b)).

[35] Assumptions 2(i)(b) and 2(i)(c) are commonly used in the study of differentiable economies. Together with Assumption 2(ii)(b), Assumption 2(i)(b) implies that prices are positive at equilibrium, while Assumption 2(i)(c) implies that an agent whose post-transfer wealth is positive will consume a positive amount of all goods (thereby eliminating inessential technicalities associated with non-negativity constraints on consumption). Assumptions 2(iii) and 2(iv) ensure that individual behavioral correspondences have the relevant continuity property required for the existence of a social (hence competitive market) equilibrium. Assumptions 2(iii)(b) and 2(iv), together with Assumptions 2(i)(c) and 2(ii)(b), are designed to imply, notably, the seemingly reasonable consequence that every agent will wish and be able to keep a positive post-transfer wealth for all positive price vectors, which ensures in turn the continuity of budget correspondences on relevant domains. The convexity of preferences of Assumption 2(iii)(a) implies then the upper hemicontinuity of behavioral correspondences.

Theorem 4 implies the characterization of Theorem 1, with an interior equilibrium distribution, for pure distributive social systems that verify Assumptions 2(ii) to 2(iv) (just let u_i be the identity map $\mathbb{R} \to \mathbb{R}$ for all i).[36]

THEOREM 4. *Let* (w, u, ω) *verify Assumption 2. Then,* (p^*, a^*) *is a social equilibrium of* (w, u, ω) *if and only if it verifies the following set of conditions*: (i) $p^* \gg 0$; (ii) $\sum_{i \in N} x_i(a^*) = (1, \dots, 1)$; (iii) *for all* i: (a) $x_i(a^*) \gg 0$; (b) $p^* x_i(a^*) = p^*(\omega_i + \Delta_i t(a^*))$; (c) *and there exist* $\lambda_i > 0$ *such that* $\partial_{x_i} u_i(x_i(a^*)) = \lambda_i p^*$; (iv) *for all* (i, j): (a) $-\partial_{u_i} w_i(u(x(a^*)))\lambda_i + \partial_{u_j} w_i(u(x(a^*)))\lambda_j \leqslant 0$; (b) *and* $(-\partial_{u_i} w_i(u(x(a^*)))\lambda_i + \partial_{u_j} w_i(u(x(a^*)))\lambda_j) t_{ij}(a^*) = 0$.

PROOF. See Appendix A.1. □

The next corollary, likewise, extends to Pareto social systems the characterization of the range and inverse of the correspondence of equilibrium distributions of pure distributive social systems given in Corollary 1.

Let the set of market-efficient allocations of (w, u) be denoted by O. Note that, as a classical application of the second fundamental theorem of welfare economics to differentiable market economies, for any (w, u) that verifies Assumption 2, $x \gg 0$ is a market optimum if and only if $\sum_{i \in N} x_i = (1, \dots, 1)$ and there exists $(p, \lambda) \gg 0$ in $\mathbb{R}^l \times \mathbb{R}^n$ such that $\partial_{x_i} u_i(x_i) = \lambda_i p$ for all i [see for instance Mercier Ythier (2000a, Lemma 3, p. 60)]. The supporting vector (p, λ) of x is unique up to a positive multiplicative constant. We denote by $(p(x), \lambda(x))$ the unique supporting vector of x such that $p \in S_l$.

For any fixed (w, u), denote by: $X(\omega)$ the set $\{x\colon \exists t$ such that $(p, a(x, t))$ is a social equilibrium of $(w, u, \omega)\}$ of equilibrium allocations of a social system (w, u, ω); M the range of correspondence X; M' the range of the restriction X' of X to $\{\omega\colon \omega_i > 0$ for all $i\}$; Ω the inverse of X, that is, the correspondence defined by $\Omega(x) = \{\omega\colon x \in X(\omega)\}$ for all x in M; Ω' the inverse of X'. And for any fixed (w, u) that verifies Assumption 2 and any $x \in O \cap \mathbb{R}^{ln}_{++}$, denote by: $\gamma(x) = \{(i, j) \in N \times N\colon -\partial_{u_i} w_i(u(x))\lambda_i(x) + \partial_{u_j} w_i(u(x))\lambda_j(x) = 0\}$; $\Gamma(x)$ the incidence matrix of digraph $\gamma(x)$.

Corollary 3(i) states that *the range of the correspondence of equilibrium allocations is the subset of market-efficient allocations such that marginal incremental bilateral transfers evaluated at supporting market prices do not increase givers' utilities*. It implies, notably, that *the first fundamental theorem of welfare economics extends to Pareto social systems*, that is, competitive exchange still yields (market) efficiency in the allocation of resources in the context of such social systems. The corollary is a simple consequence of Theorem 4 and the remark above on the supportability of market optima.

[36] Assumptions 2(iii)(b) and 2(iv) imply that the equilibrium distribution is interior, which permits to dispense with the twice differentiability of utility functions in the proof of sufficiency of first-order conditions [Arrow and Enthoven (1961: Theorem 1(b))].

COROLLARY 3. *Let (w, u) verify Assumption 2. Then:* (i) $M' = \{x \in O \cap \mathbb{R}^{ln}_{++}\colon -\partial_{u_i} w_i(u(x))\lambda_i(x) + \partial_{u_j} w_i(u(x))\lambda_j(x) \leqslant 0$ *for all* $(i, j)\}$. (ii) *For all* $x \in M'$, $\Omega'(x)$ *is the convex set* $\{\omega\colon \omega_i > 0$ *for all* i*; and* $\exists t \geqslant 0$ *such that* $g(t) \subset \gamma(x)$ *and* $p(x)x_i = p(x)(\omega_i + \Delta_i t)$ *for all* $i\}$.

4.2.3. Equivalence of money transfers and in-kind transfers

A simple but important aspect of social equilibrium is the *equivalence of cash and in-kind transfers* for non-paternalistic individuals operating on the background of perfectly competitive markets. Non-paternalistic utility interdependence implies that gifts of commodities are driven only by, and perceived only through, their consequences on the distribution of wealth. And perfect competition with free disposal implies that any beneficiary of a gift in kind can sell it at non-negative market prices, and freely spend the proceeds, without bearing any transaction cost; that is, the set of alternatives of the beneficiary of a gift is influenced by the market value of the gift, and not by its physical characteristics per se.

To make these statements precise, let $v_i : (\mathbb{R}^l_+ \backslash\{0\}) \times \mathbb{R}_+ \to \mathbb{R}$ denote individual i's indirect ophelimity function, that is, $v_i(p, r_i) = \max\{u_i(x_i)\colon x_i \in X_i$ and $px_i \leqslant r_i\}$ for all price system $p > 0$ and all (post-transfer) wealth $r_i \geqslant 0$ of individual i. Function v_i is well-defined on $(\mathbb{R}^l_+ \backslash\{0\}) \times \mathbb{R}_+$ if u_i is a continuous function $\mathbb{R}^l_+ \to \mathbb{R}$. Let: τ_{ij} be a non-negative wealth transfer (money gift) from i to j; $\tau_i = (\tau_{ij})_{j \neq i}$ denote the corresponding vector of money gifts of individual i; $\tau = (\tau_1, \ldots, \tau_n)$ be the vector of money gifts in society; $\Delta_i \tau = \sum_{j\colon j \neq i}(\tau_{ji} - \tau_{ij})$ be the net transfer of wealth accruing to individual i when the gift vector is τ; and $(\tau^*_{\backslash i}, \tau_i)$ the gift vector obtained from τ^* and τ by substituting τ_i for τ^*_i in τ^*. The social equilibrium can receive then the following alternative definition, essentially equivalent to Definition 3:

DEFINITION 3$'$. A social equilibrium with money gifts of (w, u, ω) is a vector (p^*, x^*, τ^*) such that: (i) $\sum_{i \in N} x^*_i \leqslant (1, \ldots, 1)$ and $p^*((1, \ldots, 1) - \sum_{i \in N} x^*_i) = 0$; (ii) $x^*_i \in \{x_i \in X_i\colon p^* x^*_i \leqslant p^*\omega_i + \Delta_i \tau^*$ and $u_i(x_i) = v_i(p^*, p^*\omega_i + \Delta_i \tau^*)\}$ for all i; (iii) and τ^*_i solves $\max\{w_i(v_1(p^*, p^*\omega_1 + \Delta_1(\tau^*_{\backslash i}, \tau_i)), \ldots, v_n(p^*, p^*\omega_n + \Delta_n(\tau^*_{\backslash i}, \tau_i)))\colon \tau_i \geqslant 0$ and $p^*\omega_i + \Delta_i(\tau^*_{\backslash i}, \tau_i) \geqslant 0\}$ for all i.

THEOREM 5. *Let (w, u, ω) verify Assumption 2 and suppose moreover that v_i is differentiable in $\mathbb{R}^l_{++} \times \mathbb{R}_{++}$ for all i. Then, (p^*, a^*) is a social equilibrium of (w, u, ω) if and only if $(p^*, x(a^*), p^* t(a^*))$ is a social equilibrium with money gifts of (w, u, ω).*

PROOF. See Appendix A.1. □

4.2.4. Existence of a social equilibrium

Theorem 6 and Corollary 4 below extend Theorem 2 and Corollary 2 (Section 3.4.1.2) to the social equilibria with competitive market exchange. They imply the existence properties of Theorem 2 and Corollary 2 respectively, for pure distributive social systems that verify Assumptions 2(ii) to 2(iv). They imply, also, the existence of a competitive exchange equilibrium for the standard differentiable economies of Assumption 2(i) (just let w_i be the canonical projection $\mathbb{R}^n \to \mathbb{R} : \hat{u} \to \hat{u}_i$ for all i, that is, suppose that all individuals are egoistic).

THEOREM 6. *Let (w, u) verify Assumption 2, and suppose moreover that v_i is differentiable in $\mathbb{R}^l_{++} \times \mathbb{R}_{++}$ for all i. Then: if (w, u) has no equilibrium for some ω such that $\omega_i > 0$ for all i, there exists a market optimum x and a system of market prices p supporting x such that the digraph $\{(i, j): -\partial_{u_i} w_i(u(x))\partial_{r_i} v_i(p, px_i) + \partial_{u_j} w_i(u(x))\partial_{r_j} v_j(p, px_j) > 0\}$ has a directed circuit.*

COROLLARY 4. *Let (w, u, ω) verify Assumption 2, and suppose moreover that: v_i is differentiable in $\mathbb{R}^l_{++} \times \mathbb{R}_{++}$ for all i; and either (w, u) is a Pareto–BBV social system; or (w, u) verifies* extended weak self-centredness*, meaning that $-\partial_{u_i} w_i(v(p, r))\partial_{r_i} v_i(p, r_i) + \partial_{u_j} w_i(v(p, r))\partial_{r_j} v_j(p, r_j) \leqslant 0$ for all $(p, r) \in \mathbb{R}^l_{++} \times \mathbb{R}^n_{+}$ such that $r_j \geqslant r_i > 0$ (where $r = (r_1, \ldots, r_n)$ and $v(p, r) = (v_1(p, r_1), \ldots, v_n(p, r_n))$). Then (w, u, ω) has an equilibrium.*

The proofs of Theorem 6 and Corollary 3 are built on the same pattern as those of Theorem 2 and Corollary 2, but much longer, if only because the former implies the existence of a competitive equilibrium for general exchange economies and the latter refers to the second fundamental theorem of welfare economics (that is, to the existence of price systems supporting the market optima). We will omit them therefore, and refer the reader to Mercier Ythier (2000a, Lemma 6,[37] pp. 63–64, and Theorem 3,[38] p. 52).

The introduction of market exchange generates at least two additional sources of existence failures of the general equilibrium of gifts, besides the altruistic war of gifts already analyzed in Section 3.4.1.1 above: one potentially associated with negative prices of some commodities when disposal is costly, and the other one with paternalistic motives for gift-giving (that is, preferences of the type $W_i(x_1, \ldots, x_{i-1}, u_i(x_i), x_{i+1}, \ldots, x_n)$). The owner of a commodity with a negative market price, for instance, can get rid of it at no cost simply by "giving" it. This will result in the non-existence of equilibrium if individuals are not benevolent enough to refrain from making such damaging gifts: all the temporary owners will try to "give" their bad commodities, thereby generating a non-altruistic war of gifts. Wars of gifts, which involve a direct interaction of individual gifts, are possible also in the presence of paternalistic motives, but the latter are also susceptible to generate a plague of existence failures involving an interaction of

[37] A directed circuit of the digraph $\gamma'(x)$ of Lemma 6 is a directed circuit of $\{(i, j): -\partial_{u_i} w_i(u(x))\partial_{r_i} v_i(p, px_i) + \partial_{u_j} w_i(u(x))\partial_{r_j} v_j(p, px_j) > 0\}$ by the Lemma 4(v) of the same article.

[38] Extended weak self-centredness is equivalent to the Assumption 2 of Mercier Ythier (2000a). And digraphs $\gamma'(x)$ have no circuits in Pareto–BBV social systems.

gift-giving with market exchange: paternalistic donors, notably, might relentlessly try to alter the consumption structure of beneficiaries by gifts in kind, and in so doing be systematically frustrated in their attempts, because the latter prefer to sell the gifts and use the corresponding purchasing power to achieve their own, different consumption objectives.[39]

These existence failures are intimately related to the specificities of the Cournot–Nash behavioral assumption in the case of gift-giving. The individuals of the abstract social systems of Sections 3.1 and 4.2 act as if they believed that their gifts were consumed by the beneficiaries (formally, individual i, facing action vector a^* and making gift t_i, views j's consumption as $x_j = z_j^* + \omega_j + \Delta_j(t_{\setminus i}^*, t_i)$). This can be interpreted as non-tuistic behavior in some sense of the latter: donors "believe" that their gifts are consumed by beneficiaries exactly as traders on competitive markets "believe" that they can purchase or sell any quantity at market prices. Both exhibit the same absence of concern for the actual purposes of their partners in gift and exchange, which characterizes non-tuism. An equilibrium of gifts is precisely a social state where donors' conjectures on the use of the gifts they decide to make are all validated by their beneficiaries. The non-existence of the equilibrium of gifts means, consequently, that there is, at any social state, a donor whose conjecture is invalidated by a beneficiary.

The invalidation of a donor's conjecture involves a non-expected use, by the beneficiary of the gift, of his property right on the latter (the gift received is sold, for instance, or given to somebody else, instead of being consumed). The remedy to existence failure with this type of individual giving behavior, therefore, clearly calls for adjustments in the definition of property rights, which will depend on the nature of the existence problem under consideration.[40] A natural remedy to non-paternalistic altruistic wars of gifts, for instance, is the creation of a common property right of the individuals involved, on some adequate fraction of the sum of their private ownerships. Likewise, a simple

[39] More precisely, suppose that individuals have paternalistic utilities of the type of functions W_i and define a social equilibrium of (W, u, ω) as in Section 4.2.3 (Definition 3) with obvious adaptations. One establishes straightforwardly that if (p^*, a^*) is an equilibrium, then $(p^*, x(a^*))$ is a competitive equilibrium of $(u, (\omega_i + \Delta_i t(a^*))_{i\in N})$ [Mercier Ythier (1989, P.3.1, p. 83)]. One can generate examples of non-existence of a social equilibrium in the following way: let (u, ω') have exactly one equilibrium for all ω'; and choose W so that, for all ω', there is an individual who wants to deviate from competitive equilibrium allocation by means of a paternalistic transfer to some other agent.

[40] An interesting related issue is the possibility, for the beneficiary of a gift, of *refusing* it. In the context of the present theory of gift-giving, it can happen that a gift impoverishes the "beneficiary", notably when it has a negative market value (this supposes that disposal is costly or impossible) or in the case of a transfer paradox (see Section 4.3). Gift-giving then induces encroachments of the property rights of donors (their right to make gifts) on the property rights of "beneficiaries" (the set of useful alternatives accessible to them). Gift-refusal, as a limit imposed by the beneficiaries on the freedom of action of donors, is one of the possible (spontaneous) means of regulation of such encroachments. Note that, with the assumptions of Section 4.2 (*free disposal*, which implies the non-negativity of market prices; and *individual* gift-giving, which makes the transfer paradox implausible), returning a gift to the donor or transferring it to a third person is a costless and effective way, for the beneficiary of the gift, of "refusing" it. In other words, with these assumptions, a gift can only *enlarge* the set of useful alternatives accessible to the beneficiary.

solution to the non-altruistic wars of gifts of commodities with negative prices will consist either in prohibiting such damaging gifts, or in designing adequate disincentives such as taxes on these transfers that will discourage them or at least permit appropriate compensations for the losses of "beneficiaries". Finally, if the wide spectrum of tutelary motives is incorporated in social equilibrium analysis, the whole range of restrictions on individual property rights will have to be used to solve (if at all possible) the existence problems, from the creation of common property rights to the restrictions on specific types of individual property rights, the design of adequate incentive mechanisms or the command of specific types of individual actions.

Notice, to conclude, that the existence failures discussed informally above are not related to fundamental non-convexities such as discussed by Starrett (1972): an appropriate commodification, as public goods or bads, of the externalities generated by utility interdependence, and the design of corresponding standard Lindahl pricing and equilibrium solve the existence problem, at least as long as there is no local satiation of the Pareto social preordering.[41]

4.3. Perfectly substitutable transfers and the transfer problem

The transfer problem or transfer paradox refers to the logical possibility that an agent or group of agents withholding, destroying or transferring some fraction of their initial endowment ends up better off (and/or the recipients of transfers, if any, worse off) in ophelimity terms, due to the general equilibrium effects of their endowment manipulations on market prices. This was first mentioned by Keynes (1929), and discussed later on mainly in the context of the theory of international trade and aid [see Eichengreen (1987) for a historical overview, and Kanbur's contribution to this Handbook for a well-documented review of the applications to international aid].

Let us briefly summarize here the basis of the argument. It was shown notably that: (i) there is no possibility of a transfer paradox in a Walrasian economy with two agents, two commodities, dynamically stable equilibrium and no administrative costs or waste associated with the transfer [that is, the transfer will necessarily, then, impoverish the donor and enrich the recipient: see for example Johnson (1956)]; (ii) but such a possibility appears when anyone of the former assumptions is relaxed. For example, the possibility of a transfer paradox in the presence of imperfect competition is established by Kolm (1969, pp. 529–548), for stable exchange equilibrium of two commodities (or more) between two agents, one of them (the "monopolistic" nation) able to manipulate the terms of trade with the other (the "exploited" nation) for one pair of traded goods at

[41] Local Pareto satiation due to malevolence (Section 4.1.2) does not raise any fundamental obstacle to existence either. The existence result of Theorem 6 notably, which applies to non-paternalistic preferences, supposes no restriction on individual malevolence. If all individuals are non-paternalistic malevolent, for instance, the social equilibrium allocation of (w, u, ω) is simply the competitive equilibrium allocation of the induced exchange economy (u, ω), whose existence is unrelated to the satiation (if any) of the social Pareto preordering.

least. Likewise, the possibility of a transfer paradox at stable Walrasian equilibrium is established, notably, by Bhagwati, Brecher and Hatta (1983) for economies with more than two agents [see also Gale (1974)], and by Kemp and Wong (1993) for two-agent, two-commodity economies with a cost of transfer. General possibility results are derived in Guesnerie and Laffont (1978), and Postlewaite (1979), for pure exchange economies, without explicit reference to Walrasian stability. The former, building on the Debreu–Sonnenschein theorem, establish that "nearly any group of agents can be embedded in a competitive exchange economy in which they could find it profitable to reallocate their initial endowments". And the latter proves, by direct construction of examples, that any Pareto-efficient individually rational mechanism of pure exchange economies (including, therefore, competitive market exchange) can be manipulated, notably by coalitions which could enter general exchange in an improved position by reallocating the initial endowments of their members. Sertel (1994) extends the results of Postlewaite to Lindahl equilibrium of simple public good economies of the strong BBV type.

The transfer paradox is firmly established, therefore, as a logical possibility, stemming essentially from complex interactions of substitution effects and income effects at general equilibrium. This creates in turn new possibilities of non-benevolent (egoistic or malevolent) gift-giving in general Pareto social systems, in addition to the non-altruistic gifts of market bads already discussed in Section 4.2.4. And this opens moreover the possibility of a new class of strategic behavior, where individuals or coalitions "play with the market" (that is, consciously manipulate market prices) in order to achieve their ends, benevolent or not.

The practical importance of such logical possibilities should not be overstated, nevertheless, at least in the context of the Pareto social systems of Sections 4 and 5, where transfer decisions are made by price-taking individuals, with the implicit underlying assumption that individuals are "small" relative to the economy. "Small" agents, in other words, rightfully consider that their transfer decisions have negligible effects on equilibrium market prices; and they are consequently unwilling (and also, in practice, unable) to undertake the sophisticated calculations required to make usable predictions on such effects [see Postlewaite and Roberts (1976) for an elaborate treatment of this matter]. This is not true anymore, naturally, in principle at least, when *collective* gift-giving is considered, as will be the case in the study of Pareto-efficient redistribution developed in Section 6 below. The latter will be formulated, consequently, in the simpler analytical framework of pure distributive social systems (see Footnote 53, in Section 6.1).

5. The effectiveness of public redistribution with perfectly substitutable transfers

An aspect of the perfect substitutability of transfers that has received much attention in the literature is the so-called neutrality property, which specifies general conditions under which the social equilibrium is invariant to exogenous, publicly decided redistribution of wealth.

A derivation of the neutrality property is already implicit in the two-persons bargaining triangle of Shibata (1971), but its first explicit formulations are those of Barro (1974) and Becker (1974), concentrating on the case where the social equilibrium coincides with a rational optimum (the dynastic optimum in Barro's macrosocial system[42] and the family head's optimum in Becker's microsocial system).

The study of neutrality was developed initially in the line of Barro's overlapping generations model [see notably Bernheim and Bagwell (1988), the review of Laitner (1997), and Chapter 15 of Michel et al. in the present Handbook]. Its study in the present setup received a new impulse from the contributions of Warr (1982, 1983). The latter considered a simple distributive social system with three agents, two of them rich and making altruistic gifts to the third one, an egoistic poor (that is, using the terminology of this chapter, a BBV equilibrium with two non-poor giving to a single poor). He observed that: (i) social equilibrium is not Pareto-efficient; (ii) marginal lump-sum redistribution of endowments between rich individuals or from the rich to the poor is compensated dollar for dollar by appropriate changes in equilibrium charitable contributions, and leaves therefore the equilibrium distribution of wealth unchanged as long as charitable gifts remain positive; (iii) the achievement of a Pareto-efficient distribution by means of public transfers requires the complete crowding-out of private charity. Similar contemporary statements were made, in the same basic framework of the public good theory of charity, by Sugden (1982), Cornes and Sandler (1984a), Roberts (1984, 1985), and Kemp (1984).[43]

The present account draws on the general formulations of the property provided by Mercier Ythier (2000a) for the extended distributive social system of Section 4, and by Bergstrom, Blume and Varian (1986) for strong BBV distributive social systems. It concludes with a brief account of the known extensions of the property to cases of neutral distortionary redistribution.

[42] The reference to dynasties as representative macroagents aggregating a series of altruistically linked generations remains implicit in Barro's original article but was frequently used in subsequent formulations of his result. The relation between equilibrium and dynastic optimum is partly obscured, in Barro's model, by his formulation of the utility of a generation as a function of its consumption and the *indirect utility* of the subsequent generation. This formulation combines a notion of interdependence of primitive utilities with a notion of dynamic equilibrium. These two features were disentangled in subsequent developments, thanks to the use of subgame perfect Nash equilibrium as explicitly dynamic (and dynamically consistent) equilibrium concept [see notably Bernheim and Bagwell (1988), and Chapter 15 of Michel et al. in this Handbook]. Note that the dynastic optimum need not be an equilibrium or the unique equilibrium in such frameworks, even when all generations are connected by a chain of operative transfers (see Section 4 of Chapter 15 of Michel et al., and notably their Figures 1 and 3).

[43] Sugden elicits the marginal compensation effect "dollar for dollar" with any number of contributors. Cornes and Sandler establish inefficiency and neutrality with any number of identical contributors. Robert's model and conclusions are the same as Warr's, with identical rich and an endogenous determination of public charitable transfers. And Kemp extends Warr's neutrality theorem to the case of multiple public goods.

5.1. Neutrality in general Pareto social systems

The *distributive policies* examined in this section, and in Sections 5.2.1 and 5.2.2 below, consist of *public lump-sum redistributions of individual endowments*. From now on, we let: $\theta_{ij} \in \mathbb{R}^{ln}_{+}$ denote a vector of public lump-sum transfers of endowments from i to $j \neq i$; $\theta = (\theta_{12}, \dots, \theta_{1n}, \dots, \theta_{n1}, \dots, \theta_{nn-1})$.

Distributive policy is said *locally neutral* if equilibrium distribution is not altered by public lump-sum transfers θ that maintain the resulting distribution of individual endowments $(\omega_1^0 + \Delta_1\theta, \dots, \omega_n^0 + \Delta_n\theta)$ within some relevant neighborhood of the initial distribution ω^0. It is said *globally neutral* if equilibrium distribution is not altered by any lump-sum transfers. More precisely: public lump-sum redistribution is *locally weakly* (resp. *strongly*) *neutral* at some vector $\omega^0 \in \{\omega\colon \omega_i > 0 \text{ for all } i\}$ of individual endowments if there exist a neighborhood $V(\omega^0)$ of ω^0 in $\{\omega\colon \omega_i > 0 \text{ for all } i\}$ and a vector of ophelimity levels u^0 such that u^0 is a (resp. the unique) social equilibrium vector of ophelimity levels for all ω in $V(\omega^0)$. Public lump-sum redistribution, second, is *globally* (*strongly*) *neutral* if there exists an ophelimity vector u^0 such that u^0 is the unique social equilibrium ophelimity vector for all ω in $\{\omega\colon \omega_i > 0 \text{ for all } i\}$.

We know already that, as a simple implication of Theorem 4 or Corollary 3, the set of (interior) decentralisable allocations of a Pareto social system, that is, the set of interior allocations that can be reached by a distributive policy operating by lump-sum transfers, is the set $M' = \{x \in O \cap \mathbb{R}^{ln}_{++}\colon -\partial_{u_i} w_i(u(x))\lambda_i(x) + \partial_{u_j} w_i(u(x))\lambda_j(x) \leqslant 0 \text{ for all } (i, j)\}$ of market optima such that donors' utilities are non-increasing in their marginal incremental bilateral gifts evaluated at supporting market prices.

Global neutrality, therefore, *is equivalent to set $u(M')$ being a singleton* (Theorem 7(i) below). A casual examination of the examples of distributive social systems given in Section 3 shows that global neutrality is, consequently, a very strong property, corresponding principally to the case of unanimous agreement on the best accessible distribution (i.e., with the notations of Section 3, $x^i = x^j$ for all i, j).[44]

The *local neutrality* property is stated in parts (ii) and (iii) of Theorem 7, and in the Corollaries 5 and 6 of Theorems 4 and 7 respectively (proofs in Appendix A).

Theorem 7(iii) characterizes local neutrality as *a situation where the digraph of equilibrium transfers is connected*.[45]

[44] An interesting special case of singlevaluedness of $u(M)$ is Ramsey's dynastic framework (1928), where the agents are generations and where, using Pareto's vocabulary, their ophelimities are integrated in a single utility function, common to all generations, consisting of the (non-discounted) sum of generations' ophelimities. While very close to Barro's model of 1974 (see Footnote 42 above) in several important respects, it differs nevertheless fundamentally from the latter on the neutrality property. Barro's neutrality property is local in nature: his equilibrium does not coincide, generally, with the dynastic optimum when current generations are not connected to all future generations by a chain of positive transfers.

[45] A digraph γ is *connected* if any pair of its vertices is connected by a path contained in γ (that is, by a sequence of *adjacent* darts of γ, where "adjacent" means "having at least one common vertex"). Note that the path connecting two vertices needs not be directed (a path is *directed* if: either the head-vertex of any of its darts coincides with the tail-vertex of the subsequent dart in the sequence; or the tail-vertex of any of its darts coincides with the head-vertex of the subsequent dart in the sequence).

The first part of Corollary 5 states that equilibrium survives public redistributions of endowments if and only if the corresponding wealth transfers can be *offset* by variations in equilibrium private transfers that leave unchanged the structure of the graph of private transfers which is associated with the equilibrium allocation. Note that, in the important special case of *ceteris paribus* public lump-sum redistributions between any pair of individuals (say i and j) connected by a positive equilibrium wealth transfer (say, from i to j): public transfers in the *direction opposite* to the direction of the private transfer (that is, public redistributions from j to i) *can be offset* by an equal opposite variation in the equilibrium wealth transfer from i to j (as the public transfer from j to i can be used by i to feed his private transfer to j); and public transfers in the *same direction* as the private transfer *can be offset* by an equal opposite variation in the equilibrium wealth transfer from i to j *if and only if* the latter (private transfer) is *at least as large* as the former (public transfer). Such ceteris paribus bilateral public redistributions are neutral, therefore, by Corollary 5, as is, by extension, any distributive policy analyzable in a sequence of such redistributions.

The second part of Corollary 5 states that equilibrium does not survive public redistributions involving net transfers of wealth *between* the components of the graph.[46] This result is intuitively appealing. It draws its logical strength from the fact that there is only one graph of potential equilibrium gifts associated with any potential equilibrium allocation ($\gamma(x)$, associated with $x \in M$). It points to both: a sufficient condition for the non-neutrality of distributive policy, namely, that it performs redistributions of wealth between the connected components of the graph of potential equilibrium gifts; and to its interpretation, that is, that offsetting individual counter-transfers will be incompatible then with the structure of this graph.

Corollary 6 states essentially that an equilibrium allocation survives public lump-sum transfers between the vertices of a connected component of the graph of equilibrium transfers whenever public transfers are *sufficiently small* to be offset by appropriate variations in existing private transfers.

Combining the theorem and corollaries, we end up with the following formulation of the local neutrality property of general Pareto social systems. Public lump-sum transfers *do not alter* equilibrium distribution when the net transfers they imply are *confined to the connected components* of the graph of equilibrium transfers *and can be offset* by appropriate variations in existing private transfers. Public lump-sum transfers *alter* equilibrium distribution: (i) when they imply net transfers of wealth *between the connected components* of the graph of equilibrium transfers at prior equilibrium prices; (ii) or when they imply net transfers of wealth *inside the connected components* of the graph of equilibrium transfers at prior equilibrium prices, *which cannot be offset* by appropriate variations in existing private transfers.

[46] A *connected component* of digraph γ is a connected subdigraph of γ that is a proper subdigraph of no connected subdigraph of γ.

THEOREM 7. *Suppose that* (w, u) *verifies Assumption 2.* (i) *Distributive policy is globally neutral if and only if set* $u(M')$ *is a singleton.* (ii) *For all* $x \in M'$, $\Omega'(x)$ *is a convex set of dimension* $l(n - c(\gamma(x)))$, *where* $c(\gamma(x))$ *denotes the number of connected components of graph* $\gamma(x)$. (iii) *In particular: distributive policy is locally weakly neutral at an element* ω^0 *of the interior of* $\Omega'(x)$ *in* $\{\omega\colon \omega_i > 0 \text{ for all } i\}$ *if and only if* $\gamma(x)$ *is connected.*

COROLLARY 5. *Suppose that* (w, u, ω) *verifies Assumption 2, and let* (p, a) *be an equilibrium.* (i) $(p, x(a))$ *is an equilibrium price-allocation vector of* $(w, (\omega_1 + \Delta_1\theta, \ldots, \omega_n + \Delta_n\theta))$ *if and only if there exists* t *such that:* $g(t) \subset \gamma(x(a))$*; and* $p(t_{ij} - t_{ij}(a) - (t_{ji} - t_{ji}(a))) + p(\theta_{ij} - \theta_{ji}) = 0$ *for all* (i, j). (ii) *In particular,* $(p, x(a))$ *is* not *an equilibrium price-allocation vector of* $(w, (\omega_1 + \Delta_1\theta, \ldots, \omega_n + \Delta_n\theta))$ *whenever* θ *implies net transfers of wealth between connected components of* $\gamma(x(a))$, *that is, whenever there is a connected component* γ *of* $\gamma(x(a))$ *such that* $\sum_{(i,j)\in V_\gamma \times (N\setminus V_\gamma)} p(\theta_{ij} - \theta_{ji}) < 0$, *where* V_γ *denotes the set of vertices of* γ.

COROLLARY 6. *Suppose that* (w, u, ω) *verifies Assumption 2, and let* (p, a) *be an equilibrium. Then, there exists a neighborhood* V *of* 0 *in* $\{\theta\colon \theta_{ij} = 0$ *whenever* i *and* j *are in two distinct connected components of* $g(t(a))\}$ *such that, for all* $\theta \in V$, $(p, x(a))$ *is an equilibrium price-allocation vector of* $(w, (\omega_1 + \Delta_1\theta, \ldots, \omega_n + \Delta_n\theta))$.

We conclude this account of the neutrality property of Pareto social systems by three brief remarks.

The first one concerns the structure of the digraph of equilibrium gifts. I established in Mercier Ythier (2004b, Theorems 3 and 4), that the digraph $\gamma'(x) = \{(i, j) \in \gamma(x)\colon i \neq j\}$ of potential equilibrium gifts at an equilibrium distribution x of a pure distributive social system are forests (that is, contain no circuit) generically. In other words, circuits in digraphs of equilibrium transfers are coincidental for pure distributive social systems. A consequence of this is that, generically, $\dim \Omega(x) = \#\gamma'(x) (= n - c(\gamma(x)))$ in such social systems.

Similarly, the set of equilibrium distributions that a distributive policy can reach, from a given (interior) equilibrium distribution x of a distributive social system (w, ω), by operating small lump-sum transfers in the neighborhood of ω, is, generically, a local manifold $V(x)$ of dimension $n - 1 - \#\gamma'(x)$ [that is, generically, $\dim V(x) + \dim \Omega(x) = n - 1 = \dim S_n$, as a simple consequence of Mercier Ythier (2004b, Theorems 3 and 4 and Corollary 2)].

The third remark concerns an issue raised by equilibrium multiplicity. In the presence of multiple equilibria, the same system of public lump-sum transfers can be neutral for one equilibrium and non-neutral for another, that is, *distributive policy can be weakly (locally) neutral and not strongly so*. Figure 13, adapted from Example 4 of Mercier Ythier (2004b), provides a graphical illustration of such a situation. It describes a three-agent pure distributive social system (w, ω) with two equilibrium distributions b and d such that $\gamma(b) = \{(1, 2);\ (3, 2)\} = \gamma(d)$. Sets $\Omega(b)$ and $\Omega(d)$ are triangles $bb'b''$ and

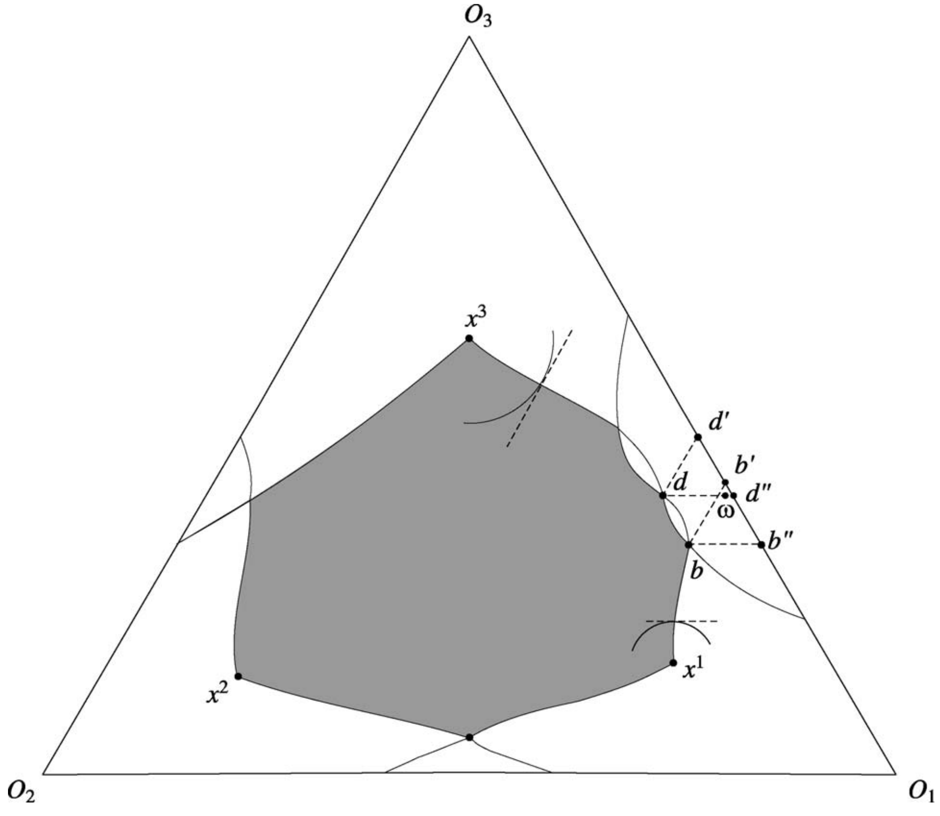

Figure 13. Neutrality with multiple equilibria.

$dd'd''$ respectively. There are two positive equilibrium gifts at b, from agents 1 and 3 to agent 2 (that is, $g(t) = \{(1, 2); (3, 2)\}$ at the associate equilibrium gift vector t), and only one positive equilibrium gift at d, from agent 1 to agent 2 (that is, $g(t') = \{(1, 2)\}$ at the corresponding equilibrium gift vector t'). One verifies easily from the figure that there is local weak neutrality with respect to b, but not with respect to d. For example, distribution d does not survive any public redistribution diminishing ω_3, while distribution b does if the redistribution is not too large. Such examples justify the distinction of a *weak* and a *strong* (local) neutrality property in the formal definitions of neutrality above.

5.2. *Neutrality in BBV distributive social systems*

The neutrality results above imply the neutrality properties of Bergstrom, Blume and Varian (1986: Theorems 1 and 7), [hence those of Warr (1983), and Kemp (1984)] as special cases, with two additional precisions following from the specificities of BBV distributive social systems.

The first precision follows from the fact that the digraphs of potential equilibrium transfers are always forests in BBV equilibrium, as subdigraphs of $\{(i, j)\colon i$ is non-poor and j is poor$\}$. Consequently, we have $\dim \Omega(x) = \#\gamma'(x) = n - c(\gamma(x))$ for all $x \in M$.

The second precision applies to strong BBV social systems where the private good (i.e. individual consumption of non-poor) and the public good (i.e. the aggregate consumption of the poor) are both strictly normal for non-poor. We know that BBV equilibrium is unique then (cf. Theorem 3, in Section 3.4.2.2 above), which implies that neutrality is strong whenever it holds.

Bergstrom and Varian (1985a), show how the neutrality property of strong BBV distributive social systems can be related to a general property of independence of Nash equilibrium from the distribution of agents' characteristics. Their result relies on the resolution of a Pexider functional equations [Aczel (1966, p. 141)], a technique already used to characterize the systems of individual preferences that imply the independence of allocative *efficiency* from distribution in the context of economies with public goods [Bergstrom and Cornes (1983)] and exchange economies [Bergstrom and Varian (1985b)]. They show that the equilibrium of a strong BBV distributive social system with at least 3 agents is independent from distribution if and only if it solves a system of equations of the type: $g_i = \alpha_i(G) + \beta_i(G)\omega_i$, $i = 1, \dots, n$, where $g_i = \sum_{j>m} t_{ij}$ is i's charitable contribution, $G = \sum_{i \in N} g_i$ denotes total contribution to charity, and α_i and β_i are continuous functions of G. Letting φ_i denote the inverse of i's unconstrained demand for the public good (Section 3.4.2.2), and supposing implicitly that φ_i is well-defined for all i (as this must be the case if charity is a normal good for all agents), Bergstrom and Varian obtain $g_i = \omega_i + G - \varphi_i(G)\omega_i$, $i = 1, \dots, n$ as a qualifying system.

5.2.1. Neutral lump-sum taxation

The neutrality property of BBV distributive social systems is local in nature. In other words, it does not hold, in general, for *any* system of lump-sum redistributions of endowments. It will generally be possible, notably, to achieve non-neutral public transfers by crowding out some of the equilibrium private transfers. In the BBV social system of Figure 11, for example: equalizing redistributions of endowments between non-poor agents 1 and 2 in segment O_1O_2 are non-neutral whenever $\omega \notin \,]a, b[$ (the equilibrium distribution runs over the broken line $\beta^1 x^* \beta^2$ when the initial distribution runs over segment O_1O_2); and any distribution of surface $\beta^1 x^* \beta^2 O_3$ (set M) is accessible by fully crowding out private transfers (status quo is the unique equilibrium for all $\omega \in M$).

Bergstrom, Blume and Varian (1986) give results of comparative statics concerning the effects of public lump-sum redistribution (neutral or non-neutral) on the provision of a public good when own consumption and the public good are strictly normal for all potential contributors. We reproduce them below without proof,[47] as Theorems 8, 9 and 10 (corresponding, respectively, to their Theorems 4, 5 and 6), with a few minor adaptations in formulation following, notably, from our interpretation of the public good as the aggregate wealth of the poor (see Section 3.3.3). The notions and notations used in the theorems have been defined above, in Sections 3.3.3 and 3.4.2.2.

Theorem 8 deals with the consequences of *redistributions of endowments among non-poor* individuals on equilibrium total charitable donations ((i), (ii) and (iv)) and on the set of contributors ((iii)).

THEOREM 8. *Let w be a strong BBV distributive social system and suppose that, for all non-poor i, there exists a single-valued (unconstrained) demand function for the public good f_i that is differentiable and such that $0 < \partial f_i(r) < 1$ for all $r > 0$. Then, in an equilibrium:* (i) *any change in the wealth distribution that leaves unchanged the aggregate wealth of current contributors will either increase or leave unchanged the equilibrium total private donation;* (ii) *any change in the distribution of wealth that increases the aggregate wealth of current contributors will necessarily increase the equilibrium total private donation;* (iii) *if a redistribution of income among current contributors increases the equilibrium total private donation, then the set of contributing consumers after the redistribution must be a proper subset of the original set of contributors;* (iv) *any simple transfer of income from one consumer to a currently contributing consumer will either increase or leave constant the equilibrium total private donation.*

Theorem 9 concentrates on the effects of *equalizing redistributions of endowments among the non-poor* on equilibrium total private donations when the latter have *identical donating preferences* (in the sense of identical demand functions f_i, which must not be confused with, and does not imply identical preferences on wealth distribution[48]).

A redistribution is equalizing in the sense of Bergstrom et al. if it is equivalent to a series of bilateral transfers in which the absolute value of the wealth difference between the two parties to the transfer is reduced.

Identical preferences in the sense above and the assumption of Theorem 8 imply that for any equilibrium total supply G^* of private donations, there is a critical wealth level $\omega^* = \varphi(G^*) - G^*$ (where φ is the inverse of individual unconstrained demand for G) such that every consumer with endowment $\omega_i \leqslant \omega^*$ contributes nothing and every consumer with endowment $\omega_i > \omega^*$ contributes $g_i = \omega_i - \omega^*$ to the public good [Bergstrom, Blume and Varian (1986, Fact 4); see also Andreoni (1988a, 2.1)]. In

[47] Proofs rely on simple properties of a function F defined from inverse demand functions φ_i by $F(G, C) = \sum_{i \in C} \varphi_i(G) + (1 - c)G$ where C denotes a set of contributors and $c = \#C$.

[48] Two non-poor agents i and j with identical demand functions $f_i = f_j$ *cannot* be said to have identical preferences relative to wealth distribution (w_i being definitely distinct from w_j) because each of them values his own wealth positively and is indifferent to the wealth of the other (w_i is increasing in x_i and independent of x_j).

particular: all contributors have greater wealth than (non-poor) non-contributors; and all contributors will consume the same amount of the private good as well as of the public good. Moreover:

THEOREM 9. *Let w be a strong BBV distributive social system and suppose that: for all non-poor i, there exists a single-valued (unconstrained) demand function for the public good f_i that is differentiable and such that $0 < \partial f_i(r) < 1$ for all $r > 0$; and $f_i = f_j$ $(= f)$ for all pairs of non-poor agents (i, j). Then:* (i) *an equalizing endowment redistribution among the non-poor will never increase the equilibrium total private donation;* (ii) *equalizing endowment redistributions among current (non-poor) non-contributors or among current contributors will leave the equilibrium supply unchanged;* (iii) *equalizing endowment redistributions that involve any transfers from contributors to non-poor non-contributors will decrease the equilibrium total private donation.*

Theorems 8 and 9 described consequences of endowment redistributions among potential and/or actual contributors to charitable donations (that is, among the non-poor). The last theorem of this section considers the effects of *endowment redistributions from non-poor to poor*, corresponding to the case where public and private actions compete in the achievement of charitable redistribution. Its part (i) characterizes neutral public actions of charitable redistribution, that is, public actions that are offset by variations in private transfers. Effective public actions of charitable redistribution are characterized in the parts (ii) and (iii) of the theorem.

THEOREM 10. *Let w be a strong BBV distributive social system and suppose that, for each non-poor i, there exists a single-valued (unconstrained) demand function for the public good f_i that is differentiable and such that $0 < \partial f_i(r) < 1$ for all $r > 0$. Suppose that starting from an initial position where non-poor consumers supply a public good voluntarily, the government supplies some amount of the public good which it pays for from lump-sum taxes on non-poor individuals. Then:* (i) *if the taxes collected from each non-poor individual do not exceed his voluntary contribution to the public good in the absence of government supply, the government's contribution results in an equal reduction in the amount of private contributions;* (ii) *if the government collects some of the taxes that pay for its contribution from non-contributors, the equilibrium total public and private supply of the public good must increase, although private contributions may decrease;* (iii) *if the government collects some of the taxes that pay for its contribution by taxing any contributor by more than the amount of his contribution, the equilibrium total public and private supply of the public good must increase.*

5.2.2. Neutral distortionary taxation

Bernheim (1986), Bernheim and Bagwell (1988), Andreoni (1988a), and Boadway, Pestieau and Wildasin (1989a), have drawn attention to the surprising fact that the neutrality property extended to a large class of "distortionary" taxes and subsidies, namely, tax-subsidy schemes in which the (net) tax paid by an individual depends on his decisions concerning labor participation (Bernheim, Bernheim and Bagwell), private consumption and saving (Bernheim and Bagwell), consumption of a local public commodity or factor (Boadway et al.) or contribution to a public good (Bernheim and Bagwell, Andreoni, Boadway et al.).

While very close in spirit to the results of Section 5.1, and notably to Theorem 7(iii), the neutrality properties of Bernheim (1986, Theorem 1), and Bernheim and Bagwell (1988, proposition), as well as the variant formulated in Game 3 of Andreoni and Bergstrom (1996, Theorem 5), are not directly comparable to them, being formulated in the dynamic setup of a subgame perfect Nash equilibrium. In Bernheim (1986), and Game 3 of Andreoni and Bergstrom, individuals contribute to a public good as in a Pareto–BBV social system with production, except that individual choices of labor participation and individual choices of gift-giving and consumption are not made simultaneously (labor participation is chosen first). And Bernheim and Bagwell (1988) consider the subgame perfect equilibria of an overlapping generations model with infinite horizon where finite-lived individuals maximize altruistic preferences on the whole stream of consumption and leisure profiles of current and future generations subject to the budget constraint determined by past choices of consumption, saving and gift-giving and by taxes based on the latter. Both setups yield the conclusion that any fiscal policy is locally weakly neutral whenever equilibrium is such that there exists, for any pair of individuals, a chain of operative transfers that connects them.

Andreoni (1988a), Andreoni and Bergstrom (1996), Boadway, Pestieau and Wildasin (1989a), and Brunner and Falkinger (1999) exhibit general properties of neutrality of distortionary taxation for the simultaneous Nash equilibrium of strong BBV social systems. A common feature of these contributions is the assumption that the government has a balanced budget for the public good, tax revenues exactly covering public spending on the latter (public provision, if any, and subsidies on private provision). There remain substantial differences between the models, nevertheless, making direct comparisons of results sometimes difficult. Details are presented in small print below.

Andreoni's (1988a) example of a neutral distortionary fiscal policy is framed in the strong BBV distributive social system, and thus allows direct comparisons with the neutrality properties of Sections 5.1 and 5.2. The present account is based on the version of the example presented as Game 2 in Andreoni and Bergstrom (1996). Let (w, ω) be a strong BBV distributive social system and consider the following three-stage game.

In stage 1, the government chooses a personalized lump-sum tax τ_i for each non-poor individual i and subsidizes private donations at rate β $(0 < \beta < 1)$. Thus a non-poor consumer who contributes $g_i (= \sum_{j>m} t_{ij})$ will receive a subsidy of βg_i and will have a net tax obligation of $\tau_i - \beta g_i$. The government spends its net revenue $\sum_{i\leqslant m}(\tau_i - \beta g_i)$ on additional units of the public good. This policy mix combines therefore two instruments of financing of the public good that have contrasted consequences on private donations: the subsidy, that encourages private donations, in the sense notably that an increase in the subsidy rate implies, ceteris paribus, an increase in individual contributions (whenever they exist); and the lump-sum tax, that competes with private donations by the neutrality property of distributive social systems.

In stage 2, individual agents play the gift game of the distributive social system (w, ω), amended to incorporate the fiscal determinants of individual behavior. Non-poor agent i faces budget constraint $x_i + g_i \leqslant \omega_i - \tau_i + \beta g_i$, and views the total supply of charitable contributions as $G = g_i + G_{-i} + \sum_{j\leqslant m}(\tau_j - \beta g_j)$ (he "sees through" the government budget constraint) where he takes $G_{-i} = \sum_{j:\, j\neq i} g_j$ and g_j, $j \neq i$, as independent of his own decisions. For any given $g^* = (g_1^*, \ldots, g_m^*)$, any $\tau = (\tau_1, \ldots, \tau_m)$ and any β, he solves therefore $\max\{v_i(x_i, G + \sum_{j>m} \omega_j)\colon G \geqslant G_{-i}^* + \sum_{j\leqslant m}(\tau_j - \beta g_j^*)$ and $x_i + G \leqslant \omega_i + \sum_{j\leqslant m:\, j\neq i}((1-\beta)g_j^* - \tau_j)\}$ where $G_{-i}^* = \sum_{j:\, j\neq i} g_j^*$. With well-defined, continuous (unconstrained) demand functions for the public good[49] and strict normality of the public good and of own private consumption for all the rich, the Cournot–Nash equilibrium exists and is unique for any subsidy rate β such that $0 \leqslant \beta < 1$ and any vector τ of individual lump-sum taxes such that $\tau_i < \omega_i$ for all $i \leqslant m$ [Andreoni and Bergstrom (1996, Theorem 3)].

Finally, in stage 3, the government observes the vector of private donations g, collects taxes $\tau_i - \beta g_i$ from each non-poor i and contributes $\sum_{i\leqslant m}(\tau_i - \beta g_i)$ to the public good.

With continuous, strictly convex preferences and strictly normal public and private goods for all $i \leqslant m$, one gets the following (local strong) neutrality property [Andreoni and Bergstrom (1996, Theorem 4)]:

THEOREM 11. *Let g^* be the vector of equilibrium private contributions if lump-sum taxes and subsidies are zero. If the government introduces taxes and subsidies such that $\tau_i \leqslant g_i^*$ for all non-poor i, then in the new equilibrium with taxes and subsidies, each consumer (poor or non-poor) will have the same private consumption as in the original equilibrium and the total amount of public good will also be unchanged.*

That is, this type of fiscal policy is neutral if (and one can add, using Theorem 10 above, only if) lump-sum taxes crowd out, in the strict sense of the word ($\tau_i > g_i^*$), none of the equilibrium private donations. Note that neutrality is a one-stage property here, corresponding to a notion of simultaneous equilibrium at the second stage of the game.

Boadway, Pestieau and Wildasin (1989a) give two neutrality properties of the simultaneous non-cooperative equilibrium for the same type of linear distortionary tax schemes as Andreoni's. They differ significantly from the latter's neutrality result, nevertheless, in assuming that agents do not see through the government budget constraint, for one of them, or that they see through this constraint but have non-zero conjectural variations, for the second result.

Boadway et al. suppose utility functions of the non-poor of the type $v_i(x_i, y_i, G)$, where y_i is interpreted as the quantity of a local public commodity (with sign convention $y_i > 0$) or factor ($y_i < 0$) consumed by agent ("locality") i. Notice that the corresponding social systems are not Pareto social systems in general, but become so with a few innocuous additional assumptions such as, for instance: utility function of rich i weakly separable in (x_i, y_i) for all $i \leqslant m$; and utility function of poor i of the type $x_i + h_i(y_i)$, that is, egoistic and quasi-linear in x_i, for all $i > m$. Boadway et al. moreover assume that the price of the local public goods is $= 1$ before tax,

[49] As implied by the continuity and strict convexity of preferences of the non-poor.

that functions v_i are strictly quasi-concave and twice differentiable, and that the private and local public goods x_i and y_i and national public good G are strictly normal for all $i \leqslant m$.

The authors concentrate on the following class of distortionary, balanced, linear tax schemes. The contributions of a locality i to the national public good are subsidized at constant rate s_i ("matching grant rate") by the government. Agent i also pays a lump-sum tax τ_i, and its consumption in the local public good is taxed at constant rate ρ_i. The balanced budget of the central government reads: $\sum_{i \leqslant m} s_i g_i = \sum_{i \leqslant m}(\tau_i + \rho_i y_i)$ for all $(g_1, \ldots, g_m, y_1, \ldots, y_m)$, where spending consists of the subsidies to localities' contributions (there is no direct contribution of the central government to the public good).

The authors restrict their study to interior equilibria, which implies positive private contributions from all potential donors ($g_i > 0$ for all $i = 1, \ldots, m$).

Their first neutrality result deals with the consequences on equilibrium of a change in lump-sum transfers $(\tau_1, \ldots, \tau_m)$. It is derived under the assumption that the implications of the budget constraint of the central government for the net tax liabilities of individual agents are not taken into account by the latter. In other words, localities do not see through the national budget constraint, that is, each i simply solves $v_i(x_i, y_i, g_i + G_{-i})$ with respect to (x_i, y_i, g_i), subject to the individual budget constraint $x_i + (1+\rho_i)y_i + (1-s_i)g_i \leqslant \omega_i - \tau_i$, for any given G_{-i}. Under the assumptions above, interior Cournot–Nash equilibrium, if any, must be unique, and the authors establish moreover that any change in lump-sum transfers that respects the budget constraint of the central government leaves unchanged the level of provision of the national public good and the private and local public good consumption of each locality (op. cit.: Theorem 1).

This neutrality property does not extend in general to changes in tax-subsidy rates $((\rho_1, \ldots, \rho_m), (s_1, \ldots, s_m))$, whether the agents see through the government budget constraint [Andreoni and Bergstrom (1996, Game 1)] or not [Boadway, Pestieau and Wildasin (1989a, Theorem 2)]. Boadway et al. obtain, nevertheless, a neutrality result for general policy changes, when agents see through the government budget constraint and have adequate non-Nash conjectures on the consequences of the policy change on the contributions of others to the national public good. Conjectural variation is specified as follows: each locality assumes that the others will respond to a change in government policy by adjusting their contributions to the national public good by an amount equal to the opposite of the variation in their individual net tax liabilities. It is proved that, then, any policy changes are fully neutralized at interior equilibrium, and that, moreover, each locality's behavior will conform exactly ex post with the conjecture of the others (op. cit.: Theorem 5).

Brunner and Falkinger (1999), finally, provide a general condition on the tax-subsidy scheme that is sufficient, and in general necessary for neutrality at interior simultaneous equilibrium when individual agents see through the government budget constraint.

They suppose the same type of utility functions as Boadway et al. above, with a different interpretation for y_i, construed as the leisure consumption of individual i. The price of leisure is the market wage rate, which will be set $= 1$ below for notational simplicity. Donors' utility functions v_i are strictly quasi-concave and differentiable. Private consumption x_i and y_i and the public good G are strictly normal for all $i \leqslant m$. The net tax liability of individual i is a differentiable function $\varphi_i(x_i, y_i, (g_1, \ldots, g_m))$ of his private consumption and the whole vector of individual contributions to the public good. It is assumed that the vector of tax functions $\varphi = (\varphi_1, \ldots, \varphi_m)$ verifies the following minimal consistency requirements: aggregate tax revenues $\sum_{i \leqslant m} \varphi_i(x_i, y_i, (g_1, \ldots, g_m))$ are equal to the (non-negative) government provision of the public good for all vectors of individual consumption and contribution; and a ceteris paribus increase in an individual contribution is never more than outweighed by a reduction in tax revenues (that is, $\sum_{i \leqslant m} \partial_{g_j} \varphi_i(x_i, y_i, (g_1, \ldots, g_m)) > -1$ for all j and all vectors of individual consumption and contribution). Finally, individual agents see through the government budget constraint, and maximize therefore $v_i(x_i, y_i, G)$ subject to the individual budget constraint $x_i + y_i + G \leqslant \omega_i + \sum_{j \leqslant m:\, j \neq i}(g_j + \varphi_j(x_j, y_j, (g_1, \ldots, g_m)))$, where G denotes total public and private provision of the public good. It is assumed that φ is such that individual budget sets are convex, a condition that is necessarily verified in the important special case where individual tax functions are linear.

Neutrality is shown to depend in a crucial way, in this setup, on the assumption that ceteris paribus variations in an individual contribution do not affect the aggregate net tax liabilities of others, that is, formally: $\sum_{j \leqslant m:\, j \neq i} \partial_{g_i} \varphi_j(x_j, y_j, (g_1, \ldots, g_m)) = 0$ for all i and all vectors of individual consumption and contribution.

The condition is sufficient for neutrality when all potential donors contribute at equilibrium. Precisely [Brunner and Falkinger (1999, Theorem 3.1)]: (i) if φ^* verifies the assumptions above, then the interior Cournot–Nash equilibria associated with $\varphi = \varphi^*$ and $\varphi = 0$ respectively are identical; (ii) if, in particular, φ_i^* is a function of the sole contribution g_i of agent i for all i, then: (a) there exists a lump-sum φ such that an (interior) equilibrium associated with φ^* is also an equilibrium for lump-sum φ; (b) and if there is a change in the tax function φ_i^* of contributor i, such that agent i keeps contributing after the change, then the associate (interior) equilibrium is unchanged. Note that the part (ii) of this theorem extends Theorem 11 above to the present setup. While the two results are not exactly comparable, due to the introduction of leisure as a strictly normal good and to the technical use of the differentiability of utility functions, the former can be viewed, nevertheless, as implying the latter, essentially at least: labor participation being free of tax by assumption, one can derive any (differentiable) social equilibrium of Andreoni–Bergstrom from some appropriate social equilibrium of Brunner–Falkinger, by making equilibrium leisure consumption fixed parameters in the latter.

The condition that ceteris paribus variations in individual contributions do not affect the aggregate net tax liabilities of others is also necessary, in general, for neutrality (op. cit.: Theorem 4.1). This point is established by means of examples of non-neutrality of linear tax-subsidy schemes (op. cit.: Section 5). The class Φ^L of linear schemes considered there are the linear φ such that $\varphi_i(x_i, y_i, (g_1, \ldots, g_m)) = \tau_i + \sum_{j \leqslant m} \beta_{ij} g_j$, $\sum_{j \leqslant m: j \neq i} \beta_{ji} \geqslant 0$ and $\beta_{ii} > -1$ for all $i \leqslant m$. Condition $\beta_{ii} > -1$ means, in particular, that individual contributions are not fully subsidized, while condition $\sum_{j \leqslant m:\, j \neq i} \beta_{ji} \geqslant 0$ states that the aggregate net tax liabilities of others is non-decreasing in i's private contributions. These tax-subsidy schemes verify the sufficient condition for neutrality above if and only if $\sum_{j \leqslant m:\, j \neq i} \beta_{ji} = 0$ for all i. The authors prove that (op. cit.: Theorem 5.1): an interior equilibrium associated with a linear $\varphi \in \Phi^L$, if any, must be unique; all linear schemes taken in $\{\varphi \in \Phi^L\colon \sum_{j \leqslant m:\, j \neq i} \beta_{ji} = 0 \text{ for all } i\}$ yield the same (unique) interior equilibrium; and if $\varphi \in \Phi^L$ is such that $\sum_{j \leqslant m:\, j \neq i} \beta_{ji} \neq 0$ for some i, then the associate interior equilibrium, if any, differs from the unique interior equilibrium associated with all elements of $\{\varphi \in \Phi^L\colon \sum_{j \leqslant m:\, j \neq i} \beta_{ji} = 0 \text{ for all } i\}$. In short, the condition that ceteris paribus variations in individual contributions do not affect the aggregate net tax liabilities of others is both necessary and sufficient for the neutrality of the linear tax-subsidy schemes of the class Φ^L with respect to the interior Cournot–Nash equilibria of the associate gift games. With the provision above relative to the assumption of strict normality of leisure, this characterization of non-neutral linear schemes implies the non-neutrality properties elicited in Falkinger (1996) (see Appendix A.2.2 below). With the same provision and the additional and more serious restriction stemming from the fact that the characterization applies to interior equilibria only, this result also implies the non-neutrality property of Game 1 of Andreoni and Bergstrom (1996) (their Theorem 2: see Appendix A.2.2 again). But it does not imply the non-neutrality property elicited by Boadway, Pestieau and Wildasin (1989a) (their Theorems 2 and 3: see Appendix A.2.2), where it is supposed that individual agents do not see through the government budget constraint.

Finally, Brunner and Falkinger provide a neutrality result analogous to the Theorem 1 of Boadway, Pestieau and Wildasin (1989a) relative to lump-sum redistributions in the presence of distortionary taxes and subsidies. They consider tax-subsidy schemes $\varphi_i(x_i, y_i, (g_1, \ldots, g_m)) = \psi_i(x_i, y_i) + \beta_{ii} g_i + \beta_i \sum_{j \leqslant m:\, j \neq i} g_j$, additively separable in private consumption and linear in private contributions, such that $\beta_{ii} + \sum_{j \leqslant m:\, j \neq i} \beta_j$ is some constant independent of i. They prove (op. cit.: Theorem 4.2) that lump-sum redistribution $(\tau_1, \ldots, \tau_m)$ from such a scheme φ is neutral whenever it verifies $\sum_{i \leqslant m} \tau_i = 0$ and leaves unchanged the equilibrium set of contributors. Note that the linear scheme of Boadway et al. violates the assumption that $\beta_{ii} + \sum_{j \leqslant m:\, j \neq i} \beta_j$ ($= s_i$, agent i's matching grant rate, in their setup) is a constant independent of i, except in the special case where all private contributions are subsidized at the same rate. Though very close in spirit, the two neutrality properties, therefore, are again not directly comparable, because the individual agents of Brunner and Falkinger see through the government budget constraint while the localities of Boadway et al. do not.

6. Efficient redistribution with perfectly substitutable transfers

Section 5 examined the feasibility of lump-sum redistribution when transfers are perfectly substitutable, with the conclusion that such redistributions are effective, essentially, if and only if they crowd out some of the equilibrium transfers. Section 6 will consider the complementary question of the normative justification of such lump-sum redistributions, based on considerations of Pareto-efficiency.

We noticed already in Section 3.2 that wealth distribution (or ophelimity distribution in general Pareto social systems) was, potentially, a pure public good in distributive social systems. Precisely, the wealth (consumption expenditure) or ophelimity (utility from consumption) of an individual is a pure public good or bad for any other individual who feels concerned about it, because the latter's distributive concerns imply that their "consumption" of the former's wealth or ophelimity are both non-excludable and nonrival ("consumption" meaning here simply the accurate perception of the individual wealth or ophelimity that makes the object of common concern). Early formulations of this simple consequence of non-paternalistic utility interdependence were made by Kolm (1968) and Hochman and Rodgers (1969).

Gift-giving generates, in Pareto social systems, two types of non-pecuniary externalities, defined in classical terms [e.g. Laffont (1988, Chapter 1)] as any effect of an individual action on other agents' utility functions or sets of alternatives at fixed market prices, namely: an effect on the budget set of the beneficiary of the gift (expansion of the budget set if the gift increases, contraction if the gift decreases); and the public good effect associated with the consequences of gift-giving on ophelimity distribution given common distributive concerns and the Cournot–Nash behavioral assumption (see Section 3.1.2 above). These external effects, which can be construed, in the manner of Meade (1973) as instances of non-contractual interactions, induce potential difficulties in the functioning of Pareto social systems, notably: the logical possibility of "wars of gifts", which involves the two types of external effects of gift-giving distinguished above (see Sections 3.4.1 and 4.2.4); and the Pareto-inefficiency of equilibrium (see the examples of Section 3.2), which results essentially from the public good externality, and falls therefore under the general class of issues known as "the public good problem" [e.g. Kolm (1964), Olson (1965)].

The sequel reviews the general theory of Pareto-efficient redistribution in pure distributive social systems and related issues relative to the influence of group size on free-riding behavior in that context, and on the design of incentive compatible mechanisms for distributive efficiency.

6.1. General theory

Redistribution has two notable specific characteristics as a public good.

One is its non-materiality, and more precisely the fact that its "consumption" and "production" do not imply, at least in the pure theory of redistribution developed here, any destruction of scarce resources: consumption merely consists of the observation of wealth distribution by concerned individuals; and production is pure and (supposed) costless[50] transfer activity. Distribution is conceptually and practically distinct, in that respect, from allocation, which usually involves, for public as well as for private goods, production and consumption activities relying on a substantial material basis. It is a *pure relational good*,[51] that is, a good (or bad) that consists of moral relations between individuals (moral sentiments, individual senses of distributive justice, ...). It is more naturally construed, consequently, as a part of an autonomous process of social exchange or social justice than as a part of the economic (and even public economic) process of allocation of resources.[52]

The second specificity can be stated as a paradox: the public good here is precisely what is usually meant by individual private wealth or welfare, in the two complementary senses of individual ownership and individual consumption of market goods and services. More precisely, the final destination of wealth in Pareto social systems is individual consumption (as opposed, notably, to collective consumption such as festivals,

[50] Real transfer activities do imply the consumption of scarce resources as inputs of a genuine production process, such as, for instance, the services of the departments or financial institutions technically in charge of their implementation. The analytical assumption of costless production is nevertheless approximately verified in practice in the sense that the production costs usually represent only a small fraction of transferred amounts.

[51] For a review of economic analyses of human relations as social "goods", see notably Chapter 9 by Sacco et al. in this Handbook.

[52] This distinction is to be related, of course, to Musgrave's distinction of the allocation and distribution branches of public finance (1959). The formal analogy of redistribution with a type of production is strongly suggested, nevertheless, by the public good theory of redistribution [see for instance its metaphorical use in the title of one of the founding contributions of the theory: "The optimal production of social justice" by Serge Kolm (1968)], and often endorsed literally in the context of BBV social systems [e.g. Cornes and Sandler (1984a), or Posnett and Sandler (1986)]. I argue below that the analogy is generally misleading when the social system is not of the BBV type.

or war effort). And these social systems are social systems of private property in the two complementary senses that: aggregate wealth is owned initially by private agents [that is, $\sum_{i \in N} \omega_i = (1, \ldots, 1)$: we have an economy of private property in the formal sense of Debreu (1959)]; and individuals enjoy a full right of usus (that is, own consumption, selling or gift-giving) and abusus (disposal) of the resources they own (that is, owned initially, purchased, or received as gifts from others). The public good problem of redistribution consists therefore of the Pareto-inefficiency of the distribution of wealth which stems from the spontaneous interaction of donors making use of their basic right of private property, caused by the two types of external effects induced by gift-giving, and notably by the public good externality generated by distributive concerns. And the natural framework for the solution of this problem is, consequently, the liberal social contract [e.g. Kolm (1985, Chapter 19, 1996, V); see also Section 2 above]. We present it below in the simple context of pure distributive social systems.[53]

6.1.1. *Collective gift-giving and social equilibrium*

This section extends the definition of the social equilibrium of a pure distributive social system (Section 3.1.2.1, Definition 1) to the case of *collective gift-giving of the contractual type*. Precisely, I allow for the possibility, for any subset of agents ("coalition"), to pool their resources (endowments and gifts received from the outside of the coalition) and decide collectively on their consumption and gifts on the basis of the (weak) unanimous preference of members (*cooperative* gift-giving). Formally:

DEFINITION 4. (i) The non-empty subset I (named *coalition*) of the set of agents *blocks* gift-vector t^* in the distributive social system (w, ω) if there exists t_I such that for all $i \in I$: $x_i(\omega, (t^*_{\backslash I}, t_I)) \geqslant 0$ and $w_i(x(\omega, (t^*_{\backslash I}, t_I))) \geqslant w_i(x(\omega, t^*))$ with a strict inequality holding for at least one i. (ii) Gift-vector t is a *strong distributive equilibrium* of (w, ω) if it is unblocked by any non-empty coalition (that is, by any non-empty subset of N).

The strong distributive equilibrium is an application to distributive social systems of a variant of the strong Nash equilibrium of Aumann (1959).[54] Aumann's equilibrium

[53] This solution is not written yet in the context of general Pareto social systems. The main difficulty with this extension lies in the basic assumption of price-taking behavior. This assumption is easy to justify in the case of individual donors (see Section 4.3 above), but no longer in the case of collective gift-giving involving agreements between many donors, possibly the population as a whole. Such coalitions of donors cannot ignore a priori the influence of their decisions on market prices and equilibrium, although one can imagine reasons why they could *decide* to do so, for instance normative reasons (if the social system performs better with price-taking behavior, relative to some normative criterion accepted by all) or practical ones (if learning the market equilibrium correspondence costs more than the benefits that can be derived from this knowledge, for example). The exploration of this question certainly is a valuable research program, though seemingly also a demanding one.

[54] Aumann's notion of strong Nash equilibrium requires that a deviation benefits all members of the coalition (strong unanimity), while I only require here that it benefits some of these members and makes none of

notion captures the simultaneous interactions of agents who can freely discuss their strategies but cannot make binding commitments [see for instance Bernheim, Peleg and Whinston (1987) for a discussion of the meaning of the concept].

The Olsonian type of characterization of the public good problem of redistribution as the Pareto-inefficiency of equilibrium in a non-cooperative game of individual gifts extends in a natural way to the case of cooperative gift-giving, namely: individual or collective free-riding on the contributions of others at distributive optimum, where "free-riding" refers to any rational action of individuals or coalitions (that is, any individual or collective action designed to increase agents' own utilities) that results in collective Pareto-inefficiency.[55] The public good problem of redistribution is then logically equivalent to the non-existence of a strong distributive equilibrium (a strong equilibrium distribution being Pareto-efficient by construction since the corresponding transfers are unblocked by the "grand coalition" N).

The right of private property plays a critical role in existence failures of strong distributive equilibrium. I analyze this point below, and recall known existence results for non-status quo and for status quo equilibria.

6.1.1.1. Private property rights and the public good problem of redistribution Private property rights cover two complementary notions in Pareto social systems, the combination of which determines individual sets of alternatives (individual budget sets). One is the rule that consists of the full right of usus and abusus of individuals over their own resources. Hereafter, it will be referred to as the Right of Private Property (in short: RPP). The other notion is individual endowment or initial right, corresponding to notation ω_i in the formal definition of Pareto social systems, and simply referred to below as individual i's *right*.

Private property rights create the possibility of free-riding in Pareto social systems by making donors' agreements non-binding. Let us make this simple but fundamental point precise.

them worse off. The definition of a strong distributive equilibrium in Mercier Ythier (1998a, 1998b, 2000b) embodies the *strong unanimous preference* of coalitions, while the definition used in Mercier Ythier (2004a) relies on *weak unanimous preference*. Strong unanimous preference is usually favored by game theory, because of the explicit causal relation it embodies, from individual incentives to the formation of coalitions. Weak unanimity, on the other hand, is the relevant notion in the liberal social contract for the decisions taken by the "grand coalition", as a normative principle of protection of individuals (or social types, depending on the interpretation that one retains for index i: see Section 6.1.2.2.2), implying an individual right of veto on the decisions of society. I adopt weak unanimity uniformly here for the sake of conceptual homogeneity.

55 This definition of free-riding implies, naturally, the usual sense, that is, the "action" (in the formal sense of game theory) of consuming a collective good without paying the contractual fare, for instance traveling for free by train or boat without permission. The extended notion in the text is designed to encompass all the various aspects of the public good problem of redistribution, such as inefficient underprovision ("too small" equilibrium transfers, free-riding in the common sense corresponding to a subcase of that type), inefficient overprovision ("too large" equilibrium transfers), or else (some equilibrium transfers "too small" and others "too large", and the cases of non-existence of Nash or strong Nash equilibrium that are related to the public good problem of redistribution, including the "wars of gifts", assimilated to a case of overprovision).

This logical consequence of private property can be understood readily from the formal representation of the right of private property (RPP) through the specification of individual and collective budget sets, namely, sets

$$B_i\left(t^*\right) = \left\{(x_i, t_i)\colon\ x_i \geqslant 0 \text{ and } x_i + \sum_{j\colon\, j \neq i} t_{ij} = \omega_i + \sum_{j\colon\, j \neq i} t^*_{ji}\right\}$$

for individuals, and

$$B_I\left(t^*\right) = \left\{(x_I, t_I)_{i \in I}\colon\ x_I \geqslant 0 \text{ and } \sum_{i \in I}\left(x_i + \sum_{j\colon\, j \neq i} t_{ij}\right) = \sum_{i \in I}\left(\omega_i + \sum_{j\colon\, j \neq i} t^*_{ji}\right)\right\}$$

for coalitions. This specification of individual and collective sets of alternatives, implied by RPP, implies in turn that the corresponding transfer decisions can always be reversed. Formally: if $(x_I, t_I) \in B_I(t^*)$, then any (x_I, t'_I) such that $0 \leqslant t'_I \leqslant t_I$ is also in $B_I(t^*)$. In such a context, an agreement between donors is binding, that is, it makes transfers irreversible for donors, if and only if the corresponding commitments are *embodied* in the individual budget sets of donors and beneficiaries, that is, if and only if the agreement achieves a *transfer of endowments* (implying a change in ω) from the former to the latter. To put it more briefly: *given the right of private property, binding donors' agreements must consist of lump-sum transfers of endowments from donors to beneficiaries*.

Donors' agreements are non-binding, therefore, in strong distributive equilibrium, as the latter represents voluntary redistribution as decisions on variables of the type t_I, which leave the vector of initial endowments ω unchanged by construction. Only exogenous public lump-sum transfers can change ω in this setup.[56]

The non-existence of equilibrium, the public good problem of redistribution, and the individual or collective free-riding on Pareto-efficient gift-giving are, in other words, in this analytical framework, three equivalent expressions of the exercise, legitimate by definition, of individual rights of private property.

6.1.1.2. Sufficient conditions for the existence of a non-trivial efficient distributive equilibrium The sufficient condition for the existence of non-trivial (that is, $\neq 0$) Pareto-efficient distributive equilibrium corresponds to the distributive equilibrium of Becker. It is derived from a result of Nakayama (1980, Proposition 2, p. 1261), adapted to the

[56] Note that the same is true, with some qualifications, for any definition of the distributive core that respects RPP and views initial rights as fixed, such as those that can be derived from Kolm (1987a, 1987b, 1987c, 1987d, 1989) for instance. Kolm's notions differ from Aumann's by allowing for a variety of types of non-cooperative interactions between coalitions involving not only conjectural variations such as Stackelberg's, but also, notably, an explicit modeling of the reactions of coalitions to the defection of some of its members ("splintering" cores, "cooperative" cores and so on). The qualifications follow from the fact that patterns of reaction to defections are susceptible to deter the latter in a variety of contexts, hence facilitating ex post stability of formally non-binding agreements [see the account of Kolm (1987a, 1987b) in Section 6.2.4 below].

present framework in Theorem 13. Nakayama's proposition states essentially that the (Nash) distributive equilibrium is Pareto-efficient whenever there is an agent who gives to all others at equilibrium, and whose utility reaches then its maximum in S_n.

THEOREM 13. *Let (w, ω) be such that for all i: $\omega_i > 0$; w_i is quasi-concave and $w_i(x) > w_i(x')$ implies $w_i(\lambda x + (1-\lambda)x') > w_i(x')$ for all $\lambda \in]0,1[$. If t^* is a distributive equilibrium of (w, ω) with a forest graph, and if there exists an agent i who makes positive gifts to all other agents at t^* (that is, $t_{ij} > 0$ for all $j \neq i$), then the associate equilibrium distribution $x(\omega, t^*)$:* (i) *is a weak distributive optimum of w;* (ii) *and maximizes i's utility in S_n (that is, $w_i(x(\omega, t^*)) = \max\{w_i(x)\colon x \in S_n\}$).*

PROOF. See Appendix A.1. □

Nakayama's proposition yields a sufficient condition for a non-trivial equilibrium solution to the public good problem of redistribution when gift-giving is individual. The following corollary extends this solution to collective gift-giving.

COROLLARY 7. *Let (w, ω) be such that $\omega_i > 0$ for all i, and $t^* \neq 0$ be a distributive equilibrium of (w, ω). Suppose that:* (a) *either there exists an agent i, with a strictly quasi-concave utility function w_i (that is, a quasi-concave w_i such that $w_i(x) \geqslant w_i(x')$ implies $w_i(\lambda x + (1-\lambda)x') > w_i(x')$ for all $\lambda \in]0,1[$ and all (x, x') such that $x \neq x'$), who makes positive gifts to all others at t^*, and all others are egoistic ($w_j : x \to x_j$ for all $j \neq i$);* (b) *or $n = 2$ and all utility functions are strictly quasi-concave. Then t^* is a strong distributive equilibrium and $w_j(x(\omega, t^*)) = \max\{w_j(x)\colon x \in S_n\}$ for every donor j.*

PROOF. See Appendix A.1. □

The condition of Becker–Nakayama is the underlying rationale for the optimism of Hochman and Rodger's original contribution (1969), which concentrates on Pareto-improving redistributions between two agents.

A variant appears also in Arrow (1981, Theorem 6) (see Section 3.3.2 above), stating that a non-trivial equilibrium of his distributive social system is Pareto-optimal if and only if: (i) there is a unique donor i, whose equilibrium wealth x_i is larger than the minimum equilibrium wealth $x_{\min}$; (ii) and the equilibrium wealth of all other agents is that minimum wealth (that is, $x_j = x_{\min}$ for all $j \neq i$, the gift j receives from i then being equal to $x_{\min} - \omega_j$). One verifies readily that the equilibrium distribution then maximizes the donor's utility in S_n, and that Arrow's condition is in fact equivalent to the following: there is a unique donor i, whose utility attains its maximum in S_n.[57]

[57] With the notations of Section 3.3.2 above and the assumptions of Arrow (1981), the equilibrium distribution x^*, such that $x_j^* = x_{\min}$ for all $j \neq i$, verifies the necessary first-order conditions: $\partial\varphi_i(x_i^*) = \partial\varphi(x_{\min})$ and $x^* \in \operatorname{Int} S_n$. And the f.o.c. characterize the (unique, interior) maximum of w_i in S_n by Arrow's assumption of strict concavity of utility functions.

Note that this condition does not imply that the donor gives to *all* other members of the social system. Corollary 8 of Section 6.1.1.3 below states that Arrow's condition is in fact necessary and sufficient for the existence of a non-trivial strong equilibrium in Arrow's distributive social systems.

The Becker–Nakayama condition is very sensitive to the number of potential donors and beneficiaries: it breaks down, in general, from two potential (net) donors [see for instance Musgrave (1970), Goldfarb (1970), and Theorem 17(iii) in Section 6.2.1 below]; and it appears very implausible when the number of potential beneficiaries is large. Mercier Ythier (2000b, Theorems 4.1 and 4.2) shows that a subset of three agents, two of them connected by a Nash equilibrium gift, and mild and natural assumptions of common distributive concerns between the three, suffice to imply the non-existence of a strong equilibrium. Such impossibilities, as well as Arrow's characterization of non-trivial efficient equilibria, leave us therefore with a simple alternative, in some respects analogous to the impossibility theorems of the theory of social choice: *either there exist integrative agents, Becker's family heads* (the analogues of the "dictator" of social choice theory), *whose individual optima make the social equilibrium, and this implies notably that distributive concerns are limited to small and closed subsets* (the "families") *of the whole set of agents; or there is no non-trivial strong equilibrium at all*.

6.1.1.3. Sufficient conditions for the existence of a status quo strong equilibrium The pervasiveness of free-riding in contexts of operative interactions (non-trivial equilibrium) does not extend, at least to the same degree, to status quo equilibrium. Let us introduce two natural assumptions relating to distributive preferences, in order to establish this point.

One is *self-centredness*. A weak variant of the assumption has already been introduced above (Section 3.4.1.2), with differentiable utility functions, as a sufficient condition for the existence of a distributive equilibrium. We now define the following, slightly stronger version, stating that an individual's distributive utility is *increasing* in bilateral progressive wealth transfers (see the definition of the latter in Footnote 7 of Section 2) from any richer individual to himself. Formally, for all (i, j) such that $i \neq j$, let e^{ij} denote the row vector of $\mathbb{R}^n$ whose entries are all $= 0$ except the ith and jth, equal respectively to -1 and 1. We say that the social system w verifies *self-centredness* if: for all (i, j) such that $i \neq j$, function $\mathbb{R}_+ \to \mathbb{R} : \tau \to w_i(x + \tau e^{ij})$ is increasing in $[0, (1/2)(x_j - x_i)]$ whenever $x_j \geqslant x_i$.

The second assumption states that individuals have no objection relative to bilateral progressive transfers as long as they are *not* involved in the transfer as donor or beneficiary. Formally: for all (i, j, k) such that $j \neq k$ and $i \neq j, k$, functions $\mathbb{R} \to \mathbb{R} : \tau \to w_i(x + \tau e^{jk})$ are non-decreasing in $[0, (1/2)(x_j - x_k)]$ whenever $x_j \geqslant x_k$. This excludes, notably, situations where individual i objects to a progressive transfer from individual j to individual k and would enjoy being the beneficiary of j's transfer in the place of k, situations of *relational envy* so to speak, induced by the relational character of wealth distribution as a public good, and where common language and psychology usually recognize a feeling of jealousy (of individual i, relative to j's gift to k).[58] By extension, I will name this second assumption *non-jealousy*, although it excludes, strictly speaking, i's jealousy relative to j's transfers to k only in situations where j is at least as rich as k.

The combination of self-centredness and non-jealousy, while compatible with any degree of individual self-centredness (the social system of the homo economicus, where $w_i : x \to x_i$ for all i, verifies both assumptions), produces a social context favorable to voluntary progressive transfers, in the sense that such transfers are vetoed neither by the beneficiaries (self-centredness), nor by the individuals who are not involved in the transfer (non-jealousy).

The Principle of Transfers, and Arrow's Assumptions 2, 4 and 5 (1981, pp. 204–205), imply them and are not implied by them.[59]

Strong BBV utility functions imply non-jealousy. They do not verify, in general, self-centredness, because each individual donor views his bilateral transfers, essentially, as gifts to a macroagent (the "sum" of all poor), who will be "richer" than him in most practical circumstances; but this does not alter the spirit and fundamental properties of these social systems, which are designed to account for progressive transfers from the rich to the very poor (see Theorem 14 below).

The next theorem shows that status quo strong equilibria abound in social systems that verify self-centredness and non-jealousy, and in strong BBV social systems: in these social systems, *the strong distributive equilibrium is a status quo* ($t = 0$) *if and only if the initial distribution is a (strong) Pareto optimum relative to distributive utilities*.

And the corollary establishes that, in the case of Arrow's distributive social system, the Becker–Nakayama condition is both sufficient *and necessary* for the achievement of a distributive optimum by means of private individual and/or collective transfers.

The (quite simple) intuition underlying the formal proof of Theorem 14 has been given in Section 2 (see the first paragraph of the presentation of the fourth characteristic property, in Section 2.2). The proofs of the theorem and corollary are detailed in Appendix A.[60]

[58] Envy is usually construed as the feeling of a person who, considering the position (say wealth, dignity, reputation ...) of another person, prefers the latter's position to his own. The position here envied by i is relational in the sense that it consists of a relation between two individuals (the position of beneficiary of the beneficence of j). Envy, and jealousy as relational envy, reduce to one and the same thing in the important special case where the envied position is a "right" guaranteed by society. These distinctions bear on the general question of the relations between moral sentiments and sentiments of justice. This note points to a conception of the sentiment of justice as a moral sentiment specifically related to comparisons of individual rights.

[59] w verifies the Principle of Transfers if $w_i(x) > w_i(x')$ for all i whenever x can be obtained from x' by a sequence of progressive transfers. Arrow's Assumptions 2 (anonymity) and 5 (convexity) together imply (and are not implied by) non-jealousy, while his Assumptions 4 ("selfishness") and 5 together imply (and are not implied by) self-centredness.

[60] Theorem 14 is adapted from Mercier Ythier (1998a). It differs from the closely similar property established in the latter reference (as Theorem 1) and in Mercier Ythier (1998b) (as Theorem 4) notably because the present chapter retains a notion of blocking coalition that involves the *weak* unanimous preference of its members (see Footnote 54), while the definition of blocking coalitions adopted in the former references

THEOREM 14. *Suppose that w either is a strong BBV distributive social system, or verifies local non-satiation of the distributive Paretian preordering (in short, local non-satiation: cf. Footnote 31), self-centredness and non-jealousy. Then,* 0 *is a strong distributive equilibrium of* (w, ω) *if and only if* ω *is a strong distributive optimum of* w.

COROLLARY 8. *If w verifies the assumptions of Arrow (1981) then a non-trivial distributive equilibrium t of (w, ω) is strong if and only if it has a unique donor i, whose utility reaches its maximum in S_n (that is, $w_i(x(\omega, t)) = \max\{w_i(x)\colon x \in S_n\}$).*

A simple but powerful consequence of Theorem 14, already noticed by Warr (1982) for individual contributions to a public good in the strong BBV setup, is the *full crowding out of private (individual or collective) transfers by any system of public lump-sum transfers achieving a distributive Pareto optimum*.[61] Likewise, *exogenous lump-sum redistributions of individual endowments within the set of concerned individuals are the only solution to the public good problem of redistribution* when the latter is raised, that is, essentially, when: the initial distribution is Pareto-inefficient (Theorem 14); and the (Nash) distributive equilibrium is non-trivial and does not verify the Becker–Nakayama condition (Section 6.1.1.2, and Corollary 8).

These consequences of Theorem 14, and the pervasiveness of the public good problem of redistribution in the presence of operative transfer motives, substantiate the definition of the distributive core below, as the set of *unblocked initial distributions* [Mercier Ythier (1998b)]. The core, in other words, is conceived here as a set of *initial conditions* (endowments) immune to individual or contractual deviations, and not, as in

supposes the *strong* unanimous preference of members. This weakening of the notion of blocking coalition, and the subsequent strengthening of the induced notion of strong distributive equilibrium, mainly result in a strengthening of the relation between status quo strong equilibrium and the Pareto-efficiency of the initial distribution: Theorem 14 establishes that the strong Pareto-efficiency of the initial distribution is *necessary and sufficient* for status quo strong equilibrium (with *weak* unanimous preference in coalitions), while my 1998 theorems establish that the strong Pareto-efficiency of the initial distribution is *sufficient* for status quo strong equilibrium (with *strong* unanimous preference in coalitions). This improvement of the property nevertheless obtains at some cost, in addition to the strengthening of the notion of strong equilibrium itself, namely, the strengthening of the assumption of self-centredness (the 1998 results only require *weak* self-centredness). Note that Theorem 14 and the 1998 results reduce to one and the same property when w is such that *weak* and *strong* distributive efficiency are equivalent. This will be the case, for example, in strong BBV distributive social systems, or in general distributive social systems such that all utility functions are strictly quasi-concave. Note that, in these cases (strong BBV, or strictly quasi-concave distributive preferences), the weak and the strong unanimous preference in coalitions are essentially equivalent (see also Footnote 73 on related subjects).

[61] Consider a social system (w, ω^0) such that ω^0 is Pareto-inefficient and w verifies the assumptions of Theorem 14, and suppose a distributive policy that operates lump-sum transfers from ω^0 in order to reach a distributive optimum ω. By Theorem 14, 0 is a strong Nash (hence Nash) equilibrium of (w, ω). And we know that 0 then is the unique Nash (hence strong Nash) equilibrium of (w, ω), generically (see Section 3.4.2.1). Therefore, the distributive policy crowds out all equilibrium transfers (individual and/or collective) existing in (w, ω^0).

conventional definitions, as a set of actions or action *outcomes* immune to such deviations given initial endowments.[62] It should not be viewed, consequently, as a solution to the public good problem of redistribution, but rather as a set where a solution, if any, must lie.

DEFINITION 5. The *distributive core* of w is the set $C(w) = \{\omega$: 0 is a strong distributive equilibrium of $(w, \omega)\}$.

The set of strong distributive optima of w is denoted by $P(w)$ in the sequel. The distributive core $C(w)$ is contained in $P(w)$ by definition, and identical to it when w verifies the assumptions of Theorem 14 (strong BBV, or local non-satiation, self-centredness and non-jealousy).

6.1.2. Distributive liberal social contract

Summarizing Section 6.1.1, the public good problem of redistribution consists of the non-existence of non-trivial strong (Nash) distributive equilibrium. This notably includes the cases of Pareto-inefficient underprovision of redistributive transfers at (non-cooperative Nash) distributive equilibrium. The public good problem appears, generally, when the initial distribution is Pareto-inefficient relative to individual distributive preferences, and there is no "family head" in the sense of Becker (that is, no individual who is able and willing to give to all others at equilibrium). Its solution supposes exogenous lump-sum redistributions of endowments within the set of concerned individuals. Any Pareto-efficient distribution can be reached by means of such exogenous transfers when

[62] Mercier Ythier (2004a, 1.2, notably Theorems 1 and 2) examines the formal relations between the distributive core as defined above, Aumann's strong equilibrium and the notion of core of Foley (1970). Foley's definitions, applied to a pure distributive social system (w, ω), yield: (i) non-empty coalition I "Foley-blocks" gift-vector t^* if there exists t_I such that for all $i \in I$: $x_i(\omega, (0_{\setminus I}, t_I)) \geqslant 0$ and $w_i(x(\omega, (0_{\setminus I}, t_I))) \geqslant w_i(x(\omega, t^*))$ with a strict inequality for at least one i; (ii) and the Foley-core of (w, ω) is then $\{x(\omega, t)$: there is no non-empty coalition that Foley-blocks $t\}$. Foley's core presents little interest as a solution concept in the context of general Pareto social systems for two reasons: the Right of Private Property (RPP); and the non-excludable character of distribution as a public good in such systems. Foley's definition implies a violation of RPP by forbidding (logically, if not normatively) individual agents and coalitions to use for their own consumption and transfers the gifts they receive from the outside. And the same "0-conjecture" on off-coalition contributions is usually interpreted as implying that deviating coalitions are able (and willing) to exclude non-members from the consumption of the public good they produce, for only then is the conjecture that non-members will react to a deviation by setting their contribution at 0 fully rationalisable in all circumstances. Note nevertheless that this "0-conjecture" characteristic of Foley's notion is rationalisable, and respects RPP, precisely at the initial distributions ω that make the distributive core in the sense of my Definition 5 (the ω such that 0 is a strong equilibrium of (w, ω)). Note also that, then, Foley's and Aumann's cores boil down essentially to the same notion (since status quo is, generically, the sole equilibrium whenever $\omega \in C(w)$: Section 3.4.2.1), and cannot account for operative redistributions by construction. See Footnotes 69 and 70, and Section 6.2.1 below for further developments on the Foley-core in the important special case of the BBV distributive social systems.

the social system is of the strong BBV type, or when it verifies local non-satiation, self-centredness and non-jealousy. And the exogenous transfers then crowd out all private (individual and/or collective) transfers.

We now endogenise endowment redistributions (the changes in ω), and the corresponding distributive policies, by means of a *distributive liberal social contract*.

General liberal social contracts were characterized, in Section 2, as *Pareto-efficient arrangements of individual rights unanimously preferred to an historical initial arrangement* (see Section 2.2). The general reason for such collective agreements is the Pareto-inefficiency of the social state that would result from individual or group interactions in the absence of them, notably because of pervasive public goods problems or externalities. The agreement can be implicit, and its implementation generally supposes some type of public intervention (see Footnote 10), such as general systems of public transfers (e.g. the Western European welfare states) or general systems of public incentives for private actions of redistribution (e.g. the North American federal systems of tax allowances for charitable contributions).

The precise (axiomatic) formulation of the distributive liberal social contract below combines two ingredients: unanimous agreement, and property rights.

Unanimous agreement (UA) simply characterizes contract as a mode of collective decision-making. It applies to any subset of agents (donors), including the whole set of them. Its formal expression is weak unanimity, corresponding practically either to an individual right to quit a "coalition" (a set of contracting donors) when the latter is a proper subset of the whole set of agents (free exit), or to an individual right of veto on collective decisions when the latter are taken by the whole set of agents.

Property rights are of two species in pure distributive social systems: individual (endowments); and constitutional. Constitutional property rights are of two types themselves: the Right of Private Property (RPP), already defined above; and its extension in the Freedom of Contracting (FC) of donors, that consists of the right of any set of individuals to pool their resources in order to decide contractually the individual consumption and gifts of the members of the resulting "coalition".

The distributive liberal social contract, finally, is required to be self-consistent (SC) in the following sense: its outcome must be immune to individual or contractual deviations of donors making use of their individual and constitutional property rights. That is: the social contract, if any, must redistribute endowments (agreement binding donors), so that the resulting distribution of individual rights lies in the distributive core. The combination of UA, RPP, FC and SC therefore yields [Mercier Ythier (1998a)]:

DEFINITION 6. ω is a distributive liberal social contract of (w, ω^0) if: (i) $w_i(\omega) \geqslant w_i(\omega^0)$ for all i; (ii) and $\omega \in C(w)$.

The remainder of this section examines the characterization, existence and determinacy of the distributive liberal social contract when distributive preferences are self-centered (or strong BBV) and non-jealous. Some fundamental intertemporal issues relative to this type of social contract are also briefly evoked at the end of the section.

6.1.2.1. Characterization, existence As a simple corollary of Theorem 14, *the set of distributive liberal social contracts of a pure distributive social system of private property* (w, ω^0) *that verifies local non-satiation, self-centredness and non-jealousy, or that is strong BBV, is the set of strong distributive optima of w that are unanimously (weakly) preferred to* ω^0. If, moreover, distributive preferences are continuous, there exists a distributive liberal social contract of (w, ω^0) for *any* initial distribution of rights ω^0 [see Mercier Ythier (1998a, Theorem 1 and 2000b, Theorem 4.3) for variants of these results]. Formally, letting $L(w, \omega^0)$ denote the set of distributive liberal social contracts of (w, ω^0):

COROLLARY 9. (i) *If* w *either verifies local non-satiation of the Paretian preordering, self-centredness and non-jealousy, or is a strong BBV distributive social system, then:* $L(w, \omega^0) = \{\omega \in P(w)\colon\ w_i(\omega) \geqslant w_i(\omega^0)$ *for all* $i\}$. (ii) *If moreover* w_i *is continuous for all* i, *then* $L(w, \omega^0)$ *is non-empty for all* $\omega^0 \in S_n$.

PROOF. See Appendix A.1. □

Mercier Ythier (1998a) gives three counterexamples to the existence of a distributive liberal social contract: the "generalized war of gifts" reproduced as Figure 12 above (Section 3.4.1.1), involving the violation of self-centredness; a paradoxical "war contract" in a two-agent social system where individuals are so malevolent that distributive efficiency implies the disposal of a fraction of aggregate wealth unless one of them owns the whole of it; and a case of jealousy.

The first two examples stem from an incompatibility of distributive preferences with the right of private property. "Wars of gifts" are suggestive of situations where the good functioning of the social system requires collective ownership of a part of total wealth, associated with collective decision making on individual consumption of collective wealth. Conversely, "war contracts" evoke situations where individual hostility is so intense that individual property rights are susceptible to collapse, because they do not receive sufficient support from the social body (whose existence itself is jeopardized, or at least subject to question in such a context).

The case of jealousy presented in Figure 14 is a variant of the Counterexample 2 of Mercier Ythier (1998a). The example emphasizes a basic problem confronting distributive liberal social contracts, namely, the rejection of welfare transfers to the poor by a sizeable fraction of the working and middle classes, typified here, metaphorically (and somewhat extremely), by the rejection of the abolition of slavery by the poor Whites on the eve of the American Secession War.

EXAMPLE 14. *On the Eve of Secession War*

The example uses a variant of the Cobb–Douglas distributive social system of Section 3.2 with three agents (types), the Abolitionist (agent 1), the poor White (agent 2) and the Slave (agent 3). Slavery is construed as a null endowment for agent 3, and

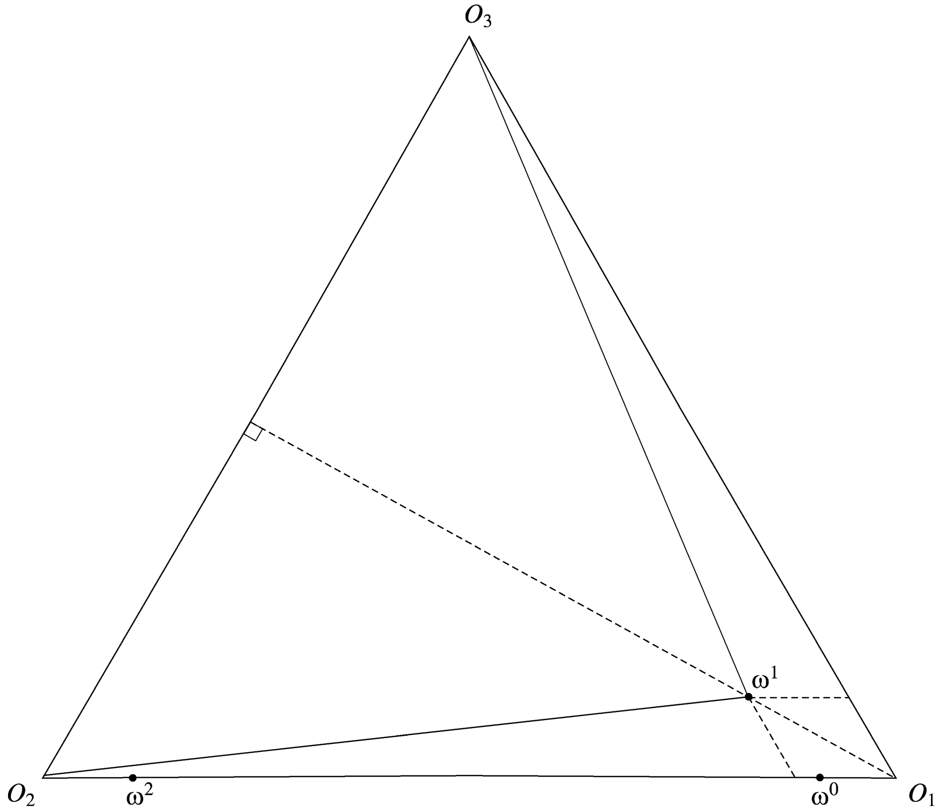

Figure 14. On the eve of the American Civil War.

implies null consumption of the latter in the absence of wealth transfers. The initial distribution of rights in this society is $\omega^0 = (9/10, 1/10, 0)$. The Abolitionist has the self-centered and non-jealous (log linear) Cobb–Douglas utility $w_1(x) = (3/4)\ln x_1 + (1/8)\ln x_2 + (1/8)\ln x_3$, that exhibits an absolute aversion to slavery. The ideal social state for this type (the distribution it prefers in S_3) is $\omega^1 = (3/4, 1/8, 1/8)$, that involves abolition and mildly progressive wealth transfers from itself to the other types, maintaining its relative dominant position and equalizing the positions of the others. The poor White has the following dichotomous utility function: his utility is $-\infty$ at any ω (or x) such that $\omega_3 > 0$, expressing an absolute aversion to abolition; and it is the log linear self-centered $w_2(x) = (1/10)\ln x_1 + (9/10)\ln x_2$ at any x (or ω) such that $x_3 = 0$. This utility function expresses jealousy relative to any wealth transfer from the Abolitionist to the Slave, and also a ("benevolent") envy relative to the (abolitionist) Rich in the sense that the associate social ideal (distribution $\omega^2 = (1/10, 9/10, 0)$) implies the permutation of his initial position with the Abolitionist's. The Slave, finally, is "egoistic": $w_3(x) = x_3$ for all x. The social system so defined verifies local non-satiation and self-centredness. One verifies readily that: $P(w) = \{\omega^2\} \cup [\omega^1, O_3]$; $C(w) = [\omega^1, O_3]$; the set of strongly efficient distributions unanimously preferred to ω^0 reduces to $\{\omega^2\}$; and therefore $L(w, \omega^0)$ is empty. The jealousy of the poor White makes the distributive liberal social contract collapse. Note that there is a unique strong equilibrium in this example, corresponding to the Beckerian equilibrium distribution ω^1: the solution to the collective issue of redistribution goes through a unilateral decision of the dominant agent ("Lincoln's policy", to pursue the metaphor[63]).

6.1.2.2. Determinacy The set of distributive liberal social contracts $L(w, \omega^0)$ of a locally non-satiated, self-centered and non-jealous social system w has the same dimension as the set of its strong distributive optima $P(w)$ when the liberal social contract induces effective redistributions.[64] It reduces generically to ω^0 otherwise, because of the generic uniqueness of status quo equilibrium. $P(w)$ is itself locally a manifold of the same dimension as S_n [generically: see Mercier Ythier (1997)]. That is, the distributive liberal social contract of Definition 6 is, generally, either fully determinate, when it does not involve any redistribution, or very indeterminate, in the sense of having the dimension of the simplex of feasible distributions, when it does involve effective redistributions (see Figure 15).

In other words, the process of individual and collective interactions of the distributive equilibrium determines fully, hence fully explains wealth distribution at the final agreements of the liberal social contract, while the requirement of weak unanimity that defines a contractual move from the initial distribution to a contractual distribution does not determine fully of course, hence does not fully explain the final agreement.

[63] It is needless to say, but nevertheless perhaps better to repeat that this example makes a *metaphorical* use of a historical event, designed to put some flesh on the abstract notions of the theory. It should not be viewed, of course as an *explanation* of the event under consideration, except perhaps through its very crude and simplistic characterization of a situation where collective action (here, the unilateral decision of the dominant agent) must, necessarily, substitute for the uncoordinated interactions of individuals or groups, in the presence of irreducible conflicts on the conception of the public good. A reference to the Secession War, even metaphorical, also makes sense in our context from another point of view: the war was the occasion of the full implementation, in the USA, of the constitutional rule of self-ownership that is a fundamental and in some sense founding part of all definitions of the liberal social contract.

[64] Precisely, we have the following: *If w verifies local non-satiation of the Paretian preordering, self-centredness and non-jealousy, if w_i is strictly quasi-concave for all i, and if $\omega^0 \notin L(w, \omega^0)$, then* $\dim L(w, \omega^0) = \dim P(w) \leqslant n - 1$.

PROOF (SKETCH). If w is locally non-satiated, self-centered and non-jealous, then $C(w) = P(w)$ by Theorem 14, so that the liberal social contract necessarily induces effective redistributions (that is, $\omega^0 \notin L(w, \omega^0)$) if and only if $\omega^0 \notin P(w)$. Set $\{\omega\colon\ w_i(\omega) \geqslant w_i(\omega^0)$ for all $i\}$ is convex by quasi-concavity of distributive utility functions. $\omega^0 \notin P(w)$, the strict quasi-concavity of utility functions and the local non-satiation of the Paretian preordering readily imply that $\{\omega\colon\ w_i(\omega) \geqslant w_i(\omega^0)$ for all $i\}$ has a non-empty interior in S_n. Hence $\dim\{\omega\colon\ w_i(\omega) \geqslant w_i(\omega^0)$ for all $i\} = \dim S_n = n - 1$. And therefore $\dim L(w, \omega^0) = \dim P(w) \leqslant n - 1$. □

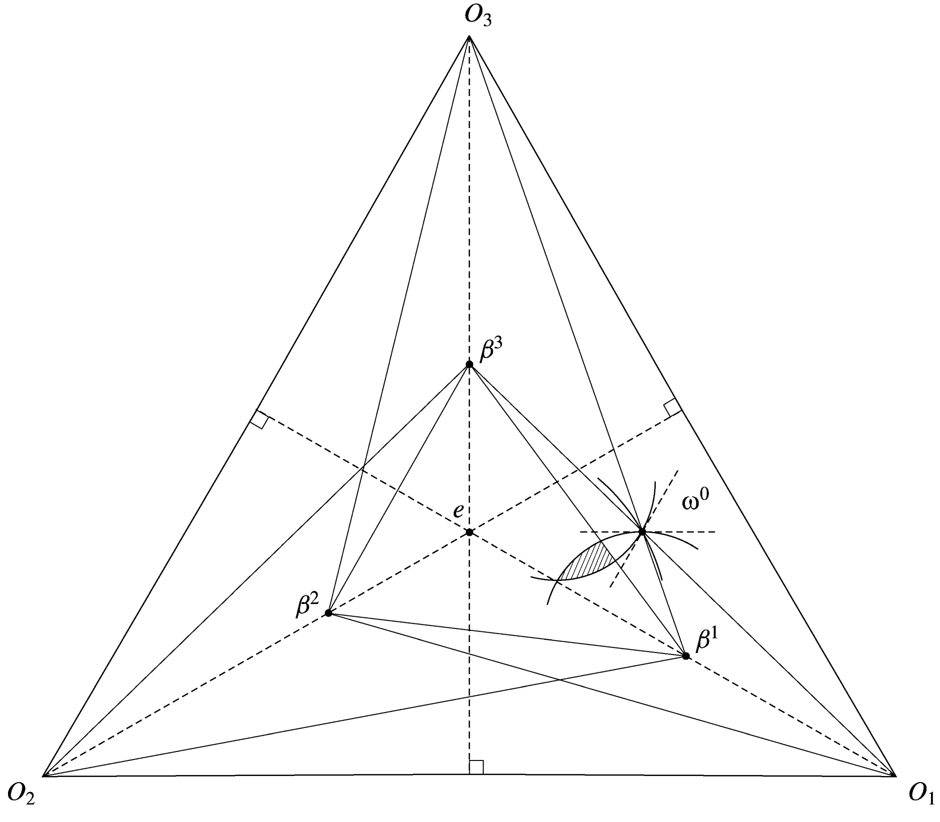

Figure 15. Distributive liberal social contracts.

This opposition between the determinacy of distributive equilibrium and the indeterminacy of distributive social contract parallels the analogous opposition, familiar in the theory market exchange, between the (local) determinacy of competitive equilibrium and the indeterminacy of the core in finite exchange economies. The solution outlined below parallels, likewise, Edgeworth's (1881) solution to core indeterminacy, and its generalization by Debreu and Scarf (1963). It consists of a process of social communication that yields essentially, in large social systems with negligible type diversity, a (generically) finite number of properly defined Lindahl equilibria.

The full description of this solution is beyond the scope of this chapter. We will provide here instead a detailed account of Lindahl equilibrium in the context of pure distributive social systems, and then give a brief literary description of the underlying "causal" process of communication.[65]

[65] See Mercier Ythier (2004a) for a precise and complete derivation.

6.1.2.2.1. Distributive Lindahl equilibrium The idea of applying the Lindahl equilibrium to the public good problem of redistribution goes back, at least, to the early precise formulations of the latter. Bergstrom (1970) analyzes in full generality the existence and efficiency properties of such an equilibrium in the context of a competitive exchange economy with non-paternalistic non-malevolent interdependence of utilities [see also Thurow (1971)]. And the fiscal application of Pareto-optimal redistribution developed by von Furstenberg and Mueller (1971) follows from the calculation of the same equilibrium in an example.

Let π_{ij} denote the value (personalized price) to individual i of individual j's consumption x_j, $\pi_i = (\pi_{i1}, \dots, \pi_{in})$ and $\pi = (\pi_1, \dots, \pi_n)$.

The sequel defines two variants of distributive Lindahl equilibrium. The first one, given in Definition 7, is designed in such a way that the equilibrium distribution is necessarily unanimously (weakly) preferred to the initial distribution of rights. This property obtains from the (purely instrumental) specification of the right-hand side of individual "budget constraints" as $\pi_i \omega^0$ (the value to i of the initial distribution), which makes the initial distribution of rights accessible to everybody at any system of Lindahl prices. This variant is named, for that reason, a *social contract equilibrium* (Theorem 15(ii)). The second notion, given in Definition 7′, corresponds to the usual version of the concept, used in the references of Bergstrom and von Furstenberg and Mueller above.[66] It is referred to as the *distributive equilibrium of Lindahl–Bergstrom* below. The associate equilibrium distribution is not, in general, unanimously preferred to the initial distribution. We give an example of a three-agent Cobb–Douglas social system that verifies the assumptions of Arrow (1981) where one or even two (that is, the majority of) agents strictly prefer the initial distribution to Bergstrom's equilibrium distribution (Figure 16 below). The two variants are equivalent, nevertheless, in the important special case of BBV social systems (Theorem 16).[67]

[66] There are two notable differences between the formal definition of Bergstrom and Definition 7′: Bergstrom's Lindahl prices are assumed non-negative, in the line of the author's assumption of non-malevolent utility interdependence; and his individual budget sets are specified as $\{x \in \mathbb{R}^n_+ \colon \pi_i x \leqslant \omega_i^0\}$, constraining individuals to choose a non-negative consumption not only for themselves but also for others. The presentation adopted here allows for malevolence, and its possible expressions through negative Lindahl prices and individual choices of negative consumption for others. The equilibrium distribution is non-negative by construction (but some equilibrium prices can be negative) when it exists. The unboundedness of prices and individual budget sets raises, naturally, potential difficulties for the existence of an equilibrium (see the discussion of the existence property at the end of Section 6.1.2.2.1).

[67] A third variant is conceivable, where the right-hand side of i's budget constraint is the *value to i of his own endowment* $\pi_{ii}\omega_i^0$, yielding

DEFINITION 7″. (π, x) is an equilibrium of (w, ω^0) if: (i) $\sum_{i \in N} \pi_i = (1, \dots, 1)$; (ii) and $w_i(x) = \max\{w_i(z) \colon z_i \geqslant 0 \text{ and } \pi_i z \leqslant \pi_{ii}\omega_i^0\}$ for all i.

This variant has not been studied in the literature. A casual examination suggests that its properties are qualitatively similar to Lindahl–Bergstrom's. Notably, the equilibrium distribution is generally not unanimously preferred to the initial distribution (but is so in the case of BBV social systems, where Definition 7″ is in fact equivalent to the other two).

DEFINITION 7. (π, ω) is a *social contract equilibrium* of (w, ω^0) if: (i) $\sum_{i \in N} \pi_i = (1, \ldots, 1)$; (ii) and $w_i(\omega) = \max\{w_i(x)\colon x_i \geqslant 0 \text{ and } \pi_i x \leqslant \pi_i \omega^0\}$ for all i.

DEFINITION 7′. (π, x) is a *Lindahl–Bergstrom equilibrium* of (w, ω^0) if: (i) $\sum_{i \in N} \pi_i = (1, \ldots, 1)$; (ii) and $w_i(x) = \max\{w_i(z)\colon z_i \geqslant 0 \text{ and } \pi_i z \leqslant \omega_i^0\}$ for all i.

The standard argument, transposed from the classical proof of the Pareto-efficiency of competitive equilibrium by Debreu (1954, Theorem 1), establishes the strong Pareto-optimality of the two variants of distributive Lindahl equilibrium when individual preferences are locally non-satiated at equilibrium [Mercier Ythier (2004a, Theorem 3), and Bergstrom (1970, Theorem 2)]. A social contract equilibrium is, therefore, a liberal social contract when the social system verifies local non-satiation of the Paretian preordering, self-centredness and non-jealousy.

THEOREM 15. *Let (π, ω) (resp. (π, x)) be a social contract equilibrium (resp. Lindahl–Bergstrom equilibrium) of (w, ω^0), such that w_i is locally non-satiated at ω (resp. x) for all i (that is, for all i and all neighborhood V of ω in $\mathbb{R}^n$, there exists $x' \in V$ such that $w_i(x') > w_i(\omega)$).* (i) *Then, ω (resp. x) is a strong distributive optimum of w.* (ii) *If, moreover, w is a strong BBV distributive social system, or if it verifies local non-satiation of the Paretian preordering, self-centredness and non-jealousy, then ω is a distributive liberal social contract of (w, ω^0).*

PROOF. See Appendix A.1. □

The Lindahl–Bergstrom equilibrium is not, generally, a distributive liberal social contract, because its equilibrium distributions are not, in general, unanimously preferred to the initial distribution of rights. This point is established through the following example, adapted from Mercier Ythier (2004a, Example 3) [a variant of this example can be found also in Bilodeau (1992), and in Section 4.7 of Bilodeau and Steinberg's Chapter 19 of this Handbook]. Let w be a three-agent Cobb–Douglas social system, with $w_i(x) = \beta_{i1} \ln x_1 + \beta_{i2} \ln x_2 + \beta_{i3} \ln x_3$, $\beta_{ii} = 1/2$ and $\beta_{ij} = 1/4$ for all i and all $j \neq i$. The social system verifies the assumptions of Arrow (1981). In particular, we have: $C(w) = P(w) = \mathrm{co}\{\beta^1, \beta^2, \beta^3\}$, the convex hull of the distributions β^i such that $\beta_i^i = 1/2 = \beta_{ii}$ and $\beta_j^i = 1/4 = \beta_{ij}$ for all i and all $j \neq i$, which maximize the agents' utilities in S_3 (see Figure 16). We let ω^0 run over $P(w)$. The strict quasi-concavity of utility functions in $\mathbb{R}^3_{++}$ readily implies then that the set of feasible distributions unanimously weakly preferred to ω^0 reduces to $\{\omega^0\}$. Therefore $L(w, \omega^0) = \{\omega^0\}$ for all ω^0 in $P(w)$. Bergstrom's equilibrium distribution of (w, ω^0) is $(\sum_{i \in N} \beta_{i1}\omega_i^0, \sum_{i \in N} \beta_{i2}\omega_i^0, \sum_{i \in N} \beta_{i3}\omega_i^0)$ [e.g. Bergstrom (1970, Example, p. 387)]. One verifies easily that this distribution is $\neq \omega^0$, hence not in $L(w, \omega^0)$,[68] and there-

[68] It is not a Nash equilibrium distribution (nor of course a strong Nash equilibrium distribution) of (w, ω^0) either: the status quo is the unique Nash and Strong Nash equilibrium of (w, ω^0) for all $\omega^0 \in P(w)$.

fore vetoed by one individual at least, unless $\omega^0 = (1/3, 1/3, 1/3) = e$. In particular, exactly two agents i and j strictly prefer the initial distribution ω^0 to the corresponding Bergstrom equilibrium distribution if and only if their initial endowments are both $> 1/3$, that is, if and only if ω^0 belongs to the interior of the hatched area of Figure 16 (relative interior in $P(w)$). Note that, as a simple consequence of the latter, the theorem of Foley (1970, 6), stating that Lindahl equilibrium, defined in the usual way, must be in the Foley-core, does not apply here.[69]

Lindahl–Bergstrom and social contract equilibrium coincide, nevertheless, in the case of BBV social systems, provided that the initial endowments of the poor are null. In other words, Definitions 7 and 7′ are equivalent when the public good problem of redistribution is framed in the standard setup of public good theory, with a list of pure public goods (consumption of the poor) "produced" from a list of pure private goods (the endowments of the rich).[70]

[69] The Foley-core of *any* distributive social system (w, ω^0) (see the definition in Footnote 62 above) is contained by construction in the set of strong distributive optima unanimously weakly preferred to ω^0 (as feasible distribution that is not Foley-blocked by any individual or by the grand coalition). It is a subset therefore [generally proper, see Mercier Ythier (2004a, Example 1)] of $L(w, \omega^0)$ whenever w verifies local non-satiation of the Paretian preordering, self-centredness and non-jealousy (or is a strong BBV social system). Letting $F(w, \omega^0)$ denote the Foley-core of (w, ω^0), we must have, in particular, $F(w, \omega^0) = L(w, \omega^0) = \{\omega^0\}$ for all strongly efficient ω^0 in the social system of Figure 16. Therefore the equilibrium distribution of Lindahl–Bergstrom is not in the Foley-core of these (w, ω^0), in contradiction with Foley's statement. Foley's property fails to hold here for a basic structural reason, already mentioned above as the second specificity of wealth distribution as a public good (see Section 6.1): there is no "private good" (in the formal sense) in Arrow's distributive social systems, because distributive concerns are ubiquitous there (everybody cares about everybody's wealth). It is essential, for Foley's property, that all public goods be "produced" (in a formal sense again) from private goods (and from them only). This structural property is verified, and Foley's property holds, in the BBV social systems where the initial endowments of the poor are $= 0$ (see Theorem 16 and Footnote 70 below).

[70] One proves, also, in the line of Footnote 69, that: *The Lindahl–Bergstrom equilibrium distributions of a BBV social system* (w, ω^0) *with endowments of the poor* $= 0$ *are in its Foley-core*.

PROOF (SKETCH). Let (π, ω) be a Lindahl–Bergstrom equilibrium of (w, ω^0), suppose that $x^* = x(\omega^0, (0_{\setminus I}, t_I))$ Foley-blocks ω, and let us derive a contradiction. Note that: $x^* \geqslant 0$ by construction; $\pi \geqslant 0$ (BBV implies non-malevolence and utility increasing in own wealth); and I can be viewed without loss of generality as a subset of the set of rich individuals (the poor being egoistic). Note also that $\sum_{i \in I} x_i^* + \sum_{j>m} x_j^* = \sum_{i \in I} \omega_i^0$, since the endowments of the poor are $= 0$ (the Foley-blocking coalition finances its consumption and the total consumption of the poor from its own resources). BBV individual utility functions verifying local non-satiation, $w_i(x^*) \geqslant w_i(\omega)$ implies $\pi_i x^* \geqslant \pi_i \omega$, so that $\sum_{i \in I} \pi_i x^* > \sum_{i \in I} \pi_i \omega$. Local non-satiation and the definition of Lindhal–Bergstrom equilibrium imply $\pi_i \omega = \omega_i^0$ for all i. We know from the proof of Theorem 16 that $\pi_{ii} = 1$ and $\pi_{ij} = 0$ for all pairs (i, j) of distinct rich individuals, so that $\pi_i x^* = x_i^* + \sum_{j>m} \pi_{ij} x_j^*$ for any rich i. And $\pi \geqslant 0$ and the definition of Lindahl prices imply $0 \leqslant \sum_{i \in I} \pi_{ij} \leqslant 1$ for all j. Thus:

$$\begin{aligned}\sum_{i \in I} \omega_i^0 = \sum_{i \in I} \pi_i \omega < \sum_{i \in I} \pi_i x^* &= \sum_{i \in I} x_i^* + \sum_{i \in I} \sum_{j>m} \pi_{ij} x_j^* \\ &= \sum_{i \in I} x_i^* + \sum_{j>m} x_j^* \sum_{i \in I} \pi_{ij} \leqslant \sum_{i \in I} x_i^* + \sum_{j>m} x_j^* = \sum_{i \in I} \omega_i^0,\end{aligned}$$

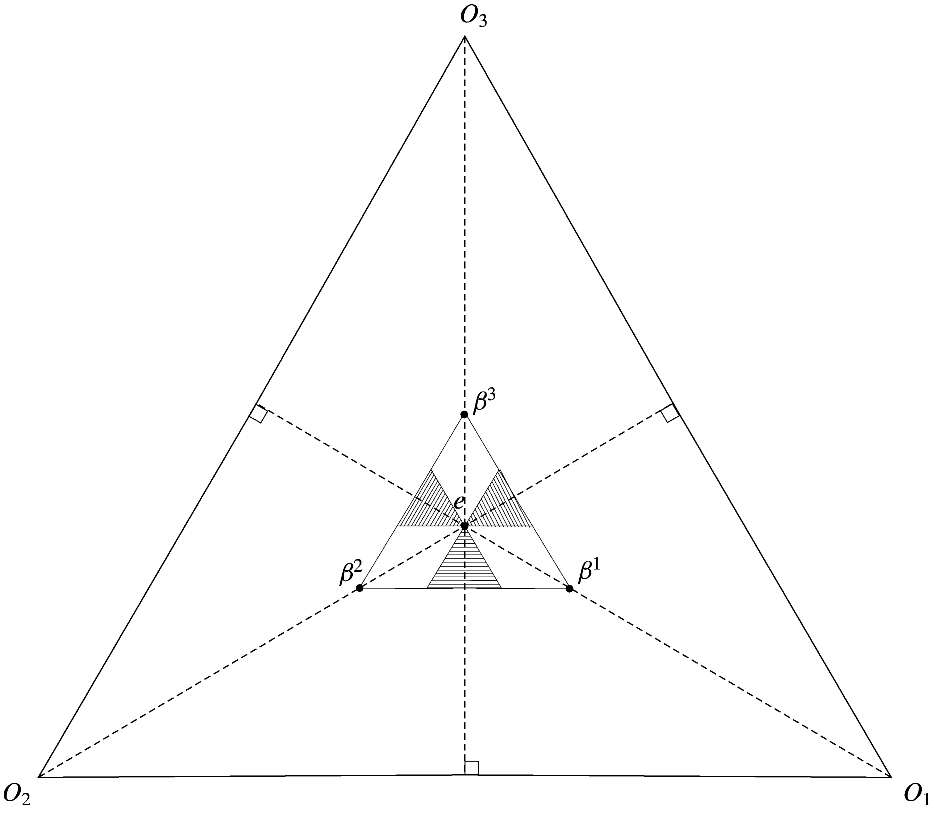

Figure 16. Majority-blocked Lindahl–Bergstrom equilibria.

THEOREM 16. *Let* (w, ω^0) *be a (weak) BBV social system such that* $\omega_i^0 = 0$ *for all* $i \geqslant m+1$ *(that is, for all poor i).* (i) (π, ω) *is a social contract equilibrium of* (w, ω^0) *if and only if it is a Lindahl–Bergstrom equilibrium of* (w, ω^0). (ii) *If, moreover, w is a strong BBV distributive social system, or if it verifies self-centredness and non-jealousy, then the equilibrium distributions of Lindahl–Bergstrom of* (w, ω^0) *are distributive liberal social contracts of the latter.*

PROOF. See Appendix A.1. □

Bergstrom (1970) and Mercier Ythier (2004a) establish, finally, that the other two fundamental properties of Lindahl equilibrium, namely, the supportability of any Pareto

the wished contradiction. □

optimum as a Lindahl equilibrium [Foley (1970, 3)], and the existence of equilibrium [Foley (1970, 4-B)], extend to distributive social systems.

Establishing these properties is necessary for two reasons. One stems from the specificities of wealth distribution as a public good. I noted above that important structural features and properties of the standard setup of public good theory do not extend to general distributive social systems (see Footnotes 69 and 70). The second reason is that malevolence, that is, the possibility that the wealth of some individuals be a "public bad" for some other individuals, cannot reasonably be discounted from distributive theory. There is of course the flat factual observation that malevolent feelings exist and interact with distributive issues. But one cannot, also, consistently derive an explanation of individual or collective voluntary redistribution from benevolent sentiments and omit taking into account opposite sentiments and their possible influence on the phenomenon under consideration. Malevolence and maleficence are in fact fundamental to this type of approach of redistribution through their potential interactions with the institution of private property. It is important for the logical and normative robustness of the construct to establish that (and to what extent) it can survive and produce reasonable results even in the presence of intense or widespread malevolent feelings [see for instance Mishan (1972) for a critique of Pareto-optimal redistributions founded, notably, on such grounds].

Supportability is established in Bergstrom (1970, Theorems 3 and 4), for convex competitive exchange economies with non-paternalistic non-malevolent distributive preferences, and in Mercier Ythier (2004a, Lemma 4) for differentiable and convex (pure) distributive social systems. The property obtains under standard conditions and minor additional restrictions designed to ensure that individual vectors of prices π_i are all $\neq 0$. As Mercier Ythier (2004a) allows for malevolence, the supportability property supposes moreover, in the latter, a variant of the local non-satiation of the Paretian preordering (differentiable local non-satiation).

Bergstrom (1970, Theorem 1) establishes the existence of distributive Lindahl equilibrium in his setup, under standard technical conditions. Mercier Ythier (2004a, Theorem 4) proves the same for social contract equilibrium in differentiable and convex (pure) distributive social systems. Existence obtains, in spite of the difficulties associated with potential malevolence (see Footnote 66 above), when individual distributive preferences verify the following property of boundedness: (i) marginal valuations of the wealth of others are bounded below, and marginal valuations of own wealth is bounded away from 0 at any solution of $\max\{w_i(x)\colon\ x_i \geqslant 0 \text{ and } \pi_i x \leqslant \pi_i \omega^0\}$; (ii) and program $\max\{w_i(x)\colon\ x_i \geqslant 0 \text{ and } \pi_i x \leqslant \pi_i \omega^0\}$ has solutions, contained in a fixed compact set independent of i, for any system of Lindahl prices verifying the boundedness condition (i). Bounded preferences rule out diverging malevolent valuations of the wealth of others, individual satiation, and unbounded choices of x in individual budget sets

$\{x\colon x_i \geqslant 0 \text{ and } \pi_i x \leqslant \pi_i \omega^0\}$[71] for systems of prices compatible with the boundedness of marginal valuations.

6.1.2.2.2. Dual distributive core Social contract equilibrium can be formulated, in the manner of Arrow and Debreu (1954), as the outcome of a process of social communication where a central agent, the Auctioneer, announces the systems of Lindahl prices that maximize the sum of individual values of individual distributive choices at any given vector of such choices [Mercier Ythier (2004a, 3.2)]. This subsection presents a brief informal analysis of the causal determination of this centralized social communication from a decentralized communication of the same type, as the field of communication opportunities expands, following the expansion of the number of agents.[72]

Decentralized communication relative to the public good (the distribution of rights ω, or equivalently the vector of net transfers of rights $\omega - \omega^0$) consists of the following process of decentralized auction. Coalitions are allowed to form and block any given strong distributive optimum ω, by proposing a vector of Lindahl shares of their members that increases the value of the public good for them (where value means, for each member i: $\min\{\pi_i z\colon w_i(\omega^0 + z) \geqslant w_i(\omega)\}$) relative to its value at supporting Lindahl shares, while maintaining their associate utility levels $w_i(\omega)$. Announcements of Lindahl shares by coalitions follow, therefore, the same type of instrumental objective as those of the central Auctioneer, namely, they tend to increase the value of the public good. But, unlike the central Auctioneer's, they embody the particular views of members about the public good, and notably their individual preferences (captured through the dual valuation functions $\min\{\pi_i z\colon w_i(\omega^0 + z) \geqslant w_i(\omega)\}$).

The *dual distributive core* is made of the strong distributive optima that are unblocked in that sense by any admissible coalition. Admissible coalitions are the coalitions allowed to express their views on the public good by blocking distributive optima. The dual distributive core is identical to: the set of social contract equilibrium distributions when all coalitions are admissible; the set of strong distributive optima when the grand coalition only is admissible (op. cit.: Theorem 5). Decentralized auction, in other words, generates a whole range of solutions to the public good problem of redistribution, from the determinate social contract equilibrium to the (very) indeterminate distributive efficiency frontier, depending on the choice of a set of admissible coalitions.

The expansion of the field of communication opportunities obtains through the replication device of Edgeworth (1881), generalized by Debreu and Scarf (1963): index i

[71] Set $\{x\colon x_i \geqslant 0 \text{ and } \pi_i x \leqslant \pi_i \omega^0\}$ is unbounded below in x_j for all $j \neq i$, and unbounded above in x_j whenever $\pi_{ij} \leqslant 0$. The boundedness of i's choices in such unbounded budget sets appears reasonable, when $\pi_{ii} > 0$, in view of the nature of the object of choice, which can be analyzed in two components: a choice of own consumption, which can be viewed reasonably as unbounded above a priori, but is bounded above by the budget constraint; and the choice of a *relative* distribution of wealth, which can be viewed as essentially bounded (relative shares mattering more than absolute consumption levels), unless of course passionate feelings dominate choices (and then it must be negative passions, for passionate benevolence will be bounded by positive Lindahl prices).

[72] The reader is referred to Mercier Ythier (2004a, 3, 4 and 5) for precise definitions, statements and proofs.

is reinterpreted as a fixed social type, and the number of agents per type is increased evenly, from 1 ("root social system") to infinity. Replication raises specific difficulties with preferences defined on the distribution of individual wealth, because the dimension of the object of preferences expands with the number of agents. They are solved by the application of a variant of the Population Principle, which maintains the structural stability of distributive preferences with expanding populations of agents, essentially by implying that the object of choice is redistribution between social types (identical individuals have identical wealth at any distributive optimum: op. cit., Theorem 6).

It is then shown that *the dual distributive core converges to the set of social contract equilibria* as the number of agents grows to infinity, even when admissible coalitions are required to be "representative", in the sense of containing at least one representative of each social type (op. cit.: Theorem 7).

This causal determination of the distribution of rights through social communication in a large society with finite type diversity faces potential difficulties that are, in many ways, symmetric to the difficulties confronting the causal determination of market prices through private communication in a large economy. The motives of agents for participating in exchange are self-evident in the latter case (participation increases own utility), and impediments to the efficacy of market communication, if any, will come, conspicuously, from informational and other practical limits to contracting and recontracting and the evolution of their relative weight with the expansion of the field of potential exchanges. The expansion of the field of social communication does not seem, symmetrically, susceptible to alter its efficacy to the same degree: convergence obtains despite conditions of representativeness of coalitions that severely restrict the set of admissible coalitions (up to which point is an open question); and the object of social communication, the distribution of rights between social types, is not substantially altered by the expansion of population size (the only significant informational issue being, there, the allocation of individuals between types). The main difficulties, if any, will appear on the motivational side, with the requirement, constitutive of this type of communication, that the relevant representative coalitions participate willingly (coalitions do not increase directly the utility of their members, only the social contract, that is, the final outcome of the participation of all representative coalitions, does) and honestly (by basing their decisions on the true preferences of their members) in social debate.

6.1.3. The distributive liberal social contract and the irreversibility of time

We conclude this review of the general theory with a brief discussion of a fundamental issue raised by the renewal of populations that results from births, deaths and migrations, in relation to the liberal social contract.

Time can be introduced in the formal representation of distributive liberal social contracts above notably by means of an intertemporal exchange economy or, more interestingly for the purposes of the present discussion, by means of an overlapping generations model [e.g. Mercier Ythier (2000b)]. The distributive liberal social contract then becomes susceptible, in principle, to account for redistribution over time, such as redistribution between past, present or future generations.

This extension is facilitated by the putative character of the social contract. The liberal social contract can remain largely imaginary, as an implicit foundation for a variety of institutions or collective decisions that might appear at first sight as expressions of pure public authority (e.g. public assistance) or, conversely, of uncoordinated private initiatives (charities for instance). It can provide, notably, a theoretical foundation for the choice of an optimal level of the public debt, understood as retro-payments from future to present generations: a large subset of concerned individuals, namely children present and future, cannot express any agreement in the present, but the government "foresees" that they will approve the transfers ex post, when they will be at an age to do so [e.g. Kolm (1985)].

This application of the logic of the liberal social contract to long-run redistributions of wealth between generations faces, nevertheless, a basic difficulty if the redistribution under consideration influences, as this actually is the case, the size and individual composition of the population, through the timing and number of births and deaths and through international migrations [e.g. Mercier Ythier (2000b: p. 107) and Footnote 9]. The social states associated with the various patterns of redistribution are not comparable then by definition on an individualistic basis. More precisely, a pattern of redistribution cannot be unanimously preferred to another if the populations of individuals determined by the two patterns differ at some point in time. A liberal social contract can still be defined in principle for this type of decision, but it should be based on the agreement of the sole set of pre-existing individuals, including, as above, relevant predictions of the future opinions of pre-existing young children, but taking due account, also, of the possible (though presumably negligible) influence of the anticipation of social contract redistributions on the size and composition of the set of pre-existing individuals. In the overlapping generations model with an endogenous population of Mercier Ythier (2000b, 4.3), for instance, a distributive liberal social contract is defined at each period of time, for the whole set of agents living at that time (but the object of the contract is instantaneous redistribution of income between social types, not intertemporal redistribution of life-cycle wealth between types or generations).

6.2. *Free-riding and population size in BBV distributive social systems*

One of the basic (and most popular) themes of Olson's *Theory of Collective Action* (1965) is the contention that the public good problem (i.e. the social suboptimality of non-cooperative equilibrium) tends to grow worse as group size increases, essentially because the increase in group size tends to weaken the link between individual contribution to, and individual benefits from, the aggregate social provision of the public good.

This conjecture of Olson has received partial confirmation in the studies of the influence of group size on the non-cooperative equilibrium of BBV social systems. It is shown, notably, that average individual contribution to the public good decreases with group size, but not total contribution, which increases with the number of agents when private goods and the public good are normal goods for donors (Section 6.2.2); and that underprovision of the public good increases with group size, relative to Lindahl equilibrium provision level (Section 6.2.3).

Two studies of Kolm (1987a, 1987b) show, on the other hand, that the increase in population size might actually facilitate the practical emergence of a cooperative solution to the public good problem by making individual benefits from free-riding negligibly small (see Section 6.2.4).

6.2.1. Inefficient underprovision of the public good at non-cooperative equilibrium

We begin this section by synthesizing useful results relative to the basic insight of Olson's theory, namely, the idea that the public good provision level tends to be too low at non-cooperative equilibrium, relative to collectively efficient levels.

Early rigorous expositions of this idea in a general setup were mainly illustrative, relying on graphical comparisons of the non-cooperative equilibria and Pareto-efficient allocations of social systems with a single pure public good produced from private goods by means of the additive technology, that is, in the terminology of the present survey, of strong BBV distributive social systems [see notably the diagram of Cornes and Sandler (1985a, Figure 6, p. 112), for symmetric equilibria and symmetric optima of social systems with any number of identical donors; and the Cornes–Sandler box in Cornes and Sandler (1986, Figure 5.3, p. 77), for general strong BBV distributive social systems with two donors].

Two recent contributions of Shitovitz and Spiegel (1998, 2001) provide general statements that confirm, in the main, Olson's insight within this framework.

Theorem 17, below, reproduces essentially the argument of the main theorem of Shitovitz and Spiegel (2001, 3, pp. 222–223) with slight improvements and an addition of my own. The first proposition (Theorem 17(i)) states that: if a strong BBV distributive social system verifies, notably, ordinal normality (defined in Section 3.4.2.2: Theorem 3), then there is an allocation in its (weak) Foley-core[73] that is unanimously *weakly* preferred by the rich to its unique social (Cournot–Nash) equilibrium. A second proposition (Theorem 17(ii)), that improves slightly upon a similar statement made by Shitovitz and Spiegel in the course of their main proof (p. 223, *Cases* 1 and 2),

[73] The weak Foley-core of distributive social system (w, ω) is defined as in Footnote 62, with the sole difference that strong unanimity (see Footnotes 54 and 60) is now required inside coalitions. Formally: (i) non-empty coalition I strongly Foley-blocks gift-vector t^* if there exists t_I such that for all $i \in I$: $x_i(\omega, (0_{\backslash I}, t_I)) \geqslant 0$ and $w_i(x(\omega, (0_{\backslash I}, t_I))) > w_i(x(\omega, t^*))$; (ii) and the weak Foley-core of (w, ω) is $\{x(\omega, t) \in S_n$: There is no non-empty coalition that strongly Foley-blocks $t\}$. We use these variants to conform the definitions and proof of Shitovitz and Spiegel. Note, nevertheless, that the strict monotonicity and continuity of functions v_i, assumed by these authors and in Theorem 17 below, readily imply the equivalence of weak and strong Foley-blocking and Foley-core in the BBV framework, and that the strict quasi-concavity of utility functions, also assumed here, has the same consequence for general distributive social systems.

states that, moreover: if the social equilibrium distribution is not in the weak Foley-core, then it is *strongly* Pareto-dominated by some distributions of the weak Foley-core, and the provision level of the public good is *strictly larger* in any of the latter. I add, as a third statement (Theorem 17(iii)), a sufficient condition for the (weak) inefficiency of distributive equilibrium that follows in a natural way from my discussion of the Becker–Nakayama condition in Section 6.1.1.2, namely, that there are at least two donors with positive private wealth at equilibrium.

THEOREM 17. *Let* (w, ω) *be a strong BBV distributive social system, and suppose that, for all* i, v_i *is* C^2*, strictly quasi-concave, and verifies ordinal normality. Denote by* $(x_1^*, \dots, x_m^*, y^*)$ *its unique equilibrium vector of individual consumption of the rich and aggregate consumption of the poor, suppose that* $x_i^* > 0$ *for all* $i \leqslant m$*, and let* x^* *be any equilibrium distribution (that is, any* $x \in S_n$ *such that* $x_i = x_i^*$ *for all* $i \leqslant m$ *and* $x_{m+1} + \dots + x_n = y^*$*).* (i) *Then, there exists a distribution* x *in the weak Foley-core of* (w, ω) *such that* $w_i(x) \geqslant w_i(x^*)$ *for all* $i \leqslant m$. (ii) *If moreover* x^* *is not in the weak Foley-core of* (w, ω)*, then:* (a) *there exists a distribution* x *in the weak Foley-core of* (w, ω) *such that* $w_i(x) > w_i(x^*)$ *for all* $i \in N$*;* (b) *and* $x_{m+1} + \dots + x_n > y^*$ *for all such* x. (iii) x^* *is not in the weak Foley-core of* (w, ω)*, nor is it a weak distributive optimum, whenever* $(x_1^*, \dots, x_m^*, y^*)$ *is such that at least two agents contribute whose private equilibrium consumption levels are both* > 0 *(that is, whenever* $0 < x_i^* < \omega_i$ *for two distinct* $i \leqslant m$ *at least).*

PROOF. See Appendix A.1. □

We have argued above (see Footnote 62) that the Foley-core is not specifically relevant as a solution concept in the context of general Pareto social systems, because it does not respect the right of private property and because the distribution of wealth is a non-excludable public good in these social systems. We gave, in particular, in Section 6.1.2.2.1, an example of a distributive social system of Arrow where the initial distribution is preferred to the Lindahl–Bergstrom equilibrium distribution by a majority of agents, and is not, consequently, in its Foley-core.

These remarks do not apply, at least to the same extent, to BBV distributive social systems. When the initial endowments of the poor are $= 0$, Lindahl–Bergstrom and social contract equilibria coincide (Theorem 16(i)) and belong to the Foley-core (see Footnote 70). Moreover the weak (resp. strong) Foley-core of (w, ω) is made, by definition, of distributions that are both weakly (resp. strongly) Pareto-efficient and unanimously weakly preferred to the initial distribution ω. It is contained, therefore, in the set of liberal social contracts of (w, ω) whenever the BBV social system is strong or verifies self-centredness and non-jealousy (Theorem 14).

Two interesting questions, following these remarks and Theorem 17, are then whether the Lindahl equilibrium distributions of a strong BBV social system are or are not unanimously preferred to its Cournot–Nash equilibrium distributions, and whether the corresponding provision levels of the public good are or are not larger than the Cournot–Nash provision levels. Shitovitz and Spiegel (1998) give partial answers to these questions, which are mainly but not entirely positive.

They are unambiguously positive in the important special case of symmetric equilibria of strong BBV distributive social systems with identical "rich" agents [Shitovitz and Spiegel (1998, Theorem 1, p. 5); and Corollary 10 below].

COROLLARY 10. *Let (w, ω) be a strong BBV distributive social system such that the initial endowments of the poor are all $= 0$ and rich individuals are identical (that is, $v_i = v$ and $\omega_i = 1/m$ for all $i \leqslant m$). Let $(z^*, y^*) \in \mathbb{R}^2$ and $(z^{**}, y^{**}) \in \mathbb{R}^2$ be respectively a Cournot–Nash and a Lindahl–Bergstrom (symmetric) equilibrium vector of private consumption of the rich and aggregate consumption of the poor (where symmetry means that the rich make identical gifts at equilibrium, equal to y^*/m and y^{**}/m respectively).* (i) *Then, $v(z^*, y^*) \leqslant v(z^{**}, y^{**})$.* (ii) *If, moreover, v is C^2 and strictly quasi-concave, and if $m \geqslant 2$ and z^* and y^* are both > 0, then: $v(z^*, y^*) < v(z^{**}, y^{**})$; and $y^* < y^{**}$ whenever v verifies the additional assumption of ordinal normality.*

PROOF. See Appendix A.1. □

Theorem 17 stated in essence that inefficient Cournot–Nash equilibria of strong BBV distributive social systems verifying ordinal normality are strongly dominated by some efficient distributions involving higher levels of provision of the public good. Corollary 10 adds the precision that Lindahl equilibria yield such distributions when the social system is made of identical donors. Unfortunately, the latter result extends only partially to strong BBV systems with multiple types of donors. Shitovitz and Spiegel (1998) give an example of a "large" strong BBV distributive social system with potential donors of two different types (one "large" agent, and a continuum of identical "small" agents), where the small donors strictly prefer Cournot–Nash to Lindahl (op. cit.: Example 5, p. 16). The same authors show that, nevertheless, Lindahl equal treatment equilibrium (where equal treatment means that identical individuals have identical private consumption levels) is unanimously preferred to Cournot–Nash equal treatment equilibrium in large social systems, when the relative weight of the set of small donors is important enough. This notably is the case for sequences of replicas of finite strong BBV distributive social systems with a finite number of fixed types of donors and fixed aggregate wealth per type: there exists an integer ι such that Lindahl equal treatment is unanimously strictly preferred to Cournot–Nash equal treatment by the rich for all replicas with numbers of donors per type $\geqslant \iota$ (op. cit.: Theorem 7, p. 12). The same holds for large strong BBV distributive social systems with a continuum of donors and fixed types of donors, including a single type of large donors (atoms) and a finite number of types of small donors (the atomless component): there exists a positive real number ρ such that Lindahl equal treatment is unanimously strictly preferred to Cournot–Nash equal treatment by the rich for all large systems made of these fixed types, such that the weight of the atomless component is $\geqslant \rho$ (op. cit.: Theorem 10: I and III, p. 15).

6.2.2. Group size and public good provision level at non-cooperative equilibrium

Chamberlin (1974), McGuire (1974), Andreoni (1988a), Fries, Golding and Romano (1991), and Shitovitz and Spiegel (1998), have studied the effect of group size on the level of individual and aggregate contribution to public good, in the context of strong BBV distributive social systems.

Chamberlin and McGuire concentrate on symmetric equilibria of social systems of identical agents (that is, agents with identical preferences and endowments), the latter in the special case of linear individual reaction functions.[74] They establish that, with fixed individual preferences and endowments (in my terms, fixed identical (w_i, ω_i) for all "rich" i), an increase in group size induces a decrease in equilibrium individual contributions, which converge to 0 as the number of agents grows to infinity. Equilibrium total contribution converges then to a finite value. Moreover, the associate sequence of equilibrium levels of provision of the public good is increasing if (and only if) the public and the private good are both (strictly) normal goods for contributors.

Andreoni (1988a) and Fries, Golding and Romano (1991) extend these results to social systems with multiple types of donors. Types may differ in preferences, endowments or both. It is assumed that individual unconstrained demands for the public good are well-defined differentiable functions (corresponding to functions f_i of Section 3.4.2.2). The private and the public good are both strictly normal. Fries et al. slightly strengthen the normality assumption by supposing moreover that the derivative of the demand for the private good has a positive lower bound for all types of donors (that is, equivalently, that $\partial f_i(r)$ is bounded above by some real number < 1 for all rich i).

The two papers differ principally in the way they model the increase in group size.

Andreoni considers finite independent random draws from a continuous distribution of types, and studies the asymptotic convergence of individual and total equilibrium provision levels as the number of draws grows to infinity, with the following conclusions (op. cit.: Theorem 1, p. 61 and Theorem 1.1, p. 66): (i) the set of contributors to the public good converges to a set containing individuals of a single type; (ii) in particular, the proportion of the population contributing to the public good, and average individual giving, decrease to 0; (iii) if all agents have identical preferences, then only the richest contribute in the limit; (iv) total donations to the public good increase to a finite asymptotic value.

Fries et al. increase the size of the social system by replication in the manner of Debreu and Scarf (1963), that is, by supposing an equal number of individuals of each type and making that number grow to infinity. They establish that: (i) the number of contributing types decreases monotonically with the size of the social system, and there is exactly one contributing type for any size of the social system larger than some well-defined, finite critical level (op. cit.: Proposition 1, p. 152); (ii) individual equilibrium

[74] Precisely, with the notations of Section 3.4.2.2, McGuire supposes that $\rho_i(G_{-i})$ is of the type $\max\{0, \alpha - \beta G_{-i}\}$, with α and $\beta > 0$.

contributions decrease monotonically to 0, and total equilibrium provision of the public good grows monotonically to a finite value, as group size grows to infinity (op. cit.: Lemmas 2 and 3, p. 151, and Proposition 1).

Shitovitz and Spiegel (1998), finally, consider large strong BBV distributive social systems with a continuum of donors of fixed types, including a single type of large donors and a finite number of types of small donors (see the last paragraph of Section 6.2.1 above). Their framework differs from Andreoni's in three main respects: it does not suppose the normality of Cournot–Nash individual demands for the private and the public good; it has a finite number of donor types, while Andreoni's has a continuum (but a finite number of types of individual preferences); and it has large donors, that is, donors of non-null relative individual weight, while Andreoni's social system is atomless. They obtain the same qualitative property on free-riding as Andreoni, namely, that only large donors contribute to the public good at Cournot–Nash equilibrium (op. cit.: Theorem 10, II, p. 15). But they do not consider the effects of population size on public good provision level, not surprisingly since they do not make the normality assumption that conditions unambiguous results concerning the latter.

6.2.3. *Group size and the suboptimality of non-cooperative equilibrium*

The contributions above make clear, and on the whole confirm Olson's view that individual free-riding, understood as individual undercontribution to the public good (including the special case of individual non-contribution), increases with population size in a non-cooperative environment. But they also introduce a qualification, by demonstrating that, despite increasing free-riding, the total provision of the public good increases with group size when, as this seems relevant in the context of strong BBV social systems, private goods and the public good are strictly normal for donors.

This qualification raises new questions for the degree of relevance of the collective side of Olson's argument, namely, the idea that social inefficiencies should worsen as population size increases [in Olson's own terms: "... the larger the group, the less it will further its common interests" (1965, p. 2). The latter have been addressed by the contributions of Cornes and Sandler (1986), Laffont (1988), Mueller (1989), Cornes and Schweinberger (1996), and Gaube (2001), which provide precise formulations and give, again, partial confirmations of this aspect of Olson's conjecture in various versions of the BBV social system.

Cornes and Sandler (1986), Laffont (1988), Mueller (1989) and Gaube (2001) share the following common features. They consider strong BBV distributive social systems, and compare non-cooperative provision with Lindahl provision as population size increases. Formally, letting s denote population size, $G(s)$ and $G^*(s)$ the non-cooperative and Lindahl public good production levels respectively, defined consistently for all values of s, these authors consider situations where the ratios $G(s)/G^*(s)$ are < 1 (underproduction of the public good), and exhibit conditions under which they are strictly decreasing in s (relative underproduction getting worse as group size increases).

The case studied by Cornes and Sandler (1986, 5.4, pp. 82–84) is the symmetric (non-cooperative and Lindahl) equilibrium of identical donors endowed with preferences linear in the private good, that is, using my notations, with quasi-linear utility functions of the type $\nu_i(x_i, x_{m+1}+\cdots+x_n) = x_i + v(x_{m+1}+\cdots+x_n)$. Population size is measured by the number of agents (potential donors, or "rich" individuals: $s = m$). They assume a strictly increasing, strictly concave, twice differentiable function v, and suppose a positive equilibrium provision level of the public good. The strict concavity of v readily implies the uniqueness of interior $G(s)$ as well as the uniqueness of $G^*(s)$.[75] The assumptions on v imply moreover that $G^*(s)$ is strictly increasing in s (op. cit.: p. 84). And the quasi-linearity of preferences and positive equilibrium provision of the public good imply that $G(s)$ is insensitive to population size: the positive wealth effects associated with an increase in the number of donors are entirely absorbed by the increase in individual demands for private goods. Ratio $G(s)/G^*(s)$, therefore, is uniquely defined for all s and decreasing in s.

Laffont (1988, p. 39) and Mueller (1989) analyze an example of symmetric non-cooperative and Lindahl equilibria of identical donors endowed with Cobb–Douglas preferences $\nu_i(x_i, x_{m+1}+\cdots+x_n) = x_i^{\beta}(x_{m+1}+\cdots+x_n)^{(1-\beta)}$ $(0 < \beta < 1)$. The ratio $G(s)/G^*(s)$, where $s = m$ denotes the number of potential donors, is, again, uniquely determined, equal to $1/(\beta s + 1 - \beta)$, therefore < 1 for all $s > 1$ and strictly decreasing in s. This example differs from Cornes and Sandler's, notably, in that individual non-cooperative demands for the public and private goods are strictly normal [that is, $0 < \partial f_i(r) = 1 - \beta < 1$, while $\partial f_i(r) = 0$ in Cornes and Sandler (1986)]. It verifies, consequently, the property, outlined by Chamberlin (1974) and McGuire (1974), of a non-cooperative provision level of the public good strictly increasing in group size (precisely, one has $G(s) = (1-\beta)se/(\beta s + 1 - \beta)$, where $e(> 0)$ denotes the identical initial endowments of donors, a function strictly increasing in s). The symmetric Lindahl equilibrium provision level $G^*(s) = (1-\beta)se$ increases with s also, but faster than non-cooperative provision ($G^*(s) > G(s)$ and $\partial G^*(s) > \partial G(s)$ for all $s > 1$).

Gaube (2001) generalizes the findings of Laffont and Mueller. In my notations and terms, he considers strong BBV distributive social systems with any number m of distinct types of "rich" individuals, strictly quasi-concave, monotonic strictly increasing BBV utility functions $\nu_i(x_i, x_{m+1} + \cdots + x_n)$, and strictly normal differentiable demands for private goods and the public good. Population size is increased by means of the Debreu–Scarf replication device. Its measure s now denotes the number of individuals per type (that is, the number of individuals per type in the s-replica of root social system $((\nu_1, \ldots, \nu_m), (\omega_1, \ldots, \omega_m))$ is s, and the total number of potential donors is

[75] Letting $\omega_i = 0$ for all poor i for simplicity, the first-order condition for interior non-cooperative equilibrium reads $\partial v(G) = 1$, which yields a unique equilibrium level for the public good since ∂v is a strictly decreasing function $\mathbb{R} \to \mathbb{R}$. Likewise, if the social system has two distinct symmetric Lindahl equilibrium provision levels of the public good, yielding necessarily the same equilibrium utility level, then any strict convex combination of equilibrium states would induce a feasible Pareto-improvement by the strict concavity of function v, contradicting the Pareto-efficiency of Lindahl equilibrium.

ms).[76] The normality assumption implies that $G(s)$ is well-defined and unique for all s. The comparison of $G(s)$ with optimal provision level is elaborated along the following lines: arbitrary lump-sum transfers are allowed between donors in the root social system, and the resulting initial distribution ω' is then maintained throughout the subsequent replicas; the corresponding Lindahl equilibrium provision level $G^*_{\omega'}(s)$ (here supposed unique for all s for simplicity) is compared with $G(s)$ as above, by means of the ratio $G(s)/G^*_{\omega'}(s)$. This procedure makes the study of relative underprovision independent of the initial distribution of wealth, as appears indispensable in the presence of multiple types of agents. Gaube proves then that: (i) $G(s+1)/G(s) < (s+1)/s$, that is, non-cooperative provision grows at a lower rate than population size (op. cit.: Lemma 1, p. 4); (ii) if the private and the public good are weak gross substitutes[77] at Lindahl equilibrium (that is, if i's Lindahl demand for the private good is non-decreasing in his Lindahl price of the public good for all i), then $G^*_{\omega'}(s+1)/G^*_{\omega'}(s) \geqslant (s+1)/s$, which means that the Lindahl provision level increases at a higher rate than population size (op. cit.: Lemma 2, p. 5); (iii) ratio $G(s)/G^*_{\omega'}(s)$ is, consequently, strictly decreasing in group size (op. cit.: Proposition, pp. 3–4).

Cornes and Schweinberger (1996), finally, consider general BBV Pareto social systems, with any finite number of private and public goods. Utility functions are strictly quasi-concave, and public goods are produced from private goods by means of concave production functions. Private goods are exchangeable on perfectly competitive markets.[78] The authors define and compare the social (Cournot–Nash) equilibrium and efficient allocation in this Pareto social system for populations that differ in size. A social system is said "more populous" than another if the latter's set of agents is a proper subset of the former's. The main result (op. cit.: Proposition 2, p. 83) states that public goods are more underproduced, at Cournot–Nash equilibrium, in the more populous social system than in the less populous social system, in the following precise sense and circumstances. Suppose that: (a) all agents contribute all factors to all public goods in a Cournot–Nash equilibrium of the more populous social system; (b) the same market value of factors is reallocated from the private to the public goods sectors in the more and in the less populous social systems; and (c) the utilities of the additional households of the more populous social system are kept unchanged in this reallocation by means

[76] We may also assume for the sake of completeness that poor types are identical, with null initial endowment, and are replicated in the same way as rich types.

[77] The examples of Laffont and Mueller verify weak gross substitutability, with a price elasticity of the Lindahl demand for the private good $= 0$. Note that *strict* gross substitutability (that is, positive price elasticity of the Lindahl demand for the private good) implies the uniqueness of Lindahl equilibrium in Gaube's setup.

[78] Their precise formulation fits in the formal definition of Pareto social systems and social equilibrium given in Section 4.2.1, with one mild qualification, and adequate interpretations of production functions. The concave production functions of public goods of Cornes and Sweinberger should be interpreted, in the framework of Section 4.2.1, as concave ophelimity functions of the poor. The qualification comes from the weak separability of donors' preferences in their own consumption of private goods, which is assumed in Section 4.2.1 as a consequence of non-paternalism, and is not supposed in Cornes and Schweinberger (1996). The latter is more general than Section 4.2.1 in this respect, and less general in all other respects.

of appropriate lump-sum taxes. Then, the gains in the more populous social system are greater than in the less populous social system, that is, the social surplus measured in terms of the numéraire in the compensated equilibrium is greater in the more populous social system than in the less populous social system. This result expresses essentially the fact that there are consumption returns to scale associated with the existence of pure public goods, simply because this type of good, by definition, "can be shared among more agents without a utility loss to anyone agent" (op. cit.: p. 83).

6.2.4. *Free-riding, population size and core solution*

Kolm (1987a, 1987b) examines the same questions in the context of cooperative gift-giving.

The main distinctive feature of the construct is the cooperative solution concept it applies, corresponding to the variant of Kolm's general notions of core with interdependent coalitions where every coalition bases its decisions relative to its own contributions on its anticipation of the best reactions of the complementary coalition (see Section 16.5.3 of the introduction Chapter 1 of the Handbook for a general presentation of the theory). Formally, using the notations of the present chapter, let $\varphi_{\backslash I}(t_I)$ denote the set $\operatorname{argmax}\{w_{\backslash I}(x(\omega,(t_I,t_{\backslash I}))){:}\ x_{\backslash I}(\omega,(t_I,t_{\backslash I}))\geqslant 0\}$, where $w_{\backslash I}=(w_i)_{i\in N\backslash I}$ is maximized with respect to $t_{\backslash I}$ (which means that if $t_{\backslash I}\in\varphi_{\backslash I}(t_I)$, then there exists no $t'_{\backslash I}$ such that $x_{\backslash I}(\omega,(t_I,t'_{\backslash I}))\geqslant 0$ and $w_{\backslash I}(x(\omega,(t_I,t'_{\backslash I})))>w_{\backslash I}(x(\omega,(t_I,t_{\backslash I})))$. A gift-vector t^* of a distributive social system (w,ω) is in the core in the sense of Kolm (1987a, 1987b) if, for all non-empty $I\subset\{1,\ldots,n\}$, there exists no t such that: $x_I(\omega,t)\geqslant 0$, $t_{\backslash I}\in\varphi_{\backslash I}(t_I)$, and $w_I(x(\omega,t))>w_I(x(\omega,t^*))$.[79]

The references above apply this general notion to a simple case, corresponding, in my terms and notations, to the strong BBV distributive social system with identical "rich" donors $i\in\{1,\ldots,m\}$ whose (ordinal) preferences admit a quasi-linear utility representation of the type $w_i(x)=x_i+v(x_{m+1}+\cdots+x_n)$ (with v twice differentiable, strictly increasing and strictly concave). The first reference (1987a) concentrates on the case of a pure public good (the relevant case in the context of the present review) while the second reference (1987b) extends the analysis to excludable public goods with any fixed degree of exclusion.

The core is characterized, in the pure (non-rival, non-excludable) public good case, as the set of Pareto-efficient individually rational states (1987a, p. 10), where individual rationality means that every donor i is satisfied with his own individual contribution given his anticipation of the best reaction of the complementary coalition $N\setminus\{i\}$ to any deviation of himself.

[79] This variant of the Kolm-core corresponds, in Kolm's terminology, to the dichotomous core with Cournot group behavior: dichotomous because each coalition I faces the best group response of complementary coalition $N\backslash I$; and Cournot group behavior in the derivation of the best responses of complementary coalitions. See Kolm (1989) for alternative definitions, extensions and refinements, of cores with interdependent coalitions.

There is no room in this construct, by definition, for free-riding as rational individual or collective behavior that would result in a Pareto-inefficient social state (see my definition of free-riding in Section 6.1.1 and Footnote 55), but there remains the possibility of core emptiness, and the possibility that some individuals contribute nothing at some core solution (which must be then a non-symmetric efficient state).

These two possibilities are logically related in the following way.

Suppose without loss of generality that the initial endowments of the poor are null, let individual preferences and endowments be fixed, let the number m of "rich" agents (that is, the number of potential donors) measure the size of the social system, and denote by G_s a provision level of the public good that maximizes the vector of utilities of potential donors in a coalition of size s. Given the quasi-linearity of utility functions, G_s maximizes, equivalently, the sum of the utilities of coalition members $-G + sv(G)$. G_s is, therefore, well-defined and unique for all $s \geqslant 1$ since v is continuous and strictly concave. For any size m of the social system, any element t of its Kolm-core and any potential donor i, let $r_i(m, t) = t_i - v(G_m) + v(G_{m-1})$. When $t_i > 0$, the latter corresponds to i's benefit from "free-riding" in the sense of Kolm, that is, from contributing 0 instead of t_i (the difference between his utility $\omega_i + v(G_{m-1})$ from contributing 0 given that the best response of the complementary coalition of donors is then to provide G_{m-1} of the public good, and his actual utility $\omega_i - t_i + v(G_m)$ at t). A potential donor of a social system of size m contributes nothing at Kolm's core solution t if and only if $r_i(m, t) > 0$. Note that the average benefit from free-riding $(G_m/m) - v(G_m) + v(G_{m-1})$ in the social system of size m only depends on m. Let it be denoted by $r(m)$.

It is notably proved, then, that: (i) the Kolm-core of a social system of size m is non-empty if and only if the associate average benefit from free-riding $r(m)$ is $\leqslant 0$, or, equivalently, if and only if there exists some efficient t that makes individual non-contribution individually non-rational for all $i \leqslant m$ (1987a, p. 15); (ii) the average benefit from free-riding vanishes (converges to 0) as m grows to infinity (1987a, p. 16).

These results state, in other words, that average and individual benefits from individual non-contribution tend to vanish, and with them potential problems of existence of a core solution in the sense of Kolm, as the number of potential (identical, quasi-linear) donors becomes large. They rely in an essential way on the basic assumption that individuals face consistent maximizing reactions of the whole group to their decisions. The point made in these studies is therefore, in many respects, complementary from the point made by Olson and his followers: the latter show how the collective inconveniences from non-cooperative private provision of the public good can increase with group size; and the former how the increase in group size can reinforce cooperative solutions (and make the sharing of cooperative surplus more equitable) when the group reacts consistently to individual defections.

6.3. Mechanism design in BBV distributive social systems

The large body of literature that designs incentive compatible mechanisms[80] in standard finite public goods economies with complete information, reviewed notably in Groves and Ledyard (1987) or Moore (1992), applies immediately, with obvious adaptations, to BBV distributive social systems.

To the best of my knowledge, incentive compatibility has, on the contrary, not yet been studied in the general distributive or Pareto social systems defined in Sections 3 and 4. The extension of known results to the latter is not straightforward, at least at first sight. The optimistic conclusion of Walker (1981), for instance, on the possibility of attaining outcomes unanimously (weakly) preferred to the initial distribution, relies on his construction of an incentive compatible mechanism that implements Lindahl equilibrium. It does not extend as such, therefore, to general Pareto social systems, for the simple reason that standard Lindahl equilibrium is generally not unanimously weakly preferred to initial distribution in such systems (see the social system of Figure 16, in Section 6.1.2.2 above).

I will not review here the general results relative to incentive compatibility in general (BBV) public goods environments. The reader is referred to Groves and Ledyard (1987) and Moore (1992) for such general presentations. I will concentrate instead on the design of mechanisms more specifically related to distributive issues, namely, the tax-subsidy schemes for private contributions to pure public goods.

This body of literature deals with three analytically distinct sets of issues. The first is implementation, which looks for the achievement of Pareto-efficiency by means of an appropriate scheme of taxes and subsidies on private (non-cooperative) actions. The second is treasury efficiency, which designs the tax-subsidy scheme with an objective of minimization of the public budget for a given equilibrium level of provision of the public good. And the third is the comparative evaluation of the distributional consequences of the different tax-subsidy schemes on the equilibrium wealth and welfare of relevant social groups.

These questions are studied in two broad classes of models. In the first one, the state is abstracted: incentive variables and individual contributions are chosen both by individual agents in a two-stage decision process, where incentive variables are determined by simultaneous non-cooperative utility-maximizing decisions in the first stage, and individual contributions to the public good by simultaneous non-cooperative utility-maximizing decisions in the second stage. The second class of models introduces, more realistically, a public authority responsible for the design of the incentive mechanism, and studies the simultaneous determination of efficient tax-subsidy schemes by the state and non-cooperative utility-maximizing contributions by individual agents.

The models and their properties are detailed in Appendix A.2.

[80] That is, incentive mechanisms that yield Pareto-efficient social states when all agents play non-cooperative (Nash equilibrium) strategies.

7. Imperfectly substitutable transfers

This section goes back over the basic common features that characterize the stream of theoretical literature reviewed in Sections 3 to 6, and examines how the theory fits to corresponding social reality.

The basic assumption of the theory is a complex set of hypotheses synthesized in a notion of perfect substitutability of transfers, whose main foundations, already enumerated in the introduction to this chapter, consist of: (i) perfect and complete competitive markets; (ii) non-paternalistic utility interdependence; (iii) and the Cournot–Nash behavioral assumption.

The implications of these assumptions which have been mainly studied in the literature are: (i) the separability and neutrality properties, the variants of which have been reviewed in Sections 2 (see Section 2.2), 4 (notably Section 4.2) and 5 above; (ii) the public good problem of redistribution, examined in detail in Section 6, whose most common expression is the Pareto-inefficient underprovision of transfers at non-cooperative equilibrium (an aspect of Olson's conjecture); (iii) and, as a joint consequence of the former two, the full crowding-out of private gifts at distributive optimum.

This theoretical construct has been tested on two complementary grounds. Logical grounds first, with numerous contributions exhibiting counterexamples to the basic properties of the theory, mainly the neutrality property, which follow from selective violations of its basic assumption. And empirical grounds, second, with two substantial streams of literature performing econometric tests of neutrality on the one hand and experimental tests of non-cooperative underprovision of a public good on the other hand.

The review of literature presented below is selective on purpose. We use it mainly as an illustrative support to a general comment on the theory. Most of its aspects are the object of more substantial developments in several chapters of the Handbook, which we will mention when appropriate in the course of the section.

7.1. Logical tests of the perfect substitutability of transfers

The tests reviewed here identify elements of the assumption of perfect substitutability that are essential in the sense that their violation involves in general the refutation of a basic property of the theory, notably the neutrality property. They contribute in eliciting the internal structure of the assumption, and they produce families of constructs derived from the central theory, which can prove useful to understand various aspects of social reality that the latter cannot grasp (see Section 7.2). The presentation below goes through the three foundations of the theory recalled above.

7.1.1. Perfect competitive markets

Papers studying the implications of market imperfections concentrate on capital market failures. The results are formulated, accordingly, in the dynamic setup of infinite horizon economies with finite-lived, altruistically related generations. Two broad classes of constructs are considered.

A first family of models [see notably Altig and Davis (1993), and the models reviewed in Laitner (1997, 3.3, pp. 222–227)] supposes that borrowing constraints impede the programming of individual life cycle consumption and saving. Individual life is divided in three periods (youth, middle age, and old age). Capital market imperfections prevent individuals from borrowing in the first two periods of their life. Parents coexist with children over the last two periods of the life cycle, and are altruistically related to them. Altruism can be one-sided [Laitner (1997)], from parents to children, or two-sided [Altig and Davis (1993)]. The coexistence of generations over two periods of time allows for a variety of patterns of intergenerational transfers, which may notably combine bequests and lifetime transfers such as middle-aged parents paying for the education of their children. It is shown that public intergenerational transfers can foster steady state Pareto improvements in the presence of operative intergenerational transfers, notably when the following conditions hold simultaneously: binding borrowing constraints; and descending intergenerational transfers, which combine null bequests with operative lifetime transfers for the education of the young [see Altig and Davis (1993) for a comprehensive classification of conceivable steady state patterns of intergenerational transfers and their implications for neutrality]. Leaving aside inessential differences associated with the dynamic features of the framework, these types of results clearly involve the violation of two characteristic properties of general Pareto social systems, namely, separability (as implied by Theorem 4) and neutrality (Theorem 7).

The capital market imperfections considered in the second family of models appear more fundamental, as they are related to the time irreversibilities that govern and constrain long-run relationships between generations, already briefly evoked in Section 6.1.3 above. These models consider, accordingly, subsequent generations (instead of overlapping ones) with descending altruism. Their main characteristic feature consists of an institutional constraint forbidding negative bequests. These contributions exhibit (steady state) equilibrium situations where binding non-negativity constraints on bequests result in (market and distributive) Pareto inefficiency. The precise interpretation of the market failure implicit in this finding depends on the type of intertemporal operations involved: credit market imperfections of the type of liquidity constraints in Nerlove, Razin and Sadka (1984, 1987, 1988), and in Becker and Murphy (1988), which concentrate on parental investments in the human wealth of children (fertility decisions and education); insurance market imperfections such as moral hazard or adverse selection problems in Barsky, Mankiw and Zeldes (1986), Feldstein (1988), Sheshinski (1988) or Strawczynski (1994), which view bequests as an insurance device against income uncertainty of current (Barsky et al., Feldstein) or future (Sheshinski, Strawczynski) generations. These sources of market failures can be related, more fundamentally, as suggested above, to the fact that current generations make their decisions at a time when future generations do not yet exist, at least as full-fledged economic agents, and cannot, consequently, enter into contractual relationships with them (a case of "fundamental market incompleteness", so to speak). They imply the possibility of Pareto-improving public redistributions from future to current generations, on both grounds of market and distributive efficiency. Note that this possibility implies a violation of the separability property, but that it implies no violation of the neutrality property since intergenerational transfers are at a corner by assumption. They point to a type of justification of public intervention which appears very important in the context of long-run economic equilibrium,[81] and is ignored by construction in the Pareto social systems above.

7.1.2. Non-paternalistic utility interdependence

Non-paternalistic utility interdependence means essentially, in the context of Pareto social systems, that the sole purpose of gift-giving is the redistribution of market money wealth.

It has been a commonplace from the very beginning of the theory to notice that neutrality does not hold in the presence of other motives of giving, such as merit wants [e.g. Becker (1974, 3.C, pp. 1085–1087)], or anyone in the large variety of motivations implying that gift-giving "matters per se", is a "consumption good" for the donor so to speak, generally of the type of a "status good" or "relational good" such as: (i) renown and prestige [the gifts of the kula ring are a famous example: see Mauss (1924)]; (ii) social rank [as in the potlatch of the Kwakiutl: Mauss (1924)]; (iii) variants of the former more directly adapted to modern individualistic psychologies such as Andreoni's "warm-glow of gift-giving" [Andreoni (1989, 1990)]; (iv) or, at the complementary opposite, the pro-gift feelings fed by modern universalistic ethics, which includes the secular ethics of the multiple variants of socialism, humanitarianism and so on [see Kolm (1984), and his contributions to the present Handbook, for fairly exhaustive pictures of the motives of giving, and notably of the modern universalistic ethics which underlie some of them].

The analysis of gift motives is a central topic of this Handbook, and it is present implicitly or explicitly in virtually all chapters, although detailed more specifically in those of Bowles et al., Elster, Fehr, Hann, Lévy-Garboua et al., Kolm, Schokkaert, Sacco et al. and Thorne. It will suffice, for the limited purposes of the present section, to refer to four early mentions of violation of the neutrality property as an elementary consequence of the assumption that gift-giving matters per se, namely, Cornes and Sandler (1984a), Posnett and Sandler (1986), and Andreoni (1989, 1990).[82] These contributions consider variants of strong BBV distributive social systems, with modified utility functions of donors of the type $v_i(x_i, g_i, G)$, where individual donation g_i appears simultaneously as an individual consumption and as an additive contribution to the public

[81] See the last section of Chapter 14 by Arrondel and Masson of this Handbook for a detailed discussion of the policy implications of Becker and Murphy (1988).

[82] The non-neutrality results of Steinberg (1987) are related in some respects with those of Cornes and Sandler and Andreoni, but more complex in structure, as they combine the assumption that gift-giving matters per se with a (non-neutral) linear distortionary tax scheme very close to the tax scheme considered by Boadway, Pestieau and Wildasin (1989a). His results convey, in other words, two independent sources of non-neutrality: warm-glow, and the tax regime (the latter including the implicit assumption that donors do not see through the budget constraint of the government).

good. The utility functions of donors are assumed strictly increasing and strictly quasi-concave. Cornes and Sandler and Posnett and Sandler show that, when private donation and the public good are Hicksian complements, an agent's donation may increase in response to increased donations of others, even when all goods are strictly normal. And Andreoni exhibits non-neutralities which contradict the neutrality properties of Theorem 10(i) above, relative to public lump-sum redistributions (1989, pp. 1454–1457, 1990, Propositions 1 and 2, pp. 467–468), and of Theorem 11 above, relative to his 1988 distortionary tax scheme (1990, Proposition 3, p. 469).

7.1.3. Cournot–Nash behavioral assumption

The Cournot–Nash behavioral assumption (that is, the assumption that the non-cooperative interactions of individuals or groups are of the non-strategic type) is a tautology when status quo is a strong distributive equilibrium, as in the type of configurations considered in Section 6.1 above (liberal social contracts of self-centered (or BBV), non-jealous distributive social systems). The same applies to the Beckerian equilibria of general Pareto social systems with a single altruistic head and $n-1$ egoistic "kids", provided that there is no paradox of transfers, that is, no (practical) possibility of strategic manipulation of market prices by the individual or collective gifts of egoistic kids [see Section 4.3 above, Mercier Ythier (2004b, 4.3.2), and Kanbur's Chapter 26 of this Handbook]. These prima facie justifications of the Cournot–Nash behavioral assumption have been further elaborated in several interesting ways by Sugden (1985), Bergstrom (1989b) and Cornes and Silva (1999).

Sugden considers strong BBV distributive social systems with identical donors, whose utility functions are continuously differentiable, strictly increasing and strictly quasi-concave. He concentrates on the comparative statics of symmetric equilibrium with arbitrary conjectures of donors relative to the individual reactions of others to variations in their own contribution [see also the companion papers of Cornes and Sandler (1984b, 1985b) on the same issue]. Considering an exogenous variation in the contribution of a donor at an interior equilibrium, and supposing that the suitable regularity condition for the application of the implicit function theorem holds, one gets the following value for the derivative of the equilibrium provision of the public good with respect to this exogenous variation of individual contribution: $-\mathrm{MRS}(x,G)/(m-1+\mathrm{MRS}(x,G))$, where $\mathrm{MRS}(x,G) = \partial_x v(x,G)/\partial_G v(x,G)$. If private consumption and the public good are both normal goods, then $\mathrm{MRS}(x,G) > 0$, implying that the derivative lies in the open interval $]-1,0[$, and therefore that individuals with consistent (i.e. self-fulfilling) conjectures should expect other people to reduce their contributions when they increase their own. Studying the behavior of the derivative as m grows to infinity, the author establishes moreover that consistent expectations of matching contributing behavior (corresponding to a derivative < -1) hold for any m only if private consumption is an inferior good, a highly implausible condition. Sugden's conclusions extend in a simple way to status quo equilibrium, where non-negativity conditions on individual contributions are binding by assumption, yielding a formal justification of

Nash conjectures as the sole plausible type of consistent expectations for this particular type of equilibrium and social system.

In the frameworks of Bergstrom (1989b) and Cornes and Silva (1999), the family head ("social planner") has a non-paternalistic benevolent utility function defined on siblings' ophelimity distribution (one of the siblings interpretable as the egoistic self of the altruistic parent). The arguments of sibling i's (indirect) ophelimity function are the vector $a = (a_1, \ldots, a_n)$ of siblings' actions and the money transfer τ_i received from the head. Equilibrium is defined as subgame perfect Nash equilibrium of the two-stage game where: in the first stage, each sibling i anticipating the head's transfer function $\tau_i(a)$ determined at the second stage, chooses his own action so as to maximize his ophelimity, given the actions of others; in the second stage, the head, observing the vector a of siblings' actions, chooses his transfers so as to maximize his utility subject to the family budget constraint $\sum_i \tau_i(a) = R(a)$, where $R(a)$ denotes the family income associated with a. Bergstrom's definitions allow for (direct) ophelimity functions depending on any number of private goods (including the numéraire) and public goods. Cornes and Silva consider the special case where: sibling i's indirect ophelimity function is the outcome of the maximization of a strong BBV utility function $v_i(x_i, g_i + G_{-i})$ with respect to his individual contribution g_i, in his budget set $\{(x_i, g_i)\colon\ x_i + g_i = \omega_i + \tau_i,\ x_i \geqslant 0,\ g_i \geqslant 0\}$, given the aggregate contribution of others G_{-i}, and the transfer τ_i he receives from the head; and the family income does not depend on the vector of contributions of siblings to the public good. The Rotten Kid Theorem holds, that is, the subgame perfect equilibrium is the Pareto optimum (relative to ophelimities) which maximizes the head's utility in the set of feasible ophelimity distributions, whenever there is *conditional transferable ophelimity*, that is, whenever the (indirect) consumption preferences of sibling i admit a functional representation of the type $\alpha_i(a) + \beta(a)\tau_i$ for all i [Bergstrom (1989b, Proposition 1)]. The Rotten Kid Theorem holds in the setup of Cornes and Silva also, provided that individual contributions of siblings to the public good are all positive (and that the utility and ophelimity functions are differentiable, quasi-concave and strictly increasing). The assumptions of Cornes and Silva do not imply conditional transferable ophelimity, but Bergstrom establishes, nevertheless, that the latter is essentially implied by the Rotten Kid Theorem when no specific assumptions are made on preferences and technology [Bergstrom (1989b, Proposition 3)]. Conditional transferable ophelimity is verified in a trivial way by our Beckerian distributive social systems with a single altruistic head and $n - 1$ egoistic kids. Bergstrom's Proposition 1 provides therefore a formal justification to the *simultaneous* interactions implied by Cournot–Nash behavior for such contexts, as it states that the agents cannot gain any individual strategic advantage by "playing first" (behaving as Stackelberg leaders) in the altruistic gift game.

There is no such univoqual case for the Cournot–Nash behavioral assumption when: (i) advantageous strategic manipulations of market prices through gift-giving become a practical possibility, as should be the case in general when collective gift-giving is considered; (ii) or Cournot–Nash equilibrium is Pareto-inefficient.

The first type of configuration has been studied in the literature essentially through the transfer paradox (see Section 4.3, Kanbur's Chapter 26, and Section 15.4.3 of the introduction Chapter 1 of the Handbook).

The second type involves a large variety of sources of coordination problems (the so-called "exchange motives" for transfers), including notably: (i) market imperfections, some of them already briefly mentioned in Section 7.1.1 above, such as fundamental market incompleteness, informational and enforcement difficulties for the design and implementation of private contracts (that is, of contracts relative to the private allocation of private commodities), and technological non-convexities; (ii) the analogous impediments to the design and implementation of local or general social contracts, that is, of contracts relative to the treatment of public goods and non-pecuniary externalities by concerned individual agents; (iii) or complex transfer motives such as merit wants (paternalistic preferences), warm-glow, joy of giving and so on.

The analysis of this second type of configurations fed three substantial strands of theoretical literature on voluntary transfers in the last twenty years or so, relative to strategic bequests, the Samaritan's dilemma, and, more recently, the theory of charitable fundraising. This is briefly illustrated in the selective account below (see also Section 2.1 above).

Bernheim, Shleifer and Summers (1985) gave the original impulse to the game-theoretic studies on strategic bequests [see Masson and Pestieau (1997), and Chapter 13 of Laferrère and Wolff of this Handbook for well-documented surveys on this subject]. The social unit ("family") they consider is made of one altruistic head and two or more egoistic children. There are two commodities: consumption, and child care. Head utility is increasing in own consumption, child care and child utility. Child utility is increasing in own consumption and decreasing in attention to head. This specification, simple as it is, implicitly involves some notion of market imperfection: there is no market equivalent for child care from the viewpoint of the head [the same holds in the more general setup of Bergstrom (1989b) above]. The authors show that there is room, then, in general, for Pareto improvements from Cournot–Nash equilibrium, all involving increases in child care and transfers to children. Moreover, the head can capture all corresponding gains from exchange by behaving strategically, namely, by precommitting to a bequest rule. The social conditions of exchange therefore support, in this case, a strategic behavior of the head, as an accessible and individually rational way to reach Pareto-efficiency. The authors show, also, that neutrality does not hold, generally, in such a context.

The "Samaritan's dilemma" [Buchanan (1975)] refers to a type of game configuration which is symmetric, in some respects, to strategic bequeathing, namely, the strategic exploitation, by the beneficiary of a gift, of the benevolence of the donor. Three representative references are the contributions of Lindbeck and Weibull (1988), Bruce and Waldman (1991), and Coate (1995) [see also the examples of the "lazy rotten kid" and the "prodigal son" in Bergstrom (1989b)]. These models have a time structure, which is given a priori, with two periods and a capital market that functions in the first period. There are two types of agents, who coexist in both periods: an altruistic parent and (possibly altruistic) child in Lindbeck and Weibull (1988); altruistic rich and egoistic poor in

Bruce and Waldman (1991) and in Coate (1995). The setup is deterministic in the first two papers and stochastic in the third, uncertainty bearing on the wealth endowment of the poor in the latter. The game is sequential: agents take their investment (i.e. savings or insurance) decisions in the first period, anticipating the second-period optimal transfer schemes of donors. It is shown that interior subgame perfect Nash equilibrium is Pareto-inefficient in general, with first period investments of donees typically too small, and second period equilibrium gifts typically too large, relative to Pareto-efficient levels. The Rotten Kid Theorem fails, therefore. But the neutrality property does not [see notably Bernheim and Bagwell (1988) on the latter]. Summarizing, the Samaritan's dilemma literature differs from the mainstream theory outlined in Sections 3 to 6 only at the margin, by introducing intertemporal strategic interactions as an additional source of inefficiency in public good provision.

Finally, recent theoretical explanations of the role of charities as intermediaries in fundraising activities for the private provision of public goods build on various sources of coordination problems, notably: warm glow [Slivinski and Steinberg (1998)], information costs and competition of donors for social status [Glazer and Konrad (1996)], fix costs in the production of the public good [Andreoni (1998)], and so on. Coordination problems translate into unexploited opportunities of Pareto-improving actions. The models construe the intermediation of charities as the implementation of such opportunities, notably through the design of appropriate fundraising strategies (see Chapter 18 by Andreoni and Chapter 19 by Bilodeau and Steinberg in this Handbook for detailed reviews on this subject). This burgeoning strand of literature shares important features with strategic bequest literature. Both build on the same variant or extension of the Coase conjecture, stating that accessible Pareto improvements should be implemented sooner or later by means of appropriate institutional design, following the initiatives ("strategic" design and move) of individuals, local intermediaries or central authorities interested in capturing for themselves some fraction of the surplus, monetary, symbolic or else, so created. Strategic behavior is generated, in particular, in such contexts, as an endogenous step in the dynamic process of exhaustion of social exchange opportunities. A "corollary" of the conjecture is, consequently, that *Cournot–Nash* (i.e. non-strategic) *behavior should prevail at long-run social equilibrium*, since there remains then, by definition, no room for advantageous strategic deviations, and, notably, no advantage from "playing first".

To finish with this presentation of alternative specifications of individual altruistic behavior, let us briefly mention the so-called "Kantian behavior". This consists of a rational (that is, utility-maximizing) individual behavior which embodies an ethical rule in its specification, namely, the Kantian imperative to "choose that action which would, if also taken by similarly motivated others, result in a good outcome" [Collard (1992); see also the seminal contributions of Laffont (1975) and Collard (1978), and the recent contribution of Bilodeau and Gravel (2004) for generalizations and up-to-date list of references]. Sugden (1984), notably, provides precise definitions and analyzes equilibrium properties in a general model of private provision of a pure public good which encompasses, and allows for direct comparisons with, strong BBV distributive social systems.

We restrict our presentation of his definitions and results to the latter, for notational simplicity. Let g_i^* denote the (supposed unique) maximum of $g \to v_i(\omega_i - g, mg)$, that is, the individual contribution that maximizes i's utility when all donors contribute the same amount. Sugden defines the Kantian rule, which he names an *obligation of reciprocity*, as the (moral) obligation, for any individual i, either to contribute at least g_i^*, or to contribute at least as much as the smallest contribution of others. An equilibrium is then a vector of contributions such that the contribution of each agent i is the smallest contribution compatible with his reciprocity obligation, given the contributions of others. It is shown, under standard assumptions, that: (i) an equilibrium exists (op. cit.: Result 1, p. 778); (ii) equilibrium is generally not unique (op. cit.: IV, p. 778); (iii) an exogenous increase in an individual contribution at equilibrium can induce an increase in the contributions of other individuals, by creating an additional obligation for them (op. cit.: Result 4, p. 780); (iv) the public good is undersupplied at equilibrium, relative to the efficiency criterion of Pareto, if and only if individual equilibrium contributions are not all identical (op. cit.: Result 5, p. 781). Note that point (iii) contradicts the perfect substitutability of transfers. One can establish easily, finally, that the Kantian rule imposes binding constraints on Cournot–Nash free-riding at Sugden's equilibrium.[83] Combined with point (iv) above, the latter statement implies that Kantian behavior improves coordination of individual actions, but not enough to achieve Pareto-efficiency [except in the case of identical individuals studied by Laffont (1975), with the additional provision, then, that only one of the (possibly) multiple equilibria of Sugden is Pareto-efficient: see Sugden (1984, IV)].

7.2. *Empirical tests of the perfect substitutability of transfers*

An empirical evaluation has been performed on two types of testable implications of the perfect substitutability of transfers, understood as the complex set of hypotheses above (see the beginning of Section 7): the Pareto-inefficient underprovision of the public good at Cournot–Nash equilibrium; and the -1 elasticity of substitution of transfers.

7.2.1. *Experimental tests of Cournot–Nash individual behavior*

Cournot–Nash free-riding of individuals in public good provision has been the object of a large number of experimental tests [e.g., among many, Andreoni (1988b, 1995), Isaac and Walker (1988), Isaac, Walker and Williams (1994), or Laury, Walker and Williams (1999); see Ledyard (1995), and Fehr's Chapter 8 in this Handbook for a

[83] Suppose that individual i makes a positive contribution g_i^* to the public good at Sugden's equilibrium and denote by G^* the equilibrium provision level. Suppose moreover that g_i^* is not a single largest contribution, i.e. that there exists some $g_j^* \geqslant g_i^*$ with $j \neq i$. Sugden's Result 2, standard first-order conditions and the strict concavity of i's utility function imply then together that $\partial_{x_i} v_i(\omega_i - g_i^*, G^*) > \partial_G v_i(\omega_i - g_i^*, G^*)$, which means that i will increase his utility by diminishing his contribution if everyone else's contribution remains unchanged.

comprehensive review], with repeated conclusions that robustly contradict the various aspects of Olson's conjecture.

Experiments are usually built on the following broad common pattern. Participants play a symmetric game of voluntary provision of a pure public good with transferable utility. Cournot–Nash equilibrium and symmetric Pareto optimum are calculable, with an equilibrium provision level of the public good smaller than the Pareto-efficient provision level [the public good equilibrium provision is generally $= 0$; see, nevertheless, Laury, Walker and Williams (1999), for an exception]. Group size is usually small (4 or 5 players), but can rise up to 100 players in some experiments [e.g. Isaac, Walker and Williams (1994)]. The game played by a given group can be single-shot or repeated, with a number of rounds then usually ranging from 10 to 20. Experiments are arranged so that players cannot communicate directly with their group fellows. They receive accurate impersonal information on the past aggregate contributions of others at each round in the case of repeated games.

It is found notably that: (i) actual public good provision is significantly larger than calculated Cournot–Nash provision, and significantly smaller than Pareto-efficient provision, both in single-shot and in repeated games; (ii) there is some tendency for a decrease in public good provision from one round to the next in repeated games, provision remaining nevertheless significantly larger than Cournot–Nash level at all rounds; (iii) actual public good provision does not decrease, and is sometimes even found to increase when group size increases [e.g. Isaac, Walker and Williams (1994)]. Overall experimental evidence tends to support the idea that these results proceed from the widespread conscious propensity of individuals to behave cooperatively in such contexts, rather than from individual misperceptions of game structure or, at the other extreme, from individual strategic sophistication [e.g. Andreoni (1988b, 1995)].

Experimental findings characterize, therefore, a phenomenon of "pure" individual propensity to cooperate in public good provision contexts, where the adjective "pure" refers to the absence, in the experiments, of any interindividual communication between participants, and of any of the various forms of social mediation by which individuals communicate and/or constrain each other in the reality of social life (except, of course, impersonal information on the past contributions of others in repeated-game experiments). A natural interpretation of the results is, then, that cooperative behavior in experiments reproduces some kind of social training:[84] participants have learnt contextual cooperative behavior in their lifelong practice of real social life, which they reproduce without much variation in the artificial (and very short run) context of laboratory experiments (there is some variation though, as pointed out in finding (ii) above).

This raises in turn interesting questions of method, relative to the relevance of such experimental results as empirical tests of the theory. The whole construct developed in

[84] A detailed discussion of the social learning of prosocial behavior can be found, for instance, in Rushton (1982). See also Chapters 10, 3, 8, 7, 9 and 2 respectively by Bardsley and Sugden, Elster, Fehr and Schmidt, Levy-Garboua et al., Sacco et al., and Schokkaert in this Handbook.

Sections 2 to 6 above consists, essentially, of a logical reconstruction of social communication and social exchange relative to the redistribution of wealth (a socially highly mediated object indeed, as one of the major objects of the debate between liberalism and socialism, which characterizes, in many respects, modern politics in developed countries), that is, a theoretical explanation of a type of phenomenon which is abstracted, by construction, from the experiments above. The heart of the theory consists of hypothetical statements such as: "The liberal social contracts are the Pareto-efficient distributions unanimously preferred to the initial distribution whenever individuals are self-centered and non-jealous" (Section 6.1.2.1); or the somewhat looser variant of the Coase conjecture stating that "Cournot–Nash behavior should prevail at long-run social equilibrium" (Section 7.1.3). Such statements are not refuted, clearly, by the experiments above, and it is debatable whether they are empirically refutable at all. The next subsection addresses the latter question and proposes, with due provisions and qualifications, a partially positive answer to it.

7.2.2. *Measures of the elasticity of substitution of transfers*

The most straightforward candidate for an empirical test of the theory is the measure of the elasticity of substitution of transfers.

Theory states that lump-sum wealth transfers between agents connected by private transfers leave, generically, the distribution of wealth locally unchanged (neutrality, see Section 5.1, Theorem 7), implying one-for-one substitution of lump-sum transfers for private equilibrium transfers, that is, an elasticity of substitution of transfers $= -1$.

A related property is the crowding-out of private transfers at distributive optimum first noticed by Warr (1982). An elaborate version states that distributive equilibrium is a status quo equilibrium, generically unique, whenever agents are self-cenetred and non-jealous or are endowed with strong BBV utility functions (see notably Section 6.1.2.1, Theorem 14). The observable consequence is the absence of private transfers at (efficient) social equilibrium.

These properties of the theory are not directly testable, in a strict sense, for two reasons. The first problem is the logical possibility of equilibrium multiplicity, which is coincidental, in the abstract sense of mathematical transversality theory, in the case of status quo equilibrium, but is not coincidental in the general framework of the neutrality property of Theorem 7. The second problem stems from the fact that the assumptions underlying the observable implications are either conspicuously counterfactual idealizations[85] of social reality (perfectly competitive markets and lump-sum taxation, for the neutrality property of Theorem 7) or very difficult to observe (distributive efficiency

[85] By idealization I mean, here, a set of features selected in a process of constrained deliberation balancing their adequacy to phenomena with the inner consistency of the system of their relations on the one hand, and with their workable value from the standpoint of the discursive intellectual operations of reasoning and calculation on the other hand.

and the assumptions on individual preferences, for the crowding-out property of Theorem 14).

The "test" will consist, therefore, in a qualitative appreciation of the distance between empirical findings and the "predictions" which follow "naturally" from a good understanding and honest reading of the theory. It will be said, notably, that the theory is refuted if the substitution elasticity of transfers has the wrong sign or is closer to 0 than to -1, or if the share of private transfers in total redistribution is closer to 1 than to 0.

A small number of empirical estimates of substitution elasticities of transfers are available for private donations to charities and family inter vivos transfers for the post-war period.

The estimates calculated from US cross-sectional data on private charitable donations range from -0.15 [Kingma (1989)] to -0.30 [Abrams and Schmitz (1978)]. Posnett and Sandler (1989), working on private charitable donations in the UK, obtain an estimate which is not significantly different from 0 (see Table 1 in Schokkaert's Chapter 2 of this Handbook).

Altonji, Hayashi and Kotlikoff (1997) obtain an elasticity of -0.13 for US panel data on family inter vivos donations. Wolff (1998), following the same methodology on French data, finds a small positive elasticity of $+0.003$.

The studies reported above are narrowly focused in terms of the time-period (the last twenty years or so) and geographic area (Western developed countries) they consider. The picture is significantly altered when the scope of the study is widened so as to situate empirical results in the process of long-run economic development. Evidence relative to the history of economic and social development in the twentieth century elicits a substantial crowding-out of private redistribution by public transfers in relative terms, that is, in proportion of aggregate wealth and redistribution, in relation to the rise of the welfare state.

Roberts (1984) observes an irreversible and almost complete crowding-out of private financial assistance to the poor by public transfers in the United States of the 1930s. Private charitable donations did not disappear of course (and could even have maintained their share in disposable income), but "underwent a fundamental transformation ... away from the relief of poverty toward other activities" such as health services and social counseling.

Lampman and Smeeding (1983) observe that the share of disposable income devoted to private interfamily transfers diminishes slightly in the US from 1935 to 1979, a fact to be contrasted with the considerable rise of the share of public transfer programs in national income during the same period.

And the results of a series of studies conducted by Cox and several co-authors on microeconomic data of Eastern European and other developing countries fit perfect substitutability (the "altruistic model", in the terminology of this literature) much better than the comparable studies on microeconomic data of Western developed countries reported above (see Section 4 of Chapter 14 of Arrondel and Masson and Section 6 of Chapter 13 of Laferrère and Wolff of this Handbook for well-documented reviews of the tests of the altruistic model of family transfers).

While still very imperfect and partial, this set of empirical findings suggests the following scenario for the historical evolution of wealth equalizing transfers during the last century: (i) public wealth equalizing transfers growing much faster than aggregate income over the period, implying a considerable growth of the former both in absolute magnitude and in terms of their share in aggregate income and redistribution; (ii) a stability of, or moderate decline in, the share of private transfers in disposable income, combined with a transformation of their composition, and notably a sharp decline in purely redistributive private wealth transfers such as financial donations to the poor; (iii) and, as a consequence of points (i) and (ii), the present low degree of substitutability (USA) or even the complementarity (Western Europe) of public redistributive transfers and residual private redistributive transfers, with differences between the USA and Western European countries related to such institutional parameters as the share of wealth equalizing transfers in aggregate income (larger in Western Europe) and the tax incentives for private donations (more vigorous in the USA).

The theoretical framework developed in Sections 2 to 6 adjusts to such (presumed) facts through the following three main channels.

The first and principal channel is the introduction of the general class of self-centered and non-jealous distributive preferences, as an ideal representation of the spreading of distributive concerns and of the extension of their object, which seem to go along with economic and social development. This general representation includes notably, but does not reduce to, the traditional concerns relative to the welfare of the poor.

The second channel is the distributive liberal social contract, which predicts the full crowding-out of private redistributive transfers as result of the combination of the public good problem of redistribution and the neutrality property. This prediction of the theory is not refuted, in the main, by the facts above, but it must be adjusted to match the observable remanence of (presumably) residual private redistributive transfers.

Adjustment is performed through the notion of imperfect substitutability of transfers. It is assumed that the bulk of remaining private transfers are complementary of public redistributive transfers. The list of the potential origins of complementarity follows from the analysis of the content of the abstract assumption of perfect substitutability. The latter suggests two main sources of complementarity at long-run social equilibrium. One is the set of the various "imperfections" in the functioning of markets (notably capital markets) and in the administration of the distribution branch of public finance. The other one is the existence of non-redistributive individual motives for transfers, including notably: for charitable donations, the reflection in the preferences of donors of the social valuation (popularity, prestige, ethical appraisal, ...) of voluntary individual participation in public good achievements; and for family gift-giving, the valuation by individuals of the transmission of their individual characteristics, and notably of their human wealth, to their descendants.

8. Conclusion

Summarizing very briefly, the theory reviewed in this chapter contributes to the explanation of the socialization of a fraction of aggregate wealth through the constitution of a redistributive welfare state in the course of long-run economic development. Its prediction of a full crowding-out of private redistributive transfers, notably, can be interpreted as a property of the long-run social equilibrium in the absence of imperfections in market and transfer activities, that is, of a social state where: all opportunities of social exchange relative to wealth distribution as a public good have been exhausted by appropriate public or private initiatives (long-run social equilibrium); and where markets are complete and competitive, and information, transaction and enforcement costs relative to market and social exchanges are negligible.

These conclusions apply to the distribution of market money wealth. Their extension to human wealth confronts many serious difficulties in the theoretical framework above. The most fundamental of them seem to be related to the succession of generations, notably the conspicuous existence of important "non-redistributive" motives of transfers in the intergenerational transmission of human wealth (reflected for example in merit wants), and also the clear case for the incompleteness and other imperfections of capital markets at the corresponding time scale (notably fundamental incompleteness). An increasingly large fraction of modern welfare states correspond to public support to the provision of education, health and social insurance services. New advances in the theoretical analysis of the process of partial socialization of income and wealth which seems to characterize modern economic development certainly require a better understanding of these specificities of human wealth, their causes, and their consequences on development and society.

Appendix A

A.1. Proofs

THEOREM 4. *Let (w, u, ω) verify Assumption 2. Then, (p^*, a^*) is a social equilibrium of (w, u, ω) if and only if it verifies the following set of conditions:* (i) $p^* \gg 0$; (ii) $\sum_{i \in N} x_i(a^*) = (1, \dots, 1)$; (iii) *for all i:* (a) $x_i(a^*) \gg 0$; (b) $p^* x_i(a^*) = p^*(\omega_i + \Delta_i t(a^*))$; (c) *and there exist $\lambda_i > 0$ such that* $\partial_{x_i} u_i(x_i(a^*)) = \lambda_i p^*$; (iv) *for all (i, j):* (a) $-\partial_{u_i} w_i(u(x(a^*)))\lambda_i + \partial_{u_j} w_i(u(x(a^*)))\lambda_j \leqslant 0$; (b) *and* $(-\partial_{u_i} w_i(u(x(a^*)))\lambda_i + \partial_{u_j} w_i(u(x(a^*)))\lambda_j) t_{ij}(a^*) = 0$.

PROOF. The set of conditions (iii) and (iv) are the first-order conditions for the solutions to $\max\{w_i(u(x((a^*_{\setminus i}, a_i)))): a_i \in B_i(p^*, a^*)\}$ such that $x_i(a^*) \gg 0$, $i = 1, \dots, n$. In view of the differentiability and quasi-concavity of functions $w_i \circ u$ implied by Assumption 2, the first-order conditions are necessary and sufficient for such solutions by Arrow and Enthoven (1961: Theorem 1(b) (sufficiency) and Theorem 2 (necessity)). It will be sufficient, therefore, to establish that if (p^*, a^*) is an equilibrium then $(p^*, x(a^*)) \gg 0$. Assumptions 2(i)(b) and 2(ii)(b) imply together that $w_i \circ u$ is monotonic strictly increasing in x_i for all i, which implies readily in turn that all prices must be > 0 at equilibrium. Hence $p^*\omega_i > 0$ for all i by Assumption 2(iv), that is, all individuals have a positive pre-transfer wealth at equilibrium. Assumptions 2(i)(c), 2(ii)(b) and 2(iii)(b) imply then together that the equilibrium allocation $x(a^*)$ is $\gg 0$. □

THEOREM 5. *Let (w, u, ω) verify Assumption 2 and suppose moreover that v_i is differentiable in $\mathbb{R}^l_{++} \times \mathbb{R}_{++}$ for all i. Then, (p^*, a^*) is a social equilibrium of (w, u, ω) if and only if $(p^*, x(a^*), p^*t(a^*))$ is a social equilibrium with money gifts of (w, u, ω).*

PROOF. Notice first that p^* must be $\gg 0$ in both definitions of social equilibrium by Assumptions 2(i)(b) (monotonic strictly increasing ophelimity) and 2(ii)(b) (utility strictly increasing in own ophelimity).

Let $(p^*, x(a^*), \tau^*)$ be a social equilibrium with money gifts. Assumptions 2(i)(c), 2(ii)(b), 2(iii)(b) and 2(iv) imply together that $x(a^*) \gg 0$ and that $p^*\omega_i + \Delta_i\tau^* > 0$ at τ_i^* solution of $\max\{w_i(v_1(p^*, p^*\omega_1 + \Delta_1(\tau^*_{\setminus i}, \tau_i)), \ldots, v_n(p^*, p^*\omega_n + \Delta_n(\tau^*_{\setminus i}, \tau_i)))$: $\tau_i \geqslant 0$ and $p^*\omega_i + \Delta_i(\tau^*_{\setminus i}, \tau_i) \geqslant 0\}$ for all i. The first-order conditions are therefore necessary [Arrow and Enthoven (1961: Theorem 2)] for the solutions of the programs above, and for the programs $\max\{u_i(x_i)$: $x_i \geqslant 0$ and $p^*x_i \leqslant p^*\omega_i + \Delta_i\tau^*\}$. Letting $r^* = (p^*\omega_1 + \Delta_1\tau^*, \ldots, p^*\omega_n + \Delta_n\tau^*)$ and $v^* = (v_1(p^*, r_1^*), \ldots, v_n(p^*, r_n^*))$, we get: for all i, $x_i(a^*) \gg 0$, $p^*x_i(a^*) = p^*(\omega_i + \Delta_i t(a^*))$, and there exists $\lambda_i > 0$ such that $\partial_{x_i} u_i(x_i(a^*)) = \lambda_i p^*$; for all (i, j), $\partial_{u_j} w_i(v^*)\partial_{r_j} v_j(p^*, r_j^*) \leqslant \partial_{u_i} w_i(v^*)\partial_{r_i} v_i(p^*, r_i^*)$ and $(-\partial_{u_i} w_i(v^*)\partial_{r_i} v_i(p^*, r_i^*) + \partial_{u_j} w_i(v^*)\partial_{r_j} v_j(p^*, r_j^*))\tau_{ij}^* = 0$. A well-known application of the envelope theorem implies moreover that $\partial_{u_i} w_i(v^*)\partial_{r_i} v_i(p^*, r_i^*) = \lambda_i$ for all i. From these conditions and Theorem 4, we deduce therefore that if (p^*, x^*, p^*t^*) is a social equilibrium with money gifts, then $(p^*, a(x^*, t^*))$ is a social equilibrium.

Conversely, let (p^*, a^*) be a social equilibrium. Then $u_i(x_i(a^*)) = v_i(p^*, p^*(\omega_i + \Delta_i t(a^*)))$ for all i as a simple consequence of the definition of equilibrium and the assumption that utility is strictly increasing in own ophelimity. Moreover, it follows readily from definitions and from the fact that $p^* > 0$, that if a_i^* solves $\max\{w_i(u(x((a^*_{\setminus i}, a_i))))$: $a_i \in B_i(p^*, a^*)\}$, then $p^*t_i(a^*)$ solves $\max\{w_i(v_1(p^*, p^*\omega_1 + \Delta_1(p^*t_{\setminus i}(a^*), \tau_i)), \ldots, v_n(p^*, p^*\omega_n + \Delta_n(p^*t_{\setminus i}(a^*), \tau_i)))$: $\tau_i \geqslant 0$ and $p^*\omega_i + \Delta_i(p^*t_{\setminus i}(a^*), \tau_i) \geqslant 0\}$. Therefore $(p^*, x(a^*), p^*t(a^*))$ is a social equilibrium with money gifts. □

THEOREM 7. *Suppose that (w, u) verifies Assumption 2.* (i) *Distributive policy is globally neutral if and only if set $u(M')$ is a singleton.* (ii) *For all $x \in M'$, $\Omega'(x)$ is a convex set of dimension $l(n - c(\gamma(x)))$, where $c(\gamma(x))$ denotes the number of connected components*[86] *of graph $\gamma(x)$.* (iii) *In particular: distributive policy is locally weakly neutral*

[86] See Footnotes 45 and 46 in Section 5.1 for the definitions of connected digraphs and connected components.

at an element ω^0 of the interior of $\Omega'(x)$ in $\{\omega\colon \omega_i > 0$ for all $i\}$ if and only if $\gamma(x)$ is connected.

The proof is adapted from Mercier Ythier (2000a, pp. 65–67). It proceeds in two steps. I establish first the following lemma (the theorem is proved next):

LEMMA. *For all $p \in \mathbb{R}^l_{++}$ and all gift vector t, there is a gift vector t' such that $g(t')$ is a forest*[87] *and $p\Delta_i t' = p\Delta_i t$ for all i.*

PROOF. Consider a circuit $\Gamma = ((i_k, j_k))_{1 \leqslant k \leqslant m}$ of $g(t)$.

Suppose without loss of generality that $pt_{i_1 j_1} = \min_k pt_{i_k j_k}$, and define recursively the following two orientation classes of the darts of Γ: dart (i_1, j_1) has positive orientation; dart (i_{k+1}, j_{k+1}) has positive (resp. negative) orientation if dart (i_k, j_k) either has positive orientation and is such that $j_k = i_{k+1}$ (resp. $j_k = j_{k+1}$), or has negative orientation and is such that $j_k = j_{k+1}$ (resp. $j_k = i_{k+1}$) (with the convention that $(i_{m+1}, j_{m+1}) = (i_1, j_1)$). The adjacent darts (i_k, j_k) and (i_{k+1}, j_{k+1}) thus have identical (resp. opposite) orientations in the circuit if the head j_k of the former coincides with the tail i_{k+1} (resp. head j_{k+1}) of the latter. This orientation is well-defined, for if a dart had simultaneously a positive and negative orientation, then this should be the case of all darts by the recursive definition above, and this would imply in turn that Γ has a single vertex i and a single dart (i, i), which contradicts the definition of $g(t)$.

There exists a gift vector t^1 such that: $pt^1_{i_k j_k} = pt_{i_k j_k} - pt_{i_1 j_1}$ whenever (i_k, j_k) has positive orientation in Γ; $pt^1_{i_k j_k} = pt_{i_k j_k} + pt_{i_1 j_1}$ whenever (i_k, j_k) has negative orientation in Γ; $t^1_{i_k j_k} = t_{i_k j_k}$ whenever i or j is not a vertex of Γ. And one verifies readily that $g(t^1)$ does not contain circuit Γ (dart (i_1, j_1) has been deleted: $pt^1_{i_1 j_1} = 0$ by construction, and $p \gg 0$ by assumption, so that $t^1_{i_1 j_1} = 0$). Moreover $p\Delta_i t^1 = p\Delta_i t$ for all i since: $\Delta_i t^1 = \Delta_i t$ whenever i is not a vertex of Γ; if i is a common vertex of two adjacent darts (j, i) and (i, k) of identical, positive (resp. negative) orientation in Γ, then $p(t^1_{ik} - t^1_{ji}) = pt_{ik} - pt_{i_1 j_1} - pt_{ji} + pt_{i_1 j_1} = p(t_{ik} - t_{ji})$ (resp. $p(t^1_{ik} - t^1_{ji}) = pt_{ik} + pt_{i_1 j_1} - pt_{ji} - pt_{i_1 j_1} = p(t_{ik} - t_{ji})$); if i is a common vertex of two adjacent darts (j, i) and (k, i) of opposite orientations in Γ, the orientation of (j, i) being positive (resp. negative), then $p(t^1_{ji} + t^1_{ki}) = pt_{ji} - pt_{i_1 j_1} + pt_{ki} + pt_{i_1 j_1} = p(t_{ji} + t_{ki})$ (resp. $p(t^1_{ji} + t^1_{ki}) = pt_{ji} + pt_{i_1 j_1} + pt_{ki} - pt_{i_1 j_1} = p(t_{ji} + t_{ki})$). The conclusion follows then from a recursive application of the algorithm above to all circuits of $g(t)$ (in finite number since $g(t)$ is finite). □

PROOF OF THEOREM 7. Part (i) of Theorem 7 is a simple corollary of Theorem 4.

Let $x \in M'$. Part (iii) is a straightforward consequence of part (ii).

Let us establish (ii).

[87] A digraph is a forest if it contains no circuit (Section 3.4.2.1).

The convexity of $\Omega'(x)$ is straightforward.

Let $p \gg 0$ be the unique price vector of S_l supporting x. Denote by Ψ the set of spanning forest subdigraphs[88] of $\gamma(x)$, and, for all $\Gamma \in \Psi$, let $\Omega_\Gamma(x)$ be the convex set $\{\omega$: $\omega_i > 0$ for all i; and $\exists t$ such that: $t_{ij} > 0$ if and only if $(i, j) \in \Gamma$; and $px_i = p(\omega_i + \Delta_i t)$ for all $i\}$. We have then $\Omega'(x) = \bigcup_{\Gamma \in \Psi} \Omega_\Gamma(x)$ since, by the lemma above, the wealth transfers associated with any gift vector can be achieved by a gift vector whose associate graph is a forest subgraph of the former. From the definition of a spanning subgraph, we know that $c(\Gamma) \geqslant c(\gamma(x))$ for all $\Gamma \in \Psi$. And from Tutte (1984, Theorem I.36) there exists a $\Gamma \in \Psi$ such that $c(\Gamma) = c(\gamma(x))$. It suffices to prove, therefore, that convex set $\Omega_\Gamma(x)$ has dimension $l(n - c(\Gamma))$ whenever Γ is a spanning forest subdigraph of $\gamma(x)$ such that $c(\Gamma) = c(\gamma(x))$.

Consider thus, from now on, a $\Gamma \in \Psi$ such that $c(\Gamma) = c(\gamma(x))$. By definition of a spanning graph, the set of vertices of Γ is N. By definition of a forest, we must have $i \neq j$ whenever $(i, j) \in \Gamma$ (loop-darts (i, i) are 1-circuits). Let the incidence matrix[89] of Γ be denoted by M_Γ. A well-known result of graph theory is then that matrix M_Γ has full rank $n - c(\Gamma)$, equal to the number of darts of Γ, if and only if Γ is a forest graph [e.g. Berge (1970, Theorem 1)].

For any t such that $t_{ij} > 0$ if and only if $(i, j) \in \Gamma$, denote by t_Γ the vector obtained from t by deleting its coordinates t_{ij} such that $(i, j) \notin \Gamma$. Let pt_Γ denote the vector of bilateral wealth transfers associated with t_Γ. The product $pt_\Gamma \cdot M_\Gamma^{\mathrm{T}}$ of the row vector pt_Γ and the transpose M_Γ^{T} of the incidence matrix of Γ is then the vector of net transfers $p\Delta t = (p\Delta_1 t, \ldots, p\Delta_n t)$. Denoting $px = (px_1, \ldots, px_n)$, we have therefore $\Omega_\Gamma(x) = \{\omega$: $\omega_i > 0$ for all i; and $\exists t$ such that: $t_{ij} > 0$ if and only if $(i, j) \in \Gamma$; and $px = p\omega + pt_\Gamma \cdot M_\Gamma^{\mathrm{T}}\}$.

Since Γ has exactly $n - c(\gamma(x))$ darts the dimension of convex set $\{t$: $t_{ij} > 0$ if and only if $(i, j) \in \Gamma\}$ is $l(n - c(\gamma(x)))$. From this and the fact that $p \neq 0$ and rank $M_\Gamma^{\mathrm{T}} = n - c(\gamma(x))$, it follows readily that the dimension of $\Omega_\Gamma(x)$ is $l(n - c(\gamma(x)))$. $\square$

COROLLARY 5. *Suppose that (w, u, ω) verifies Assumption 2, and let (p, a) be an equilibrium.* (i) *$(p, x(a))$ is an equilibrium price-allocation vector of $(w, (\omega_1 + \Delta_1\theta, \ldots, \omega_n + \Delta_n\theta))$ if and only if there exists t such that: $g(t) \subset \gamma(x(a))$; and $p(t_{ij} - t_{ij}(a) - (t_{ji} - t_{ji}(a))) + p(\theta_{ij} - \theta_{ji}) = 0$ for all (i, j).* (ii) *In particular, $(p, x(a))$ is not an equilibrium price-allocation vector of $(w, (\omega_1 + \Delta_1\theta, \ldots, \omega_n + \Delta_n\theta))$ whenever θ implies net transfers of wealth between connected components of $\gamma(x(a))$, that is, whenever there is a connected component γ of $\gamma(x(a))$ such that $\sum_{(i,j) \in V_\gamma \times (N \setminus V_\gamma)} p(\theta_{ij} - \theta_{ji}) < 0$, where V_γ denotes the set of vertices of γ.*

PROOF. The first part is a simple consequence of Theorem 4. The second part follows from the first part and the simple remark that if θ transfers wealth away from a connected

[88] A subdigraph γ' of digraph γ *spans* the latter if it has the same set of vertices.

[89] See Section 3.1.2.2 for the definition.

component γ of $\gamma(x(a))$, this aggregate wealth transfer cannot be offset by any t such that $g(t) \subset \gamma(x(a))$. □

COROLLARY 6. *Suppose that* (w, u, ω) *verifies Assumption 2, and let* (p, a) *be an equilibrium. Then, there exists a neighborhood* V *of* 0 *in* $\{\theta\colon \theta_{ij} = 0$ *whenever* i *and* j *are in two distinct connected components of* $g(t(a))\}$ *such that, for all* $\theta \in V$, $(p, x(a))$ *is an equilibrium price-allocation vector of* $(w, (\omega_1 + \Delta_1\theta, \dots, \omega_n + \Delta_n\theta))$.

PROOF. From the lemma in the proof of Theorem 7, we can restrict ourselves to systems of public net transfers θ with forest digraph $g(\theta)$. From the proof of Theorem 7, the convex sets $\{p\Delta\theta\colon\ g(\theta)$ is a forest such that $\theta_{ij} = 0$ whenever i and j are in two distinct connected components of $g(t(a))\}$ and $\{px(a) - p\omega - p\Delta t\colon\ g(t)$ is a forest subdigraph of $g(t(a))\}$ have the same dimension $n - c(g(t(a)))$. The intersection of their relative interiors contains 0 since $p\omega \gg 0$ and $px(a) \gg 0$ (that is, non-negativity conditions on endowments (resp. consumption) are not binding locally for public (resp. private) transfers). Hence the conclusion. □

THEOREM 13. *Let* (w, ω) *be such that for all* i: $\omega_i > 0$; w_i *is quasi-concave and* $w_i(x) > w_i(x')$ *implies* $w_i(\lambda x + (1-\lambda)x') > w_i(x')$ *for all* $\lambda \in]0, 1[$. *If* t^* *is a distributive equilibrium of* (w, ω) *with a forest graph, and if there exists an agent* i *who makes positive gifts to all other agents at* t^* *(that is,* $t_{ij} > 0$ *for all* $j \neq i$*), then the associate equilibrium distribution* $x(\omega, t^*)$: (i) *is a weak distributive optimum of* w; (ii) *and maximizes* i*'s utility in* S_n *(that is,* $w_i(x(\omega, t^*)) = \max\{w_i(x)\colon\ x \in S_n\}$*).*

PROOF. (i) All we have to do to prove (i) is to establish the correspondence between Nakayama's definition of equilibrium and our Definition 1. The distributive game of Nakayama differs from a Pareto social system in one respect only: the agents are not allowed to use the gifts they receive in any way other than consuming them. They cannot, in particular, use them to finance their own gifts. If, therefore, a distributive equilibrium is such that donors are not beneficiaries of gifts (that is, if $t_{ji} = 0$ for all j whenever $t_{ij} > 0$ for some j), it must be also an equilibrium of Nakayama. And this clearly is the case, in particular, if the distributive equilibrium verifies the assumption of Theorem 13. For suppose that an agent, say i, gives to all others, and that a donor j is also the beneficiary of a gift from k: then there must be a circuit connecting agents i, j and k (two of them identical at most) in the digraph associated with t^*.

(ii) Let us prove now that $w_i(x(\omega, t^*)) = \max\{w_i(x)\colon\ x \in S_n\}$. Suppose the contrary, that is, $w_i(x') > w_j(x(\omega, t^*))$ for some $x' \in S_n$. The convexity assumption on w_i implies then $w_i(\lambda x' + (1-\lambda)x(\omega, t^*)) > w_j(x(\omega, t^*))$, and the convexity of S_n implies $\lambda x' + (1-\lambda)x(\omega, t^*) \in S_n$, for all $\lambda \in]0, 1]$. The set of wealth distributions accessible to i is $A = \{x(\omega, (t^*_{\setminus i}, t_i))\colon\ x_i(\omega, (t^*_{\setminus i}, t_i)) \geqslant 0\}$. Since $t^*_{ij} > 0$ for all j, the equilibrium distribution $x(\omega, t^*)$ lies in its relative interior in S_n, so that for any $\lambda > 0$ picked close enough to 0, there exists a gift-vector t_i^λ such that $\lambda x' + (1-\lambda)x(\omega, t^*) = x(\omega, (t^*_{\setminus i}, t_i^\lambda)) \in A$, a contradiction. □

COROLLARY 7. *Let* (w, ω) *be such that* $\omega_i > 0$ *for all* i, *and* $t^* \neq 0$ *be a distributive equilibrium of* (w, ω). *Suppose that:* (a) *either there exists an agent* i, *with a strictly quasi-concave utility function* w_i (*that is, a quasi-concave* w_i *such that* $w_i(x) \geqslant w_i(x')$ *implies* $w_i(\lambda x + (1-\lambda)x') > w_i(x')$ *for all* $\lambda \in]0, 1[$ *and all* (x, x') *such that* $x \neq x'$), *who makes positive gifts to all others at* t^*, *and all others are egoistic* ($w_j : x \to x_j$ *for all* $j \neq i$); (b) *or* $n = 2$ *and all utility functions are strictly quasi-concave. Then* t^* *is a strong distributive equilibrium and* $w_j(x(\omega, t^*)) = \max\{w_j(x)\colon\ x \in S_n\}$ *for every donor* j.

PROOF. Suppose (a). Obviously we must have $t_{\setminus i} = 0$ and $t'_j = 0$ for all $j \in I \setminus \{i\}$ for all t'_I that blocks t. Moreover, $w_i(x(\omega, t^*)) = \max\{w_i(x)\colon\ x \in S_n\}$ by Theorem 13(ii), and $\operatorname{argmax}\{w_j(x)\colon\ x \in S_n\}$ reduces to $x(\omega, t^*)$ by the strict quasi-concavity of w_i, so that i does not belong to any coalition blocking t^*. Therefore, t^* is unblocked by any coalition, that is, t^* is a strong distributive equilibrium of (w, ω).

Suppose (b). Applying the lemma of the proof of Theorem 7, we know that there exists a distributive equilibrium t' of (w, ω), with a forest digraph $g(t')$, such that $x(\omega, t') = x(\omega, t)$ (just subtract $\min\{t_{12}, t_{21}\}$ to t_{12} and t_{21}). Theorem 13(i) then implies that the equilibrium distribution is a weak distributive optimum of w. And the strict quasi-concavity of utility functions readily implies the equivalence of weak and strong distributive efficiency. Since $n = 2$, the distributive equilibrium t is strong if and only if the equilibrium distribution is a strong distributive optimum. Finally, the proof of Theorem 13(ii) readily implies that $w_j(x(\omega, t^*)) = \max\{w_j(x)\colon\ x \in S_n\}$ for every donor j when $n = 2$ and utility functions have the relevant convexity property. □

THEOREM 14. *Suppose that* w *either is a strong BBV distributive social system, or verifies local non-satiation of the distributive Paretian preordering* (*in short, local non-satiation: cf. Footnote* 31), *self-centredness and non-jealousy. Then,* 0 *is a strong distributive equilibrium of* (w, ω) *if and only if* ω *is a strong distributive optimum of* w.

In the proof[90] I establish first the following lemma [adapted from Mercier Ythier, (1998b, Lemma (ii), p. 264); the theorem is proved next]:

LEMMA. *If* 0 *is blocked by coalition* I *playing gift-vector* t_I *in social system* (w, ω), *then there exist a non-empty coalition of donors* $J \subset I$, *a non-empty set of receivers* $K \subset N \setminus J$, *and a gift-vector* $t^* \neq 0$ *such that:* $x(\omega, t^*) = x(\omega, (0_{\setminus I}, t_I))$; $t^*_{jk} > 0$ *if and only if* $(j, k) \in J \times K$; *for all* $j \in J$, $\omega_j > x_j(\omega, t^*) \geqslant 0$ *(that is, the elements of* I *are net donors); for all* $k \in K$, $x_k(\omega, t^*) > \omega_k$ *(that is, the elements of* K *are net receivers).*

[90] Adapted from the proof of Mercier Ythier (1998b, Theorem 4, pp. 271–272). See Footnote 60 above for a comparison of Theorem 14 with my closely similar results of (1998a, 1998b).

PROOF. Suppose that 0 is blocked by coalition I in social system (w, ω), i.e. that there exists a gift-vector t such that, for all $i \in I$, $x_i(\omega, (0_{\setminus I}, t_I)) \geqslant 0$, and $w_i(x(\omega, (0_{\setminus I}, t_I))) \geqslant w_i(\omega)$ with a strict inequality for at least one i. Then, necessarily: $x(\omega, (0_{\setminus I}, t_I)) \neq \omega$; $x_i(\omega, (0_{\setminus I}, t_I)) = \omega_i + \sum_{j \in I} t_{ji} \geqslant \omega_i$ whenever $i \notin I$; and therefore there exist $i \in I$ such that $x_i(\omega, (0_{\setminus I}, t_I)) < \omega_i$ and $i \in N$ such that $x_i(\omega, (0_{\setminus I}, t_I)) > \omega_i$. Denote by: J the non-empty set of agents j such that $x_j(\omega, (0_{\setminus I}, t_I)) < \omega_j$, ($J \subset I$; it is the set of "net givers"); K the non-empty set of agents k such that $x_k(\omega, (0_{\setminus I}, t_I)) > \omega_k$ (K is the set of "net receivers"); $\theta = \sum_{j \in J}(\omega_j - x_j(\omega, (0_{\setminus I}, t_I))) = \sum_{k \in K}(x_k(\omega, (0_{\setminus I}, t_I)) - \omega_k) > 0$ the total amount of redistributed wealth; λ_j the share $\theta^{-1}(\omega_j - x_j(\omega, (0_{\setminus I}, t_I)))$ of agent $j \in J$ in θ; μ_k the share $\theta^{-1}(x_k(\omega, (0_{\setminus I}, t_I)) - \omega_k)$ of agent $k \in K$ in θ; t^* the gift-vector such that $t^*_{jk} = \lambda_j \mu_k \theta > 0$ whenever $(j, k) \in J \times K$, $t^*_{jk} = 0$ otherwise. We have then $x(\omega, t^*) = x(\omega, (0_{\setminus I}, t_I))$, $0 \leqslant x_j(\omega, t^*) = \omega_j - \sum_{k \in K} t^*_{jk} < \omega_j$ for all $j \in J$, $x_k(\omega, t^*) = \omega_k + \sum_{j \in J} t^*_{jk} > \omega_k$ for all $k \in K$, and the lemma is established. □

PROOF OF THEOREM 14. (a) Suppose, first, that w verifies local non-satiation, self-centredness and non-jealousy, and let P denote its set of strong distributive optima and $C = \{\omega\colon 0 \text{ is a strong equilibrium of } (w, \omega)\}$. We have $C \subset P \subset S_n$ as a simple consequence of the definition of strong equilibrium (that implies that 0 is unblocked by the grand coalition N) and local non-satiation (that implies $P \subset S_n$). It suffices, therefore, to establish that $P \subset C$.

Suppose that $\omega \notin C$, and let us prove that, then, $\omega \notin P$.

By assumption, there exists a coalition I and a gift-vector t_I such that, for all $i \in I$: $x_i(\omega, (0_{\setminus I}, t_I)) \geqslant 0$; and $w_i(x(\omega, (0_{\setminus I}, t_I))) \geqslant w_i(\omega)$, with a strict inequality for at least one i. And by the lemma, there exist a non-empty coalition of net givers $J \subset I$, a non-empty set of net receivers $K \subset N \setminus J$, and a gift-vector $t^* \neq 0$ such that: $x(\omega, t^*) = x(\omega, t)$; $t^*_{jk} > 0$ if and only if $(j, k) \in J \times K$; for all $j \in J$, $\omega_j > x_j(\omega, t^*) \geqslant 0$; and for all $k \in K$, $x_k(\omega, t^*) > \omega_k$. Let $x^* = x(\omega, t) = x(\omega, t^*)$. And suppose, without loss of generality, that $J = \{1, \ldots, m\}$, with $m < n$ and that $K = \{m+1, \ldots, m+p\}$, with $m + p \leqslant n$.

Suppose first that $x^*_j \geqslant x^*_k$ for all $(j, k) \in J \times K$. The positive components of t^*, ranked in increasing lexicographic order (see Footnote 11), make a sequence of bilateral progressive transfers from ω to x^*. Formally: let $x^0 = \omega$; and, for any given $(j, k) \in J \times K$, let $x^{(j-1)p+k-m} = x^{(j-1)p+k-m-1} + t^*_{jk} e^{jk}$. Observe that for all $(j, k) \in J \times K$ and all $x \in [x^{(j-1)p+k-m-1}, x^{(j-1)p+k-m}[$: $x_j > x_k$. Self-centredness implies that w_k is increasing along $[x^{(j-1)p+k-m-1}, x^{(j-1)p+k-m}]$ for all $(j, k) \in J \times K$. Non-jealousy implies that w_i is non-decreasing along $[x^{(j-1)p+k-m-1}, x^{(j-1)p+k-m}]$ for all $(i, j, k) \in N \times J \times K$ such that $i \in N \setminus \{j, k\}$. But $x^{mp} = x^*$ by construction. Therefore $w_i(x^*) \geqslant w_i(\omega)$ for all $i \in N \setminus J$, with a strict inequality for all $k \in K$. Since, moreover, $w_i(x^*) \geqslant w_i(\omega)$ for all $i \in I$ by assumption, and $J \subset I$, we have $\omega \notin P$.

Suppose next that $x^*_j < x^*_k$ for some $(j, k) \in J \times K$ (that is, the bilateral wealth transfer from j to k is "two large" for some net donor j, given the self-centredness assumption).

We reduce this case to the former in two steps. We first proceed to bilateral progressive transfers from social state (x^*, t^*), by diminishing the wealth transfer from j to k whenever $x_j^* < x_k^*$ for some $(j, k) \in J \times K$, until either $x_j \geqslant x_k$ or $t_{jk} = 0$ at the resulting social state (x, t). We establish next that the latter social state obtains, from $(\omega, 0)$, by a sequence of bilateral progressive transfers which do not decrease the utilities of net donors, that is, the case considered in the former paragraph.

Let $((j_q, k_q))_{1 \leqslant q \leqslant \#Q}$ denote the sequence of elements of set $Q = \{(j, k) \in J \times K\colon x_j^* < x_k^*\}$ ranked in (increasing) lexicographic order. Define the following sequence of bilateral progressive redistributions from x^*: $x^0 = x^*$; for any $q \in \{0, \ldots, \#Q - 1\}$, $x^{q+1} = x^q + t_{j_q k_q} e^{j_q k_q}$, where $t_{j_q k_q} = \min\{t^*_{j_q k_q}, (1/2)(x^*_{k_q} - x^*_{j_q})\}$. Denote $x^{**} = x^{\#Q}$, and let t^{**} be defined from t^* and the above sequence of progressive transfers $(t_{j_q k_q})_{1 \leqslant q \leqslant \#Q}$ by: $t^{**}_{j_q k_q} = t^*_{j_q k_q} - t_{j_q k_q}$ for all $(j_q, k_q) \in Q$; $t^{**}_{jk} = t^*_{jk}$ whenever $(j, k) \notin Q$. By construction: $x^{**} = x(\omega, t^{**})$; and $t^{**}_{jk} = 0$ whenever $x^{**}_j < x^{**}_k$.

We now proceed to the second step, which will conclude this part of the proof.

Self-centredness implies that w_{j_q} is increasing along $[x^q, x^{q+1}]$ for all $q \in \{0, \ldots, \#Q - 1\}$. Non-jealousy implies that w_i is non-decreasing along $[x^q, x^{q+1}]$ for all $(i, q) \in N \times \{0, \ldots, \#Q - 1\}$ such that $i \in N \setminus \{j_q, k_q\}$. Therefore, all elements of J strictly prefer x^{**} to x^*. And there is at least one $j \in J$ who is a net donor at x^{**}, for, otherwise, $x^{**} = \omega$, while $w_j(x^{**}) > w_j(x^*) \geqslant w_j(\omega)$ for all $j \in J$, a contradiction. In particular: the set $J' = \{j \in N\colon x^{**}_j < \omega_j\}$ of net donors at x^{**} is non-empty, and its elements all strictly prefer x^{**} to ω.

Let $K' = \{k \in N\colon x^{**}_k > \omega_j\}$ denote the set of net receivers at x^{**} (a subset of K by construction, which is non-empty since J' is non-empty), and $Q' = \{(j, k) \in N \times N\colon t^{**}_{jk} > 0\}$ denote the set of pairs of agents linked by a net wealth transfer at t^{**} (Q' is non-empty, and is contained in $\{(j, k) \in J' \times K'\colon x^{**}_j \geqslant x^{**}_k\}$ by construction). The positive components of t^{**}, ranked in increasing lexicographic order, make a sequence of bilateral progressive transfers from ω to x^{**}. Formally, let $((j_q, k_q))_{1 \leqslant q \leqslant \#Q'}$ denote the sequence of elements of Q' ranked in increasing lexicographic order, and define, as above, the following sequence of bilateral progressive redistributions from ω: $x^0 = \omega$; for any $q \in \{0, \ldots, \#Q' - 1\}$, $x^{q+1} = x^q + t^{**}_{j_q k_q} e^{j_q k_q}$. Observe that for all $q \in \{0, \ldots, \#Q' - 1\}$ and all $x \in [x^q, x^{q+1}[$: $x_{j_q} > x_{k_q}$. Self-centredness implies that w_{k_q} is increasing along $[x^q, x^{q+1}]$ for all $q \in \{0, \ldots, \#Q' - 1\}$. Non-jealousy implies that w_i is non-decreasing along $[x^q, x^{q+1}]$ for all $(i, q) \in N \times \{0, \ldots, \#Q' - 1\}$ such that $i \in N \setminus \{j_q, k_q\}$. But $x^{\#Q'} = x^{**}$ by construction. Therefore $w_i(x^{**}) \geqslant w_i(\omega)$ for all $i \in N \setminus J'$, with a strict inequality for all $k \in K'$. Since, moreover, $w_i(x^*) > w_i(\omega)$ for all $i \in J'$, we have $\omega \notin P$.

(b) Suppose, finally, that w is a strong BBV distributive social system. One verifies readily from the definitions that BBV social systems verify local non-satiation. It will suffice, therefore, to establish that $\omega \notin C$ implies $\omega \notin P$. By the lemma, combined with the structure of distributive preferences particular to strong BBV social systems (egoistic poor, and individual utility of the rich strictly increasing in own wealth and in aggregate wealth of the poor and independent of the wealth of the other rich), $\omega \notin C$

readily implies the existence of a non-empty subset J of the set of rich individuals $\{1, \ldots, m\}$, and of a gift-vector $t_J > 0$ of coalition J such that, for all $j \in J$: $\omega_j > x_j(\omega, (0_{\setminus J}, t_J)) \geqslant 0$; $w_j(x(\omega, (0_{\setminus J}, t_J))) > w_j(\omega)$; $t_{jk} = 0$ whenever $k \leqslant m$ (the rich do not receive gifts); and, therefore, $\omega_k < x_k(\omega, (0_{\setminus J}, t_J))$ only if $k > m$ (net receivers are poor), and $\omega_j > x_j(\omega, (0_{\setminus J}, t_J))$ if and only if $j \in J$ (the net donors are the members of J). Let: $t^* = (0_{\setminus J}, t_J)$; $x^* = x(\omega, t^*)$; $K = \{k \in N\colon t^*_{jk} > 0$ for some $j \in J\}$. The assumptions on distributive preferences readily imply then: $w_i(x^*) > w_i(\omega)$ for all $i \in \{1, \ldots, m\} \setminus J$; and $w_i(x^*) \geqslant w_i(\omega)$ for all $i > m$, with a strict inequality whenever $i \in K$. Therefore $\omega \notin P$, and the proof is completed. □

COROLLARY 8. *If w verifies the assumptions of Arrow (1981) then a non-trivial distributive equilibrium t of (w, ω) is strong if and only if it has a unique donor i, whose utility reaches its maximum in S_n (that is, $w_i(x(\omega, t)) = \max\{w_i(x)\colon x \in S_n\}$).*

PROOF. One verifies readily from the definitions that Arrow's distributive social systems verify local non-satiation of the Paretian preordering, self-centredness and non-jealousy. Necessity then immediately follows from the definition of a strong equilibrium, the Theorem 6 of Arrow (1981), and the Footnote 57 of this chapter. Let us establish sufficiency. Let $t \neq 0$ be a distributive equilibrium with a unique donor i ($t_j = 0$ for all $j \neq i$) such that $w_i(x(\omega, t)) = \max\{w_i(x)\colon x \in S_n\}$, suppose that t is blocked by a coalition I playing t^*_I, and let us derive a contradiction. By definition of a blocking coalition, for all $j \in I$, $x_j(\omega, (t_{\setminus I}, t^*_I)) \geqslant 0$ and $w_j(x(\omega, (t_{\setminus I}, t^*_I))) \geqslant w_j(x(\omega, t))$ with a strict inequality holding for at least one j. This implies in turn that coalition I playing t^*_I blocks 0 in the social system $(w, x(\omega, t))$. Let $\omega' = x(\omega, t)$. The maximum ω' of w_i in S_n being unique by strict concavity of w_i, we have $\omega' \in P$, so that $\omega' \in C$ by Theorem 14, the desired contradiction. □

COROLLARY 9. (i) *If w is a strong BBV distributive social system, or if it verifies local non-satiation of the Paretian preordering, self-centredness and non-jealousy, then:* $L(w, \omega^0) = \{\omega \in P(w)\colon w_i(\omega) \geqslant w_i(\omega^0)$ for all $i\}$. (ii) *If moreover w_i is continuous for all i, then $L(w, \omega^0)$ is non-empty for all $\omega^0 \in S_n$.*

PROOF. (i) follows immediately from Theorem 14, Corollary 8 and the definition of $L(w, \omega^0)$. Let us prove (ii). Set $X(\omega^0) = \{\omega \in S_n\colon w(\omega) \geqslant w(\omega^0)$ for all $i\}$ is a non-empty ($\omega^0 \in X(\omega^0)$) and closed (by continuity of utility functions) subset of compact set S_n. It is therefore a non-empty compact set. Function $\sum_i \alpha_i w_i$, where α_i denotes a positive real number for all i, is continuous and attains therefore a maximum at some $\omega^* \in X(\omega^0)$. ω^* is a strong distributive optimum by construction, unanimously preferred to ω^0 by definition of $X(\omega^0)$. It is therefore a distributive liberal social contract of (w, ω^0) by Corollary 9(i). □

THEOREM 15. *Let (π, ω) (resp. (π, x)) be a social contract equilibrium (resp. Lindahl–Bergstrom equilibrium) of (w, ω^0), such that w_i is locally non-satiated at ω*

(resp. x) for all i (that is, for all i and all neighborhood V of ω in $\mathbb{R}^n$, there exists $x' \in V$ such that $w_i(x') > w_i(\omega)$). (i) *Then, ω (resp. x) is a strong distributive optimum of w.* (ii) *If, moreover, w is a strong BBV distributive social system, or if it verifies local non-satiation of the Paretian preordering, self-centredness and non-jealousy, then ω is a distributive liberal social contract of (w, ω^0).*

PROOF. (i) By Definition 7(ii) (resp. 7′(ii)): $w_i(z) > w_i(\omega)$ (resp. $w_i(z) > w_i(x)$) implies $\pi_i z > \pi_i \omega^0$ (resp. $\pi_i z > \omega_i^0$) whenever $z_i \geqslant 0$. By local non-satiation of individual preferences: $w_i(z) \geqslant w_i(\omega)$ (resp. $w_i(z) \geqslant w_i(x)$) implies $\pi_i z \geqslant \pi_i \omega^0$ (resp. $\pi_i z \geqslant \omega_i^0$) whenever $z_i \geqslant 0$. If, therefore, there exists $z \geqslant 0$ that is Pareto-superior to ω (resp. x), i.e. such that $w_i(z) \geqslant w_i(\omega)$ (resp. $w_i(z) \geqslant w_i(x)$) for all i with a strict inequality for at least one i, then $\sum_{i \in N} \pi_i z > \sum_{i \in N} \pi_i \omega^0$ (resp. $\sum_{i \in N} \pi_i z > \sum_{i \in N} \omega_i^0$), while

$$\sum_{i \in N} \pi_i \omega^0 = \left(\sum_{i \in N} \pi_i\right) \omega^0 = (1, \ldots, 1) \cdot \omega^0 = \sum_{i \in N} \omega_i^0 = 1.$$

Therefore z is not feasible and the first part of the theorem is established.

(ii) We know from the first part of the proof that ω is a strong distributive optimum. It suffices therefore, from Corollary 9, to establish that ω is unanimously weakly preferred to ω^0. But ω^0 belongs to $\{z \in \mathbb{R}^n\colon z_i \geqslant 0 \text{ and } \pi_i z \leqslant \pi_i \omega^0\}$ for all i. We have therefore $w_i(\omega) \geqslant w_i(\omega^0)$ for all i by Definition 7(ii). □

THEOREM 16. *Let (w, ω^0) be a (weak) BBV social system such that $\omega_i^0 = 0$ for all $i \geqslant m+1$ (that is, for all poor i).* (i) *(π, ω) is a social contract equilibrium of (w, ω^0) if and only if it is a Lindahl–Bergstrom equilibrium of (w, ω^0).* (ii) *If, moreover, w is a strong BBV distributive social system, or if it verifies self-centredness and non-jealousy, then the equilibrium distributions of Lindahl–Bergstrom of (w, ω^0) are distributive liberal social contracts of the latter.*

PROOF. The second part of the theorem is a simple consequence of the first part, Theorem 15(ii), and the obvious remark that BBV social systems verify local non-satiation of the Paretian preordering. Let us prove the first part.

In view of Definitions 7 and 7′, it suffices to prove that $\pi_i \omega^0 = \omega_i^0$ for all i when (π, ω) is a social contract equilibrium and when it is a Lindahl–Bergstrom equilibrium of (w, ω^0). Let (π, ω) be either a social contract equilibrium or a Lindahl–Bergstrom equilibrium of (w, ω^0) from now on.

Note first that $\pi_{ii} > 0$ for all i, for if $\pi_{ii} \leqslant 0$ individual i can increase his utility indefinitely in $\{x \in \mathbb{R}^n\colon x_i \geqslant 0 \text{ and } \pi_i x \leqslant \pi_i \omega^0\}$ and in $\{x \in \mathbb{R}^n\colon x_i \geqslant 0$ and $\pi_i x \leqslant \omega_i^0\}$ simply by increasing his consumption (the utility function of a BBV agent being strictly increasing in his own consumption).

Consider now a pair of distinct agents i and j such that either i is poor or i and j are rich, and let us prove that $\pi_{ij} = 0$. Suppose $\pi_{ij} \neq 0$, let $\varepsilon > 0$ be a positive real number, and define distribution x from equilibrium distribution ω by: $x_i = \omega_i + \varepsilon$; $x_j = \omega_j - (\pi_{ii}/\pi_{ij})\varepsilon$; $x_k = \omega_k$ for all k distinct from i and j. Then $\pi_i x = \pi_i \omega$, so that x belongs to $\{z \in \mathbb{R}^n\colon\ z_i \geqslant 0 \text{ and } \pi_i z \leqslant \pi_i \omega^0\}$ if ω is a social contract equilibrium distribution and to $\{z \in \mathbb{R}^n\colon\ z_i \geqslant 0 \text{ and } \pi_i z \leqslant \omega_i^0\}$ if ω is a Lindahl–Bergstrom equilibrium distribution. But agent i is indifferent to j (that is, his utility does not depend on j's consumption) by BBV assumptions, so that $w_i(x) > w_i(\omega)$, a contradiction.

From the above result and the assumption that the endowments of the poor are $= 0$, we deduce that $\pi_i \omega^0 = \pi_{ii}\omega_i^0$ for all i. If i is poor, then $\pi_{ii}\omega_i^0 = 0 = \omega_i^0$, so that $\pi_i \omega^0 = \omega_i^0$ as expected. If i is rich, then $\pi_{ji} = 0$ for all $j \neq i$ and the definition of equilibrium Lindahl prices ($\sum_{i \in I} \pi_i = (1, \ldots, 1)$) implies therefore that $\pi_{ii} = 1$, so that $\pi_{ii}\omega^0 = \omega_i^0$, and finally $\pi_i \omega^0 = \omega_i^0$. □

THEOREM 17. *Let (w, ω) be a strong BBV distributive social system, and suppose that, for all i, v_i is C^2, strictly quasi-concave, and verifies ordinal normality. Denote by $(x_1^*, \ldots, x_m^*, y^*)$ its unique equilibrium vector of individual consumption of the rich and aggregate consumption of the poor, suppose that $x_i^* > 0$ for all $i \leqslant m$, and let x^* be any equilibrium distribution (that is, any $x \in S_n$ such that $x_i = x_i^*$ for all $i \leqslant m$ and $x_{m+1} + \cdots + x_n = y^*$).* (i) *Then, there exists a distribution x in the weak Foley-core of (w, ω) such that $w_i(x) \geqslant w_i(x^*)$ for all $i \leqslant m$.* (ii) *If moreover x^* is not in the weak Foley-core of (w, ω), then:* (a) *there exists a distribution x in the weak Foley-core of (w, ω) such that $w_i(x) > w_i(x^*)$ for all $i \in N$;* (b) *and $x_{m+1} + \cdots + x_n > y^*$ for all such x.* (iii) *x^* is not in the weak Foley-core of (w, ω), nor is it a weak distributive optimum, whenever $(x_1^*, \ldots, x_m^*, y^*)$ is such that at least two agents contribute whose private equilibrium consumption levels are both > 0 (that is, whenever $0 < x_i^* < \omega_i$ for two distinct $i \leqslant m$ at least).*

PROOF. I first establish parts (i) and (ii) of the theorem, and then turn to the proof of part (iii).

(i) The proof of parts (i) and (ii) follows Shitovitz and Spiegel (2001, 3, pp. 222–223) with minor adaptations. Part (i) is a simple consequence of part (ii). Let us establish the latter.

Let (M, V) be the cooperative non-transferable utility game such that $M = \{1, \ldots, m\}$ and $V(I) = \{v \in \mathbb{R}^m\colon$ There exists $((x_i)_{i \in I}, y) \in \mathbb{R}_+^{\#I} \times \mathbb{R}_+$ such that $\sum_{i \in I} x_i + y \leqslant \sum_{i \in I} \omega_i$, $x_i \leqslant \omega_i$ for all $i \in I$, and $v_i \leqslant v_i(x_i, y)$ for all $i \in I\}$ for any $I \subset M$. Define the core of (M, V) by: $C(M, V) = \{v \in V(M)\colon$ There is no non-empty coalition $I \subset M$ and $v' \in V(I)$ such that $v_i' > v_i$ for all $i \in I\}$. We know from Shitovitz and Spiegel (2001, 3.1) that $C(M, V)$ has the following external stability property: $v \in V(M) \setminus C(M, V)$ implies that there exists $v' \in C(M, V)$ and a non-empty coalition $I \subset M$ such that $v' \in V(I)$ and for each $i \in I$, $v_i > v_i'$.

One verifies readily, from the definition of strong BBV distributive social systems, that $V(M) \setminus C(M, V)$ contains $\{(w_i(x))_{i \leqslant m}\colon\ x \in S_n$ and is not in the weak Foley-core of $(w, \omega)\}$, the set of utility vectors of the rich associated with the feasible distributions that are strongly Foley-blocked.

Since, by assumption, equilibrium distribution x^* is not in the weak Foley-core of (w, ω), there must exist therefore, by the external stability property recalled above, a utility vector $v \in C(M, V)$ and a non-empty coalition $I \subset M$ such that $v \in V(I)$ and, for each $i \in I$, $v_i > w_i(x^*) = \nu_i(x_i^*, y^*)$. And by the definition of $V(I)$ there exists $((x_i)_{i \in I}, y) \in \mathbb{R}_+^{\#I} \times \mathbb{R}_+$ such that $\sum_{i \in I} x_i + y \leqslant \sum_{i \in I} \omega_i$, $x_i \leqslant \omega_i$ for all $i \in I$, and $v_i \leqslant \nu_i(x_i, y)$ for all $i \in I$.

I prove that $y > y^*$. Suppose, on the contrary, that $y \leqslant y^*$, and let us derive a contradiction. Since $\nu_i(x_i, y) \geqslant v_i > \nu_i(x_i^*, y^*)$ for all $i \in I$, it follows that $x_i > x_i^*$ for all $i \in I$, by strict monotonicity of functions ν_i. But then $g_i^* = \omega_i - x_i^* > \omega_i - x_i \geqslant 0$ for all $i \in I$, which implies that all agents in I are contributing to the public good at distributive equilibrium. And therefore y^* is > 0, since I is non-empty. Let i be, from there on, a fixed element of non-empty coalition I. From the first-order conditions for distributive equilibrium (Section 3.1.2.2, Theorem 1), $y^* > 0$ and $\partial_y \nu_i(x_i^*, y^*) \neq 0$ (> 0), we have $\partial_{x_i} \nu_i(x_i^*, y^*)/\partial_y \nu_i(x_i^*, y^*) \leqslant 1$. Ordinal normality, $y \leqslant y^*$ and $x_i > x_i^*$ imply that $\partial_{x_i} \nu_i(x_i, y)/\partial_y \nu_i(x_i, y) \leqslant \partial_{x_i} \nu_i(x_i, y^*)/\partial_y \nu_i(x_i, y^*) < \partial_{x_i} \nu_i(x_i^*, y^*)/\partial_y \nu_i(x_i^*, y^*)$. Therefore $\partial_{x_i} \nu_i(x_i, y)/\partial_y \nu_i(x_i, y) < 1$. And $x_i > 0$ since $x_i > x_i^*$. This implies in turn that there is a real number ε such that $0 < \varepsilon < x_i$ and $\nu_i(x_i - \varepsilon, y + \varepsilon) > \nu_i(x_i, y) \geqslant v_i$. Strict monotonicity implies then that $\nu_j(x_j, y + \varepsilon) > v_j$ for all $j \in I \setminus \{i\}$, while $(x_i - \varepsilon + \sum_{j \in I:\ j \neq i} x_i) + (y + \varepsilon) = \sum_{i \in I} \omega_i$ by construction. Therefore the utility vector v is not in $C(M, V)$, the desired contradiction.

I now establish that $v_i > \nu_i(x_i^*, y^*)$ for all $i \in M \setminus I$. Suppose, on the contrary, that $v_i \leqslant \nu_i(x_i^*, y^*)$ for some $i \in M \setminus I$, and let us derive a contradiction. Inequalities $x_i^* \leqslant \omega_i$ and $y > y^*$ and strict monotonicity clearly imply then $\nu_i(\omega_i, y) > \nu_i(x_i^*, y^*) \geqslant v_i$. Since $\omega_i > 0$ (see Section 3.3.3) and ν_i is continuous, there exists, therefore, a real number ε such that $0 < \varepsilon < \omega_i$ and $\nu_i(\omega_i - \varepsilon, y + \varepsilon) > v_i$. And $\nu_j(x_j, y + \varepsilon)$ is then strictly greater than $\nu_j(x_j, y) \geqslant v_j$ for all $j \in I$ by strict monotonicity. Let v' be any element of $\mathbb{R}^m$ such that $v_i' = \nu_i(\omega_i - \varepsilon, y + \varepsilon)$ and $v_j' = \nu_j(x_j, y + \varepsilon)$ for all $j \in I$. Clearly, $v' \in V(I \cup \{i\})$, and non-empty coalition $I \cup \{i\}$ is such that $v_j' > v_j$ for all j in $I \cup \{i\}$, so that v is not in $C(M, V)$, the desired contradiction.

Since $v \in V(M)$ is $\gg (\nu_i(x_i^*, y^*))_{i \leqslant m}$, there exists $((x_i')_{i \leqslant m}, y') \in \mathbb{R}_+^m \times \mathbb{R}_+$ such that $\sum_{i \in M} x_i' + y' \leqslant \sum_{i \in M} \omega_i$, $x_i' \leqslant \omega_i$ for all $i \in M$, and $\nu_i(x_i', y') \geqslant v_i > \nu_i(x_i^*, y^*)$ for all $i \in M$. Any $((x_i)_{i \leqslant m}, y) \in \mathbb{R}_+^m \times \mathbb{R}_+$ such that $\sum_{i \in M} x_i + y < \sum_{i \in M} \omega_i$ must be blocked, utilities being strictly increasing; therefore we must have $\sum_{i \in M} x_i' + y' = \sum_{i \in M} \omega_i$. One establishes as above that, necessarily, $y' > y^*$. And it follows then immediately from the latter that there exists a wealth distribution of the poor $(x_i')_{i > m}$ such that $x_{m+1}' + \cdots + x_n' = y'$, $x_i' > x_i^*$ for all $i > m$, and $x' \in S_n$. Distribution x', being feasible, must be in the weak Foley-core of (w, ω), for otherwise v would be blocked. And we have $w_i(x') > w_i(x^*)$ for all $i \leqslant m$ by construction, and also $w_i(x') = x_i' > w_i(x^*) = x_i^*$ for all $i > m$ by definition of BBV distributive social systems.

Finally, inequalities $(w_i(x))_{i>m} \gg (w_i(x^*))_{i>m}$ readily imply $x_{m+1}+\cdots+x_n > y^*$, and this remark completes the proof of parts (i) and (ii) of the theorem.

(ii) The result of part (iii) is quite simple and largely independent of the results of parts (i) and (ii). Notably, it does not suppose ordinal normality. Suppose that there are two distinct agents i and j in $\{1, \dots, m\}$ whose equilibrium contributions verify: $0 < \omega_k - x_k^* < \omega_k$, $k = i, j$. Then y^* must be > 0, and the first-order equilibrium conditions and the monotonicity assumptions on functions ν_k imply that $\partial_{x_k} \nu_i(x_k^*, y^*) = \partial_y \nu_i(x_k^*, y^*) > 0$, $k = i, j$. Function $\xi_k : \mathbb{R} \to \mathbb{R}$ defined by $\xi_k(\varepsilon) = \nu_k(x_k^* - (\varepsilon/2), y^* + \varepsilon)$ has derivative $\partial \xi_k(\varepsilon) = -(1/2)\partial_{x_k} \nu_k(x_k^* - (\varepsilon/2), y^* + \varepsilon) + \partial_y \nu_k(x_k^* - (\varepsilon/2), y^* + \varepsilon)$, which is continuous by the smoothness assumption on ν_k, and > 0 at $\varepsilon = 0$ for $k = i, j$ by the first-order condition above. Therefore, function $\mathbb{R} \to \mathbb{R}^2$ defined by $\varepsilon \to (\xi_i(\varepsilon), \xi_j(\varepsilon))$ is monotonic strictly increasing in an open neighborhood V of 0 in $\mathbb{R}$. Let ε in V be such that $0 < \varepsilon < \min\{x_i^*, x_j^*\}$, and define distribution x such that: $x_k = x_k^* - (\varepsilon/2)$ if $k = i$ or j; $x_k = x_k^*$ if $k \in \{1, \dots, m\} \setminus \{i, j\}$; and $x_k = x_k^* + (\varepsilon/(n - m))$ for all $k > m$. Obviously, $x \in S_n$. And we have $w_k(x) > w_k(x^*)$ for $k = i, j$ by construction, and $w_k(x) > w_k(x^*)$ for all $k \neq i, j$ by the monotonicity properties of preferences in strong BBV distributive social systems. Therefore x^* is strongly Pareto-dominated, implying that it is neither weakly Pareto-efficient nor in the weak Foley-core of (w, ω). □

COROLLARY 10. *Let (w, ω) be a strong BBV distributive social system such that the initial endowments of the poor are all $= 0$ and rich individuals are identical (that is, $\nu_i = \nu$ and $\omega_i = 1/m$ for all $i \leqslant m$). Let $(z^*, y^*) \in \mathbb{R}^2$ and $(z^{**}, y^{**}) \in \mathbb{R}^2$ be respectively a Cournot–Nash and a Lindahl–Bergstrom (symmetric) equilibrium vector of private consumption of the rich and aggregate consumption of the poor (where symmetry means that the rich make identical gifts at equilibrium, equal to y^*/m and y^{**}/m respectively).* (i) *Then, $\nu(z^*, y^*) \leqslant \nu(z^{**}, y^{**})$.* (ii) *If, moreover, ν is C^2 and strictly quasi-concave, and if $m \geqslant 2$ and z^* and y^* are both > 0, then: $\nu(z^*, y^*) < \nu(z^{**}, y^{**})$; and $y^* < y^{**}$ whenever ν verifies the additional assumption of ordinal normality.*

PROOF. We know from Footnote 70 that the Lindahl equilibrium distributions are in the (strong) Foley-core of BBV (w, ω). Part (i) of Corollary 10 follows readily from this fact, for $\nu(z^*, y^*) > \nu(z^{**}, y^{**})$ implies, clearly, that Lindahl equilibrium distribution is strongly Foley-blocked by the coalition of the rich playing the Cournot–Nash equilibrium transfers. Inequality $\nu(z^*, y^*) < \nu(z^{**}, y^{**})$, in the second part of the corollary, follows from the same fact and Theorem 17(iii). Combined with ordinal normality, it implies $y^* < y^{**}$ by the reasoning developed in §5 of the proof of Theorem 17(ii). □

A.2. Mechanisms for private contributions to public goods

A.2.1. Two-stage mechanisms

The mechanisms reviewed here develop the original idea of Guttman (1978, 1987). All of them suppose that individual agents are perfectly informed about the preferences and endowments of others.

Guttman sets his mechanism in the framework, already described several times above, of a strong BBV distributive social system of identical ("rich") agents endowed with distributive utility functions that are quasi-linear in the private good, strictly concave in the public good and differentiable. A donor's contribution to the public good is made of two parts: a flat contribution g_i; and a matching grant $s_i \sum_{j \leqslant m:\ j \neq i} g_j$ proportional to the sum of the flat contributions of the other donors. In the first stage of the game, donors choose simultaneously the matching rates s_i that maximize their utility given the matching rates announced by the others and the flat contributions that they anticipate for the second stage of the game as functions of the whole vector of matching rates. In the second stage (subgame), they choose simultaneously the flat contributions that maximize their utility, given the flat contributions of others and the vector of matching rates $(s_1, \dots, s_m)$ determined in the first stage. Each player's strategy consists therefore of a matching rate and a flat contribution as a function of all matching rates. An m-tuple of strategies is a subgame perfect equilibrium of this sequential game if it makes a Nash equilibrium at both stages of the game. Attention is restricted to symmetric equilibria. Guttman states that the equilibrium provision level of the public good is the (unique) Pareto-efficient provision level. This property is no longer verified in the presence of income effects [Guttman (1987)] or preference heterogeneity. In the latter case, nevertheless, (that is, in the case of heterogeneous quasi-linear preferences), the equilibrium provision level of the public good induced by Guttman's mechanism remains larger than the non-cooperative level.

Danziger and Schnytzer (1991) develop a variant of Guttman's mechanism where donors choose the rate at which they subsidize the flat contributions of others. Formally, let s_i ($\geqslant 0$) denote a subsidy rate chosen by agent i. His total contribution to the public good is made of the following two parts: his subsidies to the other donors $s_i \sum_{j \leqslant m:\ j \neq i} g_j$; and his own subsidized flat contribution $(1 - \sum_{j \leqslant m:\ j \neq i} s_j) g_i$. And the sequential game is specified as the following variant of Guttman's implementation game. In the first stage, donors choose simultaneously the subsidy rates that maximize their utility given the subsidy rates announced by the others and their flat contributions as functions of the vector of subsidy rates. In the second stage, they choose simultaneously the flat contributions that maximize their utility, given the flat contributions of others and the vector of subsidy rates determined in the first stage. Donors have general (strong) BBV preferences, which are assumed differentiable and strictly concave. The private and the public goods are both strictly normal goods for all of them, and the marginal rates of substitution $\partial_{x_i} v_i(x_i, y)/\partial_y v_i(x_i, y) \rightarrow \infty$ as $x_i \rightarrow 0$. Danziger and Schnytzer prove, in their Theorems 1 and 2 [op. cit.: pp. 59

and 61 respectively; see also the remarks of Althammer and Buchholz (1993)], that interior sequential equilibria exist and are in one-to-one correspondence with interior Lindahl equilibria (and therefore Pareto-efficient) when Pareto-efficiency requires a positive provision of the public good. If, however, the latter condition is verified, there exist also sequential equilibria with zero provision of the public good (therefore inefficient) if and only if there is no pair of individuals who want to make stand-alone positive joint contributions with equal cost-sharing (that is, formally, if and only if $(\partial_y \nu_i(\omega_i, 0)/\partial_{x_i} \nu_i(\omega_i, 0)) + (\partial_y \nu_j(\omega_j, 0)/\partial_{x_j} \nu_j(\omega_j, 0)) \leqslant 1$ for all (i, j), $i \neq j$). If, finally, non-provision of the public good is Pareto-efficient, then: all sequential equilibria imply non-provision; there exists at least one such equilibrium; and the sequential equilibria are Pareto- efficient.

Varian (1994a, 1994b), to finish with, designs three alternative variants of Guttman's original mechanism.

The first one is simply the subsidy-setting mechanism of Danziger and Schnytzer above, applied to a context that does not fit in the technical assumptions of the latter, namely, a strong BBV distributive social systems with only two donors, endowed with utility functions quasi-linear in the private good, strictly concave in the public good and differentiable (violating, therefore, notably, the strict concavity and normality assumptions of Danziger and Schnytzer). Varian (1994a, Theorem 1) establishes that the subsidy-setting game of Danziger and Schnytzer has a unique equilibrium in such social systems and that the associate allocation is a Lindahl equilibrium allocation.

In the second mechanism, the rate at which a donor i subsidizes the flat contribution of a donor j ($\neq i$) is chosen by a third agent k picked in $\{1, \ldots, m\} \setminus \{i, j\}$ according to some fixed rule assigning one and only one k to any pair (i, j) (e.g. $k = \min\{1, \ldots, m\} \setminus \{i, j\}$). This supposes, naturally, that $m \geqslant 3$. In order to facilitate exposition, I will suppose, following Varian (1994a), that $m = 3$. Then $\{1, \ldots, m\} \setminus \{i, j\}$ reduces to a single agent, and we can denote therefore, without ambiguity, by s_{kj} the subsidy rate facing agent j as set by agent k (and paid by the single remaining agent in $\{1, 2, 3\} \setminus \{k, j\}$). The total contribution of a donor, say agent 1, to the public good consists of the following two components: his subsidized flat contribution $(1 - s_{31} - s_{21}) g_1$; and the sum $s_{32} g_2 + s_{23} g_3$ of his subsidies to the other agents. The sequential game is defined along the same lines as above, with a first stage setting subsidy rates and a second stage setting individual flat contributions. The author establishes (1994a, Theorem 3) that, in a strong BBV distributive social system with continuous convex preferences and locally invertible individual (Lindahl) demand functions for the public good, the subgame perfect equilibria of this subsidy-setting game yield Lindahl equilibrium allocations. Demand functions are locally invertible, in particular, when distributive utility functions verify ordinal normality or are quasi-linear in the private good, differentiable and strictly concave in the public good.

Varian's third mechanism is specified as follows. In the first stage, each agent i announces a number $1 - s_i$, which will turn out, in equilibrium, to be both the rate at which agent i's contributions are subsidized (agent i being paid subsidy $(1 - s_i) g_i$) and the rate at which he subsidizes the contributions of everyone else (agent i paying the subsidy

$(1 - s_i)g_j$ to every $j \neq i$). In the second stage, each agent i chooses his private consumption and flat contribution, given the vector $(s_1, \ldots, s_m)$ and the flat contributions of others, to maximize his utility subject to the budget constraint

$$x_i + \left(1 - \sum_{j \leqslant m:\ j \neq i} s_j\right) g_i$$
$$= \omega_i - \left(1 - \sum_{j \leqslant m:\ j \neq i} s_j\right) \sum_{j \leqslant m:\ j \neq i} g_j - Q(s_1, \ldots, s_m),$$

where $Q(s_1, \ldots, s_m)$ is a quadratic penalty term defined as $(\sum_{j \leqslant m:\ j \neq i} s_j)^2$. The subsidy rate $\sum_{j \leqslant m:\ j \neq i} s_j$ facing agent i in the second stage of the game is set by the other agents as in Varian's second mechanism, but each agent announces now a single number as in the mechanism of Danziger and Schnytzer. Varian's third mechanism differs from the latter also by the penalty term introduced in its second stage. If utility functions of donors are differentiable and quasi-concave, and if the subgame perfect equilibrium allocation $(x_1, \ldots, x_m, G)$ is an interior Pareto optimum, then the penalty term must be $= 0$ by Samuelson's first-order conditions for Pareto-efficiency. This implies in turn that $\sum_{j \leqslant m} s_j = 1$, so the equilibrium budget constraints read $x_i + s_i G = \omega_i$ and we have, clearly, a Lindahl equilibrium. Therefore, any interior Lindahl allocation is a subgame perfect equilibrium allocation of the mechanism [Varian (1994a, 1994b)].

A.2.2. *One-stage mechanisms*

The interest in one-stage mechanisms was fostered, notably, by an original study of Roberts (1987). This article examines the relative treasury-efficiency and distributional consequences of direct taxation and subsidy as alternative means of financing of a pure public good. The underlying (largely implicit) setup is the strong BBV distributive social system. Two paradoxical statements are made in this context.

A first proposition can be viewed as a simple consequence of the neutrality property of Andreoni (1988a) (see Section 5.2.2 above: an increase in public spending on the public good corresponding to a budget-balanced increase in lump-sum taxes and/or flat subsidy rate on private contributions leaves the equilibrium provision level of the public good unchanged as long as it does not push any existing contribution to 0). Roberts' proposition states that a flat (proportional) subsidy is more treasury-efficient than (lump-sum) direct taxation *irrespective* of the price elasticity of private contributions to the public goods: the neutrality property implies that the equilibrium provision of the public good G is invariant to the tax-subsidy scheme; and the corresponding public spending is G if provision is entirely financed by direct taxation and $\beta G < G$ (where $\beta < 1$ denotes the uniform subsidy rate facing all donors) if the same provision level is financed by subsidies. This property stands in sharp contrast with the conventional statement that the subsidy is more efficient if and only if the price elasticity of the sum of private contributions is larger than 1 (the partial equilibrium condition for an increase in the subsidy rate to prompt an increase in aggregate *net* private contributions).

The second proposition states that an increase in the individual subsidy rate facing a donor i makes him worse off, ceteris paribus, at equilibrium, if the agent's contribution to the public good is price elastic: unchanged provision level of the public good (as implied by the ceteris paribus proviso) and increased individual net contribution (as implied by the elasticity assumption) together result in a fall in i's private consumption x_i and utility $v_i(x_i, G)$. As a consequence, paradoxically, rich donors, whose contributions are more susceptible to be price elastic, will presumably prefer flat subsidy schemes, involving a uniform subsidy rate for all agents, to skewed schemes involving subsidy rates increasing in wealth or contribution. Similarly, rich donors might end up better off with uniform lump-sum taxation than with flat proportional subsidy. Note that, although this is not explicitly stated by Roberts, unchanged G is an assumption here, not a consequence of neutrality (a change in the subsidy rate of a single agent is non-neutral in general: see the account of Boadway, Pestieau and Wildasin (1989a) below).

While essentially correct, Roberts' findings suffer from some imprecision, due to some ambiguity in the way they combine general equilibrium analysis (the neutrality property) and partial equilibrium comparative statics. They were further elaborated in explicit general equilibrium setup, and, moreover, explicitly related to implementation theory, in two broad classes of models, namely, the models where individuals ignore the budget constraint of the government when taking their decisions (they ignore, for instance, that the subsidies they receive individually must be financed endogenously by appropriate taxes), and the models where they fully integrate the consequences of this budget constraint for themselves (they "see through" the government budget constraint: see Section 5.2.2 above).

Models of the first type are studied by Bergstrom (1989a), Boadway, Pestieau and Wildasin (1989a, 1989b), and Roberts (1992) [see also Kaplow (1995)]. The present account follows the elaborate presentation of Boadway, Pestieau and Wildasin (1989a, 1989b). They consider strong BBV distributive social systems,[91] whose main features are briefly summarized as follows (see Section 5.2.2 for details): agent i's budget constraint reads: $x_i + (1 - s_i)g_i = \omega_i - \tau_i$, where s_i is his matching grant rate and τ_i a lump-sum tax; the balanced budget of the central government reads $\sum_{i \leqslant m} s_i g_i = \sum_{i \leqslant m} \tau_i$ for all $(g_1, \ldots, g_m)$; agents do not see through the government budget constraint, and maximize therefore $v_i(x_i, g_i + G_{-i})$ with respect to (x_i, g_i), subject to the individual budget constraint above for any given G_{-i}. Attention is restricted to (Cournot–Nash) equilibria that involve positive contributions of all potential donors (that is, equilibria such that $g_i > 0$ for all $i = 1, \ldots, m$).

[91] They suppose, actually, as mentioned already in Section 5.2.2 above, utility functions of the type $v_i(x_i, y_i, G)$, where y_i is a quantity of a local public commodity or factor consumed by agent ("locality") i and taxed at a given fixed rate by the central government. Nevertheless, local public goods and associate taxes play a role only in their analysis of the neutrality property (see Section 5.2.2). Tax rates on local public goods are set $= 0$ and equilibrium levels of y_i can be viewed essentially as fixed parameters in their treatment of non-neutral fiscal policy and related implementation theory. The same remarks apply, essentially, to the article of Brunner and Falkinger (1999), with a change in the interpretation of y_i (the leisure or the labor participation of agent i in their setup).

The authors obtain the following three sets of remarkable results relative to the efficiency and distributional implications of non-neutral changes in matching rates. First, a (marginal) budget-balanced increase in the subsidy rate s_i of a single agent i (financed, therefore, by any appropriate marginal change in lump-sum transfers): (i) increases the equilibrium provision level of the public good; (ii) increases i's equilibrium contribution, and lowers his equilibrium private consumption and welfare; (iii) lowers the contributions of the other agents and increases their private consumption and welfare [Boadway, Pestieau and Wildasin (1989a, Theorem 2 and proof); see also Bergstrom (1989a, Puzzle 2, parts 1 and 3), for the distributional aspects of this property]. Second, a (marginal) budget-balanced increase in a uniform subsidy rate $\beta = s_1 = \cdots = s_m$ raises individual and total contributions to the public good, with ambiguous consequences on individual welfare in general, except in the case of two-agent social systems with identical individual preferences, where individual welfare raises if $\beta < 1/2$ and attains a local maximum at $\beta = 1/2$ [Boadway, Pestieau and Wildasin (1989a, Theorem 3 and proof)]. Third: (i) there is a one-to-one mapping from (interior) Lindahl allocations (hence interior Pareto-efficient allocations) to the vectors of matching rates $(s_1, \ldots, s_m) \gg 0$ satisfying $\sum_{i \leqslant m} s_i = m - 1$; (ii) the price facing any agent i at the Lindahl allocation associated with such a vector $(s_1, \ldots, s_m)$ is $1 - s_i$; (iii) in particular: (a) the unique uniform matching rate that yields a Pareto-efficient allocation at Cournot–Nash equilibrium is $\beta = s_1 = \cdots = s_m = (m - 1)/m$; (b) and a (marginal) budget-balanced increase in the subsidy rate s_i of agent i, from a vector $(s_1, \ldots, s_m) \gg 0$ satisfying $\sum_{i \leqslant m} s_i = m - 1$, lowers his Lindahl private consumption and welfare and increases the Lindahl private consumption and welfare of others [Boadway, Pestieau and Wildasin (1989a, Theorem 4, 1989b); see also Roberts (1992), Kaplow (1995) and Brunner and Falkinger (1999, Lemma 6.1), for similar characterizations of Pareto-efficient linear tax-subsidy schemes with positive individual contributions in one-stage equilibrium setups].

A comparison with the results of Roberts (1987) is instructive in two respects.

First, Boadway et al. make full general equilibrium comparative statics, which was not the case of Roberts. The heart of their argument lies in the derivation of the Sultsky equations for equilibrium individual contributions [Boadway, Pestieau and Wildasin (1989a, Equation (13))]. These equations combine exogenous (compensated) substitution effects associated with policy manipulations of matching rates, and endogenous wealth effects associated with the variation in the equilibrium provision level of the public good determined by these manipulations (this second type of effects ignored in Roberts' study). This combination, and the normality assumption which implies that substitution and wealth effects work in the same direction, drive all their other results [including an interesting confirmation, not mentioned above, of a standard result in the literature on grants, namely, that matching grants stimulate private contributions more than lump-sum grants do: Boadway, Pestieau and Wildasin (1989a, Equation (14))].

Second, while the type of policy considered by Boadway et al. is not neutral (notably because their individual agents do not "see through" the government's budget constraint: see Section 5.2.2 above), their statement that the Pareto-efficient level of a uniform matching rate is $\beta = (m-1)/m$ implies, nevertheless, similar conclusions as Roberts' on comparative treasury-efficiency of (flat) matching grants and lump-sum direct taxation, namely: public spending associated with efficient equilibrium provision G is smaller with flat subsidy ($= ((m-1)/m)G$) than with direct taxation ($= G$). Moreover, the relative advantage of flat subsidy in terms of treasury-efficiency decreases in the number of donors and vanishes in the limit as this number grows to infinity, a property that parallels Roberts' remarks that the subsidy rate will be close to 1 in the case of genuine pure public goods such as national defense or public assistance and that the treasury-efficiency advantage of flat subsidy will be negligible then.

We now turn, to finish with, to the class of one-stage implementation models where it is assumed that the agents see through the budget constraint of the government. We will review the contributions of Andreoni and Bergstrom (1996),[92] Falkinger (1996), Kirchsteiger and Puppe (1997), and Brunner and Falkinger (1999) successively. All study variants of the linear tax-subsidy scheme.

We follow more particularly Brunner and Falkinger for the formulation of the analytical framework.[93] General budget-balanced tax-subsidy schemes are specified in the setup of the strong BBV distributive social system with differentiable, strictly quasi-concave utility functions of donors and strictly normal private and public goods. The net tax paid by contributor i when the gift-vector is $(g_1, \ldots, g_m)$ reads: $\tau_i + \sum_{j \leqslant m} \beta_{ij} g_j$, where τ_i is lump-sum, $\beta_{ii} > -1$ is the subsidy rate facing i for his own contribution and β_{ij} is the tax rate facing i for j's contribution, $j \neq i$. It is assumed that taxes levied on any agent's contribution are non-negative on the aggregate (that is: $\sum_{j \leqslant m:\ j \neq i} \beta_{ji} \geqslant 0$ for all i) and that the government balances its budget (that is: $\sum_{i \leqslant m}(\tau_i + \sum_{j \leqslant m} \beta_{ij} g_j)$ covers exactly the direct contribution of the government to the public good). Individual budget constraints read: $x_i + (1 - \beta_{ii}) g_i = \omega_i - \tau_i - \sum_{j \leqslant m:\ j \neq i} \beta_{ij} g_j$. Finally, individual agents see through the government budget constraint, which means that each agent i maximizes $v_i(x_i, g_i + G_{-i} + \sum_{i \leqslant m}(\tau_i + \sum_{j \leqslant m} \beta_{ij} g_j))$ subject to the individual budget constraint above.

Two subclasses of linear schemes are considered and studied in this literature.

The first one is the class of uniform tax schemes, where, by definition, each agent i faces a unique tax rate β_i on the contributions of others (that is: $\beta_{ij} = \beta_i$ for all i and all $j \neq i$). The distortionary tax paid by an individual is then completely determined by the aggregate contribution of others. A non-cooperative equilibrium where all agents contribute is Pareto-efficient if and only if it verifies the condition of Boadway et al. above, that is: $-\sum_{i \leqslant m} \beta_{ii} = m - 1$ [Brunner and Falkinger (1999, Lemma 6.1)]. Each uniform linear tax-subsidy scheme that verifies this condition yields a unique equilibrium allocation $(x_1, \ldots, x_m, G)$, which is efficient and $\gg 0$ (op. cit.: part (a) of Theorem 5.1). If all

[92] As already noticed in Section 5.2.2 above, Andreoni and Bergstrom present their games in a multi-stage setting, but the equilibria that they study in their first two games are, actually, simultaneous equilibria played at one of these multiple stages only (at stage 2, to be precise). Their results apply to, therefore, and are actually viewed in the literature as relative to, one-stage implementation theory.

[93] With the provision on the specification of individual utility functions mentioned in Footnote 91 above.

agents contribute, nevertheless, the efficient non-cooperative equilibrium is not unique, for there exists then an infinite number of equilibrium vectors of positive private contributions $(g_1, \ldots, g_m)$ that yield the unique efficient allocation (op. cit.: Theorem 6.1).

Two examples of uniform linear tax-subsidy schemes have been studied in some detail in the literature.

Game 1 of Andreoni and Bergstrom (1996), first, lets individuals be subsidized at a flat rate β and pay taxes proportional to aggregate government subsidies βG at an individual tax rate s_i. Formally, they let $\tau_i = 0$, $\beta_{ii} = -\beta(1 - s_i)$, $\beta_{ij} = s_i \beta$ for all $j \neq i$, and $\sum_{i \leqslant m} s_i = 1$ in the general scheme above, with resulting individual budget constraints of the type: $x_i + (1 - \beta) g_i = \omega_i - s_i \beta G$. They prove that: the Cournot–Nash equilibrium is unique whenever $(\beta, s_1, \ldots, s_m)$ is such that $0 \leqslant \beta < 1$ and $0 \leqslant s_i < 1$ for all i [Andreoni and Bergstrom (1996, Theorem 1)]; moreover, the equilibrium provision level of the public good is then strictly increasing in $\beta (< 1)$ (op. cit.: Theorem 2). Kirchsteiger and Puppe (1997) establish in the same setup, under the additional assumption of continuous differentiability of utility functions, that: there always exists $\beta \leqslant 1$ such that the associate equilibrium supply of the public good is Pareto-efficient; and an efficient equilibrium is interior if and only if $\beta = 1$, the associate individual prices $1 - \beta + \beta s_i = s_i$ then being the Lindahl equilibrium prices, such that $\sum_{i \leqslant m} s_i = m - 1$ [Kirchsteiger and Puppe (1997, Theorem 1)]. The tax-subsidy scheme of Andreoni and Bergstrom, therefore, yields an efficient interior equilibrium that is not unique and implies the full financing of the public good by government budget. This was noticed by Falkinger (1996) who proposed an alternative uniform linear scheme with better treasury-efficiency [see also Falkinger et al. (2000)]. The tax paid by individual i is now proportional to the deviation $g_i - (G_{-i}/(m - 1))$ of his contributions from the average contribution of others. There is a unique tax rate β, so that individual budget constraints read: $x_i + (1 - \beta) g_i = \omega_i - \beta(G_{-i}/(m - 1))$. Falkinger establishes that: the Cournot–Nash equilibrium is unique whenever $0 \leqslant \beta < 1 - (1/m)$, and the associate provision level of the public good is then increasing in β (op. cit.: Proposition 1(i)); an interior equilibrium is efficient if and only if $\beta = 1 - (1/m)$ (op. cit.: pp. 417–418). The interior efficient equilibrium is not unique [op. cit.: Proposition 1(ii), or Brunner and Falkinger (1999, Lemma 6.1)], but the associate supply of the public good is [Brunner and Falkinger (1999, Theorem 5.1)]. The latter can be therefore achieved approximately by a unique non-cooperative equilibrium, by taking $\beta < 1 - (1/m)$ arbitrarily close to $1 - (1/m)$. The Pareto-efficient scheme of Falkinger is more treasury-efficient than Andreoni and Bergstrom's, in relative terms, because the associate public spending covers only a fraction of public good provision.[94]

The second subclass of mechanisms examined by this literature is the class of non-uniform linear tax-subsidy schemes. The linear tax-subsidy scheme associated with the

[94] Comparison can only be made in relative terms, that is, on the basis of total public spending per unit of supply of the public good, because the efficient provision levels corresponding to the two schemes will, in general, be different.

family $(\beta_{ij})_{i\leqslant m, j\leqslant m}$ of tax-subsidy rates is non-uniform if, by definition, $\beta_{ij} \neq \beta_{ik}$ for some triplet of distinct agents i, $j \neq i$ and $k \neq i, j$. The tax paid by an agent whose tax rates are non-uniform depends on the whole vector of contributions of others, and not only on their sum. An example is the case of uniformity of the tax rate with respect to a partition of the set of donors, imagined by Falkinger (1996). The set of donors $\{1, \ldots, m\}$ is partitioned first in a family $(I_r)_{1\leqslant r\leqslant \rho}$ of ρ non-empty subsets, with $\rho \geqslant 2$ and $\#I_r \geqslant 2$ for all r (implying that $m \geqslant 4$). The tax paid by member i of group r is then proportional to the deviation $g_i - ((1/(\#I_r - 1)) \sum_{j\in I_r} g_j)$ of his contributions from the average contribution of the group, with the same tax-subsidy rate β^r for all. Formally, we have, for any pair of distinct agents i and j: $\beta_{ij} = \beta^r (= -\beta_{ii}/(\#I_r - 1))$ if i and j are members of group r; $\beta_{ij} = 0$ otherwise; so the budget constraint of the members of group r read $x_i + (1 - \beta^r)g_i = \omega_i - \beta^r((1/(\#I_r - 1)) \sum_{j\in I_r:\ j\neq i} g_j)$. It is then shown that: interior non-cooperative equilibrium is unique whenever $\beta^r \neq 1 - (1/\#I_r)$ for all r [as a consequence of Brunner and Falkinger (1999, Theorem 6.2); see also Falkinger (1996, Proposition 2) as a special case]; it is both unique and efficient, in particular, whenever $\beta^r = (1/(\#I_r - 1))(1 - (1/m))$ for all r [as a consequence of Brunner and Falkinger (1999, Lemma 6.1)]. Kirchsteiger and Puppe (1997), nevertheless, exhibit examples of strong BBV distributive social systems with Cobb–Douglas utility functions, which have a Pareto-inefficient boundary equilibrium, in addition to their unique interior efficient equilibrium (op. cit.: 4, notably Theorem 2 and proof); moreover, the efficient interior equilibrium is unstable in some of their examples. Note, finally, that the Pareto-efficient non-uniform scheme of Falkinger is superior, in terms of relative treasury-efficiency (public spending per unit of equilibrium supply of the public good), to his Pareto-efficient uniform scheme, since $\beta^r < 1 - (1/m)$ for all r.

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