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The distribution of wealth in the liberal social contract

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Abstract

I want to consider several systems of private property, where individual agents can redistribute private wealth, individually or collectively, according to moral sentiments. The 'Distributive Lindahl Equilibrium' is considered first and it is proven unstable and ethically questionable. Then, a definition of the 'Distributive Liberal Social Contract' is proposed which appears ethically and practically acceptable. The logical consistency of the liberal social contract is established in a theorem which proves the existence of such a contract for all initial distributions of wealth, when individual agents share the common opinions that wealth should be consumed by individuals rather than disposed of, and that gifts should flow down the scale of wealth. The distributive liberal social contracts are then the Pareto efficient distributions that are unanimously preferred to the initial distribution of rights.

Keywords: Moral sentiments; Social contract; Private wealth

0. Introduction

This article develops a pure theory of the distribution of private wealth, founded on moral sentiments and the right of private property.

I am interested in the phenomena of individual gifts (bequests, individual charitable gifts, ...) or collective gifts by private institutions, or even by public ones if the corresponding redistributive transfers are widely desired by tax payers.

My purpose is thus to develop a conceptual framework that will account for these facts, and, more precisely, for those gifts which can be traced back to moral

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sentiments like benevolence, compassion, or an individual sense of distributive justice, in a context of private property.

An early attempt to integrate moral sentiments into the modern theory of value can be found in Pareto's article, "Il Massimo di Utilità per Una Collettività in Sociologià" (Pareto, 1913), where he recognizes that the utility of an individual can depend on the ophelimity of the others (i.e., on the satisfaction they derive from their private consumptions) and extends accordingly to these interdependent utilities the definition of his famous optimum.

The distribution of private wealth is then analogous to a public good (Kolm, 1966; Hochman and Rodgers, 1969), but a "public good" of a specific and somewhat paradoxical nature. I propose here, in the spirit of Kolm's liberal social contract (1985, Chap. 19; 1987), a theoretical framework that integrates the ideal feature of Pareto efficiency with the institutional fact of private property, in a way that accounts for the existence of individual or collective gifts stemming from moral sentiments.

This article is organized as follows: Section 1 lays a simple foundation for an explanation of the distribution of wealth based on moral sentiments and private property, Section 2 analyzes the distributive Lindahl equilibrium, Section 3 defines the distributive liberal social contract, Section 4 studies the existence of the distributive liberal social contract.

1. Definitions of wealth

Wealth is defined as the money value of private consumption commodities. We want, in other words, to concentrate on that part of community's wealth that is available for both individual consumption and individual ownership, and will name it private wealth. We consider that the total amount of private wealth is given once and for all, and equal to one unit. We suppose, also, that wealth is owned by individual agents. The social systems that we consider here are, in that sense, social sytems of private property. Agent i 's share of the unit of available wealth is his initial share, endowment or right. It is denoted ω_i and takes a real value between 0 and 1. The private ownership of consumption commodities translates then into the following: $\sum_i \omega_i = 1$. [1]

While private consumption by individuals seems to be the natural destination of wealth as defined above, the agents can nevertheless contemplate, individually or collectively, at least two alternatives. First, gift, defined as a decision of an agent to transfer his property right on some part of his own wealth to some

[1] These definitions and assumptions are consistent with the representation of wealth implied in Walrasian exchange economies.

other agent.[2] Second, disposal, defined as a decision of an agent to renounce to his property right on some part of his own wealth (transfer it to nature, so to speak, i.e., cancel the right).

We concentrate here on these three possible uses of wealth, and ignore the fourth conceivable one, that is, the use of consumption commodities as inputs of production processes ruled by individual agents (home production can be safely neglected here, and market production is considered as given).

We assume also that the money costs induced by the activities of consuming, giving or disposing are negligible. This implies that we ignore here distortionary taxes based on consumed, given or disposed wealth;[3] and contracting costs as well, whenever these decisions are part of some private or social contract.

The money value of individual i 's private consumption, his gift to agent j, and the money value of his disposal of consumption commodities are non-negative real numbers, denoted respectively x_i, t_{ij} and d_i. Since we decided to neglect household production activities as well as costs generated by consumption, gift, and disposal activities, individual decisions must verify the following accounting identities: $x_i + d_i + \sum_{j \neq i} t_{ij} = \omega_i + \sum_{j \neq i} t_{ji}$, connecting wealth outflows to wealth inflows in individual ownership.[4]

2. Unstable Pareto optima

A simple way of introducing moral sentiments in our representation of individual agents is to suppose that each of them has opinions on the

[2] Gifts, so defined, belong to the more general category of wealth transfers. To avoid notational inflation, wealth transfers from i to j will be denoted t_{ij}. The latter can be a positive quantity (wealth flowing out i's estate) or a negative one (flowing in). Wealth transfers can be the result of an individual or a collective decision, voluntary or compulsory, legal or illegal... A gift from i to j is then a non-negative wealth transfer, decided by agent i, either isolated ('individual gift', see for instance Mercier Ythier, 1989, 1992, 1993) or as part of a collective decision process ('social' or 'general' gift, see Kolm, 1984, 1985), within the limits of some individual or collective right (as the right of private property, for instance).

[3] Such taxes are directed, in practice, to specific types of consumption commodities (e.g. luxury goods) or gifts (e.g. bequests), rather than to consumption or gift as a whole. These phenemona belong therefore to price theory, rather than to the pure theory of wealth distribution that is our object here.

[4] While our terms are defined in money value, the reader might find it more comfortable to decompose values into quantities and prices. Suppose that there are l consumption commodities, and denote X_i, D_i, T_{ij}, Ω_i and Z_i the vectors of quantities respectively consumed (X_i), disposed of (D_i), given to individual j (T_{ij}), owned initially (Ω_i), and exchanged (net trade Z_i) by individual i. Ignoring production activities, these elements of the commodity space $\mathbb{R}^l$ must verify the physical accounting identities: $X_i + D_i + \sum_{j \neq i} T_{ij} = \Omega_i + Z_i + \sum_{j \neq i} T_{ji}$. Denoting p as the vector of market prices, one observes then that the money accounting identities $pZ_i = 0$ are logically equivalent to the identities of the text above.

distribution of private consumption expenditures among households, summarized in utility functions $(x_1, \ldots, x_n) \to w_i(x_1, \ldots, x_n)$ (where n denotes the number of agents).

The distribution of wealth is then formally analogous to a public good (Kolm, 1966; Hochman and Rodgers, 1969). And the price mechanism of the distributive Lindahl equilibrium (Bergstrom, 1970) might appear, consequently, as a natural, theoretical if not practical way of defining a social functioning ensuring the achievement of Pareto efficiency. Section 2 challenges this point of view, on the grounds that distributive Lindahl equilibrium can prove 'unstable' and ethically questionable.

2.1. Distributive Lindahl equilibrium

Let me introduce a small number of formal definitions and assumptions.

The *space of consumption distributions* is $\mathbb{R}^n$. Elements of its positive orthant $\mathbb{R}^n_+$ are *a priori feasible* consumption distributions. Elements of set $K_n = \{x \in \mathbb{R}^n_+ | \sum_i x_i \leq 1\}$ are *feasible* consumption distributions. Elements of unit simplex $S_n = \{x \in K_n | \sum_i x_i = 1\}$ are the feasible distributions that exhaust available wealth. Utility functions $w_1, \ldots, w_n$ are defined on the space of consumption distributions $\mathbb{R}^n$.

A *distributive* (*strong*) *Pareto optimum* is an undominated feasible distribution, that is, an element x of K_n such that x' is not feasible whenever $w_i(x') \geq w_i(x)$ for all i and $w_i(x') > w_i(x)$ for at least one i. [5]

Denote: w the distribution of preferences $(w_1, \ldots, w_n)$; ω the initial distribution of wealth $(\omega_1, \ldots, \omega_n)$; π_{ij} the money value of one unit of j's wealth to individual i; $\Pi = (\pi_{ij})_{ij}$ the (n,n)-matrix of personalized prices; and $\pi = \max(0, \sum_i \pi_{i1}, \ldots, \sum_i \pi_{in})$ the social value of wealth. A *free disposal distributive Lindahl equilibrium* of the social system of private property (w, ω) is then a pair (Π^*, x^*) such that: (i) x^* is feasible; (ii) $\pi^* (\sum_i x_i^* - 1) = 0$; (iii) $(\pi^* - \sum_i \pi_{ij}^*)\, x_j^* = 0$ for all j; (iv) for all i, x^* is a maximum of w_i in budget set $B_i(\Pi^*, \omega_i) = \{x \in \mathbb{R}^n \mid x_i \geq 0, \sum_j \pi_{ij} x_j \leq \omega_i\}$.

The following correspondence between distributive optima and distributive equilibria is then easily established, under standard assumptions on utility functions (cf. Appendix A, Theorem A.1). A distributive Lindahl equilibrium distribution is a distributive Pareto optimum. If x^* is a distributive Pareto optimum, there exists some matrix of individual values Π^* such that (Π^*, x^*) is

[5] A distributive weak Pareto optimum is, likewise, an element x of K_n such that x' is unfeasible whenever $w_i(x') > w_i(x)$ for all i. Except for an explicit warning to the contrary, a distributive Pareto optimum will always be a strong optimum in the text below.

a distributive Lindahl equilibrium of the social system of private property $(w, \Pi^* x^*)$.[6]

The distributive Lindahl equilibria of social system (w, ω) are therefore the Pareto optima that are consistent with the initial distribution of rights ω, consistency meaning here that individual equilibrium consumptions x_i^* and transfers[7] $t_{ij}^* = \pi_{ij}^* x_j^*$ $(i \neq j)$ verify budget constraints $x_i^* + \sum_{j \neq i} t_{ij}^* \leq \omega_i + \sum_{j \neq i} t_{ji}^*$ or equivalently $\sum_j \pi_{ij}^* x_j^* \leq \omega_i$.[8]

2.2. *Unstable distributive Lindahl equilibrium*

Example one. Malevolence.

We consider a social system of three agents $(n = 3)$. The initial distribution of wealth is $\omega^0 = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$. Agent i has a linear utility function w_i: $(x_1, x_2, x_3) \rightarrow \sum_j \beta_{ij} x_j$, where $\beta_{ii} > 0$. Agents 1 and 2 are 'unsympathetically isolated' (Edgeworth, 1881), which means that $w_i(x) = \beta_{ii} x_i$ for all x whenever $i = 1, 2$; we set, without loss of generality, $\beta_{11} = \beta_{22} = 1$. Agent 3 is malevolent to agents 1 and 2 in the following technical sense: w_3 is strictly decreasing in x_1 and x_2 (or equivalently β_{31} and $\beta_{32} < 0$).[9] We set, more precisely, $\beta_{33} = 1$ and $\beta_{31} = \beta_{32} = -1$.

Easy calculations yield then the following results. The set of distributive Pareto optima is the simplex S_3. The unique equilibrium distribution is distribution $x^* = (\frac{1}{4}, \frac{1}{8}, \frac{5}{8})$, and the corresponding equilibrium transfers $t_{ij}^* = \pi_{ij}^* x_j^*$ $(i \neq j)$ are all equal to 0 except $t_{31}^* = -\frac{1}{4}$ and $t_{32}^* = -\frac{1}{8}$.

Malevolent agent 3, in particular, is tied to equilibrium distribution x^* by the cost-advantage relationship associated with personalized prices π_{31}^* and π_{32}^*. He receives, more precisely, a money compensation proportional to agent 1's and agent 2's consumptions, that exactly balances his unhappiness at the latter: a marginal decrease $-\varepsilon < 0$ of agent 1's or agent 2's consumption diminishes his own income and consumption in such a way $(dx_3 = \pi_{31}^* \varepsilon = -\varepsilon)$ that his utility is left unchanged $(\partial w_3(x^*) \cdot dx = w_3(x^* + dx) = -\varepsilon + \varepsilon = 0)$.

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[6] Similar results are established in Bergstrom (1970) under more general technical assumptions, but more particular psychological ones since he assumes malevolence away. We do not want to adopt such an optimistic view of human nature here.

[7] These transfers can be negative as well as positive numbers (see for instance example one below). They must be distinguished, therefore, from gifts, that are non-negative transfers by definition (cf. footnote 2 above).

[8] Budget constraints differ from the money accounting identities written down in Section 1 in omitting disposal decisions. The latter account for possible discrepancies between the left and right-hand sides of the constraints.

[9] This should be understood as an interpretation, in psychological terms, of a technical property of w_3, rather than as a definition of malevolence.

The equilibrium of this example seems very unstable, and appears moreover, to paraphrase Ramsey (1928), ethically indefensible, for the following three reasons.

First, common sense suggests that agents 1 and 2 will in fact refuse to pay the transfer, and simply decide on their own to consume their initial endowments (their utility is larger then), thereby ignoring this Paretian social contract (technically, this Lindahl equilibrium is not a Nash equilibrium, cf. Section 3 below). The ability to consume one's own endowment seems moreover implied by any reasonable definition of the right of private property. This example exhibits, therefore, a case of inconsistency of this distributive mechanism with the right of private property (agent 1 and agent 2's equilibrium budget constraints do not allow them to consume their endowments) that results in a potential instability of its outcome (agents 1 and 2 are willing to consume their endowments).[10]

Second, it might be difficult to find an ethical justification for the money transfers received by agent 3 from agents 1 and 2, at least if we interpret his distributive preferences as reflecting a genuine malevolence. The price mechanism allows these negative feelings to influence the distribution of wealth, at the expense of the other two agents (the majority, moreover, in this simple social system). These money transfers hurt, therefore, common sense individual ethics, which suggest that malevolence should not be rewarded.

This distributive mechanism, paying money transfers to malevolent agents, might eventually open Pandora's box, and generate a difficult preference revelation problem, since it implies a clear incentive for all agents, whether malevolent or not, to declare or exaggerate malevolent feelings.

3. Distributive liberal social contract

The cost-advantage mechanism of the distributive Lindahl equilibrium appeared as a dead-end in Section 2, despite obviously seductive properties intimately associated with the functioning of a price mechanism, like unanimous agreement, efficiency and the existence of equilibrium.

Some of the difficulties outlined above find their origin in the fact that the 'public good' (in the sense of a 'good' or 'bad' which is common concern to several agents) is, here, the private wealth of individual agents. The allocation of this particular public good by means of the usual price mechanism is inconsistent with the usual definition of the right of private property, and

[10] This inconsistency of the distributive Lindahl equilibrium with the right of private property, and the subsequent potential instability of 'equilibrium', follow from the specification of budget sets (cf. Section 3.1 below), and does not hinge, therefore, upon the presence of a malevolent agent.

therefore lacks logical foundation in undermining the notion of private wealth itself.

I will therefore build directly, in this section, on the right of private property, as logical and practical foundation of private wealth.

3.1. Right of private property

In our simple context, where individual uses of wealth are restricted to consumption, gift and disposal, the usual definition of the right of private property merely states that an agent can consume, give or dispose of any fraction of his own wealth, that is, of his initial wealth (endowment) augmented by the gifts received from other agents.

Let me formulate this definition more precisely. An *action* of agent i is a consumption-gift vector $a_i = (x_i, t_i)$ where t_i denotes vector $(t_{i1}, \dots, t_{in})$ whose ith component t_{ii} is conventionally set equal to 0 (using the accounting identity of Section 1, we may define it equivalently as a disposal-gift vector $((d_i, t_i))$. We denote: a action-vector $(a_1, \dots, a_n)$; $a_{n\setminus i}$ action-vector a deprived of its ith component a_i; $(a_{n\setminus i}, a_i^*)$ the action-vector built from a by replacing its ith component a_i by a_i^*; $x(a)$ the consumption-distribution built from action-vector a by extracting the consumption components of a; $t(a)$ the gift-vector built from action-vector a by extracting the gift components of a. We name then *social state* associated with a the pair $(x(a), t(a))$, and say that a social state $(x(a), t(a))$ of the social system of private property (w, ω), or the corresponding action-vector, are *feasible* if distribution $x(a)$ is feasible and if there is some disposal-vector d of $\mathbb{R}^n_+$ such that $(x(a), t(a), d)$ verifies the accounting identities of Section 1.

Agent i's *budget set* associated with action-vector a^* in the social system of private property (w,ω) is by definition the set:

$$\mathrm{B}_i(a^*, \omega_i) = \left\{(x_i, t_i) \in \mathbb{R}^{n+1}_+ \mid x_i + \sum_{j \neq i} t_{ij} \leq \omega_i + \sum_{j \neq i} t_{ji}(a^*)\right\}$$

(that must be distinguished from set $B_i(\Pi, \omega_i)$ of Section 2).

We say that the social system of private property (w, ω) *respects the right of private property* if every agent i can choose any action a_i in his budget set $\mathrm{B}_i(a^*, \omega_i)$ for all action-vectors a^*. This means that the agents of such a social system freely allocate their own wealth (i.e. initial wealth and gifts received from other agents) between the three alternative uses of private consumption, gift and disposal. [11]

As noticed above, the social systems of Section 2, endowed with the distributive Lindahl equilibrium, violate the right of private property as just defined.

[11] The freedom of exchange, of buying and selling quantities of consumption commodities at given market prices, is implicit in this definition of the right of private property, as observed in footnote 4.

Individual values transmitted by personalized prices distort budget sets, making some individual consumptions or gifts inaccessible while they are consistent with the individual budget constraint written above (for instance: consumption ω_1^0 for individual 1 of example one). [12]

3.2. *Gift equilibrium*

A *coalition* is any non-empty subset of the set of agents (possibly reduced to a single agent). Let I be a coalition, and denote: ω_I vector $(\omega_i)_{i\in I}$; a_I vector $(a_i)_{i\in I}$; $a_{\setminus I}$ vector $(a_i)_{i\notin I}$; $(a_{\setminus I}, a_I^*)$ the action-vector built from a by replacing a_i by a_i^* for all i in I. The *budget set of coalition I*, associated with action-vector a^* in social system (w, ω) is then the set:

$$\mathrm{B}_I(a^*, \omega_I)$$
$$= \left\{ a_I = ((x_i, t_i))_{i\in I} \in \mathbb{R}_+^{(n+1)\mathrm{card}I} \middle| \sum_{i\in I} \left(x_i + \sum_j t_{ij} \right) \right.$$
$$\left. \leq \sum_{i\in I} \left(\omega_i + \sum_{j\in I} t_{ji} + \sum_{j\notin I} t_{ji}(a^*) \right) \right\}.$$

We say that feasible action-vector a or state $(x(a), t(a))$ are *blocked* by coalition I in social system (w, ω) if there is some a_I^* in $\mathrm{B}_I(a, \omega_I)$ that is strictly preferred by all agents in coalition I (formally, $w_i(x(a_{\setminus I}, a_I^*)) > w_i(x(a))$ for all i in I).

A *distributive core equilibrium* of social system (w,ω) is then an action-vector that is both feasible and unblocked in this social system. [13]

The following facts are immediate consequences of the definitions. A social system endowed with the distributive core equilibrium respects the right of private property. A distributive core equilibrium is a distributive Nash equilibrium (it is unblocked by single-agent coalitions). A core equilibrium distribution is a distributive weak Pareto optimum (it is unblocked by the coalition of all agents). A Lindahl equilibrium distribution is not, in general, a core equilibrium one (it might be blocked, for instance, as noticed in example one above, by single-agent coalitions).

This definition of gift equilibrium allows for the existence of a wide range of voluntary redistributions, including individual gifts (grasped in Nash equilibrium), and collective or cooperative ones resulting from contractual decisions taken in groups of agents pooling their resources for distributive purposes. Since I want to be able to account for all these phenomena, because of both their factual existence and practical importance in actual voluntary redistributions,

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[12] As already mentioned in footnote 10, this will happen quite generally, even if all agents are non-malevolent.

[13] This type of equilibrium is sometimes known as a 'strong equilibrium' (e.g. Moulin, 1981, p. 85).

I will retain the distributive core equilibrium as main definition of gift equilibrium below.

3.3. Distributive liberal social contract

The distributive core equilibrium synthesizes two fundamental principles of economic liberalism, the right of private property (implied in the specification of individual budget sets) and free contracting (implied in the definition of blocking coalitions). It can be viewed therefore as part of a distributive liberal social contract.

Such a contract (Kolm, 1985, chap. 19; Kolm, 1987) consists in a decision on the distribution of individual rights $(\omega_1, \ldots, \omega_n)$, taken in common by all members of the society, and respecting some liberal constitutional principles. I suggest the following formal definition:

Definition. The distribution of rights $\omega^* \in S_n$ is a distributive liberal social contract in the social system of private property (w, ω^0) if: (i) $w_i(\omega^*) \geq w_i(\omega^0)$ for all i; and (ii) ω^* is a core equilibrium distribution of social system (w, ω^*).

Condition (i) merely states the unanimity principle that the liberal social contract must be unanimously preferred to the initial distribution of rights. Condition (ii) draws the consequences of the right of private property and free contracting: it would be pointless to choose, in the social contract, a distribution of rights that would then be rejected by isolated or united individuals using their constitutional rights of private property and free contracting.

The distributive liberal social contract appears therefore, in this definition, as the outcome of a social decision process that consists in the maximization of the usual Paretian partial preordering, subject to constraints induced by constitutional rules and the initial distribution of rights. Society (more precisely, the set of individuals $\{1, \ldots, n\}$) selects distributions of rights that are both unanimously preferred to the initial one, and stable in the sense of core equilibrium.

The distributive liberal social contract captures in one single concept several interesting aspects of voluntary redistribution, namely, private gifts stemming from autonomous initiatives of isolated or united individuals (core equilibrium), and social or public gifts resulting from a conscious deliberation of all members of society (or, in first approximation, of their elected representatives).

4. Existence of distributive liberal social contracts

Section 3 established the adequacy of the distributive liberal social contract to the object of this study. Section 4 explores the internal consistency of the concept through the study of its existence property.

4.1. Existence theorem

Denote e_{ij} the vector of $\mathbb{R}^n$ whose components are all equal to 0 except the ith one, equal to -1, and the jth one, equal to $+1$.

Consider then the following assumptions on utility functions:

Assumption 1. *Function $\tau \to w_i(x + \tau e_{ij})$, defined on $\mathbb{R}_+$, is non-increasing in some neighborhood of* 0 *whenever $x_j \geq x_i$.*
Assumption 2. *Function $\tau \to w_i(x + \tau e_{jk})$, defined on $\mathbb{R}_+$, is non-decreasing in some neighborhood of* 0 *whenever j and k are distinct from i and $x_j \geq x_k$.*
Assumption 3. *For all coalitions I, all a priori feasible distributions $x \in \mathbb{R}^n_+$, and all neighborhoods V of x in $\mathbb{R}^n_+$, there exists some x' in V such that $w_i(x') > w_i(x)$ for every i in I.*

Assumption 1 means that agent i does not individually desire to redistribute wealth from himself to wealthier agents. Assumption 2 says that he does not object to wealth redistributions from agent j to agent k (j, $k \neq i$) as long as the former is at least as rich as the latter. Assumptions 1 and 2 together imply that the agents share the common opinion that transfers should flow down the scale of wealth. [14] Assumption 3 supposes local non-satiation, extended to coalitions, in the manner of Rader, 1980, for instance, where local non-satiation is assumed for the coalition of all agents.

These assumptions imply that any Pareto efficient initial distribution of rights is a core equilibrium. The existence of a distributive liberal social contract for all initial distributions of rights follows from this result and from the additional assumption of continuity of utility functions.

Theorem 1. *Suppose that social system w verifies assumptions* 1–3. *Then, any (strong) Pareto optimum ω is a core equilibrium distribution of the social system of private property (w, ω). If, moreover, w_i is continuous for all i, then, for every $\omega \in S_n$, there is at least one distributive liberal social contract in the social system of private property (w, ω).*

Proof. Suppose that the first part of theorem 1 is true, and let then prove the second part. Set $X(\omega^0) = \{\omega \in S_n | w(\omega) \geq w(\omega^0)\}$ is a non-empty (it contains ω^0)

[14] What is essential for the results of Theorem 1 is that the agents share a common opinion on the acceptable direction of wealth transfers, not the particular common opinion contained in Assumptions 1 and 2 (that transfers should flow down the scale of wealth). Suppose, for instance, that index i, designating individuals in this model, is, in fact, an index of social rank (a 'degree of nobility', so to speak). The common opinion that wealth transfers should flow down the scale of nobility would imply the same consequences as the assumptions above (simply replace inequalities $x_i \geq x_j$ by inequalities $i \geq j$ in assumptions and proof). I nevertheless selected these particular assumptions, because I believe in their practical and factual relevance for the object of this inquiry.

and closed (continuity of w) subset of compact set S_n. It is therefore a non-empty and compact set. Function $\sum_i \alpha_i w_i$, where α_i denotes a strictly positive real number for all i, is continuous, and has therefore at least one maximum ω^* in $X(\omega^0)$. ω^* is a strong Pareto optimum by construction, unanimously preferred to ω^0 by definition of $X(\omega^0)$, and is a core equilibrium distribution of (w, ω^*) by the first part of Theorem 1. It is therefore a distributive liberal social contract of (w, ω^0), and the second part of the theorem is proved.

Let us prove now the first part of Theorem 1. Consider a distribution of rights ω^0, suppose that state $(\omega^0, 0)$ is blocked by some coalition I in the social system of private property (w, ω^0), and let us prove that it is not, then, a strong Pareto optimum.

For any given state (x, t), let $a(x, t)$ be the associate action-vector $((x_i, t_i))_{i \in N}$. Since $(\omega^0, 0)$ is blocked by coalition I, there exists some a_I^* in budget set $B_I(a(\omega^0, 0), \omega_I^0)$ of coalition I, such that $w_i(x(a_I^*, a_{\setminus I}(\omega^0, 0))) > w_i(\omega^0)$ for all i in I.

Denote $a^0 = a(\omega^0, 0)$ and $a^* = (a_I^*, a_{\setminus I}^0)$. It follows immediately from Assumption 3 that a_I^* can be chosen, without loss of generality, such that $x_i(a^*) = \omega_i^0 - \sum_j t_{ij}(a^*)$ for all $i \in I$, i.e. such that $d_i = 0$ for all $i \in I$ (no disposal in coalition I).

Denote by H the set of agents i such that $x_i(a^*) < \omega_i^0$ ($H \subset I$ is non-empty; it is the set of "net givers"); K the set of agents i such that $x_i(a^*) > \omega_i^0$ (K is the set of 'net receivers'); $\theta = \sum_{i \in H} (\omega_i^0 - x_i(a^*)) = \sum_{i \in K} (x_i(a^*) - \omega_i^0) > 0$ the total amount of redistributed wealth; λ_i the share $\theta^{-1}(\omega_i^0 - x_i(a^*))$ of agent $i \in H$ in θ; μ_i the share $\theta^{-1}(x_i(a^*) - \omega_i^0)$ of agent $i \in K$ in θ; t^* gift-vector such that $t_{ij}^* = \lambda_i \mu_j \theta > 0$ whenever $(i, j) \in H \times K$, $t_{ij}^* = 0$ otherwise; $x^* = x(a^*)$. It follows then readily from definitions that: $x(a(x^*, t^*)) = x^*$; $a_H(x^*, t^*) \in \mathrm{B}_H(a^0, \omega_H^0)$; $w_i(x^*) > w_i(\omega^0)$ for all i in H. We will therefore assume below, without loss of generality, that $a^* = a(x^*, t^*)$ and $I = H$.

It follows from Assumptions 1 and 2 that, if $t_{ij}^* > 0$ (i.e. if $(i, j) \in H \times K$) and if $x_i^* \leq x_j^*$, the utilities of the members of coalition H (agent i included) are not diminished if the wealth transfer from i to j is decreased by some small enough amount. We will assume therefore, without loss of generality again, that $x_i^* > x_j^*$ whenever $t_{ij}^* > 0$.

State (x^*, t^*) is thus such that: (i) $t_{ij}^* > 0$ if and only if $(i, j) \in H \times K$; (ii) $x_i^* = \omega_i^0 - \sum_{j \in K} t_{ij}^*$ if $i \in H$; (iii) $x_i^* = \omega_i^0 + \sum_{j \in H} t_{ji}^*$ if $i \in K$; (iv) $x_i^* = \omega_i^0$ if $i \notin H \cup K$; (v) $\omega_i^0 > x_i^* > x_j^* > \omega_j^0$ whenever $(i, j) \in H \times K$. But Assumptions 1 and 2 readily imply then that $w_i(x^*) \geq w_i(\omega^0)$ whenever $i \notin H$. Since, moreover, x^* is strictly preferred to ω^0 by the elements of $I = H$, ω^0 is not a strong Pareto optimum. □

4.2. Examples

This subsection gives examples of simple utility functions verifying Assumptions 1–3 above. It provides, next, the detailed analysis of three social systems.

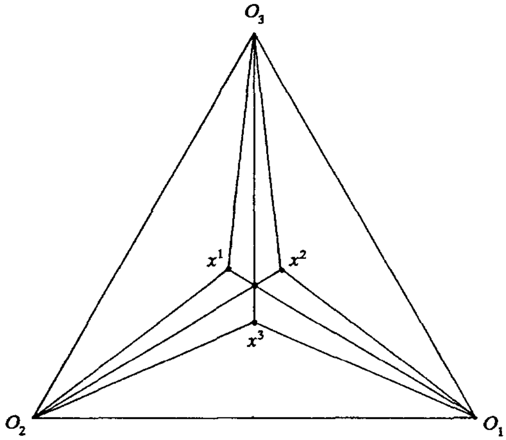

Fig. 1.

Each of these systems violates one and only one of assumptions 1, 2 and 3, and none of them have distributive liberal social contracts (counterexamples to existence 1–3).

Consider, first, a social system w whose members are endowed with linear (resp. Cobb-Douglas) utility functions, that is, functions of the type $w_i : x \to \sum_j \beta_{ij} x_j$ (resp. $\prod_j x_j^{\beta_{ij}}$), where β_{ii} is assumed strictly positive for all i. The following facts obtain then immediately: w verifies Assumption 1 if and only if $\beta_{ii} \geq \beta_{ij}$ for all (i, j); it verifies Assumption 2 if and only if $\beta_{ij} = \beta_{ik}$ for all i and all pairs of agents (j, k) both distinct from i; w verifies assumption 3 whenever $\beta_{ij} \geq 0$ for all i, j.

Let me turn now to the detailed analysis of particular social systems.

Counterexample 1. *War of gifts.* [15]

This social system is made of three benevolent Cobb–Douglas agents, endowed with the following symmetrical preferences: $\beta_{ij} > \beta_{ii} > 0$ and $\beta_{ij} = \beta_{ji} = \beta$ for all i and all $j \neq i$; $\beta^i = (\beta_{i1}, \beta_{i2}, \beta_{i3}) \in S_3$ for all i. It is illustrated in Fig. 1, where: O_i is the element of S_3 where agent i owns or consumes the entire unit of wealth; triangle $O_1 O_2 O_3$ represents S_3.

These utility functions verify all the assumptions of Theorem 1, except Assumption 1. This example exhibits a severe case of non-existence, since there is, here, no distributive Nash equilibrium, and therefore no distributive core

[15] An early version of this example is given in Mercier Ythier (1992).

equilibrium and no distributive liberal social contract whatever the initial distribution of rights.

One notices more precisely that the set M_{ij} of (i, j)-maximal distributions (defined in Appendix B) is surface $O_k x^i O_l$ where k and l are distinct agents different from j, and where x^i denotes the maximum of w_i in S_n ($x^i = \beta^i$ for all i). Set $M = \cap_i M_{ii}$ is therefore empty (cf. Fig. 1). This means that there is, at every distribution x in S_n, some agent who desires and can transfer some of his own wealth to some other agent (that is, some pair of distinct agents (i, j) such that $x_i > 0$ and $\partial_{x_j} w_i(x) > \partial_{x_i} w_i(x)$). Every distribution of S_n is therefore destabilized (blocked) by some individual agent using his right of private property. Since any Nash equilibrium distribution must be in S_n (utility being strictly increasing in own consumption), the sets of distributive Nash and core equilibria must be empty whatever the initial distribution of rights. This implies, in turn, the non-existence of liberal social contracts for all such distributions.

Counterexample 2. *Jealousy.*

The agents have Cobb–Douglas utility functions again. Agent 1 is unsympathetically isolated ($\beta^1 = (\beta_{11}, \beta_{12}, \beta_{13}) = (1, 0, 0)$). Agent 2 is indifferent to 3, benevolent to 1, with the following vector of marginal elasticities of individual wealth: $\beta^2 = (\beta_{21}, \beta_{22}, \beta_{23}) = (\frac{1}{2}, \frac{1}{2}, 0)$. Agent 3 is indifferent to 1, benevolent to 2, and $\beta^3 = (\beta_{31}, \beta_{32}, \beta_{33}) = (0, \frac{1}{2}, \frac{1}{2})$. The initial distribution of rights is point $\omega^0 = (\frac{1}{8}, \frac{3}{8}, \frac{1}{2})$ (cf. Fig. 2).

The utility functions verify all the assumptions of Theorem 1, except Assumption 2. Here, there is no distributive liberal social contract in (w, ω^0). The source

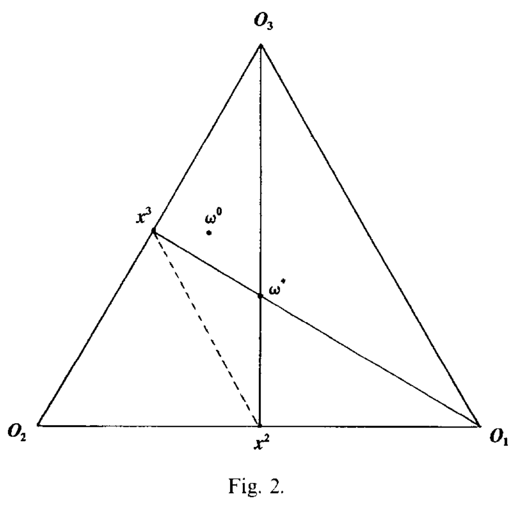

Fig. 2.

of the existence failure lies in the fact that the Pareto efficient distributions of rights that are unanimously preferred to ω^0 are not stable (they are not Nash equilibria).

Let us establish this.

Notice first that M_{ij} is surface $O_k x^i O_l$, where k and l are both distinct from j. Set $M = \cap_i M_{ii}$ is therefore surface $x^2\omega^* O_1$, where $\omega^* = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. This means that the Nash equilibrium distribution runs into surface $x^2\omega^* O_1$ when the initial distribution runs into S_n.

Notice moreover that $\partial_{x_1} w_2(x) > \partial_{x_2} w_2(x) > 0$ and $\partial_{x_3} w_3(x) \geq \partial_{x_2} w_3(x) > \partial_{x_1} w_3(x) = 0$ whenever x lies in surface $x^2\omega^* x^3$ deprived of segment $x^2\omega^*$. It implies that, at any such points: (i) agent 2 desires to give more to individual 1, in the sense that a marginal "one-to-one" transfer of ε units of wealth from agent 2 to individual 1 increases the utility of the former by $\partial w_2(x)\cdot(\varepsilon, -\varepsilon, 0) > 0$; (ii) agent 3 is jealous of marginal "one-to-one" transfers of ε units of wealth from individual 2 to individual 1, in the sense that he would like to be the beneficiary of such transfers in the place of individual 1 ($\partial w_3(x)\cdot(0, -\varepsilon, \varepsilon) > 0 > \partial w_3(x)\cdot(\varepsilon, -\varepsilon, 0)$).[16]

Denoting P the set of Pareto efficient distributions (surface $x^2 O_1 x^3$), it follows from (i) that the distributions of $P\backslash M$ (surface $x^2\omega^* x^3$ deprived of segment $x^2\omega^*$) are destabilized by the desire and ability of agent 2 to redistribute wealth from himself to agent 1. Since the Pareto efficient distributions that agent 3 prefers to ω^0 clearly all belong to set $P\backslash M$, there is no distributive liberal social contract in this social system of private property. The unanimously preferred efficient distributions are destabilized by agent 2's benevolence to agent 1, while the stable distributions are rejected by 'jealous' agent 3.

Counterexample 3. *Contract of war.*

We consider here the following social system of two agents: individual 1 (resp. individual 2) has the linear utility function w_1: $x \to x_1 + \gamma x_2$ (resp. w_2: $x \to \gamma x_1 + x_2$), with $\gamma < -1$, expressing intense malevolence to individual 2 (resp. 1). ω^0 denotes, as usual, the initial distribution of rights.

This social system verifies all the assumptions of Theorem 1, except Assumption 3. Here, there is no distributive liberal social contract, unless agent 1's or agent 2's initial endowment is 0.

One notices more precisely that: (i) any distribution of wealth in set $A = \{x \in K_n | x_1 x_2 > 0\}$ is blocked by coalition $\{1, 2\}$ since $\partial w_1(x)\cdot dx = \partial w_2(x)\cdot dx = -(\gamma + 1)\varepsilon > 0$ whenever $dx = (-\varepsilon, -\varepsilon)$ and x is in set A;

[16] This, again, should be considered as an interpretation, rather than as a definition of jealousy. All we need here is to be able to develop such an interpretation without hurting common sense definitions of this feeling.

(ii) a distribution of set $\{x \in K_n \backslash A | x_i = 0\}$ is blocked by agent i if and only if $\omega_i^0 > 0$, since consumption ω_i^0 is always accessible to individual i (private property right) and always preferred by him to zero consumption; (iii) a necessary condition for the existence of a liberal social contract is therefore that ω^0 belongs to $S_n \backslash A$; (iv) one verifies immediately that this condition is in fact sufficient and that the liberal social contract is then ω^0 itself.

The existence problem can be analyzed here as a cyclical inconsistency in the distributive desires of agent 1 (or agent 2), who wishes, as an isolated person, to consume a strictly positive fraction of total wealth, but prefers, as a member of some larger coalition, to dispose of it. These agents, who remain sensible as long as they take their decisions alone, indulge, by mutual malevolence, in a wasteful and paradoxical 'contract of war' if they happen to make collective decisions.

5. Conclusions

The theory of the distribution of wealth founded on moral sentiments that is outlined in this article stands in sharp contrast with the theory of competitive exchange equilibrium, although the same equilibrium concept (core equilibrium) and implied institutional foundations (private property and free contracting) lie at the heart of both.

Competitive exchange involves individual agents who are 'unsympathetically isolated' (Edgeworth, 1881) on, and in some sense by, markets. The social link is, there, perfect competition itself, characterized by the fact that each commodity has a single price and that the agents 'take' these single prices. A fundamental result of this theory, established in full generality by Arrow and Debreu (1952) and Debreu (1953), is the existence and efficiency of competitive equilibrium.

Moral sentiments expel, symmetrically, perfect competition as defined above, from the determination of the distribution of wealth. The relevant prices are now individual ones (marginal money value to an agent of some individual wealth). The agents, moreover, do not take prices any more, in the sense, first, that all of them have incentives to alter individual prices by disguising their preferences, and second, that some of them might be able and willing to use their right of private property to reject some or even all distributive Pareto optima. The system of individual prices, thus split from perfect competition, and inconsistent with the right of private property, collapses simultaneously on issues of morality, preference revelation, and right. The existence of a distributive liberal social contract relies then, in Theorem 1, on conditions on distributive preferences assuming, in essence, that individuals share the common opinions that wealth should be consumed by individuals rather than disposed of, and that redistributive transfers should flow down the scale of wealth.

Moral sentiments and perfect competition appear here as mutually exclusive social links, and my remarks only express with some technical precision a fact that already framed Adam Smith's work, and in particular the historical sequence of his Theory of Moral Sentiments (Smith, 1759) and Inquiry into the Nature and Causes of the Wealth of Nations (Smith, 1776). I believe, and prove here, that these mutually exclusive concepts can be merged into the definition of a social equilibrium, and fruitfully be applied to the analysis of complex social systems involving the distribution of private property rights, market exchange and production.

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Appendix A. Distributive Pareto optimum and Lindahl equilibrium

Suppose that, for all i, w_i is strictly increasing in its ith argument x_i; differentiable and quasi-concave in the positive orthant $\mathbb{R}^n_+$. Suppose moreover that, for all $x^* \in K_n$ and all i, the constraint set $A_i(x^*) = \{x \in K_n \mid w_j(x) \geq w_j(x^*)$ for all $j \neq i\}$ verifies constraint qualification (Kuhn and Tucker, 1951) in the program $\mathrm{Max}\{w_i(x) | x \in A_i(x^*)\}$.[17] We then have the following standard theorem:

Theorem A.1. If (Π^*, x^*) is a free disposal distributive Lindahl equilibrium of (w,ω), distribution x^* must then be a strong distributive Pareto optimum. If, conversely, x^* is a strong distributive Pareto optimum, there then exists a matrix Π^* of individual values such that (Π^*, x^*) is a free disposal distributive Lindahl equilibrium of $(w, \Pi^* \cdot x^*)$.

Proof. Let us establish the first part of Theorem 2. Consider a free disposal Lindahl equilibrium (Π^*, x^*) of (w, ω) and suppose that x^* is not a distributive strong Pareto optimum, i.e. that there exists $x^{**} \in K_n$ such that $w(x^{**}) > w(x^*)$.

[17] This condition eliminates coincidental colinearities of the gradients of binding constraints.

We must have then $\sum_j \pi^*_{ij} x^{**}_j \geq \sum_j \pi^*_{ij} x^*_j$ for all i, the inequality being strict whenever $w_i(x^{**}) > w_i(x^*)$ (if $w_i(x^{**}) > w_i(x^*)$ and $\sum_j \pi^*_{ij} x^{**}_j \leq \sum_j \pi^*_{ij} x^*_j$, x^* is not a maximum of w_i in $B_i(\Pi^*, \omega_i)$ and therefore not an equilibrium; if $w_i(x^{**}) \geq w_i(x^*)$ and $\sum_j \pi^*_{ij} x^{**}_j < \sum_j \pi^*_{ij} x^*_j$, one can increase i's utility while remaining in i's budget set by increasing the ith component of x^{**} by a small enough amount, and x^* is therefore not a Lindahl equilibrium). Since w_i is strictly increasing in x_i, we must have moreover $\sum_j \pi^*_{ij} x^*_j = \omega_i$ for all i, and therefore, by definition of a free disposal equilibrium:

$$1 = \sum_i \omega_i = \sum_i \sum_j \pi^*_{ij} x^*_j = \sum_j x^*_j \sum_i \pi^*_{ij} = \pi^* \sum_j x^*_j,$$

where π^* denotes $\max(0, \sum_i \pi^*_{i1}, \ldots, \sum_i \pi^*_{in})$. Free disposal implying moreover $\pi^*(\sum_i x^*_i - 1) = 0$, we must have in fact $\pi^* = \sum_i x^*_i = 1$, and this implies in turn:

$$\sum_i x^{**}_i = \pi^* \sum_i x^{**}_i \geq \sum_i \sum_j \pi^*_{ij} x^{**}_j > \sum_i \sum_j \pi^*_{ij} x^*_j = 1$$

which contradicts the feasibility of x^{**}.[18]

Let us establish the second part of Theorem 2. Consider a distributive strong Pareto optimum x^*. It must then be a maximum of w_i in $A_i(x^*)$ for all i, and must therefore verify the following necessary first order conditions (Kuhn and Tucker, 1951): for all i, there are $(\lambda_i, \mu_i) \in \mathbb{R}_+ \times \mathbb{R}^n_+$, ith $\mu^i_i = 1$, such that: (i) $\lambda_i(\sum_k x^*_k - 1) = 0$; (ii) $\sum_k \mu^i_k \partial_{x_j} w_k(x^*) \leq \lambda_i$ for all j; and $(\lambda_i - \sum_k \mu^i_k \partial_{x_j} w_k(x^*)) x^*_j = 0$ for all j. Pick some arbitrary $\alpha \gg 0$ in S_n, define $\lambda = \sum_i \alpha_i \lambda_i$ and $\mu = \sum_i \alpha_i \mu_i \gg 0$, and notice that (λ, μ) verifies the Kuhn and Tucker inequalities (i)–(iii) for all i. Denote then by π^* the number $\max(0, \sum_i \mu_i \partial_{x_1} w_i(x^*), \ldots, \sum_i \mu_i \partial_{x_n} w_i(x^*))$; Π^* the (n,n)-matrix whose generic element is $\pi^*_{ij} = \mu_i \partial_{x_j} w_i(x^*)$. The Kuhn and Tucker inequalities above imply in particular that: $\pi^*(\sum_i x^*_i - 1) = 0$; and $(\pi^* - \sum_i \pi^*_{ij}) x^*_j = 0$ for all j. In view of the definition of a distributive Lindahl equilibrium, we have only to prove, therefore, that x^* is a maximum of w_i in $B_i(\Pi^*, \sum_j \pi^*_{ij} x^*_j)$ for all i. But observe that identities $\partial_{x_j} w_i(x^*) = \pi^*_{ij}/\mu_i$, with $j = 1, \ldots, n$, can be viewed as the Kuhn and Tucker conditions for the maximization of w_i in $B_i(\Pi^*, \sum_j \pi^*_{ij} x^*_j)$. Since Kuhn and Tucker conditions are sufficient by our assumptions and those of Arrow and Enthoven (1961) (Theorem 1, (b)), x^* must be a maximum of w_i in $B_i(\Pi^*, \sum_j \pi^*_{ij} x^*_j)$ for all i, and the proof is completed. □

[18] We adapted here, of course, the reasoning developed in Debreu (1953) (proof of Theorem 1).

Appendix B. Distributive Nash equilibrium

An *(i, j)-maximum* (Mercier Ythier, 1989, 1992, 1993) is an element x of S_n such that either $x_j = 0$ or $\partial_{x_j} w_i(x) \geq \partial_{x_k} w_i(x)$ for all k.

One establishes easily that the social systems of Counterexamples to existence 1 and 2 verify the following characterization (Mercier Ythier, 1989, 1992, 1993) of distributive Nash equilibrium: a feasible action-vector a^* is a distributive Nash equilibrium if and only if $x(a^*)$ is (i, i)-maximal for all i and (i, j)-maximal whenever $t_{ij}(a^*) > 0$.

Denoting by M_{ij} the set of (i, j)-maximal distributions and M the intersection $\cap_i M_{ii}$, it follows then immediately from the characterization above that the Nash equilibrium distributions run into M when the initial distribution of rights ω runs into S_n.

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