Two-sided legal systems of borders, international migration of adults and social welfare

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Abstract

In a 3-period OLG model with two countries, endogenous education of young and endogenous labor of adults and old, we study the impact of differences in returns to education combined with differences in the qualities of the social security on incentives for international migration. In each country, the social planner uses the optimal migration flow as instrument to reach the maximum of social welfare. The steady-state welfare maximizing capital differs across countries. Therefore, prices, wages and interest rates never equalize across countries. Incentives for illegal migration exist. Simulations illustrate real migration flows (North-North, South-North, South-South) [1].

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1 INTRODUCTION

In 2014 international migration flows represent about 230 million individuals, i.e., about 3% of the world population[2]. As a consequence, international migration has become one of the most topical issues from a political point of view. Economists have contributed to the debate by their theoretical as well as empirical research, especially in a context where the interest in migration policy is increasingly keen. The economic literature on the topic mainly focuses on both the design and the implementation of migration policies as well as their effect on the social welfare.

The empirical literature concentrates on selective immigration policies. It is an important investigation since it represents the only component on which the policymakers have direct control. It affects both the labor market and the population growth, consequently the economic growth. Several empirical studies analyze the consequence of the choice for a country of some specific criteria that help determine a welfare maximizing international migration policy. Cobb-Clark and Conolly (1997) empirically show that the qualifications of those wanting to enter Australia are influenced by a set of factors, some of them are internal to Australia (i.e., economic conditions), others are external (i.e., the immigration policy of other countries). These factors may have an important impact on the quality of future immigrants. Due to international competition among receiving countries, attracting the best is the major priority of the main destination countries, Abramitzy and Braggion (2006). For that reason, the study and the understanding of the international market of qualified workers is an important area of research.

Considering the set of countries who compete for selecting the best individuals on economic criteria, the comparative study of Antecol, Cobb-Clark and Trejo (2001) shows that selective immigration policies of Canada and Australia, based on the point system of weighted individual criteria, provides a better quality workforce compared to the United States. The reason is that the USA implement a "proximity immigration" policy and not an immigration policy based on selected criteria. Chojnick, Docquier and Ragot (2005) show that selective immigration is a solution for the aging problem in some countries and could even be a solution, at least partially, for the tax burden of an aging population.

Theoretical research on international migration is extensive and focuses mostly on exogenous or endogenous quotas of migrants. In this line of research, there are many papers relative to the role of human capital as a discriminant criterion for policy makers to let migrants enter in their country.

Epstein and Nitzan (2001) analyze the determination of endogenous quotas of migrants, which is assimilated to an outcome of a political struggle between two groups: those in favor and those against endogenous migrants quota. The theoretical effects for local policy depend on the existence (or not) of a lobbying between the two groups of natives. Mayr (2012) uses a framework of an economic policy analysis with endogenous prices and determines the specific occupation of the immigration quota and compares it with the social optimum. The author shows that a positive quota for a specific occupation could be a political result, even when it diminishes the global welfare. Two main findings should be emphasized. The (unique) voting outcome on immigration quotas is positive when workers are immobile across occupations. It is negative (positive) for occupations where the native labor supply is sufficiently large (small), when workers are mobile across

[2]Source: OECD international Migration Outlook, 2014.

occupations. In the context of game theory, Stark and *al*. proposed a model in which countries implement endogenous bilateral agreements that are welfare improving.

In a static one-sector model without capital accumulation, where the social planner is a Stackelberg leader, Benhabib and Jovanovic (2007) determine the world-welfare maximizing level of migrants. Calibration underlines that such an optimal level of migration hugely surpass the observed level of migration. Extension to a 2-sector model does not change the results.

International migration in overlapping generations (OLG) models has been initiated by Galor (1986). A huge literature has followed and developed in the direction of post-migration market equilibrium, where wages and interest rates always equalize in post-migration steady state equilibrium[3]. A strong characteristic of these models is that the role of the social planner is not analyzed.

Surprisingly, there is no literature on dynamic OLG models with both a social security system and human capital accumulation. However, Geide-Stevenson (1998) shows that the existence of a social security system alone does not pre-determine the direction of international migration. We ask the following question: is human capital or the social security the most important criterion for a migrant to migrate, or for a social planner to accept migrants ?

This paper contributes to the research on international migration policies by studying theoretically various legal systems of two-sided borders, where the social planner of a given country maximizes social welfare. One of the particularities of this model is that the host country 2 reaches the maximizing social welfare level by choosing the optimal immigration flow. This is encapsulated into the concept of explicit border. The departure country chooses its optimal emigration level by the same procedure. This is encapsulated into the concept of implicit border. This paper extends the model of Chaabane and Gaumont (2015) by allowing only adults to migrate and by considering that only one of the two countries is endowed with a social security system. In this framework, we examine the consequences of a selective immigration policy via welfare maximizing criteria on steady-state equilibrium in each country: a novelty.

In the proposed 3-period OLG model, individuals train endogenously in the first period of their life-cycle, work endogenously when adults and choose their retirement date when old. When borders are open in the autarkic steady-state equilibrium, incentives for migration exist.

There is a social planner in each country, who uses the country-specific migration flow as an instrument for guiding the market economy to the social welfare maximizing steady-state equilibrium. The emergence of asymmetric borders across countries comes from the differences in the return to education and in social security systems. Indeed, when a country exhibits a higher return to education, and its when social security system is different than the other country, the optimal level of selected migrants by each country necessarily differs. Each country designs its own policy to cross its out-going border. Consequently, borders have two sides. In that sense, the concept of dual borders is taken into account. Such a result is new and open various areas of research, especially in the direction of international trade where the quality of goods differs across countries. In our labor international migration framework, it is of interest for a policy maker to understand

[3]In Gaumont and MacDissi 2012 neither the wages nor the interest rates equalize.

the two-sided characteristic of a border since it helps determine an optimal migration policy.

Social planners who implement optimal legal systems of international migration view education as being one of the most important criteria for selecting immigrants, even if immigrants do not. Indeed, the efficiency of education relies heavily on indirect utilities, ultimately influencing the direction of incentives for permanent international migration. So does the social security system, but it will be shown under which condition it is dominated by the return to education.

To simplify the analysis, we suppose that there are two types of countries. The first type captures all countries with a poor pension system: those classified C and D according to the Melbourne Mercer Global Pension Index, (see Appendix D for more details) and a low level of education. The second type represents all countries with a high quality pension system (classified A and B) and a high level of education. We also classified countries according to there return to education. As a proxy of it, we used the Shanghai ranking of universities.

This kind of classification is important for the understanding of the design of immigration policies. The choice of the framework we use in this paper is motivated by a series of empirical facts that we present hereafter. Indeed, The United States, Australia, Canada and more recently Germany, have implemented selective programs which aims at improving the proportion of qualified immigrants. Usually, this kind of program uses a point system, based on the potential economic integration abilities of immigrants. The primary goal is to find a legal framework for the labor market to be regulated, due to the existence of various qualification disequilibria. In our paper, a selective migration policy allows the social planner of each country to maximize the steady-state post-migration social welfare of his own country. As an interesting result, there is no price equalization in the post-migration steady-state equilibrium. Moreover, asymmetric borders emerge endogenously.

Moreover, compared with usual OLG models, there are some nontrivial technical details to be worked out here — e.g., Lemma 3 — which is not surprising, but surprisingly difficult to prove. Having said that, once the technical results are established, the framework is very tractable. In particular, it is tractable enough that we can generate simulations of the model for reasonable values of parameters. Indeed, it is shown that incentives for international migration are directed differently depending on all the possible combinations of high (low) return to education and high (low) quality of the social security system. The mix of all these cases makes very interesting interpretations of the real flows of migrants between countries. Contrary to intuition, the solely social security system fails at explaining by itself incentives for international migration, except for similar return to education levels across countries.

The remainder of the paper is organized as follows: Section 2 presents the model, Section 3 studies the temporary equilibrium in autarky, Section 4 analyses the autarkic perfect foresight inter-temporal equilibrium. Section 5 is devoted to the international migration. Section 6 sums up the theoretical results of the paper and discusses them. Section 7 exposes some simulations and empirical evidence. Section 8 concludes.

2 THE MODEL

In a perfectly competitive international world, the model operates with two countries, $i = 1, 2$ and over infinite discrete time, $t = 0, 1, 2, \ldots, \infty$. A new generation of individuals N_t^i is born in each period. For simplicity, in autarky $N_{t+1}^i = N_t^i = N^i$ where $N^i = 1$. Three

factors of production are used to produce a single tradable good: capital, adult efficient labor and old efficient labor. Capital depreciates fully after one period. An individual and the representative firm make rational decisions under perfect foresight.

2.1 THE INDIVIDUAL

Prior to migration, individuals are identical within as well as across generations in a country. Individuals born in country $i = 1, 2$ live three periods: the youth, adulthood and old age. During the first period, when young, they bear the total cost of training ae^i_{t-1} financed by E^i_{t-1}, where a is the price of one unit of education e^i_{t-1} in each country and where $0 < e^i_{t-1} < 1$. The young borrow E^i_{t-1} on their future saving s^i_t when adult. Education is an individual's choice. During the second period, adults consume c^i_t, save s^i_t and reimburse their education cost $R^i_t E^i_{t-1}$, where $R^i = 1 + r^i_t$ is the given competitive factor of interest and r^i_t the competitive interest rate in country i during period t. Adults finance their current consumption, savings and previous education with their current labor income, w^i_t. They supply ℓ^i_t subunits of labor, which is paid at the given competitive wage w^i_t, so that the total earning of an adult is $w^i_t \ell^i_t (e^i_{t-1})^{\varepsilon^i}$, where $0 < \varepsilon^i < 1$ is the country specific return to education.

The labor income tax $-1 < \tau^i < 1$ is constant over time. In the third period when old, individuals consume d^i_{t+1} — a function of the first period education e^i_{t-1} — financed through the return on the second period saving $R^i_{t+1} s^i_t$, their third period labor supply at the given competitive wage p^i_{t+1} during θ^i_{t+1} subunits of time, where R^i_{t+1} and p^i_{t+1} are perfectly anticipated and the retirement pension is z^i_{t+1} during $(1 - \theta^i_{t+1})$ subunits of time. Rational individuals maximize their log-linear utility function and solve the following problem where β is the time preference and γ is the preference for leisure

$$\max_{c^i_t, e^i_t, \ell^i_t, d^i_{t+1}, \theta^i_{t+1}} \log c^i_t + \gamma \log(1 - \ell^i_t) + \beta \log \left[d^i_{t+1}(e^i_{t-1}) \right] + \beta\gamma \log(1 - \theta^i_{t+1})$$

subject to

$$\left\{ \begin{array}{rcl} a e^i_{t-1} & = & E^i_{t-1}, \\ c^i_t + s^i_t + R^i_t a e^i_{t-1} & = & (1 - \tau^i) w^i_t \ell^i_t (e^i_{t-1})^{\varepsilon^i}, \\ d^i_{t+1} & = & R^i_{t+1} s^i_t + (1 - \tau^i) p^i_{t+1} \theta^i_{t+1} + z^i_{t+1}(1 - \theta^i_{t+1}). \end{array} \right. \tag{1}$$

2.2 THE FIRM

In each period, production occurs according to a constant returns to scale production technology. The representative competitive firm produces the output Q^i_t of the single good using three factors of production: capital K^i, young efficient labor $\ell^i_t (e^i_{t-1})^{\varepsilon^i}$ and old efficient labor θ^i_t. The production technology is a Cobb-Douglas production function $Q^i_t = {K^i_t}^{1-\sigma-\nu} \left[\ell^i_t (e^i_{t-1})^{\varepsilon^i} \right]^{\sigma} {\theta^i_t}^{\nu}$, knowing that $0 < \sigma < 1$ is the elasticity of young efficient labor and $0 < \nu < 1$ is the elasticity of old efficient labor. The rational representative competitive firm maximizes its profit

$$\max_{K^i_t, \ell^i_t, \theta^i_t} {K^i_t}^{1-\sigma-\nu} \left[\ell^i_t (e^i_{t-1})^{\varepsilon^i} \right]^{\sigma} {\theta^i_t}^{\nu} - w^i_t \ell^i_t (e^i_{t-1})^{\varepsilon^i} - p^i_t \theta^i_t - R^i_t K^i_t. \tag{2}$$

2.3 THE GOVERNMENT

The government determines the labor income tax that equates the receipts and the uses of the social security system. The total receipt is the return of the labor income tax $\tau^i w_t^i \ell_t^i e_{t-1}^{i\varepsilon^i} + \tau^i p_t^i \theta_t^i$. The total is the pension $z_t^i(1-\theta_t^i)$. The Government equilibrium requires the following condition to be satisfied

$$\tau^i \left[w_t^i \ell_t^i (e_{t-1}^i)^{\varepsilon i} + p_t^i \theta_t^i \right] = z_t^i (1-\theta_t^i). \tag{3}$$

We now turn to the study of the temporary equilibrium, which is the solution of the three previous problems, the one of the individual, the one of the firm and the one of the government.

3 TEMPORARY EQUILIBRIUM IN AUTARKY

This section is devoted to the study of the temporary equilibrium of the economy in autarky. Let us first recall the definition.

DEFINITION **1** *A competitive equilibrium is a price vector $P = (p, w, R)$, where p is the price of consumption, w is the price of labor, and R is the price of capital such that all markets clear*[4].

DEFINITION **2** *In country i, the temporary equilibrium of period t is a competitive equilibrium given perfect anticipation of prices, R_{t+1}^i, p_{t+1}^i and z_{t+1}^i, given past variables, s_{t-1}^i and $I_{t-1}^i = N_{t-1}^i s_{t-1}^i$, or equivalently $K_t = s_{t-1}$, and the clearing condition on the labor market is $L_t = N_{t-1}\ell_t$ and $\Theta_t = N_{t-2}\theta_t$.*

Consider the individual's problem 1. Solving the first period budget constraint for s_t^i and replacing its new expression into the second period budget constraint gives

$$d_{t+1}^i = R_{t+1}^i \left[(1-\tau^i) w_t^i \ell_t^i (e_{t-1}^2)^{\varepsilon^i} - R_t^i a e_{t-1}^2 - c_t^i \right] + (1-\tau^i) p_{t+1}^i \theta_{t+1}^i + z_{t+1}^i (1-\theta_{t+1}^i). \tag{4}$$

Replacing (4) into the corresponding objective function, individuals solve the following problem

$$\max_{c_t^i, e_{t-1}^i, \ell_t^i, \theta_{t+1}^i} \log c_t^i + \gamma \log(1-\ell_t^i) + \beta \log \left[R_{t+1}^i \left[(1-\tau^i) w_t^i \ell_t^i (e_{t-1}^i)^{\varepsilon^i} - R_t^i a e_{t-1}^i - c_t^i \right] \right.$$
$$\left. + (1-\tau^i) p_{t+1}^i \theta_{t+1}^i + z_{t+1}^i (1-\theta_{t+1}^i) \right] + \beta\gamma \log(1-\theta_{t+1}^i).$$

The first order condition gives the following relations

$$\frac{1}{c_t^i} = \frac{\beta R_{t+1}^i}{d_{t+1}^i}, \tag{5}$$

$$\varepsilon^i (1-\tau^i) w_t^i \ell_t^i \varepsilon^i (1-\tau^i) w_t^i \ell_t^i (e_{t-1})^{\varepsilon^i} = R_t^i a e_{t-1}^i, \tag{6}$$

$$\frac{\gamma}{1-\ell_t^i} = \frac{\beta R_{t+1}^i (1-\tau^i) w_t^i (e_{t-1}^i)^{\varepsilon^i}}{d_{t+1}^i}, \tag{7}$$

[4]Note that in our paper the price of consumption is a numeraire and $p_t = 1$.

$$\frac{(1-\tau^i)p^i_{t+1}-z^i_{t+1}}{d^i_{t+1}}=\frac{\gamma}{1-\theta^i_{t+1}}. \tag{8}$$

In each country $i=1,2$, the representative rational competitive firm maximizes its profit. The first order condition is

$$(1-\sigma-\nu)Q^i_t=R^i_tK^i_t, \tag{9}$$

$$\sigma Q^i_t=w^i_t\ell^i_te^{\varepsilon^i}_{t-1}, \tag{10}$$

$$\nu Q^i_t=p^i_t\theta^i_t. \tag{11}$$

With the firm's solutions, the government program becomes[5]

$$\tau^i(\sigma+\nu)Q^i_t=z^i_t(1-\theta^i_t). \tag{12}$$

LEMMA **1** *In temporary equilibrium, the adult efficient labor supply and the old efficient labor supply are both constant. We have* $\ell^i_{t+1}=\ell^i_t=\ell^i$ *and* $\theta^i_{t+1}=\theta^i_t=\theta^i$*, where*

$$\ell^i=\frac{1-\sigma(1-\tau^i)+\beta(1-\sigma-\nu)}{\gamma[1-\sigma(1-\tau^i)](1-\varepsilon^i)(1-\tau^i)+\beta(1-\sigma-\nu)+1-\sigma(1-\tau^i)}, \tag{13}$$

$$\theta^i=\frac{\nu(1-\tau^i)}{\gamma[1-\sigma(1-\tau^i)]+\tau^i\sigma+\nu}, \tag{14}$$

For $\tau_i\in[-1,1]\mid 0<\ell(\tau^i)<1$ and $0<\theta(\tau^i)<1$.

The proof of Lemma 1 is given in Appendix B.

Using (6), (9) and (10), we get the level of education

$$e^i_{t-1}=\frac{\varepsilon^i\sigma(1-\tau^i)K^i_t}{a(1-\sigma-\nu)}\iff e^i_t=\frac{\varepsilon^i\sigma(1-\tau)K^i_{t+1}}{(1-\sigma-\nu)a}. \tag{15}$$

PROPERTY **1** *The level of education is an increasing linear function of capital and of the return to education, as well as a decreasing function of the education cost,* a*.*

4 THE PERFECT-FORESIGHT INTER-TEMPORAL EQUILIBRIUM IN AUTARKY.

In order to study the perfect-foresight inter-temporal equilibrium in each country $i=1,2$ in autarky, we use the capital dynamics $K^i_{t+1}=s^i_t$ and the constant population growth.

[5] A condition on elasticities for a positive relation between the income taxation rate and the retirement pension is given in Appendix A.

LEMMA **2** *The dynamics of the capital stock*

$$K_{t+1}^i = \frac{\beta(1-\sigma-\nu)(1-\tau^i)}{[1-\sigma(1-\tau^i)]+\beta(1-\sigma-\nu)}\sigma(1-\varepsilon^i)\ell^{i\sigma}\left[\frac{\varepsilon^i\sigma}{(1-\sigma-\nu)a}\right]^{\varepsilon^i\sigma}\theta^{i\nu}K_t^{i\,1-(1-\varepsilon^i)\sigma-\nu},$$

converges to an unique steady-state equilibrium in each country $i = 1, 2$

$$\overline{K}^i = \left[\frac{\beta(1-\sigma-\nu)\sigma(1-\varepsilon^i)(1-\tau^i)}{[1-\sigma(1-\tau^i)]+\beta(1-\sigma-\nu)}\left[\frac{\varepsilon^i\sigma(1-\tau^i)}{a(1-\sigma-\nu)}\right]^{\varepsilon^i\sigma}\ell^{i\sigma}\theta^{i\nu}\right]^{\frac{1}{(1-\varepsilon^i)\sigma+\nu}}.$$

Proof.

By Lemma 1, the adult and old efficient labor are constant over time. Replacing their expression into the production function of the current period t gives

$$Q_t^i = K_t^{i\,1-\sigma-\nu}(\ell^i e_{t-1}^{i\,\varepsilon^i})^\sigma\theta^{i\nu}.$$

Using $K_{t+1}^i = s_t^i$ into the first period budget constraint, the dynamics of the economy are

$$K_{t+1}^i = (1-\tau^i)w_t^i\ell_t^i(e_{t-1}^{i\,\varepsilon^i}) - R_t^i a e_{t-1} - c_t^i.$$

Using the first order condition of the firm (10) and of the individuals (6) and (61 (see A)), we have

$$K_{t+1}^i = \frac{\beta(1-\sigma-\nu)}{[1-\sigma(1-\tau^i)]+\beta(1-\sigma-\nu)}\sigma(1-\varepsilon^i)(1-\tau^i)Q_t^i.$$

Replacing the production by its corresponding expression, we get

$$K_{t+1}^i = \frac{\beta(1-\sigma-\nu)}{[1-\sigma(1-\tau^i)]+\beta(1-\sigma-\nu)}\sigma(1-\varepsilon^i)(1-\tau^i)(\ell^i e_{t-1}^{i\,\varepsilon^i})^\sigma\theta^{i\nu}(K_t^i)^{1-\sigma-\nu}.$$

The steady-state equilibrium[6] in each country is

$$\overline{K}^i = \left[\frac{\beta(1-\sigma-\nu)\sigma(1-\varepsilon^i)(1-\tau^i)}{[1-\sigma(1-\tau^i)]+\beta(1-\sigma-\nu)}\left[\frac{\varepsilon^i\sigma(1-\tau^i)}{a(1-\sigma-\nu)}\right]^{\varepsilon^i\sigma}\ell^{i\sigma}\theta^{i\nu}\right]^{\frac{1}{(1-\varepsilon^i)\sigma+\nu}}. \tag{16}$$

□

5 INTERNATIONAL LABOR MIGRATION

For the remainder of the paper, we consider the case where country 1 has no social security. Consequently, $\tau^1 = 0$ and $z^1 = 0$. Without any loss of generality, let us assume that the following inequality holds, $\varepsilon^1 > \varepsilon^2$ for the rest of the paper. Note that the return to education in higher in country 2 than in country 1, since $\varepsilon^i \in [0,1]$ and $e_{t-1}^i \in [0,1]$. The steady-state indirect utility V^i of an individual born in country i depends on $V^i := V(\varepsilon^i, \tau^i)$, while the indirect utility of this individuals who has migrated in country j is $V^{ij} := V(\varepsilon^i, \tau^j)$. Consequently, there are configuration of parameters for which incentives for migration exists. Note that τ^j finances the pension z^j. Contrary to intuition, it will be shown in Subsection 7.1 that ε^i is more important in the determination of the direction of international migration than τ^j.

[6] Using (15) into the previous relation, factorize k_t, we get the following relation: $K_{t+1}^i = ZK_t^{i\mu}$, where

$$Z = \frac{\beta(1 \quad \sigma \quad \nu)\sigma(1 \quad \varepsilon^i)(1 \quad \tau^i)}{[1 \quad \sigma(1 \quad \tau^i)]+\beta(1 \quad \sigma \quad \nu)}\left(\frac{\varepsilon^i\sigma(1 \quad \tau)}{(1 \quad \sigma \quad \nu)a}\right)(\ell^i)^\sigma\theta^{i\nu} \text{ and } \mu = (1 \quad \varepsilon)\sigma + \nu.$$

Consequently, the capital converges to the steady-state equilibrium $\overline{K}^i = Z^{1/\mu}$.

5.1 INCENTIVES FOR PERMANENT INTERNATIONAL MIGRATION

Let us now consider that labor is permitted to migrate internationally. We assume that only adult can permanently migrate. Migrants spend their education time in their home country and spend their working time as well as their retirement time over the two periods in the immigration country. Borders between countries are supposed to be opened at time $t = 0$.

PROPOSITION **1** *Incentives for international migration of adults always exist and are unilateral from country* 1 *to country* 2*.*

Proof. Rational adult individuals born in country 1 have an incentive for permanent migration in country 2 if their indirect utility evaluated at the steady-state price system of country 2 over their life-cycle is higher than their indirect utility evaluated at the steady-state prices of country 1. The condition is

$$\log c^{11} + \gamma \log\left(1-\ell^{11}\right) + \beta \log\left(d^{11}\right) + \beta\gamma \log\left(1-\theta^{11}\right)$$
$$< \log c^{12} + \gamma \log\left(1-\ell^{12}\right) + \beta \log\left(d^{12}\right) + \beta\gamma \log\left(1-\theta^{12}\right).$$

Knowing that $\theta^{11} = \theta^{12}$ we have

$$\log\left[\frac{c^{11}}{c^{12}}\right] + \gamma \log\left[\frac{1-\ell^{11}}{1-\ell^{12}}\right] < \beta \log\left[\frac{d^{12}}{d^{11}}\right].$$

Using (15), (60), (61) (see A), and simplifying we get

$$\log\left[\frac{(1-\sigma)(1-\tau^2)}{[1-\sigma(1-\tau^2)]}\right] + \gamma \log\left[\frac{1-\ell^{11}}{1-\ell^{12}}\right] < \beta \log\left[\frac{1-\sigma(1-\tau^2)Q^2}{(1-\tau^2)Q^1}\right],$$

which means

$$\log\left[\frac{(1-\sigma)(1-\tau^2)}{[1-\sigma(1-\tau^2)]}\right] + \gamma \log\left[\frac{1-\ell^{11}}{1-\ell^{12}}\right] + \beta \log\left[\frac{1-\tau^2}{[1-\sigma(1-\tau^2)]}\right] < \beta \log\left[\frac{Q^2}{Q^1}\right],$$

$$\text{If } \ \sigma < 1/2 \ \text{ and } \ \frac{\sigma}{1-\sigma} < \tau < \frac{1-\sigma}{1-\sigma+\beta(1-\sigma-\nu)} \ \text{ then } \ V^{12} > V^{11}.$$

Indeed the left hand side is negative, so that the condition is satisfied, considering that in the right hand side, the ratio of productions is greater than one[7]. □

5.2 DYNAMICS WITH PERMANENT INTERNATIONAL MIGRATION

This subsection is devoted to the study of the dynamics of capital in each countries 1 and 2. For the remaining of the paper, the notation is as follows: upper script i corresponds to the native country of an individual, and upper script j to his living country. For example, all the variables of an individual born in country 1 and living in country 2 have upper scripts 12

As long as incentives for migration are directed from country 1 to country 2, we consider the case where only adults are permitted to permanently migrate. In the autarkic

[7]Simulations show that for reasonable values of parameters $\beta = 0.67$, $\sigma = \nu = 1/3$, $\gamma = 2/3$ and $e = 1/3$, we have $\partial Q/\partial\varepsilon < 0$, which means that $Q^2 > Q^1$. Consequently, the condition is always true.

steady-state equilibrium, period $t = 0$, borders are open. A fraction m^i of adults is allowed to migrate. Consequently, $m^1 > 0$ is the level of individuals who which to emigrate from country 1 to country 2, while country 2 accepts a level $m^2 > 0$ of immigrants. It will be shown that these two levels are not equal.

In this particular case, after migration, individuals are identical in country 1 but are heterogeneous in country 2 as they train in their home country. Indeed, after migration, an immigrant keeps his home country's return to education in the host country, ε^1, while home workers have a return to education ε^2. In a given period $t \geq 2$ post migration, the population in country 1 is $\ell_t^1 = (1 - m^1)\ell_t^{11}$, while the population in country 2 is $\ell_t^2 = \ell_t^{22} + m\ell_t^{12}$. Consequently, in each country efficient labor is defined as $L_t^1 = (1 - m^1)\ell_t^1 e_{t-1}^{\varepsilon^1}$ and $L_t^2 = \ell_t^{22} e_{t-1}^{\varepsilon^2} + m^2 \ell_t^{12} e_{t-1}^{\varepsilon^1}$. The production function of the country 1 is[8]

$$Q_t^1 = (K_t^1)^{1-\sigma-\nu}(1 - m^1)^{\sigma}(\ell_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}(1 - m^1)^{\nu}\theta_t^{\nu}$$

$$\iff \quad Q_t^1 = (1 - m^1)^{\sigma+\nu}(K_t^1)^{1-\sigma-\nu}(l_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}\theta_t^{\nu}.$$

The production function of the country 2 is

$$Q_t^2 = \left(K_t^2\right)^{1-\sigma-\nu}\left[\ell_t^{22}(e_{t-1}^2)^{\varepsilon 2} + m^2\ell_t^{12}(e_{t-1}^1)^{\varepsilon 1}\right]^{\sigma}\left[\theta_t^{22} + m^2\theta_t^{12}\right]^{\nu}.$$

Consequently, the population of country 2 is heterogenous, since permanent adult migrants born and educated in country 1 preserve their own return to education ε^1 in country 2. Considering the migration flows, the individual's first and second budget constraints are modified. For simplicity, an individual in the departure country 1 who lives in country 1 gets an upper script of 1. Replace $\tau^1 = 0$ into the set of constraints. It follows for the (sending) country 1

$$\left\{\begin{array}{rcl} ae_{t-1}^1 & = & E_{t-1}, \\ c_t^1 + \frac{k_{t+1}^1}{1-m^1} + R_t^1 ae_{t-1}^1 & = & w_t^1 \ell_t^1 (e_{t-1}^1)^{\varepsilon^1}, \\ d_{t+1}^1 & = & R_{t+1}^1 \frac{k_{t+1}^1}{1-m^1} + p_{t+1}^1 \theta_{t+1}^1. \end{array}\right.$$

for the receiving country 2

$$\left\{\begin{array}{rcl} ae_{t-1}^2 & = & E_{t-1}, \\ c_t^{ij} + s_t^{ij} + R_t^2 ae_{t-1}^i & = & (1-\tau^2) w_t^2 \ell_t^{ij} (e_{t-1}^i)^{\varepsilon^i}, \\ d_{t+1}^{ij} & = & R_{t+1}^{ij} s_t^{ij} + (1-\tau^2) p_{t+1}^2 \theta_{t+1}^{ij} + z_{t+1}^2 (1 - \theta_{t+1}^{ij}). \end{array}\right.$$

Modify the first order condition of country 1 obtained in autarky according to $\tau^1 = 0$ and obtain for country 2, the first order condition

$$\frac{1}{c^{ij}} \quad = \quad \frac{\beta R_{t+1}^2}{d^{ij}}, \tag{17}$$

$$\frac{\gamma}{1 - \ell_t^{ij}} \quad = \quad \frac{\beta R_{t+1}^2 (1-\tau^2) w_t^2 e_{t-1}^{\varepsilon^i}}{d_{t+1}^{ij}}, \tag{18}$$

[8]Notation: the index (11) means an individual born in country 1 and lives in his home country 1; the index (12) means an individual born in country 1 and immigrates to country 2 when adult. As well, the index (22) means an individual born in country 2 and still leaving in hi country.

$$\frac{\gamma}{1-\theta_t^{ij}} = \frac{(1-\tau^2)p_{t+1}^2 - z_{t+1}^2}{d_{t+1}^{ij}}, \tag{19}$$

$$\varepsilon^i(1-\tau^2)w_t^2\ell_t^{ij}(e_{t-1}^i)^{\varepsilon^i} = R_t^2 a e_{t-1}^i. \tag{20}$$

In each country the representative rational firm maximizes its profit. In the sending country we have

$$\max_{K_t^1,\ell_t^1,\theta_t^1}(1-m^1)^{\sigma+\nu}(K_t^1)^{1-\sigma-\nu}(\ell_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}\theta_t^{\nu} - w_t^1(1-m^1)\ell_t^1 e_{t-1}^{\varepsilon^1} - p_t^1(1-m^1)\theta_t^1 - R_t^1 K_t^1.$$

The first order condition for country 1 is

$$(1-\sigma-\nu)\frac{Q_t^1}{1-m^1} = R_t^1\frac{K_t^1}{1-m^1}, \tag{21}$$

$$\sigma\frac{Q_t^1}{1-m^1} = w_t^1\ell_t^1 e_{t-1}^{\varepsilon^1}, \tag{22}$$

$$\nu\frac{Q_t^1}{1-m^1} = p_t^1\theta_t^1. \tag{23}$$

Let us denote $\Theta_t^2 = \theta_t^{22} + m\theta_t^{12}$. In the receiving country we have

$$\max_{K_t^2,L_t^2,\Theta_t^2}(K_t^2)^{1-\sigma-\nu}(L_t^2)^{\sigma}(\Theta_t^2)^{\nu} - w_t^2 L_t^2 - p_t^2\Theta_t^2 - R_t^2 K_t^2.$$

Let's rewrite the efficient labor of the country 2 in vector form.

$$\begin{bmatrix} w^2 & m^2 w^2 \end{bmatrix}\begin{bmatrix} \ell_t^{22}(e_{t-1}^2)^{\varepsilon 2} \\ \\ \ell_t^{12}(e_{t-1}^1)^{\varepsilon 1} \end{bmatrix}.$$

Consequently, we will necessarily have $\ell_t^{22}(e_{t-1}^2)^{\varepsilon 2} = \ell_t^{12}(e_{t-1}^1)^{\varepsilon 1}$. Therefore, the efficient labor in the receiving country can be rewritten as $L_t^2 = (1+m^2)\ell_t^{22}(e_{t-1}^2)^{\varepsilon 2}$. We note $\mathcal{L}_t^2 = \ell_t^{22}(e_{t-1}^2)^{\varepsilon 2}$, thus $L_t^2 = (1+m^2)\mathcal{L}_t^2$. The profit of the firm in country 2 becomes

$$\max_{K_t^2,\mathcal{L}_t^2,\Theta_t^2}(K_t^2)^{1-\sigma-\nu}\left[(1+m^2)\mathcal{L}_t^2\right]^{\sigma}(\Theta_t^2)^{\nu} - w_t^2\left[(1+m^2)\mathcal{L}_t^2\right] - p_t^2\Theta_t^2 - R_t^2 K_t^2.$$

The first order condition for country 2 is

$$(1-\sigma-\nu)Q_t^2 = R_t^2 K_t^2, \tag{24}$$

$$\sigma Q_t^2 = w_t^2(1+m^2)\mathcal{L}_t^2, \tag{25}$$

$$\nu Q_t^2 = p_t^2\left[\theta^{22} + m^2\theta^{12}\right]^{\nu}. \tag{26}$$

The government's social security program in the country 2 is modified as follows

$$\tau^2 w_t^2\left[(1+m^2)\mathcal{L}_t^2\right] + \tau^2 p_t^2\left[\theta^{22} + m^2\theta^{12}\right] = z_t^2(1+m^2) - z_t^2\left[\theta^{22} + m^2\theta^{12}\right].$$

By using the new solutions of the firm of country 2 and the previous expressions, we get

$$\tau^2(\sigma+\nu)Q_t^2 = z_t^2\left[1+m^2-\theta_t^{22}-m^2\theta_t^{12}\right]. \tag{27}$$

Post-migration, the population can no longer be normalized to unity as it was in the case in autarky. The dynamics of country 1 and of country 2 are

$$K_{t+1}^1 = (1-m^1)s_t^1,$$
$$K_{t+1}^2 = s_t^{22}+m^2 s_t^{12}.$$

Using exactly the same procedure as in autarky, we obtain the new expressions for the consumption of the old in the sending country

$$d_{t+1}^1 = (1-\sigma)\frac{Q_{t+1}^1}{1-m^1},$$

and in the receiving country

$$D_{t+1}^2 = d_{t+1}^{22}+md_{t+1}^{12} = [1-\sigma(1-\tau^2)]Q_{t+1}^2.$$

Consumption of the adults in the sending country is

$$c_t^1 = \left[\frac{1-\sigma}{\beta(1-\sigma-\nu)}\right]\frac{K_{t+1}^1}{1-m^1},$$

and consumption of adults in the receiving country is

$$C_t^2 = c_t^{22}+m^2c_t^{12} = \left[\frac{1-\sigma(1-\tau^2)}{\beta(1-\sigma-\nu)}\right]K_{t+1}^2. \tag{28}$$

Let's now sum up the budget constraints of each type of individual in the host country

$$C_t^2+K_{t+1}^2+R_t^2(1+m^2)ae_{t-1} = (1-\tau^2)w_t^2\left[(1+m^2)\mathcal{L}_t^2\right]. \tag{29}$$

Put (28) into (29), simplify and isolate K_{t+1}^2

$$K_{t+1}^2 = \frac{\beta(1-\sigma-\nu)(1-\tau^2)\left[(1+m^2)-(\varepsilon^2+m^2\varepsilon^1)\right]}{1-\sigma(1-\tau^2)+\beta(1-\sigma-\nu)}w_t^2\mathcal{L}_t^2. \tag{30}$$

Let's rewrite the specific expression (20) for each type of individual in the receiving country, and isolate $e_{t-1}^i, i=1,2$.

$$e_{t-1}^2 = \frac{\varepsilon^2(1-\tau^2)w_t^2\ell_t^{22}(e_{t-1}^2)^{\varepsilon^2}}{R_t^2a},$$
$$e_{t-1}^1 = \frac{\varepsilon^1(1-\tau^1)w_t^2\ell_t^{12}(e_{t-1}^1)^{\varepsilon^1}}{R_t^2a}.$$

Then, we sum $e_{t-1}^2+m^2e_{t-1}^1$ to get

$$e_{t-1}^2+m^2e_{t-1}^1 = \frac{(1-\tau^2)\sigma(\varepsilon^2+m^2\varepsilon^1)}{(1+m^2)a(1-\sigma-\nu)}K_t^2 \iff e_{t-1}^2+m^2e_{t-1}^1 = E_{t-1}^2. \tag{31}$$

By using the same reasoning as in autarky we find the expression of the post migration level of education in the sending country 1

$$e_{t-1}^1 = \frac{\varepsilon^1\sigma}{a(1-\sigma-\nu)}K_t^1 \iff e_t^1 = \frac{\varepsilon^1\sigma}{a(1-\sigma-\nu)}K_{t+1}^1.$$

5.3 POST-MIGRATION EQUILIBRIA

This subsection studies the convergence of the post-migration economy to a country specific steady-state equilibrium. Let us first define terms.

DEFINITION **3** *In country i with opened borders, given perfect anticipation of prices, R^i_{t+1}, p^i_{t+1} and z^i_{t+1}, given past variables, s^i_{t-1} and $I^i_{t-1} = N^i_{t-1}s^i_{t-1}$, or equivalently $K_t = s_{t-1}$, and the clearing condition on the labor market is $L_t = (1+m^i)\ell_t$ and $\Theta_t = (1+m^i)\theta_t$, a post-migration temporary equilibrium of period t is a particular value of the migration flow m^i such that all endogenous variables are welfare maximizing.*

Note that in that case, the migration flow m^i is endogenously determined by the country-specific social planner, as it will become clearer in Section 5.4.

LEMMA **3** *In post migration temporary equilibrium of a given country, the adult efficient labor supply is constant and the old efficient labor supply is constant too. We have $\ell^1_{t+1} = \ell^1_t = \ell^1$, $\theta^1_{t+1} = \theta^1_t = \theta^1$, $\mathcal{L}^2_{t+1} = \mathcal{L}^2_t = \mathcal{L}^2$ and $\Theta^2_{t+1} = \Theta^2_t = \Theta^2$.*

Proof. Recall that the adult labor supply and the old labor supply in the sending country 1 are unchanged since they are independent of the migration rate. Let's now study the adult labor supply in the receiving country. We sum up the individual's solution (18) for both the natives and the migrants living in the host country, and we find

$$\gamma D^2_{t+1} = \beta R^2_{t+1}(1-\tau^2)w^2_t\left[\left[(e^2_{t-1})^{\varepsilon 2} + m^2(e^1_{t-1})^{\varepsilon 1}\right] - \left[\ell^{22}_t(e^2_{t-1})^{\varepsilon 2} + m^2\ell^{12}_t(e^1_{t-1})^{\varepsilon 1}\right]\right].$$

Let us define $\mathcal{E}^2_{t-1} = (e^2_{t-1})^{\varepsilon 2} + m^2(e^1_{t-1})^{\varepsilon 1}$. After replacing the consumption of old by its expression and using the first order condition of the firm we get

$$\frac{\gamma[1-\sigma(1-\tau^2)]}{(1-\tau^2)\beta(1-\sigma-\nu)} = \frac{\sigma Q^2_t}{K^2_{t+1}}\left[\frac{\mathcal{E}^2_{t-1}}{L^2_t} - 1\right]. \tag{32}$$

In order to prove that $\frac{\sigma Q^2_t}{K^2_{t+1}}$ is a constant, we use the individual's and the firm's first order conditions in (30) we get

$$K^2_{t+1} = \frac{\beta(1-\sigma-\nu)(1-\tau^2)\left[(1+m^2) - (\varepsilon^2 + m^2\varepsilon^1)\right]}{1-\sigma(1-\tau^2) + \beta(1-\sigma-\nu)(1+m^2)}\sigma Q^2_t.$$

Then we have

$$\frac{\sigma Q^2_t}{K^2_{t+1}} = \frac{1-\sigma(1-\tau^2) + \beta(1-\sigma-\nu)(1+m^2)}{\beta(1-\sigma-\nu)(1-\tau^2)\left[(1+m^2) - (\varepsilon^2 + m^2\varepsilon^1)\right]}. \tag{33}$$

Put (33) into (32)

$$\mathcal{E}^2_{t-1} = \frac{(1+m^2)\left[1-\sigma(1-\tau^2)+\beta(1-\sigma-\nu)\right] + \gamma[1-\sigma(1-\tau^2)]\left[(1+m^2) - (\varepsilon^2+m^2\varepsilon^1)\right]}{(1+m^2)\left[1-\sigma(1-\tau^2)+\beta(1-\sigma-\nu)\right]}L^2_t. \tag{34}$$

Let's rewrite the sum of (18) as follows

$$D^2_{t+1} = \frac{\beta}{\gamma}R^2_{t+1}(1-\tau^2)w^2_t\left[\frac{\mathcal{E}_{t-1}}{L^2_t} - 1\right].$$

Then rewrite the sum of (17) as follows

$$D^2_{t+1} = \beta R^2_{t+1}\left[\frac{1-\sigma(1-\tau^2)}{\beta(1-\sigma-\nu)}\right]K^2_{t+1}.$$

Equalize the two latter expressions

$$\frac{\beta}{\gamma}R_{t+1}^2(1-\tau^2)w_t^2\left[\frac{\mathcal{E}_{t-1}^2}{L_t^2}-1\right]=\beta R_{t+1}^2\left[\frac{1-\sigma(1-\tau^2)}{\beta(1-\sigma-\nu)}\right]K_{t+1}^2.$$

Replace $\left[\frac{\mathcal{E}_{t-1}^2}{L_t^2}-1\right]$ by its expression, use the first order condition of the firm and simplify

$$\mathcal{L}^2=\frac{1}{1+m^2}\iff L^2=1. \tag{35}$$

Let's now study the old labor supply in the receiving country 2.

Rewrite the sum of (17) for both types of individuals

$$\frac{d_{t+1}^{22}+m^2d_{t+1}^{12}}{p_{t+1}^2}=\frac{\beta R_{t+1}^2}{p_{t+1}^2}[c_t^{22}+m^2c_t^{12}].$$

By definition, $d_{t+1}^{22}+m^2d_{t+1}^{12}=D_{t+1}^2$ and that $c_t^{22}+c_t^{12}=C_t^2$, replace and simplify

$$\frac{Q_{t+1}^2}{p_{t+1}^2}(1-\sigma-\nu)=\frac{R_{t+1}^2K_{t+1}^2}{p_{t+1}^2}. \tag{36}$$

Using (24) and (26) we have

$$\frac{(1-\sigma-\nu)Q_{t+1}^2}{\nu Q_{t+1}^2}=\frac{R_{t+1}^2K_{t+1}^2}{p_{t+1}^2[\theta^{22}+m^2\theta^{12}]}.$$

Isolating $\theta_{t+1}^{22}+m^2\theta_{t+1}^{12}=\Theta_{t+1}^2$ to have

$$\Theta_{t+1}^2=\frac{\nu Q_{t+1}^2}{p_{t+1}^2}.$$

Rewrite the sum of (19), use the first order condition of the firm, use the government's equilibrium relation, and simplify to obtain

$$\frac{Q_{t+1}^2}{p_{t+1}^2}=\frac{(1-\tau^2)(1+m^2)}{\nu+\sigma\tau^2+\gamma[1-\sigma(1-\tau^2)]}.$$

Replace this expression into the previous one to get a constant expression of the old labor supply

$$\Theta^2=\frac{\nu(1-\tau^2)(1+m^2)}{\nu+\sigma\tau^2+\gamma[1-\sigma(1-\tau^2)]}.$$

□

PROPERTY **2** *The old efficient labor supply is independent of the returns to education, ε^i.*

Economically, migrants are identical when old in the host country from a labor market point of view, i.e., the old labor supply is homogenous. No migrants carry any stigma.

PROPOSITION **2** *The dynamics of the post-migration economy of each country are convergent.*

Proof.

As it has been noticed above, the adult labor supply and the old labor supply are unchanged for the sending country 1. From the second period budget constraint, it is easy to compute the steady-state capital per worker in country 1

$$\hat{K}^1 = \left[\frac{\beta(1-\sigma-\nu)\sigma(1-\varepsilon^1)(1-m^1)^{\nu+\sigma(1-\varepsilon^1)}}{(1-\sigma)+\beta(1-\sigma-\nu)} \left[\frac{\varepsilon^1 \sigma}{a(1-\sigma-\nu)} \right]^{\varepsilon^1 \sigma} \ell^{1\sigma} \theta^{1\nu} \right]^{\frac{1}{\nu+\sigma(1-\varepsilon^1)}}. \quad (37)$$

Rewrite (30) as follows

$$K^2_{t+1} = \left[\frac{\beta(1-\sigma-\nu)(1-\tau^2)\left[(1+m^2)-(\varepsilon^2+m^2\varepsilon^1)\right]}{1-\sigma(1-\tau^2)+\beta(1-\sigma-\nu)} \right] \sigma Q^2_t.$$

Replace the production function by its expression and simplify

$$K^2_{t+1} = \left[\frac{\beta(1-\sigma-\nu)(1-\tau^2)\left[(1+m^2)-(\varepsilon^2+m^2\varepsilon^1)\right]}{1-\sigma(1-\tau^2)+\beta(1-\sigma-\nu)} \right] (L^2)^\sigma (\Theta^2)^\nu (K^2_t)^{1-\sigma-\nu}.$$

In the receiving country 2 the post migration dynamics are convergent and the steady-state equilibrium is unique. Its expression is the following

$$\hat{K}^2 = \left[\frac{\beta(1-\sigma-\nu)(1-\tau^2)\left[(1+m^2)-(\varepsilon^2+m^2\varepsilon^1)\right]}{1-\sigma(1-\tau^2)+\beta(1-\sigma-\nu)} (L^2)^\sigma (\Theta^2)^\nu \right]^{\frac{1}{1-\sigma-\nu}}. \quad (38)$$

□

Both economies converge to a country specific steady-state equilibrium. The question now is: how can the social planner lead his country to the best static-welfare optimum? Usually in OLG models, there exists a tax system that leads the static per capita capital to the Golden Rule, i.e., this particular capital per worker maximizes total consumption. Our problem is slightly different, since it is multidimensional. The government uses the migration rate as a policy instrument, in order to choose the static welfare maximizing level of education, adult and old labor and consumption, as well as capital per worker ratio. Therefore, we must reformulate the social planner's problem. This is the objective of the next subsection.

5.4 The Static Welfare Optimum with Permanent International Migration of Adults

Let's define the static welfare optimum of each economy as being the stationary state that a social planner would select to maximize welfare under the feasibility constraint. The welfare criterion a social planner must choose in order to rank all possible steady-states has usually been described —following Samuelson (1958) — as the one that maximizes aggregate consumption. In standard models, this is called the Golden Rule and the government would calculate the static per capita capital that achieves this. Our problem is slightly different in the sense that now the social planner of each country $i = 1, 2$ maximizes the country specific static welfare, and by doing this, he chooses the optimal levels of education e^i_w — where the subscript w captures the welfare maximizing solution

of each variable —, adult labor ℓ_w^i and old labor θ_w^i, adult and old consumptions c_w^i and d_w^i, and the capital per worker k_w^i. The social planner uses the level of migration m^i as an instrument to guide his country toward the static welfare optimum, taking into account the macroeconomic equilibrium constraint of his country.

Each social planner solves his own country specific problem. In the sending country

$$\max_{K_w^1,\ell_w^1,e_w^1,\theta_w^1,c_w^1,d_w^1} \log[c_w^1] + \gamma \log(1-\ell_w^1) + \beta \log[d_w^1] + \beta\gamma \log(1-\theta_w^1),$$

subject to the macroeconomic equilibrium constraint

$$ae_w^1 + c_w^1 + d_w^1 + K_w^1 = {K_w^1}^{1-\sigma-\nu}(\ell_w^1 e_w^{\varepsilon^1})^\sigma \theta_w^\nu.$$

Isolate c_w^1 and put the new expression into the objective function. The first order condition of the country 1 is

$$(1-\sigma-\nu)Q_w^1 = K_w^1, \tag{39}$$

$$\frac{\sigma Q_w^1}{c_w^1 \ell_w^1} = \frac{\gamma}{1-\ell_w^1}, \tag{40}$$

$$ae_w^i = \varepsilon^1 \sigma Q_w^1, \tag{41}$$

$$\frac{\nu Q_w^1}{c_w^1 \theta_w^1} = \frac{\beta\gamma}{1-\theta_w^1}, \tag{42}$$

$$d_w^1 = \beta c_w^1. \tag{43}$$

In the receiving country the social planner determines the legal number of working hours. For ethic reasons, a given social planner is assumed not to discriminate between inhabitants and migrants. Each type of worker is supposed to work the same number of hours. From that we deduce that $L^2 = \ell^{22}(e^2)^{\varepsilon^2} + m^2\ell^{12}(e^1)^{\varepsilon^1}$. Using the previous argument we have $\ell^{22} = \ell^{12} = \ell^2$, consequently $L^2 = \ell^2\left[(e^2)^{\varepsilon^2} + m^2(e^1)^{\varepsilon^1}\right]$, which can be rewritten as $L^2 = \ell^2\mathcal{E}^2$. Since only adults migrate, we define $e^2 = \alpha\mathcal{E}^2$, where α captures the share of native population educated home.

The social planner of the receiving country solves the following problem

$$\max_{K_w^2,\ell_w^2,\mathcal{E}_w^2,m^2,\theta_w^2,D_w^2,C_w^2} \log\left[(1+m^2)C_w^2\right] + \gamma\log(1-\ell_w^2) + \beta\log\left[(1+m^2)D_w^2\right] + \beta\gamma\log(1-\theta_w^2),$$

subject to the macroeconomic equilibrium constraint

$$a\alpha\mathcal{E}_w^2 + (1+m^2)C_w^2 + (1+m^2)D_w^2 + K_w^2 = (K^2)_w^{1-\sigma-\nu}(\ell^2)_w^\sigma[\mathcal{E}_w^2]^\sigma(1+m^2)^\nu\theta_w^\nu.$$

Isolate $(1+m^2)c_w^2$ from the previous macroeconomic constraint and put the new expression into the objective function. The first order condition of the country 2 is

$$(1-\sigma-\nu)Q_w^2 = K_w^2, \tag{44}$$

$$\frac{\sigma Q_w^2}{(1+m^2)C_w^2\ell_w^2} = \frac{\gamma}{1-\ell_w^2}, \tag{45}$$

$$\alpha a \mathcal{E}^2 = \sigma Q_w^2, \tag{46}$$

$$\frac{\nu Q_w^2}{(1+m^2)^2 C_w^2} - \frac{D_w^2}{(1+m^2)C_w^2} + \frac{\beta}{(1+m^2)} = 0, \tag{47}$$

$$\frac{\nu Q_w^2}{(1+m^2)C_w^2\theta_w^2} = \frac{\beta\gamma}{1-\theta_w^2}, \tag{48}$$

$$D_w^2 = \beta C_w^2. \tag{49}$$

Use the first order condition of both countries and rewrite the macroeconomic constraint of each country. For country 1, use (39), (41) and (43), and isolate $\frac{Q_w^1}{c_w^1}$ to have

$$\frac{Q_w^1}{c_w^1} = \frac{(1+\beta)}{\nu + (1-\varepsilon^1)\sigma}. \tag{50}$$

For country 2, we use (44), (46) and (49), and isolate $\frac{Q_w^2}{(1+m^2)C_w^2}$

$$\frac{Q_w^2}{(1+m^2)C_w^2} = \frac{1+\beta}{\nu}. \tag{51}$$

Put the two latter expressions (50) and (51) into (40) and into (45) respectively, then isolate ℓ_w^i to find the optimal adult labor in each country $i = 1, 2$

$$\ell_w^1 = \frac{\sigma(1+\beta)}{\sigma(1+\beta) + \gamma[\nu + \sigma(1-\varepsilon^1)]}, \tag{52}$$

$$\ell_w^2 = \frac{\sigma(1+\beta)}{\gamma\nu + \sigma(1+\beta)}. \tag{53}$$

Put (50) and (51) into (42) and into (48) and isolate θ_w^i to find the optimal old labor in each country $i = 1, 2$

$$\theta_w^1 = \frac{\nu(1+\beta)}{\beta\gamma[\nu + \sigma(1-\varepsilon^1)] + \nu(1+\beta)}, \tag{54}$$

$$\theta_w^2 = \frac{1+\beta}{\beta\gamma + 1 + \beta}. \tag{55}$$

Use (39) and (44) into (41) and into (46) respectively to find the optimal level of education in the sending country

$$e_w^1 = \frac{\varepsilon^1 \sigma K_w^1}{(1-\sigma-\nu)a}, \tag{56}$$

and the optimal level of education in the receiving country

$$\mathcal{E}_w^2 = \frac{\sigma K_w^2}{(1-\sigma-\nu)a\alpha}. \tag{57}$$

From expressions (39) and (44) respectively, deduce the optimal capital per worker that maximizes the welfare in each country

$$K_w^1 = \left[(1-\sigma-\nu)\left(\frac{\varepsilon^1\sigma}{(1-\sigma-\nu)a}\right)^{\varepsilon^1\sigma}(\ell_w^1)^\sigma(\theta_w^1)^\nu\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^1)}}, \tag{58}$$

$$K_w^2 = \left[(1-\sigma-\nu)(1+m^{2\star})\left(\frac{\sigma}{(1-\sigma-\nu)a\alpha}\right)^{\sigma}(\ell_w^2)^\sigma(\theta_w^2)^\nu\right]^{\frac{1}{\nu}}. \tag{59}$$

PROPOSITION **3** *As long as incentives for adult's permanent migration exist from country* 1 *to country* 2*, the level of migrants chosen by the social planner of country* 2 *is bigger than the one chosen by the social planner of country* 1*.*

Proof. To find the country specific optimal level of migrants that lead to the first-best static welfare optimum, we proceed as follows. For the sending country 1, the social planner equalizes $\hat{K}^1(m^1) = K_w^1$ so that $m^{1\star} = \Psi^{-1}(K_w^1)$.

$$m^{1\star} = 1 - \left[\left[\frac{1-\sigma+\beta(1-\sigma-\nu)}{\beta\sigma(1-\varepsilon^1)}\right]\frac{(\ell_w^1)^\sigma}{(\ell^1)^\sigma}\frac{(\theta_w^1)^\nu}{(\theta^1)^\nu}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^1)}}.$$

It is necessary that the migration rate satisfy $-1 \leq m^{1\star} \leq 1$, which gives the conditions

$$-1 \leq m^{1\star} \iff \left[\left[\frac{1-\sigma+\beta(1-\sigma-\nu)}{\beta\sigma(1-\varepsilon^1)}\right]\frac{(\ell_w^1)^\sigma}{(\ell^1)^\sigma}\frac{(\theta_w^1)^\nu}{(\theta^1)^\nu}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^1)}} < 2,$$

and

$$-\left[\left[\frac{1-\sigma+\beta(1-\sigma-\nu)}{\beta\sigma(1-\varepsilon^1)}\right]\frac{(\ell_w^1)^\sigma}{(\ell^1)^\sigma}\frac{(\theta_w^1)^\nu}{(\theta^1)^\nu}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^1)}} \leq 0, \quad \text{which is always true.}$$

For the receiving country 2, the social planner chooses the optimal level of immigrants that leads to the welfare optimum. Note that with (49) into (47), the remaining expression of (47) is always positive. Consequently, the social planner always chooses the maximum of immigrants $m^{2\star} = 1$. The social planner chooses also the optimal level of taxation in the receiving country 2, $\tau^{2\star}$, by equalizing $\Theta^2 = (1+m^2)\theta_w^2$ which is equivalent to

$$\frac{\nu(1-\tau^2)}{\nu+\sigma\tau^2+\gamma[1-\sigma(1-\tau^2)]} = \frac{1+\beta}{\beta\gamma+1+\beta}.$$

Solving for $\tau^{2\star}$

$$\tau^{2\star} = \frac{\nu\left[\beta(1+\gamma)+1\right]+(1+\beta)\left[\gamma(\sigma-1)-\nu\right]}{\nu\left[\beta(1+\gamma)+1\right]+(1+\beta)\sigma(1+\gamma)}.$$

The condition for $-1 < \tau^{2\star} < 1$ is shown in Appendix C. □

The intuition of the result is the following. The theoretical result that $m^{2\star} = 1$ should not be interpreted per se in face of the real world, but simply understood as the social planner of country 2 wants more migrants than those sent by the social planner of country 1. The reason is that country two has an explicit social security system. The more individuals contribute to it, the higher the utility of retired. Indeed their utility increases with the number of immigrants (and consequently of their investment in education in their home country when young) since the second period consumption is a linear increasing function of production, the latter being increasing with migrants. Migrants are beneficial to the social security system.

6 THEORETICAL RESULTS: DISCUSSION

This theoretical paper provides a rationale for the emergence of selective immigration policies. There are various kind of legal systems of migration: those which are explicit and those which are implicit, see Chaabane and Gaumont (2015). It is well understood that most countries do not allow every immigrant to enter their country (by explicit selection devices), but is not well understood that many countries discourage emigrants from leaving their country (by implicit stay-home incentives), even if border are legally open[9]. One of the particularities of this model is that the host country 2 reaches the maximizing social welfare level by choosing the optimal immigration flow. This is encapsulated into the concept of explicit border. The departure country chooses its optimal emigration level by the same procedure. This is encapsulated into the concept of implicit border. In this section, the main theoretical results are exposed, and the link with the reality is enlightened. This paper contributes to the theoretical literature by providing evidence on the relative importance of the education system compared to the social security system in setting migration policies.

6.1 MAIN THEORETICAL RESULTS

In each country, the social planner maximizes the social welfare utility, therefore all education, consumption, labor and capital are set at their respective welfare maximizing levels.

PROPOSITION **4** *In the post-migration steady-state equilibria, there is no price equalization across countries.*

Proof.

Since the returns to education differ across countries, the optimal migration decisions lead to different steady-state equilibria. In post-migration perfect foresight equilibria, the post-migration flow is defined by $m = \min\{m^1, m^2\}$ which is perfectly anticipated by each country. Using the previous results, $m^{1\star} < m^{2\star}$, the social planner of country 2 is constrained by the choice of the social planner of country 1, and $m^{2\star\star} = \min\{m^{1\star}, m^{2\star}\} = m^{1\star}$. Consequently, the effective real movement of people from 1 to 2 is $m^{1\star}$. Let us interpret such a result as if the country 1 "reaches" the optimal level "before" country 2. In such a case $\hat{K}^1(m^{1\star}) = K_w^1$ and $\hat{K}^2(m^{1\star}) \neq K_w^2$. Immediately after having reached his optimal migration flow, country 1 closes its borders. Country 2 has an incentive for illegal migration, and can try to favor arrival of migrant from country 1 on his territory.

[9] In the context of game theory, Stark and *al.* proposed a model in which countries implement endogenous bilateral agreements that are welfare improving.

A natural consequence of such a differences in post-migration steady-state capital is that there are no prices equalization across countries. There always remains a wage differential $w^2 \neq w^1$ and $p^2 \neq p^1$, as well as a difference in the interest rate across countries, $R^2 \neq R^1$.

□

6.2 MIGRATION WITH EXPLICIT AND IMPLICIT BORDERS

The less educated country chooses a much smaller migration flow than the other country. The sending country closes its borders prior to the other country and does not let its individuals leave the home country. Figure 1 illustrates this phenomenon.

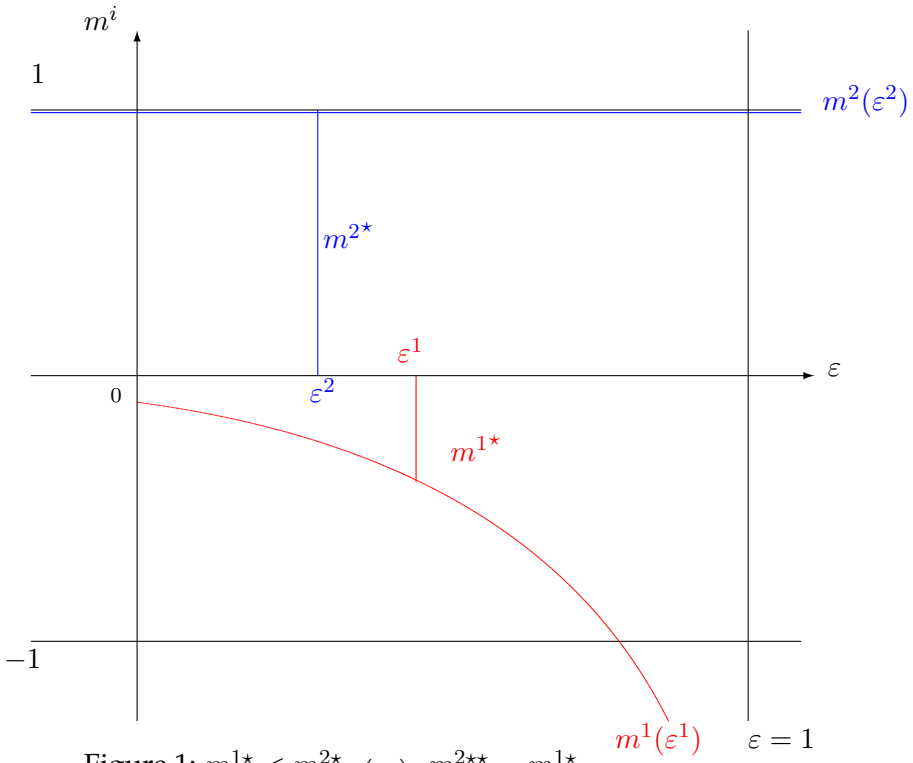


Figure 1: $m^{1\star} < m^{2\star} \iff m^{2\star\star} = m^{1\star}$.

Application: the migration flow directed from France to Canada

It's not a secret that French adults are more likely to leave their home country, France. Obviously, the best known destination country is Canada. Indeed, the young graduated adults are the first to migrate, because their individual' characteristics perfectly match the selective legal system of migration, optimally designed by the Canadian immigration policy. This policy is based on a system of points. Thereby, Canada has a great economic interest to attract French educated adults.

France has an incentive to let some of its well educated adults migrate up to its optimal level of emigrants, $m_1^\star$. After this threshold, France obviously loses too much social welfare in this type of exchange, and has an incentive for keeping individuals in their homeland. Indeed, in adulthood, individuals start to contribute to the welfare as entrepreneurs. In our model, old are the owner of the firm. Consequently, if individuals leave the country when adult, then there are less firms. The big challenge is to find a way to encourage adults to stay

in France. For that reason, empirically, the social planner provides adults with stay-home incentives. Indeed, there exists various implicit legal systems that provide individuals (or firms) with incentive not to migrate. Among them we find industrial policies that protect low qualified labor from delocalization, fiscal competition on social security, education policies (scientific research, vocational training) that sustain growth and slow international migrations by providing individuals with a higher level of welfare. Legal working time is also a welfare criterion that implicitly induces individuals not to migrate. Moreover, attractive fiscal policies are implemented in order to promote investment opportunities.

6.3 THE LINK BETWEEN REALITY AND OUR THEORETICAL RESULTS

Selective immigration policy, in the main receiving countries, is usually based on a legal system of points[10]. A set of criteria is established by the country, and the would-be migrant must reach the minimum threshold of points with his own characteristics. Since the main selection criteria are education, age and languages abilities, the threshold is rapidly reached for a well educated individual with a high degree. Canada and Australia are the best known countries using such an instrument of selection, and given the rather beneficial results, other countries have decided to implement this efficient system (Great Britain, Germany for examples). Obviously the most important criterion for the social planner is education. This is the reason why we build a 3-period model with education.

The social planner chooses these criteria to select migrants. The selected individuals contribute to the social welfare of the receiving economy. In a broad sense, the utility of native individuals is maximized with the contribution of the future migrants in terms of labor, consumption, saving and taxes. Consequently, the capital per capita is expected to increase in these receiving countries (especially if the migrant brings more than the local per capital saving). By doing so, the legal system of points— by determining the optimal migration flows — guides the economy in the direction of our theoretical concept of static welfare optimum. The model developed above shows that in the case of unilateral international migration of heterogenous labor from country 1 — low return to education — to country 2 — high return to education —, the optimal level of migrants chosen by the receiving country 2 is much greater than the optimal level chosen by the sending country 1. This means that the effective movement of migrants is the optimal flow of the country 1, therefore, the host country is still under its own optimal social welfare level.

In the real life, this result is reflected by a rising and perpetual demand for skilled immigrants coming from the less educated countries and toward the main host countries with selective immigration policies. Since the results show that the sending country has successfully reached its welfare optimum, it has no more interest in letting the local individuals leave the country. Indeed, the departure of more than the optimal chosen flow of emigrants could upset its final social welfare optimum.

There exist incentives for permanent international migration in country 1 for all adults, but the social planner refrain them to leave, while the social planner of country 2 welcome every one to migrate from 1 to 2. Consequently, there exists incentive for illegal migration. This point is very important and have some counterpart in the real world. Indeed, during the 2000-2010 decades, more than 11 million Mexicans migrated to the USA. In 2002, the USA built a 1300km long wall along its border to officially stop illegal migration. Actually, discussion between US and Mexican governments were directed to define how many holes the wall will contain, and where these holes will be located. The goal of this discussion

[10]For more details, see Chaabane 2011

was to set up a migration policy that lets the US government be in a position to control illegal migration flows.

Generally, implicit migration policies have some economic effects.

1. The protection of low qualified labor from delocalization (emigration) is usually obtained by any industrial policy that favors wage increase. Among these policies, education policies (scientific research, vocational training) help sustain growth. In our model, since adults are not taxed, and since a fraction of the population has already migrated in post-migration steady-state equilibrium, the capital per worker has increased, compared with the autarkic steady-state equilibrium. Consequently, wage increase provides a protection against the migration of low skilled labor. Moreover, interest rate decreases, providing incentive for investment, another implicit migration border. Those policies are successful in refraining international migrations by providing individuals with a higher level of welfare. An objective that is reached by the social planner of the departure country.

2. Fiscal competition on social security is a huge question. On the one hand for government, Feinleib and Warner (2005) estimate that an increase in legal immigration of about a quarter of a million would reduce the 75-year actuarial deficit of the Social Security program by about 5 percent under the current set of assumptions. On the other hand, Teitelbaum (in Feinleib and Warner (2005)) provided evidence that immigrant flows are powerfully affected by government policies and the things that governments do. There are a lot of migrants for which social security is an important criterion while making their migration decision. In our model, the departure country has no retirement fund and a low return to education. Consequently, on the one hand, incentive for migration is directed toward the country which exhibits a higher quality of social security, on the other hand, stay-home incentives are also playing a role since adults are not taxed at all. Two-sided borders are active simultaneously.

3. Legal working time is also a welfare criterion that implicitly induces individuals not to migrate. In our model, comparing the value of the autarkic steady-state equilibrium of old labor with post-migration welfare maximizing level of the old labor, there are values of the return to education ε for which old work less post-migration, and reciprocally work more, another implicit emigration policy.

7 Simulations and Empirical Evidence: Discussion

It is commonly known that pension systems are an important issue for international migration both for the government in terms of public finance and / or for the individuals in terms of incentives. In fact, the role of the pension system is highly valued by government and many countries use it to support their migration policies.

According to the "Melbourne Mercer Global Pension Index"[11], our 2 country model mimics the real world since the theoretical country 2 characterizes all countries with a high quality pension system, ranked A and B, while country 1 represents all countries with a low quality pension system, ranked C and D. This section emphasizes the relation between a given pension system and international migration rates. For the remainder of the paper, parameters values of the theoretical model are set in order to reflect the real world. The

[11] Details of the study are given in Appendix D.

discussion is organized as follows. First we consider 2 countries: one is characterized by a high return to education, say $\varepsilon = 0.1$, while the other one is characterized by a low return to education, say $\varepsilon = 0.9$. We then discuss various legal social security systems for $\tau \in [-1, 1]$ or no social security system, $\tau = 0$. Second we consider the case where both countries experience similar returns to education. All other parameters are as follows: $\sigma = \nu = 1/3$, the rate of interest is 1% consequently the time preference is $\beta = 0.671653$, the preference for leisure is 0.66. One can change the parameters and see that our results are fairly robust.

7.1 Education differentials and Indirect Utility

In order to make a migration decision, an adult compares his inter-temporal indirect utility in his home country with his inter-temporal indirect utility in the foreign country at the steady-state system of prices of each country. Incentives for international migration exist if his indirect utility is higher abroad.

7.1.1 No social security System and Indirect Utility

Suppose that only education differentials exist across countries. None of them exhibits a high quality social security system. Such a framework characterizes an international migration flows between 2 countries being ranked C or D by Mercer Melbourne Global Pension Index, see Appendix D. In blue is the utility in country with higher returns to education, in yellow the other one.

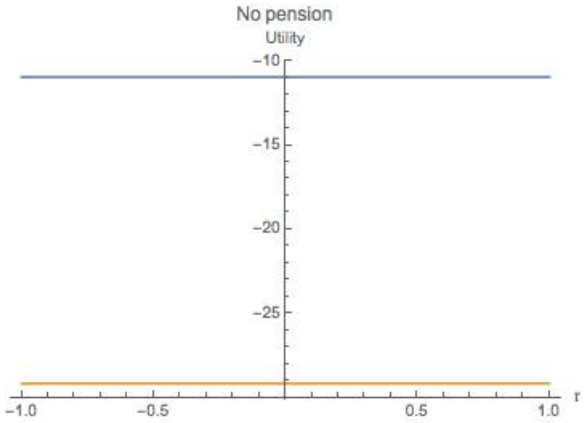


Figure 2

International migration is directed toward the country with the higher return to education. In the real world, the previous case captures for example the incentives to migrate from South Korea to Japan.[12]

7.1.2 Social Security System in Only one Country

We start by comparing the indirect utility levels in the case where the social security exists only in the country endowed with the higher return to education.

[12]To illustrate our examples of migration in real life, we refer to the Shanghai Academic rankings of World Universities, summarized in the table 2 in appendix E.

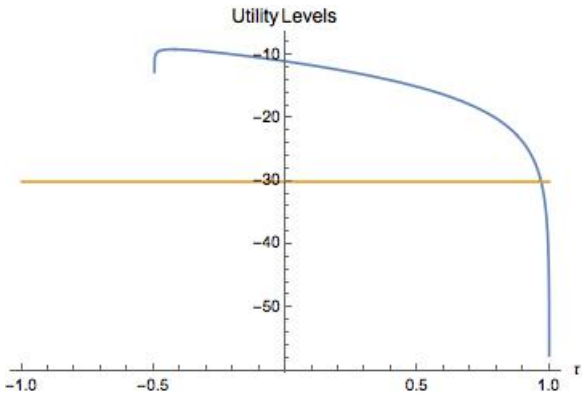


Figure 3

Figure 3 shows the impact of a social security system on utilities when countries crucially differ in terms of educational performance. In blue the utility level of adults in a country with a high return to education. In yellow, the utility level of adults in a country with a low return to education and no pension system $\tau = 0$.

For reasonable rates of taxation/subsidy on wages, the utility level is much higher in the country providing a social security system. Adults living in a country without social security have an incentive to permanently migrate to the country that offers one. Out of these reasonable rates of taxation/subsidy on wages, the relation reverses and migration flows are in the opposite direction. Too much taxation or too much subsidy alter the benefits of the social security in terms of incentives for migration. As examples, see the incentives to migrate from China to Canada or Switzerland.

This is no longer the case when the country with the pension system has a low education performance. Indeed, comparing individual's utility in that case reverses incentives for international migration. Figure 4 illustrates it.

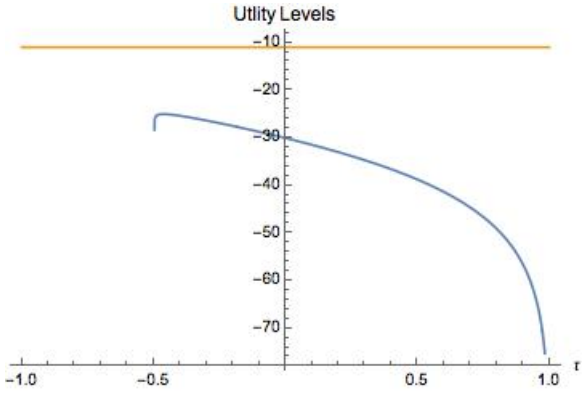


Figure 4

Note that contrary to the previous case of Figure 3, the no-pension country dominates the social security country over approximatively $[-0.5, 1]$ an interval to which the rate of taxation on wages belongs to. Outside this interval, the $\log$ utility is not defined. There is no rate of subsidy that accommodates the social security system. Since the utility level is greater in the country with a higher performance of education, adults have incentives to permanently migrate even if there is a retirement pension in their home country. In other words, the effects of the return to education on utility dominates those of the social security. According to Appendix D and E, this illustrates for example incentives for international migration from Singapore to France.

7.1.3 SOCIAL SECURITY SYSTEM IN BOTH COUNTRIES

Let us investigate the differences in levels of indirect utility between two countries with a social security system. We investigates various cases where the differential of the returns to education varies across countries. All the following Figures exhibit the level of utility of each country according to various possible differences of their return to education.

Figures 5 characterizes the case of a high differential in the return to education: $\varepsilon = 0.1$ and $\varepsilon = 0.9$. Figure 6 depicts a low differential combined with low level of the return to education in each country: $\varepsilon = 0.8$ and $\varepsilon = 0.9$. Figure 7 exhibits a similar case but the level of the return to education are high in each country: $\varepsilon = 0.1$ and $\varepsilon = 0.2$. The blue curves illustrate the country with the higher return to education.

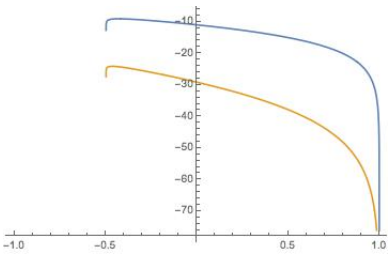


Figure 5

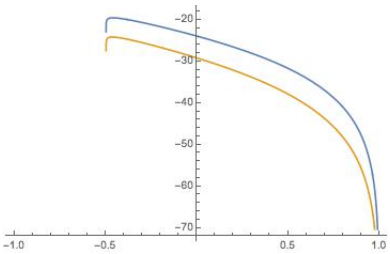
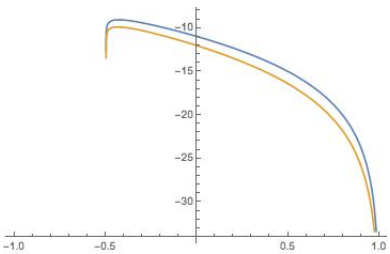


Figure 6



Figue 7

As these Figures illustrate, the difference in the return to education implies disparities in terms of indirect utility. Indeed, as long as the rate of taxation on wages are the same across countries, the efficiency of education strongly dominates the utilitarian benefits of social security. Consequently the direction of international migration is unilateral from the lowest to the highest return to education country. Note that if the rate of taxation on wages differ across countries, then two cases arise. Consider Figure 6 for example. Suppose that the rate of taxation of the country with higher return to education is higher than the one in the other country, then it is possible that individuals prefer to migrate from this country to the low quality social security.

Figure 5 illustrates the South-North migration, like migration from Singapore to Switzerland for instance. Consider as a starting point that both countries have no social security $\tau_i = 0, i = 1, 2$. Suppose that governments install a social security associated with a positive rate of taxation on wages. It appears that the indirect utility loss in the country with the highest return to education is smaller than the one of other country. To pay for having the right of retirement pension costs less in indirect utility in the country with the highest ε. However, this is no longer the case for very high level of the rate of taxation on wages.

Note that as long as the rate of taxation on wages are the same across countries, for the same difference in the return to education, $\delta = 0.9 - 0.8 = 0.1$ on Figure 6 and $\delta = 0.2 - 0.1 = 0.1$ on Figure 7, the differential of indirect utility is smaller for high return to education. Incentives for international migration are directed toward the country providing a better return to education. It perfectly illustrates the South-North migration in the real world. Figure 6 illustrates the South-South migration flows, like migration from Chile to Singapore. Figure 7 corresponds to the North North migration flows, like from Denmark to UK.

7.2 SIMILAR EDUCATION AND INDIRECT UTILITIES

This subsection is devoted to the study of indirect utility differentials when countries exhibit similar returns to education, but with various social security systems. Obviously,

if they have the same return to education and the same social security, both countries are identical so that there are no migration flows. More interestingly, Figure 8 compares various possible social security systems. The blue curve captures all possible social security systems.

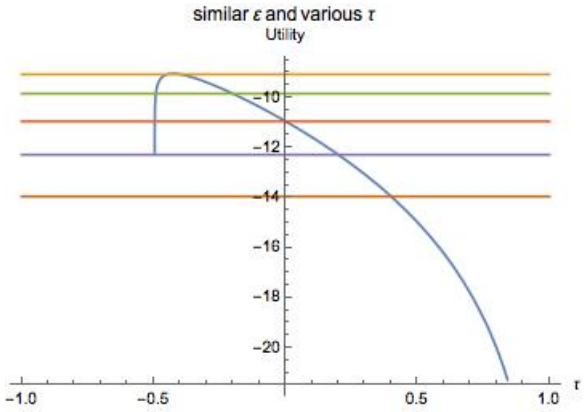


Figure 8

Note that the rate of taxation on wages is not defined for low values, since preferences are represented by a log linear utility function. The highest line, which is yellow, characterizes a country that subsidies the young at an arbitrary level of $\tau = -40\%$. Incentives for international migration are directed from the yellow to the blue country as long as $\tau_{\text{blue}} \in [-0.447, -0.40]$. Over this zone, higher taxation rates provide inhabitants with higher utility in the blue country. The relation reverses elsewhere. This phenomenon repeats whatever the given taxation rate, there exists a zone of lower rate of taxation on wages that supports higher level of indirect utility, but the incentives for migration reverse out of this interval.

It is of interest to note that for identical increases in the rate of taxation on wages, countries experience various differentials in utility, as the utility is concave with respect to taxes. This suggests that the higher the taxation rate, the higher the marginal loss in utility.

After studying the role of pension systems, our analysis is extended one step further: how can we understand that some countries economically select migrants?

7.3 Extension: Only The Best

This sub-section relaxes the assumption of two countries. Let us consider the case where each country i faces a continuum of countries j differentiated only with respect to the return to education, so that for country i we have $\varepsilon^j \in]\underline{\varepsilon}; \overline{\varepsilon}[$. In that case, a given country can choose the quality of his optimal potential migrants. Among high educated countries, the competition to attract the best individuals is very intense and the selection of migrants among all the would-be migrants has become a major government decision.

Indeed, the immigration policies are implemented according to the level of immigrant education which represents an important criterion for social planners. The broad range of possible returns to education makes possible to implement a new kind of selection. Indeed, the host country i has the possibility to choose the optimal level of return of education, across the wide range of options. The receiving country obviously chooses a lower level of education than the local one. Whereby, $\varepsilon^j \in]\varepsilon^i, 0,9[$.

It is possible to show that the level of education is a concave function in the return to education. Since e is concave in ε, we have $Max_{\varepsilon^1}\ e(\varepsilon^1)$. The solution provides the "best quality" of immigrants. Figure 9 illustrates this particular case.

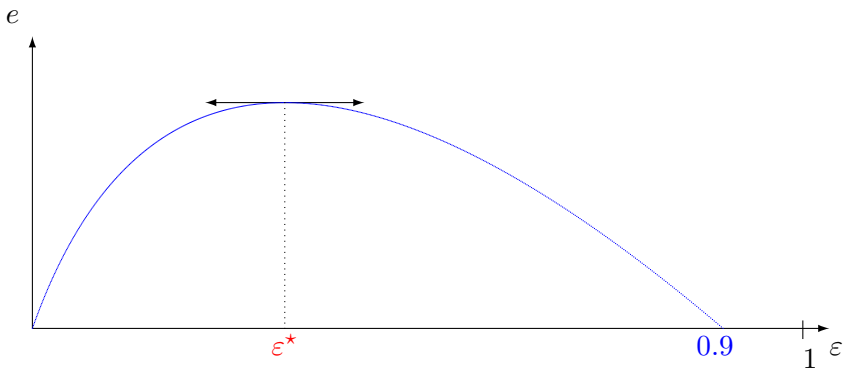
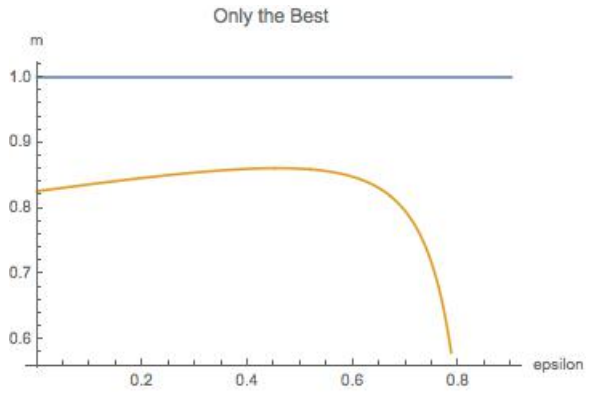


Figure 9: $Max_{\varepsilon^1}\ e(\varepsilon^1) = \varepsilon^\star$.

More interestingly, it can be shown that the optimal migration rate $m^\star$ is a concave function of the returns to education ε, so that a social planner can select " only the best" migrants, as depicted in the following Graph where returns to education are on the horizontal axis, the migration rate on the vertical axis.



On this Graph, we selected the parameters as follows: there is no pension $\tau = 0$, adults are less productive ($\sigma = 0.9$) than old ($\nu = 0.1$) workers, the rate of interest per annum is $r_t = 0.01$ over a period of 40 years consequently the time preferences rate is $\beta = 0.6712653$, the preference for leisure is $2/3$. Note that for low values of $\varepsilon \in [0, 0.5]$ the relation between the return to education and the rate of migration is decreasing. A country endowed with a very high return to education selects more migrants with a lower return to education than the native population.

8 Conclusion

Using a 3-period overlapping generations model with two countries differentiated with respect to their returns to education and their social security system, this paper analyzes the behavior of a country-specific social planner who optimally selects the rate of migration that leads his country to the optimal post-migration social welfare. Due to a difference in the returns to education across countries, each social planner chooses a country-specific level of migrants that is social welfare maximizing. Consequently, since the optimal level of migrants differ between countries, an optimal legal system of borders emerges in each country. This generates endogenous two-sided borders across countries since borders are asymmetric. A first direct consequence of a such a framework is the non equalization of factor-price. Both wages and interest rates still remain different in post-migration steady-state equilibria with optimal legal system of international migration. A second

direct consequence is that unlike the traditional literature, international labor migration is constrained by a social planner's decision.

On the one hand, the differences in returns to education plays a role on the incentive for migration, on the other hand, the social security systems plays also a role in the incentives for permanent international migration. The question is to know which one is the most important. This is a major matter for destination countries. Indeed, a high efficiency of education combined with an efficient retirement system, allows the country to attract more immigrants. Consequently, enabling it to reach the social welfare optimum. After simulations of North-North, South-North and South-South international migrations, this paper has shown that the education motive dominates social security systems in general, but obviously not when education is similar across countries.

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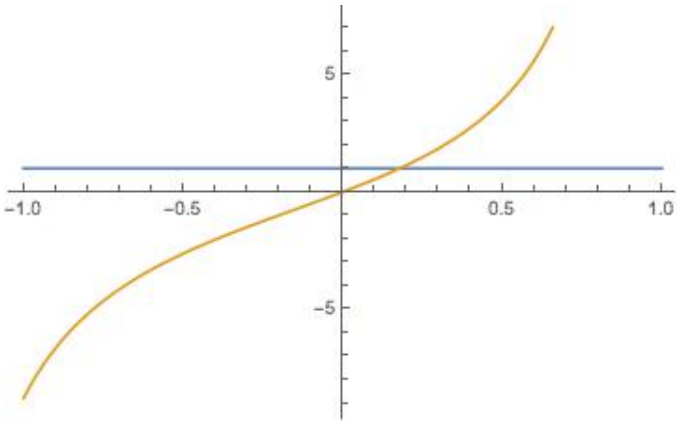
A APPENDIX

Condition on elasticities for a positive relation between the income taxation rate and the retirement pension is below the blue horizontal line in the following Figure.



The tax rate must not exceed 20%. In fact, beyond this level, the increase the tax rate no longer increases pensions.

B APPENDIX

Proof. Proof of Lemma 1. Using Definition 2 forward, $K^i_{t+1} = s^i_t$, rewrite the second period budget constraint

$$d^i_{t+1} = R^i_{t+1}K^i_{t+1} + (1-\tau^i)p^i_{t+1}\theta^i_{t+1} + z^i_{t+1}(1-\theta^i_{t+1}).$$

Use the first order condition of the firm and (12) in the previous expressions

$$d^i_{t+1} = [1-\sigma(1-\tau^i)]Q^i_{t+1}, \tag{60}$$

put it into (5) the first order condition of the individual to have

$$\frac{1}{c^i_t} = \beta\frac{(1-\sigma-\nu)Q^i_{t+1}}{[1-\sigma(1-\tau^i)]Q^i_{t+1}K^i_{t+1}} \iff c^i_t = \frac{[1-\sigma(1-\tau^i)]}{\beta(1-\sigma-\nu)}K^i_{t+1}, \tag{61}$$

and put (60) into (7)

$$\frac{\gamma}{1-\ell^i_t} = \frac{\beta(1-\sigma-\nu)\sigma Q^i_t}{[1-\sigma(1-\tau^i)]K^i_{t+1}\ell^i_t}. \tag{62}$$

By using (10) and (61) into the individual's budget constraint, we have

$$\frac{\sigma Q^i_t}{K^i_{t+1}} = \frac{\beta(1-\sigma-\nu)+1-\sigma(1-\tau^i)}{\beta(1-\sigma-\nu)(1-\varepsilon^i)(1-\tau^i)}. \tag{63}$$

In order to have $\ell^i_{t+1} = \ell^i_t = \ell^i$ for each country $i = 1, 2$, we replace (63) into (62)

$$\ell^i = \frac{1-\sigma(1-\tau^i)+\beta(1-\sigma-\nu)}{\gamma[1-\sigma(1-\tau^i)](1-\varepsilon^i)(1-\tau^i)+\beta(1-\sigma-\nu)+1-\sigma(1-\tau^i)}.$$

Using (5), (8) can be rewrite as

$$\frac{\beta\gamma c^i_t}{1-\theta^i_{t+1}} = \frac{(1-\tau)p^i_{t+1}-z^i_{t+1}}{R^i_{t+1}}. \tag{64}$$

In order to have $\theta^i_{t+1} = \theta^i_t = \theta^i$ for each country $i = 1, 2$, we put (9), (11) and (61) into (64)

$$\theta^i = \frac{\nu(1-\tau^i)}{\gamma[1-\sigma(1-\tau^i)]+\tau^i\sigma+\nu}.$$

□

C APPENDIX

The optimal level of taxation must satisfy the following condition $-1 < \tau^{2\star} < 1$.

Let us start the analysis with $\tau^{2\star} < 1 \iff (1+\beta)\sigma(1+\gamma) > (1+\beta)\left[\gamma(\sigma-1)-\nu\right]$.

After simplification we get $\sigma > -(\gamma+\nu)$. Let us now turn to the analysis of $\tau^{2\star} > -1$, which is equivalent to study

$$\nu\left[\beta(1+\gamma)+1\right] + (1+\beta)\left[\gamma(\sigma-1)-\nu\right] > -\left[\nu[\beta(1+\gamma)+1] + (1+\beta)\sigma(1+\gamma)\right].$$

After simplification we have the following condition

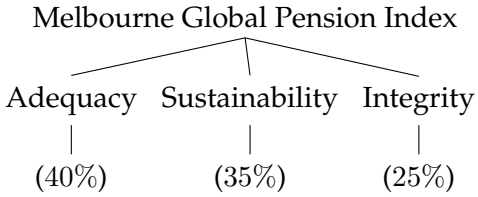
$$\beta > \frac{-\nu-\sigma(1+\gamma)-\gamma(\sigma-1)}{(1+\gamma)(2\nu+\sigma)+\gamma(\sigma-1)-\nu} \text{ which is always true.}$$

If the two previous conditions are satisfied then the previous conditions holds and we have $\tau^{2\star} \in \,]-1, 1\,[$.

D APPENDIX

The objective of the Melbourne Mercer Global Pension Index is to benchmark each country's system using more than 50 questions. It's well known that a comparison of the diverse retirement systems around the world is not straight forward. *"Retirement-income systems are diverse and often involve a number of different programs. Classifying pensions systems and different retirement-income schemes is consequently difficult."*[13]

The Melbourne Global Pension Index (M.G.P.I) uses three sub-indexes: Adequacy, Sustainability and Integrity, to measure each country's retirement income system. The following schema describe these sub-indices.



Each sub-index contains various indicators, as follows:

$$\text{Adequacy} \Rightarrow \begin{cases} \text{Benefits} \\ \text{Savings} \\ \text{Tax support} \\ \text{Benefit design} \\ \text{Growth assets} \end{cases}$$

$$\text{Sustainability} \Rightarrow \begin{cases} \text{Coverage} \\ \text{Total assets} \\ \text{Contributions} \\ \text{Demography} \\ \text{Government debt} \end{cases}$$

$$\text{Integrity} \Rightarrow \begin{cases} \text{Regulation} \\ \text{Governance} \\ \text{Protection} \\ \text{Communication} \\ \text{Costs} \end{cases}$$

The entire index valurit0

the retirement systems around the world. The study relates 26 countries and scores range from 43.5 for India to 82.4 for Denmark.

The table below summarizes the results of the ranking study. It shows that none of the studied countries has a E-grade with an index value under 35. The D-grade indicates that the pension system has some sound features but also has some major omissions and / or weaknesses. This classification grade may include countries whose retirement system is in its early stages of development, such as China and Indonesia.

Grade	Index Value	Countries	Description
A	**>80**	**Denmark**	**First class and robust retirement system.**
B^+	**75-80**	**Australia** **Netherlands**	
B	**65-75**	**Finland** **Switzerland** **Sweden** **Canada** **Chile** **UK** **Singapore**	**System with sound structure, many good features but some areas for improvement.**
C^+	**60-65**	**Germany** **Ireland**	
C	**50-60**	**USA** **France** **Poland** **South Africa** **Austria** **Brazil**	**Some good features but major risks and/or shortcomings.**
D	**35-50**	**Italy** **Mexico** **China** **Indonisia** **Poland** **Japan** **Korea (South)** **India**	**Some desirable features but major weaknesses and /or omissions.**
E	**<35**	**Nil**	**A very poor or a non-existent systems.**

Table 1: The Melbourne Mercer Global Pension Index

E APPENDIX

Ranking Criteria and Weights:

"Universities are ranked by several indicators of academic or research performance, including alumni and staff winning Nobel Prizes and Fields Medals, highly cited researchers, papers published in Nature and Science, papers indexed in major citation indices, and the per capita academic

performance of an institution. For each indicator, the highest scoring institution is assigned a score of 100, and other institutions are calculated as a percentage of the top score. The distribution of data for each indicator is examined for any significant distorting effect; standard statistical techniques are used to adjust the indicator if necessary. Scores for each indicator are weighted as shown below to arrive at a final overall score for an institution. The highest scoring institution is assigned a score of 100, and other institutions are calculated as a percentage of the top score. An institution's rank reflects the number of institutions that sit above it[15] *"*.

Rank	Country	University
1	**USA**	**Harvard University**
5	**UK**	**University of Cambridge**
19	**Switzerland**	**Swiss Federal Institute of Technology Zurich**
21	**Japan**	**The University of Tokyo**
24	**Canada**	**University of Toronto**
35	**France**	**Pierre and Marie Curie University - Paris 6**
39	**Danmark**	**University of Copenhagen**
44	**Australia**	**The University of Melbourne**
47	**Sweden**	**Karolinska Institute**
49	**Germany**	**Heidelberg University**
57	**Netherlands**	**Utrecht University**
69	**Norway**	**University of Oslo**
70	**Israel**	**The Hebrew University of Jerusalem**
73	**Finland**	**University of Helsinki**
84	**Russia**	**Moscow State University**
101-150	**Singapore**	**National University of Singapore**
101-150	**China**	**Peking University**
101-150	**South Korea**	**Seoul National University**
401-500	**Chile**	**Catholic University of Chile**

Table 2: Shanghai Academic Ranking Of World Universities

[15] Source: www.shanghairanking.com