## Optimal International Migration Policies with endogenous borders

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#### Abstract

In an OLG model with 2-periods and two countries, I first analyze the optimal migration policies a country specific social planner implements, and second the one a world social planner implements. Social planners use the migration flows as a welfare maximizing instrument. Differences in time preference rates across countries imply differences in optimal migration policies. In the post-migration welfare maximizing steady-state equilibrium, there is no prices equalization. A world social planner respects individual's incentive to migration, while a country-specific social planner does not. Calibrations of migration rates are provided. An extension to costly borders is made.

JEL Classification: K37, D91, F22

**Key words:** International Migration, Overlapping Generations Models, Legal System of Borders.

## **1** INTRODUCTION

For decades, international borders are under a constantly growing stress of migration. Conditions relatively favorable in "rich" countries, conditions relatively unfavorable in "poor" countries, in addition of a high population growth in these latter, probably generate incentives for international migration toward lands where life is better.

Mass migration arrivals pouches destination countries to establish control systems in their borders. The immigration control is still ruled by a single method: the immigration quotas using immigration laws as an instrument to implement such quotas. Immigration laws define quotas as being a boundary on the number of admitted individuals in a country. For that reason, these quotas are mainly exogenously set up and very inefficient. In fact, limiting the number of admitted migrants will fuel tensions on borders . Consequently, incentives for illegal migration emerges in direction to these countries.

The immigration always had a central role in the receiving economies, but it is obvious that the migration does not fill the income gap between the two borders, as assumed in several former research. For example, Mexico and US have the greatest income gap among all contiguous countries. And a difference between the income of Eastern Europe countries and EU is about 10 times, OECD (2006).

There is a large literature on economic immigration policy, and almost all models use the theory of the median voter, along with papers like Benhabib (1996). The author uses the median voter to determine the quality of immigrants. The inconvenience of this model is that the preferences of voters are not considered. Amegashie (2004) incorporates lobbying into his immigration model, between firms and a labor union. The author studies how the reservation wage of immigrants, the lobbying cost and the price of goods affect the number of allowed migrants: the immigration quota.

Boeri & Brücker (2005) studied the European migration and show that legal immigration rules implemented by the European Union toward arrivals from the third world countries became increasingly tight: since 1990, there were 92 reforms in national migration policies into the EU-15, with more than 5 reforms per year. Moreover, these reforms are only procedures-tighten to immigration at borders. For instance, by increasing the obstacles and lengthening procedures for visas applicants, by reducing the duration of work permits and even by making family reunification more difficult. All this go through exogenous quotas.

Various factors influence the level of migrants allowed in a country: the excess of labor demand, the competition with other countries, humanitarian reasons, etc. Like the case of migration policy of Canada, Borjas (1994).

Despite the manifest tend toward an increasing closure of borders, there is a paradoxical side. In fact, on the flip side, there is also an increase in regulation so that the European policies become more flexible. What kind of migration policy is better suited for each country? Is it better to strengthen or to reduce legal systems of international migration?

Actually, there is an incentive to implement economically-based international migration policies. A better immigration control system — through the use of immigration tools and emigration subsidies — allows to exploit the gains of human exchanges existing between two countries, Myers & Papageorgiou (2002).

We notice that a much greater importance is allocated to the registration of arrivals than to the registration of departures. This asymmetrical interest in one way of migration processes -arrivals- is more visible. In Hungary, and less visible in the Czech Republic, the departures are monitored too, Golinowska (2008). The analysis of the of the departure side of migration processes is extremely important. Generally, studies on sending countries focus on the "Brain Drain" and "Brain Waste". It does not exist any literature on the two-sided character of borders

International migration in overlapping generations (OLG) models has been initiated by Galor (1986). A huge literature has followed and was developed in the direction of post-migration market equilibrium, where wages and interest rates always equalize in post-migration steady state equilibrium. A strong characteristic of these models is that the role of the social planner is not analyzed.

In parallel, there is a wide theoretical literature on international migration in OLG models. Initiation was made with Galor (1986) with the study of the Golden Rule. This literature studies post-migration equilibrium where wages and interest rates always equalize in post-migration steady state equilibrium. The broadening of this trend was performed with Gaumont & Macdissi (2012). Authors show that neither the wages nor the interest rates never equalize. Due to uncertainty, in their model this is the expected wages and the expected interest rates that equalize, not the real wage or the real interest rate per se. The characteristic of these models is that the role of the social planner is not analyzed. Without any uncertainty, Chaabane & Gaumont (2015) introduce in a 3-period OLG model the role of social planners by using the migration flows as an instrument to maximize the social welfare in each country. Countries differ with respect to the return to education, and young individuals migrate for ever.

The model presented below follows this line of literature and proposes an alternative way to study the optimal migration levels for both, sending and departure countries, in the context where countries differ with respect to their time preference.

In a simple two-period OLG model and two countries, this paper proposes a modeling alternative of international migration legal systems, where countries are solely differentiated by their time preference. The migration flow, in each border-side, is optimally chosen by the social planner to maximize the overall welfare of the country. Since the Golden Rule level is determined, the optimal level of migrants is the number of immigrants (emigrants) allowing the receiving (departure) country to reach the maximum level of social welfare.

Several legal systems are possible. Indeed, there are legal systems controlled by the country specific benevolent social planner and others controlled by an unique world social planner.

In the first type of legal system, the country specific social planner chooses the optimal level of migrants allowing his own country to reach the Golden Rule level. Since countries are different in terms of time preference rate, the two optimal levels of international migration are different. Consequently, the real movement of people is the smallest of the two levels. The international legal system of borders is the optimal migration policy optimally chosen by the social planner of the correspondent country.

In the second type of legal system, the world social planner chooses the optimal levels of migrants for both countries, which allows to release a new international migration policy. Of course, the optimal levels of migrant funded by the first type of legal system and the second type of legal system are substantially different. The model shows the economic impact for a country depending of which type of legal system is implemented. The first type of legal system generates a migration policy that goes against the individuals' incentives for international migration. On the contrary, the second type of legal system implements a migration policy that is consistent with the individuals' inventive for migration. The study is extended to the case of costly borders. Regardless the country, once the borders cost is small enough, the first type of legal system generates optimal migration policies are consistent with incentives for international migration.

The remainder of the paper is organized as follows. Section 2 exposes the theoretical model, section 3 presents the temporary equilibrium in autarky, section 4 exposes the inter-temporal equilibrium in autarky and section 5 the international migration. The section 6 presents an alternative modeling and finally an extension o social cost of borders is given in the section 7. Section 8 concludes.

## 2 THE MODEL

The model operates in a perfectly competitive world with no uncertainty, two representative countries, i = 1, 2, which only differ with respect to their saving rate. Each country operates over infinite discrete time,  $t = 0, 1, 2, ..., \infty$ . In every period, a new generation of individuals  $N_t^i$  is born. For simplicity, in autarky  $N_{t+1}^i = N_t^i = N^i$ , where  $N^i = 1 > 0$ . In each country, a single tradable good is produced using two factors of production: the capital and the efficient labor. Capital depreciates fully after one period. Individuals and firms make rational decisions under perfect foresights.

#### **2.1** The Individual

In a country, individuals are identical within as well as across generations. As in the standard overlapping generations models, each individual lives two periods. In the first period, an individual works and earns the competitive market wage rate  $w_t$ . This wage allows him to consume  $c_t$  and to save  $s_t$ . During the second period of his life cycle, an individual is retired and consumes the return of his savings  $d_{t+1} = R_{t+1}s_t$ . where  $R^i = 1 + r_t^i$  is the given competitive factor of interest and  $r_t^i$  the competitive interest rate in country *i* during period *t*. Rational individuals maximize their log-linear utility function and solve the following program where  $\beta^i$  is the country specific time preference rate

$$\max_{c_t, d_{t+1}} \log c_t^i + \beta^i \log d_{t+1}^i$$

subject to

$$\begin{cases} c_t^i + s_t^i &= w_t^i \\ d_{t+1}^i &= R_{t+1}^i s_t^i. \end{cases}$$
(1)

#### 2.2 The firm

In each period and each country, production occurs according to a constant returns to scale technology. The representative firm produces a single output  $Q_t^i$  with two factors of production, capital  $K_t^i$  and labor  $L_t^i$ . The production technology is given by the following Cobb-Douglas production function  $Q_t^i = K_t^{i\alpha} L_t^{i1-\alpha}$ , where  $0 < \alpha < 1$  is the elasticity of the capital. Labor market equilibrium imposes  $L_t^i = N^i = L^i$ . Thus let's first define  $k_t^i = \frac{K_t^i}{L^i}$ ,  $q_t^i = \frac{Q_t^i}{L^i}$ , and we have  $Q_t^i = K_t^{i\alpha} L^{i1-\alpha}$ , so  $q_t^i = k_t^i$ .

The representative competitive firm maximizes its profit

$$\max_{k_t^i} \quad (k_t^i)^{\alpha} - w_t^i - R_t^i k_t^i.$$
(2)

We now turn to the study of the temporary equilibrium, which is the solution of the two previous problems, the one of the individual and the one of the firm.

## **3** TEMPORARY EQUILIBRIUM OF THE ECONOMY IN AUTARKY

In this section, the temporary equilibrium of the economy in autarky is determined. First, let's recall the definition.

DEFINITION **1** In country *i*, the temporary equilibrium of period *t* is a competitive equilibrium given perfect anticipations on prices,  $w_t^i$  and  $R_{t+1}^i$ , given past variables,  $s_{t-1}^i$  and  $I_{t-1}^i = N_{t-1}^i s_{t-1}^i$ , or equivalently  $K_t = s_{t-1}$  as well as  $N_t = L_t$ .

Consider the individual's problem 1. Solving the first period budget constraint for  $s_t^i$  and replacing its new expression into the second period budget constraint gives

$$d_{t+1}^i = R_{t+1}^i (w_t^i - c_t^i).$$
(3)

Replacing (3) into the objective function, an individual solves the following program

$$\max_{c_t^i} \log c_t^i + \beta^i \log \left[ R_t^i (w_t^i - c_t^i) \right].$$

The first order condition gives the following relation

$$\frac{1}{c_t^i} = \frac{\beta^i R_{t+1}^i}{d_{t+1}^i}.$$
(4)

We rewrite the second period budget constraint as follows

$$s_t^i = \frac{d_{t+1}^i}{R_{t+1}^i}.$$

Rewrite the first period budget constraint using the previous expression to get

$$c_t^i = \frac{w_t^i}{1 + \beta^i}.$$

Combine the two previous expressions to find a new relation between saving and wage

$$s_t^i = \frac{\beta^i}{1+\beta^i} w_t^i. \tag{5}$$

In each country, the representative rational firm maximizes its profit. The first order condition gives

$$R_t^i = \alpha(k_t^i)^{\alpha - 1},\tag{6}$$

$$w_t^i = (1 - \alpha)(k_t^i)^{\alpha}.$$
(7)

Using the second period budget constraint,  $k_{t+1}^i = s_t^i$  and (6), we obtain

$$d_{t+1}^{i} = \alpha k_{t+1}^{i}.$$
 (8)

Puting the previous expression into (4) we have

$$c_t^i = \frac{1}{\beta^i} k_{t+1}^i. \tag{9}$$

## **4** The perfect-foresight inter-temporal equilibrium in Autarky

In order to study the perfect-foresight inter-temporal equilibrium in each country i = 1, 2, we use  $L_t^i = N^i = 1$  and the capital dynamics is  $k_{t+1}^i = s_t^i$ .

LEMMA 1 The dynamics of the economy

$$k_{t+1}^{i} = (1 - \alpha) \frac{\beta^{i}}{1 + \beta^{i}} (k_{t+1}^{i})^{\alpha}$$

are convergent to a unique steady-state equilibrium in each country i = 1, 2

$$\overline{k}^{i} = \left[ (1-\alpha) \frac{\beta^{i}}{1+\beta^{i}} \right]^{\frac{1}{1-\alpha}}$$

**Proof.** Using  $K_{t+1}^i = s_t^i$ , (6) and (8) into the first period budget constraint, we have

$$k_{t+1}^i = (1-\alpha)\frac{\beta^i}{1+\beta^i}k_t^i$$

In steady-state equilibrium,  $k_{t+1}^i = k_t^i = k^i$ . Isolating  $k^i$ , the dynamics of the economy are convergent to a unique steady-state equilibrium.

$$\overline{k}^{i} = \left[ (1-\alpha) \frac{\beta^{i}}{1+\beta^{i}} \right]^{\frac{1}{1-\alpha}}.$$
(10)

Note that the higher the discount rate, the higher the steady-state capital per worker.  $\Box$ 

## **5** INTERNATIONAL MIGRATION

Countries 1 and 2 are solely characterized by a difference in the time preference. Without any loss of generality, let us assume that the following inequality holds  $\beta^1 < \beta^2$  for the rest of the paper. In the country 2 the time preference is higher than in the country 1, and  $\beta^i \in [0, 1]$ .

#### 5.1 INCENTIVES FOR PERMANENT INTERNATIONAL MIGRATION

In this paper, labor is permitted to migrate internationally. We assume that individuals can permanently migrate. Migrants spend their entire life-cycle in the immigration country.

**PROPOSITION 1** Incentives for international migration of individuals always exist and are unilateral from country 1 to country 2.

**Proof.** Rational individual born in country 1 has an incentive to definitely migrate to country 2 if his indirect utility evaluated at the steady-state price system of country 2 is higher than their indirect utility evaluated at the steady-state prices of the country 1. The condition is

$$\log c^{11} + \beta^1 \log d^{11} < \log c^{12} + \beta^1 \log d^{12}.$$

Replacing the expressions of the consumption of the two periods (8) and (9) into the previous inequality and simplify we get

$$\log \left[ \frac{\overline{k}^1}{\overline{k}^2} \right] < \beta^1 \log \left[ \frac{(\overline{k}^2)^\alpha}{(\overline{k}^1)^\alpha} \right] \iff k^1 < k^2.$$

With the previous inequality, incentives for international migration from country 1 to country 2 always exist, and are unilateral.

#### **5.2** Dynamics with permanent International Migration

This subsection is devoted to the study of the dynamics of capital in both countries. Incentives for international migration are unilaterally directed from country 1 to country 2. Borders are open in steady-state equilibrium, say at period t = 0. A fraction  $m^i$ , i = 1, 2 of individuals is allowed to definitely migrate. After migration, individuals are still identical in the sending country. Note that the post-migration population of country 2 is heterogeneous. As the population is initially normalized to unity, the post-migration population in the departure country is  $(1 - m^1)$  and the post-migration population in the host country is  $(1 + m^2)$ . As each migrant migrates with his own time preference rate, the post migration dynamics are as follows, in the departure country 1:  $k_{t+1}^1 = (1 - m^1)s_t^i$ , in the host country 2:  $k_{t+1}^2 = s_t^2 + m^2 s_t^1$ . Knowing from (5) that  $s_t^i = \frac{\beta^i}{1+\beta^i}w_t^i$  and using (7) we rewrite the new expressions of dynamics. The dynamics in the departure country 1 are

$$k_{t+1}^{1} = (1 - m^{1}) \frac{\beta^{1}}{1 + \beta^{1}} (1 - \alpha) (k_{t}^{1})^{\alpha},$$

the dynamics in the host country 2 are

$$k_{t+1}^2 = \left[\frac{\beta^2}{1+\beta^2} + m^2 \frac{\beta^1}{1+\beta^1}\right] (1-\alpha) (k_t^2)^{\alpha}.$$

Isole  $k^i$ , we can easily compute the steady-state capital in each country.

$$\hat{k}^{1} = \left[ (1-\alpha) \frac{\beta^{1}}{1+\beta^{1}} (1-m^{1}) \right]^{\frac{1}{1-\alpha}},$$
(11)

$$\hat{k}^{2} = \left[ (1-\alpha) \left( \frac{\beta^{2}}{1+\beta^{2}} + m^{2} \frac{\beta^{1}}{1+\beta^{1}} \right) \right]^{\frac{1}{1-\alpha}}.$$
(12)

Both countries converge to a country-specific steady-state market equilibrium. We now study the country-specific post-migration welfare.

#### **5.3** The country-specific post-migration Golden Rule

Subsection 5.3 is dedicated to the study of the Golden Rule in country 1 and in country 2. The social welfare optimum of the economy is the stationary state that the benevolent social planner would select to maximize the welfare under the following country specific

feasibility constraint  $c^i + d^i + k^i = q^i$ . The welfare criterion a country chooses, to rank all possible steady states - following Samuelson (1958) - is the one that maximizes the aggregate consumption. The reference model is called Golden Rule, in which the government calculates the static capital per capita achieving this goal.

There is no discrimination between individuals, even if the post-migration population is heterogenous, the social planner treat them identically within the same country.

In the post-migration economy, the benevolent social planner in each country i = 1, 2 maximizes the steady-state social welfare by solving the following problem

$$\max_{k^i,d^i}\ \log c^i + \beta^i \log d^i$$

subject to

$$c^i + d^i + k^i = q^i.$$

The first order condition gives the following relations

$$\alpha q^i = k^i,\tag{13}$$

$$d^i = \beta^i c^i. \tag{14}$$

Knowing that  $q^i = (k^i)^{\alpha}$ , and replacing it into (13) we get

$$k^i = \alpha (k^i)^{\alpha}$$

From what we deduce the expression of the Golden Rule

$$k_{GR}^i = [\alpha]^{\frac{1}{1-\alpha}} \,. \tag{15}$$

#### 5.4 OPTIMAL LEGAL SYSTEM OF INTERNATIONAL MIGRATION

This sub-section sets the optimal legal system of international migration determined in each country i = 1, 2. The optimal level of migrants is optimally determined in order to reach the post-migration Golden Rule.

**PROPOSITION 2** In post-migration steady-state equilibrium, if the per capita capital of country 2 is greater than per capita capital of country 1, then the optimal level of migrants differs between countries.

**Proof.** Due to differences in time preference rates between countries, country 2 is endowed by a greater per capita capital than country 1. Once borders are open, the optimal level of migrants is determined by each social planner in order to enable the economy to reach the post-migration Golden Rule, i.e.,  $\hat{k}^i(m^i) = k_{GR}^i$ . Solving this equation for  $m^i$ , with i = 1, 2, allows us to determine the expression of the welfare maximizing level of migrants for each country

$$m^{1\star} = 1 - \frac{\alpha(1+\beta^1)}{\beta^1(1-\alpha)},$$
 (16)

$$m^{2\star} = \frac{1+\beta^1}{\beta^1} \left[ \frac{\alpha(1+\beta^2) - (1-\alpha)\beta^2}{(1-\alpha)(1+\beta^2)} \right].$$
 (17)

LEMMA **2** For some reasonable values of parameters, no country totally collapses into the other one. Both country are populated in post-migration steady-state.

**Proof.** The proof is given in Appendix A.

# 5.5 THE EMERGENCE OF AN OPTIMAL PRICE DIFFERENTIAL BETWEEN COUNTRIES

In each country, the social planner chooses the optimal level of migrants allowing his country to reach the maximum level of welfare. Due to differences between both countries, the optimal migration policies lead the economies to different steady-state equilibria. The two optimal levels of migrants are different, thus the real movement of people is the smaller of the two levels, i.e.,  $m = \min\{m^1, m^2\}$ .

**PROPOSITION 3** *There are no prices equilization between countries in the post-migration steadystate equilibrium.* 

#### Proof.

- 1.  $m^{1\star} \ge m^{2\star}$ : country 2 reaches the Golden Rule unlike the country 1. In this case,  $\hat{k}^2(m^{2\star}) = k_{GR}^2$  and  $\hat{k}^1(m^{2\star}) \ne k_{GR}^1$ .
- 2.  $m^{2\star} \ge m^{1\star}$ : country 1 reaches the Golden Rule level unlike the country 2. In this case  $\hat{k}^1(m^{1\star}) = k_{GR}^1$  and  $\hat{k}^2(m^{1\star}) \ne k_{GR}^2$ .

Due to difference in steady-state capital per worker, post-migration prices still remain different. There are always a wage differential  $w^1 \neq w^2$ , as well as an interest rate differential,  $R^1 \neq R^2$ .

It is interesting to note, as explained hereafter, that depending on parameters value, it could be country one which is constrained by the migration policy of country 2 or the reverse.

If case 1 holds, the receiving country 2 reaches its optimal welfare level, it will no longer have any incentives in letting people entering. The receiving country closes its borders, while the departure country has not yet reached its optimal level of welfare, since the real movement of migrants is the optimal level chosen by the host country 2. Accordingly, the country 1 has an incentive to let more people migrate, in order to reach its Golden Rule.

If case 2 holds, the departure country 1 reaches its his optimal social welfare, hence removing any incentives in letting its people move. That is, the country 1 closes its borders, while country 2 is still far off its Golden Rule, as the real movement of people is the chosen level by country 1.

## 6 ALTERNATIVE MODELING

In this section, the analysis is carried further. Indeed, unlike the previous section where the optimal social welfare is determined by a country-specific social planner, we consider the case where a world social planner determines the optimal migration policy for each country when borders are open.

#### 6.1 THE WORLD SOCIAL PLANNER

The world Social planner maximizes the global welfare program under the macroeconomic equilibrium constraint. This program determines the world social planner's optimal level of migrants in each country.

The world social planner solves the following problem

$$\max_{m^1,m^2} \log c^1(m^1) + \beta^1 \log d^1(m^1) + \log c^2(m^2) + \beta^2 \log d^2(m^2),$$

subject to the macroeconomic constraint

$$c^{1}(m^{1}) + c^{2}(m^{2}) + d^{1}(m^{1}) + d^{2}(m^{2}) + \hat{k}^{1} + \hat{k}^{2} = q^{1}(m^{1}) + q^{2}(m^{2}).$$

Isolate consumptions from the previous macroeconomic constraint. In order to maximize consumptions, the world social planner solves the following problem

$$\max_{m^1,m^2} (\hat{k}^1)^{\alpha} + (\hat{k}^2)^{\alpha} - \hat{k}^1 + \hat{k}^2.$$

Using the post migration steady-state equilibria, (11) and (12) we obtain

$$\max_{m^{1},m^{2}} \left[ (1-\alpha)\frac{\beta^{1}}{1+\beta^{1}}(1-m^{1}) \right]^{\frac{\alpha}{1-\alpha}} + \left[ (1-\alpha)\left(\frac{\beta^{2}}{1+\beta^{2}}+m^{2}\frac{\beta^{1}}{1+\beta^{1}}\right) \right]^{\frac{\alpha}{1-\alpha}} \\ - \left[ (1-\alpha)\frac{\beta^{1}}{1+\beta^{1}}(1-m^{1}) \right]^{\frac{1}{1-\alpha}} - \left[ (1-\alpha)\left(\frac{\beta^{2}}{1+\beta^{2}}+m^{2}\frac{\beta^{1}}{1+\beta^{1}}\right) \right]^{\frac{1}{1-\alpha}}.$$

The first order condition yields

$$\alpha \left[\frac{\beta^1 (1+m^1)(1-\alpha)}{1+\beta^1}\right]^{\frac{2\alpha-1}{\alpha-1}} = \left[\frac{\beta^1 (1+m^1)(1-\alpha)}{1+\beta^1}\right]^{\frac{\alpha}{1-\alpha}},$$
(18)

$$\alpha \left[ \frac{\beta^2}{1+\beta^2} + m^2 \frac{\beta^1}{1+\beta^1} \right]^{\frac{2\alpha-1}{1-\alpha}} = \left[ \frac{\beta^2}{1+\beta^2} + m^2 \frac{\beta^1}{1+\beta^1} \right]^{\frac{\alpha}{1-\alpha}}.$$
 (19)

Simplifying the two previous expressions (18) and (19), the optimal legal system for international migration emerges. The optimal level of migrants chosen by the world social planner for each country i = 1, 2 is

$$m_w^1 = \frac{\alpha(1+\beta^1)}{\beta^1(1-\alpha)} - 1,$$
(20)

$$m_w^2 = \left[\alpha - \frac{\beta^2}{1+\beta^2}\right] \frac{1+\beta^1}{\beta^1}.$$
 (21)

Note that there are various cases, for which either both migration rates have the same sign, or have an opposite sign. We now turn to study the legal systems of borders.

#### 6.2 LEGAL SYSTEMS OF BORDERS

This subsection discusses the different migration policies that emerge from the previous subsections. Subsection 5.4 studies the optimal legal systems of international migration, where the optimal migration levels are determined separately by each country specific social planner. Sub-section 6.1 exposes the case where the world social planner chooses the optimal levels of migration for each country, by maximizing the social welfare defined as the sum of all consumptions.

Comparing the optimal migration flows, it is possible to show that  $m^{1\star} \ge m^{2\star}$  if and only if the following condition is satisfied

$$\frac{\beta^2}{\beta^1} \ge \frac{3\alpha - 1}{1 - 3\alpha + 2\beta^1 (1 - 2\alpha)}.$$

A high level of capital per worker involves an low level of the optimal migration flow.

**PROPOSITION 4** *The world social planner's optimal legal system of international migration generically differs from the optimal legal system chosen by each social planner.* 

#### Proof.

Let us start by comparing  $m^{1\star}$  and  $m_w^1$  using relations (16) and (20). Note that we always have  $m^{1\star} = -m_w^1$ . Comparing  $m^{2\star}$  and  $m_w^2$  for country 2 leads to the following inequality

$$|m^{2\star}| \neq |m_w^2|$$

To sum up, the world social planner always chooses the reverse direction of migration flow in comparison with a given country-specific social planner choice. Moreover, the world social planner chooses the same direction of migration flow of individuals born in country 1, the low capital per worker steady-state equilibrium in autarky, but he chooses the opposite direction for the high capital per worker steady-state equilibrium in autarky. One can conclude that the optimal legal systems of borders differ and depend on the migration policy of a given country or the group of countries. We now investigate various reasonable values for parameters in order to better estimate different possible optimal legal systems.

We now investigate various reasonable values for parameters in order to better estimate different possible optimal legal systems.

#### **6.2.1** CALIBRATIONS AND SIMULATIONS

The simulation of the optimal levels of migrants is made after having calibrated the values of parameters. The value of the elasticity of substitution  $\alpha$  in the production function has been the subject of many empirical studies. In the context of overlapping generations models Benhabib & Jovanovic (1991) estimates  $\alpha = 1/3$ . We now simulate the difference optimal migration levels.

The following figures illustrate the behavior of the policy maker in terms of international migration policy within a country. Depending on whether it is a specific social planner in one country who decides to implement a migration policy or it is the world planner who decides to implement a world migration policy, the legal system of international migration differs.

Figure 1 shows the different optimal migration flows for  $\alpha = 1/3$ , the rate of interest *r* is already fixed at  $r \sim 2.04$  per annum for the 30-year French Treasury Bond, see the "Banque de France") from what the corresponding time preference for country 2 is  $\beta^2 = 0.45$ . Figure 1 plots the optimal rates of migration *m* against the time preference  $\beta^1$ .

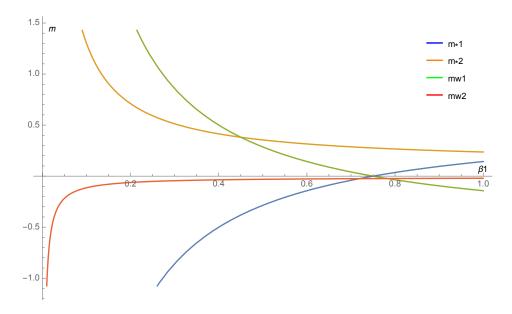


Figure 1: Different Optimal migration flows: Case 1

#### Country 1:

Note that for country 1, whatever the parameter values, the international migration policy of the country specific social planner is the opposite to the one implemented by the world social planner. The country specific social planner optimally chooses an immigration policy for his country 1 as long as the time preference is below  $\beta^1 = 0.75$ . Beyond this threshold, the chosen policy is a departure policy (see the blue line in Figure 1). Knowing that incentives for international migration are directed toward the high time preference country, migration are directed toward country 2. The migration policy implemented by the country-1 social planner goes against individuals' incentives to migrate. It is interesting to note that the world planner chooses a policy that aligns with the will of individuals. Indeed the world social planner optimally chooses an emigration policy for the country 1 when the country has a low time preference. However, for quite high time preference values, the world social planner choses the opposite policy (see green line in Figure 1).

Interestingly, when the time preference of the country 1 is exactly at the critical level  $\beta^1 = 0.75$ , both the country specific social planner and the world social planner optimally choose to stop all migration flows from and toward this country. No migration flow is needed to achieve the optimal welfare (see the intersection of the two green and bleu lines on the axis).

#### Country 2:

Concerning the country 2, note that the immigration policy is always optimal regardless of the value of  $\beta^1$ . However, when  $\beta^1$  value increases, the optimal migration flow chosen by the specific social planner  $m^{2*}$  decreases, (see orange line in Figure 1). This is no longer true when the world social planner decides all migration policies. In this case, he

implements an emigration policy for country 2 whatever the value of  $\beta^1$ . There are case where the optimal value of *m* is very small.

#### Empirical fact:

These calibrations aligns with the case where country 1 corresponds to China , and country 2 to France. The Chinese rate of interest r is already fixed at  $r \sim 5$  per annum for the 30-year Chinese Treasury Bond, (see the World Bank statistics) which implies ( $\beta = 0.142$ ). Chinese individuals have an incentive to migrate toward European countries endowed with a higher time preference, like France ( $\beta = 0.445$ ) for instance.

Let us now simulate the case of the different optimal migration flows from the USA to the EU, for  $\alpha = 1/3$ , see Figure 2. According to the US 30-year Treasury Bond, the rate of interest is  $r \sim 3, 2$  per annum which implies a time preference for country 1 of  $\beta^1 = 0.283$ . Figure 2 plots the optimal migration flows against the time preference  $\beta^2$ .

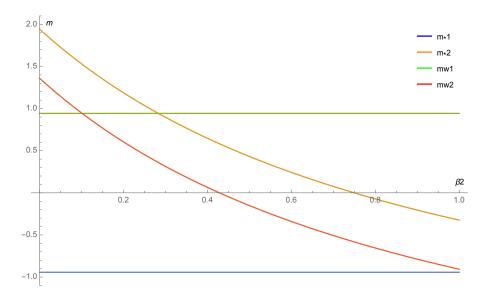


Figure 2: Different Optimal migration flows: Case 2

#### Country 1:

Note that the optimal migration levels of country 1 are independent of the value of the time preference rate of country 2. Consequently, the optimal migration flow has a zero-slope, (see blue and green lines in figure 2). The country specific social planner of country 1 always implements an immigration policy for his country, whereas the world social planner always implements an emigration policy. The social planner chooses a policy that goes against the individuals' incentives for international migration. Therefore, a world social planner implements a migration policy that is voted by country-1 individuals.

#### Country 2:

In country 2, as long as the time preference is relatively low, both the country specific social planner and the world social planner optimally choose an immigration policy. Moreover if  $\beta^2$  is around 0.25 - 0.4, the optimal immigration flow chosen by the country specific social planner is quite high, especially if the time preference of the departure country is low. Once  $\beta^2 = 0.75$ , the country specific social planner chooses an optimal zero migration flow to maximize the social welfare. Borders are closed in that country and incentives for illegal migration emerge endogenously. If  $\beta^2 > 0.75$  the direction of the

optimal migration policy implemented by the social planner is reversed. The world social planner chooses a migration policy that is characterized by the same shape, see Figure 2, but for the same corresponding values of  $\beta^2$  the migration flows differ.

If the time preference of country 2 is under 0.4, the optimal migration policy is an immigration policy. If  $\beta^2 \sim 0.425$ , the world social planner chooses a zero migration flow in country 2. Above this value, the migration policy is reversed. Whoever the country specific or the world social planner, when  $\beta^2$  increases, the optimal migration flows in the country 2 decrease. Consequently, the migration policy is consistent with individuals' incentives for international migration.

Empirical fact: All these scenarios match the international migration flows from Unites States ( $\beta = 0.283$ ) to the European Union states ( $\beta \sim 0.4$  or 0.5).

We now turn to an interesting extension in the direction of costly borders. How would the previous results change if we consider that the implementation of a border cost a fraction of the total production ?

## 7 EXTENSION: THE SOCIAL COST OF BORDERS

Section 7 presents an extension of the model in the case where the border is costly, Ethier (1986). Indeed, the analysis is deepened by the study of migration policies when the borders have a cost. Such cost can be motivated by the control cost of citizen at borders (in equilibrium it could be a cost for controlling citizen that exit a country or that enter this country), the cost of the administration (labor and capital), the cost of maintenance of all buildings or capital that are affected to the legal system of borders. In few words, this cost is supposed to be proportional to the production of each country,  $\delta^i m^i q^i$ , where  $\delta^i$  is a fraction belonging to [0, 1]. This fraction is country specific, i = 1, 2. Consequently, the new macroeconomic feasibility constraint become  $c^i + d^i + k^i = q^i - \delta^i m^i q^i$ .

#### 7.1 THE OPTIMAL PER CAPITA CAPITAL WITH SOCIAL COSTS OF BORDER

In each country, the social planner selects the optimal migration rate that maximizes the social welfare under his new country specific feasibility constraint by solving the following problem

$$\max_{k^i, d^i} \log c^i + \beta \log d^i$$

subject to

$$c^i + d^i + k^i = q^i (1 - \delta^i m^i)$$

Solving for consumption, we have  $c^i = q^i(1 - \delta^i m^i) - k^i - d^i$ . Maximizing the consumption is equivalent to maximize the right hand side of the previous relation. Knowing that  $q^i = (k^i)^{\alpha}$ , the first order condition gives the following relations

$$\alpha k^{i\alpha} (1 - \delta^i m^i) = k^i, \tag{22}$$

$$d^i = \beta c^i. \tag{23}$$

From what we deduce the new expression of the optimal per capita capital, when borders have a control cost

$$\tilde{k}^{i} = \left[\alpha(1-\delta^{i}m^{i})\right]^{\frac{1}{1-\alpha}}.$$
(24)

As long as  $m^i$  is positive (negative), the steady-state capital per worker decreases (increases).

#### 7.2 Optimal Migration Levels

In this subsection, the optimal migration rate of each country when borders have a cost is defined. There is no world social planner in this subsection. Consequently, the social planner of each country determines the optimal migration rate that allows his own country to reach the welfare maximizing per-capita capital level. For this purpose, each country specific social planner finds the optimal migration flows that leads the steady-state per capita capital to the welfare maximizing steady-state capital per worker. Solving the following equation  $\hat{k}^i(m^i) = \tilde{k}^i$  for  $m^i$ , i = 1, 2, allows us to determine the modified expression of the optimal migration level in the case of costly borders.

$$\tilde{m}_{i} = \frac{\alpha - (1 - \alpha) \frac{\beta^{1}}{1 + \beta^{1}}}{\alpha \delta^{1} - (1 - \alpha) \frac{\beta^{1}}{1 + \beta^{1}}},$$
(25)

$$\tilde{m}_{2} = \frac{\alpha - (1 - \alpha)\frac{\beta^{2}}{1 + \beta^{2}}}{\alpha \delta^{2} + (1 - \alpha)\frac{\beta^{1}}{1 + \beta^{1}}}.$$
(26)

We now turn to study the optimal migration policy in each country when the control of borders is costly.

#### 7.3 Optimal Migration Policies

In this subsection, the migration flows are illustrated for the capital elasticity of substitution  $\alpha = 0.3$ , the time preference of country 1, the country 1 30-year interest rate of about  $r \sim 0.028$  (Lebanon, Switzerland, Namibia) or  $r \sim 0.022$  (Hungry, Switzerland) which implies  $\beta^1 \sim 0.3$  or 0.4. For country 2 the 30-year interest rate is  $r \sim 0.017$  (Iceland, Malaysia) which implies a time preference of  $\beta^2 \sim 0.5$ . In all cases, the borders control cost is chosen so as  $\delta^i \in [0, 1]$ . Recall that the optimal migration policy is a function of  $\delta^i$ .

Figures 3 illustrates the country 1 optimal emigration / immigration policy in the case of a social cost of border.

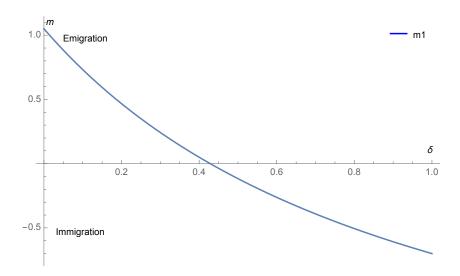


Figure 3: Optimal migration flow with costly borders in country 1

This figure shows that if the social cost of border is fairly low, the country specific social planner optimally implements an emigration policy for his country. For  $\delta^1 > 0.425$  the welfare improving migration policy is inverted and becomes an immigration policy.

Figures 4 illustrates the optimal immigration / emigration policy of the country 2 in the case of a social cost of border.

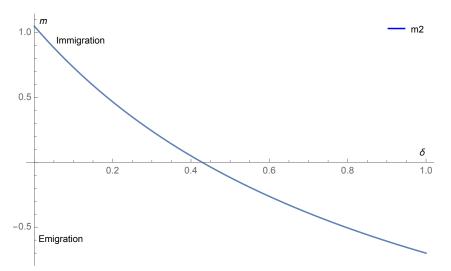


Figure 4: Optimal migration flow with costly borders in country 2

This figure exhibits the same shape as the previous one. Indeed, as long as  $\delta^2 < 0.425$ , the country specific social planner optimally chooses an immigration policy for his country 2. If  $\delta^2 > 0.425$  the welfare improving migration policy reverses and becomes an emigration policy.

To sum up, whether we are in a country or in another, as long as the share of the borders cost is under  $\delta = 0.425$ , the optimal migration policies implemented by both country specific social planners are consistent with individuals incentives for international migration, and are welfare improving. In equilibrium, there still exists a wage differential and an interest rate differential, as well as incentives for illegal migration for high values

of  $\delta^i$ . There are a lot of examples where building a wall between countries provides high incentive to illegally cross the wall.

## 8 CONCLUSION

In a 2-period overlapping generations model and two countries differentiated with respect to their time preferences, this paper provides an alternative modeling for international migration policies. Following Galor (1986), we compute the steady-state equilibrium of each country in autarky. Opening borders in steady-state equilibrium, we provide an alternative analysis of the international migration policies. We first study the welfare maximizing migration flows set up by each country social planner. It is shown that the country specific level of welfare maximizing migrants differs across countries. From what we define the two-sided borders concept. Second, we extend the analysis to the case where only one world social planner makes decision in order to maximize the global social welfare. It is shown that the global welfare maximizing migration policy differs from the one each country specific social planner would implement. Whoever leads a country or the two countries, the migration flows are constrained by legal decisions.

Since countries differ in their time preference, the 2 steady-state equilibria in autarky differ. After the opening of the borders, the economy converges to a post-migration steady-state equilibrium. Each social planner chooses a migration flow that leads the economy from its post-migration steady-state equilibrium to the post-migration Golden Rule. The optimal migration flows differ across countries. Accordingly, optimal legal systems for international migration and endogenous two-sided borders across countries emerge. In such a situation, a discrepancy between the legal system of migration implemented by the social planner and the individual's incentives for international migration arises.

The legal system of international migration implemented by a world social planner who maximizes the global social welfare is also studied. As a consequence, the legal system chosen by the world social planner for each country is different from the one chosen by the country specific social planner.

An extension of the model for costly borders is provided. The behavior of both countries specific social planners is modified and a new optimal migration levels are determined in order to maximize the social welfare. In this feature, the migration policies are more compatible with individuals incentives for international migration. In post-migration equilibrium, whoever leads a country or the two countries, there still exist a wage differential and a interest rate differential across countries. For high value of the time preference, incentive for illegal migration exist.

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## A APPENDIX

In order to keep both countries populated, the level of migrants must satisfy the following condition:  $0 < m^{i\star} < 1$ .

Conditions on the immigration optimal level  $m^{2\star}$ :

$$m^{2\star} \ge 0 \iff \left\{ \begin{array}{c} \beta^1 (1-\alpha)(1+\beta^2) \ge 0\\ (1+\beta^1) \left[ \alpha(1+\beta^2) - \beta^2(1-\alpha) \right] \ge 0 \end{array} \right.$$

Afetr simplification, the condition is:

$$\begin{cases} \beta^1 \ge \frac{1-\alpha(1+\alpha\beta^2)}{\alpha(1+\alpha\beta^2)-1} \\ \beta^1 \ge 0 \end{cases}$$
$$m^{2\star} < 1 \iff \beta^1(1-\alpha)(1+\beta^2) > (1+\beta^1) \left[\alpha(1+\beta^2) - \beta^2(1-\alpha)\right]$$

After simplifications, the condition is:

$$\beta^2 < \frac{\beta^1 (1-\alpha) - \alpha (1+\beta^1)}{(1+\beta^1)(2\alpha-1) - \beta^1 (1-\alpha)}.$$

Conditions on the emmigration optimal level  $m^{1\star}$ :

$$m^{1\star} > 0 \iff \frac{\alpha(1+\beta^1)}{\beta^1(1-\alpha)} < 1$$

Which means

$$\begin{split} \beta^1 &> \frac{\alpha}{1-2\alpha}. \\ m^{1\star} &< 1 \iff 1 - \frac{\alpha(1+\beta^1)}{\beta 1(1-\alpha)} < 1, \end{split}$$

always true.