Chapitre 3

Current Account Sustainability and Determination

3.1 Introduction

Alors que le premier chapitre présente les outils (issus de la comptabilité nationale) permettant de décrire les flux commerciaux et les flux de capitaux entre le pays domestique et le reste du monde, le **deuxième chapitre** présente les outils théoriques permettant de déterminer le solde courant à l'aide d'un modèle simple à deux périodes.

Nous débuterons le chapitre en déterminant la condition de solvabilité intertemporelle et en rappelant son principe. On se posera deux questions : i) est-ce possible d'enregistrer un déficit commercial récurrent ?, et ii) est-ce possible d'enregistrer un déficit courant de manière perpétuelle ? Il est possible d'enregistrer un déficit commercial récurrent à condition d'avoir une position extérieure nette positive. Il est possible d'enregistre un déficit courant tant que le pays enregistre un solde commercial positif permettant de rembourser une partie des intérêts de la dette extérieure.

Nous poursuivrons en développant un modèle à deux périodes d'une petite économie ouverte avec capital. En économie fermée, l'équilibre sur le marché des capitaux est garanti par le taux d'intérêt qui assure l'égalité entre l'épargne et l'investissement. En économie ouverte, le taux d'intérêt est fixe et tout décalage entre épargne et investissement est assuré par le solde courant dont la contrepartie est un flux de capitaux sortant ou entrant. En économie ouverte, les possibilités de consommation s'élargissent car le taux d'intérêt mondial est fixe : doubler l'épargne permet de doubler la consommation future et le prix de la consommation présente ne va pas augmenter lorsque l'individu diminue son épargne ce qui lui permet de consommer davantage. Dit autrement, le fait que le taux d'intérêt auquel il emprunte n'est plus croissant avec la consommation présente (car une hausse de la consommation présente diminue l'épargne et élève le taux d'intérêt) implique que les possibilités de consommation s'élargissent : l'accès au marché mondial des capitaux ouvre la possibilité d'emprunter à un taux fixe et non plus à un taux croissant. L'autre avantage de l'ouverture au marché des capitaux est que l'économie peut limiter les effets de choc négatif du PIB sur la consommation tout en maintenant constant l'investissement en enregistrant un déficit courant. Le rôle de la balance couante va donc être d'éviter une réduction brutale de la consommation face à un choc négatif et donc de lisser la consommation au cours du temps.

A côté du comportement de lissage intertemporel, il existe un autre comportement susceptible d'affecter la balance courante : c'est l'épargne de précaution. Lorsque le revenu est incertain et si l'individu est averse au risque, alors une baisse de revenu va provoquer une perte d'utilité qui est supérieure au gain d'utilité provoquée par une hausse du revenu du même montant. Le simple fait que le revenu soit incertain implique donc que l'espérance d'utilité (moyenne pondérée des utilités lorsque le revenu est bas et haut) est inférieure à l'utilité du revenu espéré. En d'autres termes, lorsque l'on a le choix entre un revenu certain et une loterie qui rapporte un revenu espéré équivalent au revenu certain, au chosira le revenu certain car la perte d'utilité dans la loterie l'emporte sur le gain d'utilité (ce qui traduit l'aversion pour le risque). Maintenant, au lieu de proposer à l'individu le choix entre un revenu certain et une loterie lui rapportant un revenu espéré égal au revenu certain, on lui demande le revenu certain qu'il serait prêt à accepter pour échapper à la loterie. Ce montant certain appelé équivalent certain qu'il accepterait pour échapper à cette loterie sera moins élevé que le revenu espéré en raison de son aversion au risque et la différence entre le revenu espéré et l'équivalent certain mesure la prime de risque. Finalement, comme l'individu est averse au risque, il va renoncer à consommer une partie de son revenu à la première et va donc constituer une épargne de précaution. Lorsque l'incertitude diminue ce qui est reflété par une réduction de la variance du revenu (le revenu devient moins volatile), l'individu va réduire son épargne de précaution et donc le solde courant va se détériorer. Ce résultat, comme nous le verrons, pourrait expliquer la dégradation du solde courant aux USA à partir du début des années 1980 qui coïncide avec la baisse de la volatilité du revenu.

Jusqu'à présent, le chapitre a permis de déterminer les conditions de solvabilité intertemporelle et de présenter le principe de lissage intertemporel et d'épargne de précaution : l'individu répartit une baisse du revenu courant sur plusieurs périodes par le jeu de l'endettement extérieur. Toutefois, ce modèle de détermination du solde courant n'est pas approprié pour analyser l'origine du déficit courant américain car les Etats-Unis sont d'une taille suffisamment élevée pour influencer le taux d'intérêt mondial. Il s'agira dans ce chapitre de développer un modèle simple d'une économie mondiale composée de deux pays ou de deux régions permettant d'analyser les causes du déficit extérieur des Etats-Unis. La première prédiction de ce modèle est qu'un pays qui connaît une croissance économique forte enregistrera un déficit courant entraîné par la baisse de l'épargne (et la hausse de l'investissement). Toutefois, les flux de capitaux entre les Etats-Unis et la Chine suggèrent l'inverse : ce sont les Etats-Unis qui sont emprunteur net et la Chine qui est prêteur net. Plusieurs explications ont été avancées dont celle de Caballero, Farhi et Gourinchas (2008). Les auteurs suggèrent que la qualité moindre du système financier des économies asiatiques et les dysfonctionnements qui sont apparus à la suite de la crise asiatique de 1997, ont réduit à la fois la demande de financement (car dans un pays où le système financier est moins développé, le financement externe est plus coûteux, l'allocation du capital est moins efficiente - des institutions de mauvaise qualité agissent comme un impôt) et l'offre de financement (car les frictions financières rendent l'économie plus intensive en travail ce qui élève mécaniquement l'épargne) ce qui a provoqué un excès d'épargne qui s'est reporté sur le marché des capitaux américains. La raison est qu'en baissant le taux d'intérêt mondial, l'afflux d'épargne a stimulé l'investissement et diminué l'épargne aux Etats-Unis. En d'autres termes, l'attrait pour les titres émis sur le marché des capitaux américains a pour origine la qualité moindre du système financier des pays asiatiques (aggravée par la crise asiatique qui a mis en lumière des dysfonctionnements de gouvernance, des relations entre la sphère réelle et bancaire, des financements de projets peu rentables, le manque de régulation du secteur bancaire). L'autre explication de l'excès d'épargne sur l'investissement dans les économiques asiatiques en forte croissance est liée à l'épargne 'forcée' des ménages (par le biais d'un secteur bancaire détenu par la sphère publique) permettant une accumulation de réserves internationales, ainsi qu'un contrôle des capitaux qui empêche l'entrée de capitaux étrangers.

Nous terminerons ce chapitre en évoquant le paradoxe de Feldstein et Horioka mis en évidence en 1980 selon lequel, l'ouverture au marché des capitaux n'aurait par affaibli la relation étroite entre l'épargne et l'investissement. Ce paradoxe a été ré-examiné notamment par Blanchard et Giavazzi (2002) pour les pays en rattrapage économique dans la zone euro comme le Portugal et la Grèce. Les deux auteurs développent un modèle simple d'une petite économie ouverte qui prédit que l'intégration économique (qui réduit la baisse de prix nécessaire pour exporter davantage et assurer le remboursement des intérêts de la dette externe), financière (qui diminue le taux d'intérêt en rendant les règles plus transparentes et en levant le contrôle des capitaux) et monétaire (en baissant la prime de risque de change) devraient favoriser l'émergence de déficits courants dans les pays en rattrapage. Les résultats des deux auteurs affaiblissent le paradoxe de Feldstein et Horioka en montrant que l'intégration économique et financière a accru la dispersion des balances courantes des membres de la zone euro et a considérablement affaibli la relation étroite entre épargne et investissement.

3.2 Current Account Sustainability

A natural question that arises from our description of the recent history of the U.S. external accounts is whether the observed trade balance and current account deficits are sustainable in the long run. In this chapter, we develop a simple framework to address this question.

3.2.1 Intertemporal Solvency Condition

In this subsection, we address the following question : can a country run a perpetual trade balance deficit ? The answer to this question depends on the sign of a country's initial net international investment position. A negative net international investment position means that the country as a whole is a debtor to the rest of the world. Thus, the country must generate trade balance surpluses either currently or at some point in the future in order to service its foreign debt. Conversely a positive net international investment position means that the country is a net creditor of the rest of the world. The country can therefore afford to run trade balance deficits forever and finance them with the interest revenue generated by its credit position with the rest of the world.

Let's analyze this idea more formally. Consider an economy that lasts for only two periods, period 1 and period 2. Let TB_1 denote the trade balance in period 1, CA_1 the current account balance in period 1, and B_1 the country's net international investment position (or net foreign asset position) at the end of period 1. Let r denote the interest rate paid on investments held for one period and B_0 denote the net foreign asset position at the end of period 0. Then, the country's net investment income in period 1 can be positive if $B_0 > 0$ or negative if $B_0 < 0$, putting aside the interest differential between assets and liabilities :

$$\text{Net investment income in period } 1 = r \, .B_0. \tag{3.1}$$

This expression says that net investment income (NII) in period 1 is equal to the return on net foreign assets held by the country's residents between periods 0 and 1. Keeping in mind that the net foreign asset position B is equal to the difference between assets A and liabilities L, the underlying assumption of (3.1) is that $r^A = r^L = r$.

In what follows, we ignore net international payments to employees and net unilateral transfers by assuming that they are always equal to zero. Then, the current account equals the sum of net investment income $r \, .B_0$ - which can be positive or negative - and the trade balance TB_1, that is,

$$CA_1 = r \, .B_0 + TB_1. \tag{3.2}$$

If the current account CA_1 is positive, the net foreign asset position improves because the home countries accumulate foreign assets, $B_1 - B_0 > 0$: in other words, the current account, in turn, represents the change in the net foreign asset position changes in period 1, that is,

$$CA_1 = B_1 - B_0. \tag{3.3}$$

Here we are abstracting from valuation changes.

Combining equations (3.2) and (3.3) to eliminate CA_1 allows us to define the net foreign asset position at the end of period 1 which is equal to the initial net foreign asset position plus the trade balance plus the net investment income :

$$B_1 = (1 + r) \times B_0 + TB_1. \tag{3.4}$$

A relation similar to this one must also hold in period 2. So we have at the end of periode 2 :

$$B_2 = (1 + r) \, B_1 + TB_2. \tag{3.5}$$

Combining the last two equations (3.4)-(3.5) to eliminate B_1, i.e., $B_1 = \frac{B_2 - TB_2}{1+r}$ we obtain an inverse relationship between the initial net foreign asset position and the present discounted value of trade balance :

$$(1 + r) \, B_0 = \frac{B_2}{(1 + r)} - TB_1 - \frac{TB_2}{(1 + r)}. \tag{3.6}$$

Now consider the possible values that the net foreign asset position at the end of period 2, B_2, can take :

– If B_2 is negative ($B_2 < 0$), it means that in period 2 the country is acquiring debt to be paid off in period 3. However, because the world ends in period 2, the country does not reimburse its debt. Thus, the rest of the world will not be willing to lend to the debtor country in period 2. This means that B_2 cannot be negative, or that B_2 must at least zero or positive $B_2 \geq 0$. This restriction is known as the no-Ponzi-game.

– Can B_2 be strictly positive ? The answer is no. A positive value of B_2 means that the country is lending to the rest of the world in period 2. But clearly the country will be unable to collect this debt in period 3 because, again, the world ends in period 2. Thus, the country will never choose to hold a positive net foreign asset position at the end of period 2, that is, it must be the case that $B_2 \leq 0$.

If B_2 can be neither positive nor negative, it must be equal to zero :

$$B_2 = 0. \tag{3.7}$$

This condition is known as the **transversality condition**. Using this expression (3.7), the budget constraint (3.6) can be rewritten as the **intertemporal solvency condition** :

$$(1 + r)\, B_0 = -TB_1 - \frac{TB_2}{(1 + r)}. \tag{3.8}$$

This equation states that the initial net foreign asset position must equal the present discounted value of its future trade deficits. Our claim that a negative initial net foreign wealth position implies that the country must generate trade balance surpluses, either currently or at some point in the future, can be easily verified using equation (3.8). Suppose that the country is initially a net debtor to the rest of the world ($B_0 < 0$). Clearly, if it never runs a trade balance surplus ($TB_1 < 0$ and $TB_2 < 0$), then the left-hand side of (3.8) is negative while the right-hand side is positive, so 3.7) would be violated. In this case, the country would be running a Ponzi scheme against the rest of the world.

3.2.2 Can a Country Run a Perpetual Current Account Deficit in Infinite Horizon ?

We can address the question by relating the initial net foreign asset position with the current account. Additionally, we study how these results change in a more realistic setting in which the economy lasts for an infinite number of periods.

The current account CA_t is equal to the trade balance TB_t plus the net investment income $r^\star B_t$ due to foreign bonds holdign (B_t = Assets − Liabilities) ; the NII is positive if the country is a net creditor, i.e., if assets are larger than liabilities. We assume that assets and liabilities bear the same interest rate, $r^\star$. The current account thus reads as :

$$CA_t = r^\star \,.B_{t-1} + TB_t \equiv B_t - B_{t-1}. \tag{3.9}$$

The net external asset position at time t denoted by B_t is equal to the period $t-1$ external asset position plus the current account. Plugging the BoP definition of the current account, i.e., $CA_t = r^\star B_{t-1} + TB_t$, the NIIP at time t is equal to the trade balance plus the period $t-1$ NIIP including NII :

$$B_t = CA_t + B_{t-1} = (1 + r^\star)\, B_{t-1} + TB_t. \tag{3.10}$$

To solve the first difference equation(3.10), we first evaluate (3.10) at time $t = 1$:

$$B_1 = (1 + r^\star)\, B_0 + TB_1, \quad B_0 = \frac{B_1}{1 + r^\star} - \frac{TB_1}{1 + r^\star}. \tag{3.11}$$

Because this equality holds at time $t = 2$, we thus have :

$$B_2 = (1 + r^\star)\, B_1 + TB_2, \quad B_1 = \frac{B_2}{1 + r^\star} - \frac{TB_2}{1 + r^\star}. \tag{3.12}$$

Substituting (3.11) into the expression of B_1 given by (3.12), one obtains (we solve the difference equation forward) :

$$B_0 = \frac{B_2}{(1+r^\star)^2} - \frac{TB_1}{1+r^\star} - \frac{TB_2}{(1+r^\star)^2}.$$

Iterating forward at time T, we get :

$$B_0 = \frac{B_T}{(1+r^\star)^T} - \frac{TB_1}{1+r^\star} - \frac{TB_2}{(1+r^\star)^2} - ... - \frac{TB_T}{(1+r^\star)^T}. \tag{3.13}$$

The general solution (3.13) of the first difference equation (3.10) can be rewritten in a more compact form :

$$B_0 = \frac{B_T}{(1+r^\star)^T} - \sum_{t=1}^{T} \frac{TB_t}{(1+r^\star)^t}. \tag{3.14}$$

In an infinite-horizon economy, the transversality condition (3.7) becomes

$$\lim_{T\to\infty} \frac{B_T}{(1+r)^T} = 0. \tag{3.15}$$

This expression says that the net foreign debt of a country must grow at a rate less than r. If the debt grows at a rate higher than the interest rate r, it means that you can finance the principal and the interest payments by rolling over the debt perpetually : hence, it is a scheme whereby the debt is never paid off. The no-Ponzi-game constraint precludes this type of situations. At the same time, the country will not want to have a net credit with a country which never reimburses the principal and the interest when the debt is growing at a rate r or higher : that would mean that the rest of the world forever rolls over its debt with the country in question. In brief, whether $B_t > 0$ or $B_t < 0$, the net external asset position must grow ($\frac{B_t}{B_{t-1}} = 1+g$) less rapidly than the principal plus interest rate which allows us to eliminate explosive trajectories. Note that assuming that the NIIP rises at a constant growth rate, using the fact that $B_T = B_0 \,.\, (1+g)^T$, the transversality condition can be rewritten as follows :

$$\lim_{T\to\infty} B_0 \,.\, \left(\frac{1+g}{1+r}\right)^T = 0.$$

This equation holds as long as $g < r$.

Imposing the transversality condition (3.184) in the general solution of the first difference equation (3.14), we obtain the intertemporal solvency condition for a small open economy (recall that $r^\star$ is exogenous) :

$$B_0 = -\sum_{t=1}^{T} \frac{TB_t}{(1+r^\star)^t}. \tag{3.16}$$

This equation says that if a country is initially a net debtor, it must run a trade balance surplus at some point for (3.16) to hold, otherwise the transversality condition (3.184) would not be satisfied.

We next revisit the question of whether a country can run perpetual current account deficits. We can write out the net foreign asset position at time t in terms of the net foreign asset position at time $t-1$ plus the trade balance :

$$B_t = (1+r^\star)\ .B_{t-1} + TB_t. \tag{3.17}$$

We set two assumptions :

– We first assume that the initial net foreign asset position of the country, B_0, is negative. That is, the country starts out as a net debtor to the rest of the world.
– We consider a situation where the country generates a trade balance surplus sufficient to pay a fraction $0 < \alpha < 1$ of its interest obligations. That is :

$$TB_t = -\alpha \times r^\star .B_{t-1}. \tag{3.18}$$

Note that according to this expression (3.18), whenever the country is a net debtor to the rest of the world, i.e., whenever $B_{t-1} < 0$, it generates a trade balance surplus.

Using the fact that $TB_t = -\alpha \times r^\star .B_{t-1}$ (see eq. (3.18)), the law of motion of the net foreign asset position (3.17) can be rewritten as follows :

$$\begin{aligned} B_t &= (1+r^\star) .B_{t-1} + TB_t, \\ &= (1+r^\star - \alpha .r^\star) .B_{t-1}. \end{aligned} \tag{3.19}$$

To answer the question, we proceed in five steps :
– We write out the next external asset position at time t ;
– we solve the first difference equation $B_t = (1+r^\star - \alpha .r^\star) .B_{t-1}$;
– we show that the external asset position is perpetually negative and that the country will run a recurring negative current account $CA_t < 0$;
– when expressing the solution in present value terms, we show that the transversality condition holds ;
– finally, we determine the trade balance adjustment which is consistent with the intertemporal solvency condition.

Writing out the net foreign asset position at $t-1$ leads to :

$$\begin{aligned} B_{t-1} &= (1+r^\star) B_{t-2} + TB_{t-1}, \\ &= (1+r^\star - r^\star\alpha) B_{t-2}, \end{aligned}$$

and iterating backward, eq. (3.19) can be rewritten as follows :

$$\begin{aligned} B_t &= (1+r^\star - \alpha r^\star) .(1+r^\star - \alpha r^\star) .B_{t-2}, \\ &= (1+r^\star - \alpha r^\star)^2 .B_{t-2}. \end{aligned} \tag{3.20}$$

Iterating backward until $t = \tau$, one obtains the general solution for the net foreign asset position :

$$\begin{aligned} B_t &= (1+r^\star - \alpha r^\star)^\tau .B_{t-\tau}, \\ &= (1+r^\star - \alpha r^\star)^t .B_0. \end{aligned} \tag{3.21}$$

where $(1+r^\star - \alpha r^\star) > 0$ and $B_0 < 0$; since the country is initially a net debtor and the trade surplus only allows the country to pay a fraction of interest payments on the net external debt, the country stays a net debtor, i.e. $B_t < 0$

Using the fact that $TB_t = -\alpha r^\star B_{t-1}$, the country also runs a current account deficit :

$$\begin{aligned} CA_t &= r^\star .B_{t-1} + TB_t = r^\star .(1-\alpha) B_{t-1} < 0, \\ &= r^\star .(1-\alpha) .(1+r^\star - \alpha r^\star)^{t-1} .B_0, \end{aligned} \tag{3.22}$$

où $B_{t-1} < 0$ puisque $B_0 < 0$, et $0 < (1-\alpha) < 1$.

To determine the net foreign asset position adjustment which is consistent with the transversality condition (which implies that the country satisfies its intertemporal solvency condition), we pre-multiply eq. (3.188b) by the discount factor $\frac{1}{(1+r^{\star})^t}$ which allows us to express the amount in terms of $t = 0$ units :

$$\frac{B_t}{(1+r^{\star})^t} = \left(\frac{1+r^{\star}-\alpha r^{\star}}{1+r}\right)^t .B_0. \tag{3.23}$$

For the transversality condition to hold, the present of the net foreign asset position must tend toward zero :

$$\lim_{t\to\infty} \frac{B_t}{(1+r^{\star})^t} = 0. \tag{3.24}$$

Apllying (3.24) to (3.23), we find the following condition

$$\begin{aligned} \lim_{t\to\infty} \left(\frac{1+r^{\star}-\alpha r^{\star}}{1+r}\right)^t B_0 = 0, \\ \text{if } \left(\frac{1+r^{\star}-\alpha r^{\star}}{1+r}\right) < 1, \end{aligned}$$

for the trajectory to be stable. The condition above can be rewritten as follows : $1 + r^{\star} > 1 + r^{\star} \ .(1-\alpha)$; it holds when $0 < \alpha < 1$; in this case, the country runs a trade balance surplus which amounts to a fraction α of interest payments ; even if α is low, the condition holds ; in other words, for the country to stay solvent, it must pay a share of interest payments over the oustading debt while rolling the debt to reimburse the debt and to pay a fraction $1-\alpha$ of interest rate payments that are due to creditors.

To be consistent with what we said when discussing the implications of the intertemporal solvency condition, let us calculate the growth rate of the net foreign asset position :

$$\begin{aligned} B_t - B_{t-1} &= \left(1+r^{\star}-\alpha r^{\star}\right) .B_{t-1} - B_{t-1}, \\ \frac{B_t - B_{t-1}}{B_{t-1}} &= r^{\star} .(1-\alpha) < r^{\star}. \end{aligned} \tag{3.25}$$

As long as the net external debt grows at a lower rate than $r^{\star}$, the transversality condition holds so that the country satisfies ther nation's intertemporal solvency condition.

The trade balance adjustment is obtained by combining $TB_t = -\alpha r^{\star} B_{t-1}$ with $B_{t-1} = \left(1+r^{\star}-\alpha r^{\star}\right)^{t-1} B_0$ given by eq. (3.21)

$$TB_t = -\alpha r^{\star} \left(1+r^{\star}-\alpha r^{\star}\right)^{t-1} .B_0 > 0. \tag{3.26}$$

Since $\left[1+r^{\star}(1-\alpha)\right] > 1$, the trade surplus must be larger as time passes. In brief, the country may borrow abroad an amount which increases over time while running a trade surplus which rises over time.

How does the country run a trade surplus which increases over time ? We must recall that GDP Y_t grows over time. De denote the growth rate by g with $Y_t = (1+g)\, Y_{t-1}$. Setting $r^{\star}(1-\alpha) = g$, eq. (3.26) can be rewritten as follows :

$$TB_t = -\alpha .r^{\star} .(1+g)^{t-1} .B_0 > 0. \tag{3.27}$$

the ratio TB_t/Y_t will be constant while the trade surplus grows at a constant rate $g_Y = r^{\star}(1-\alpha) > 0$ which allows the country to satisfy its solvency condition. To see it, divide both sides by Y_t and note that $Y_t = (1+g)^t \; .Y_0$, denoting the trade balance-to-GDP ratio $tb_t = TB_t/Y_t$ et and the debt-to-GDP ratio $b_0 = B_0/Y_0$ which allows us to :

$$\begin{aligned} tb_t &= -\alpha r^\star \left(1+r^\star-\alpha r^\star\right)^{t-1} \times \frac{Y_0}{Y_t} \times b_0, \\ &= -\alpha r^\star \left(1+r^\star-\alpha r^\star\right)^{t-1} \times \frac{1}{(1+g)^t} \times b_0, \\ &= -\frac{\alpha r^\star}{1+r^\star-\alpha r^\star} \left(1+r^\star-\alpha r^\star\right)^t \times \frac{1}{(1+g)^t} \times b_0, \\ &= -\frac{\alpha r^\star}{1+r^\star-\alpha r^\star} \left[\frac{1+r^\star(1-\alpha)}{1+g}\right]^t \times b_0. \end{aligned} \tag{3.28}$$

Setting $1+g = 1+r^\star(1-\alpha)$ or alternatively $g = r^\star(1-\alpha)$, the solvency condition is consistent with a constant trade surplus in % of GDP as long as the economy grows at a rate $r^\star(1-\alpha)$: as as the country pays a smaller fraction of interest payments, the growth rate must be larger in order to stabilize the debt-to-GDP ratio since the economy rolls over a growing external debt B_t and thus interest payments $r^\star \; .B_t$ increase faster.

3.2.3 Empirical Evidence : Lane and Milesi-Ferretti (2002)

We have shown that according to the intertemporal solvency condition, a positive steady-state net external asset position enables a country to run persistent trade deficits. In turn, all else equal, the capability to sustain a negative net export balance in equilibrium is associated with an appreciated real exchange rate. Conversely, a debtor country that must run trade surpluses to service its external liabilities may require a more depreciated real exchange rate.

Consider a semi-small open economy without physical capital which has an endowment of $Y(t)$ which is consumed domestically $C^H(t)$ and the rest is exported to the rest of the world. Exports correspond to the share of GDP which is not consumed : $EX(t) = Y(t) - C^H(t)$. Denoting by $P(t) \equiv \frac{P^H}{P^F}$ the terms of trade or the real exchange rate which is defined as the price of domestic goods in terms of foreign goods, and denoting by $C^F(t)$ imports, the trade balance expressed in terms of the foreign goods can be written as follows :

$$TB(t) \equiv P(t) \; .EX(t) - IM(t) = P(t) \; . \left(Y(t) - C^H(t)\right) - C^F(t). \tag{3.29}$$

When abstracting from physical capital, the stock of financial wealth of the economy denoted $A(t)$ consists only of $B(t)$. Holding a stock of foreign assets allows to receive interest receipts equal to $r^\star \; .B(t)$ where $r^\star$ is the world interest rate. In addition, the economy produces a quantity $Y(t)$. Hence, $r^\star \; .B(t) + P(t) \; .Y(t)$ corresponds to the real disposable income. Savings correspond to the share of the real disposable income $r^\star \; .B(t) + P(t)Y(t)$ which is not consumed $P(t) \; .C^H(t) + C^F(t)$. Savings are invested in the accumulation of foreign bonds which represent the assets available in the economy. Hence, savings represent the accumulation of foreign bonds over time, i.e., $B(t+dt) - B_t$ in discrete time. Letting the differential between period $t+dt$ and t tend toward zero, $dt \to 0$, by applying the derivative principle, we get :

$$\lim_{dt \to 0} = \frac{B(t+dt) - B(t)}{dt} = \dot{B}(t), \tag{3.30}$$

where we denote by a dot the time derivative of a variable, i.e., $\dot{B}(t) = \frac{dB}{dt}$. The budget constraint can be rewritten as the real

$$\dot{B}(t) = r^\star B(t) + P(t) \; .EX(t) - C^F(t). \tag{3.31}$$

This expression allows us to make the distinction between three types of variables : i) a state variable which is accumulated over time with an initial value which is predetermined, $B(0) = B_0$, ii) a control variable $C(t)$ which can jump at any instant of time and with an initial value not predetermined, iii) an exogenous variable which is given in the model. The term $Y(t) - C^H(t)$ corresponds to exports $EX(P(t))$ which depend negatively on the real exchange rate as an appreciation in the real exchange rate (a rise in $P(t)$) makes the home hoods more expensive. Because it also makes the foreign goods less expensive, a real exchange rate appreciation deteriorates the trade balance as long a we impose the Marshall-Lerner condition.

At each instant the representative household consumes domestic goods and foreign goods denoted by C^H and C^F, respectively, which are aggregated by means of a CES function :

$$C(t) = \left[\varphi^{\frac{1}{\phi}} \left(C^H(t)\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(C^F(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{3.32}$$

where φ is the weight of the domestic good in the overall consumption bundle ($0 < \varphi < 1$) and ϕ corresponds to the intratemporal elasticity of substitution between domestically produced and imported goods. For given C, agents minimize expenditure $P\,.C^H + C^F$ or alternatively maximize C for given $P\,.C^H + C^F$. The agents choose a basket (C^H, C^F) so that the MRS is equal to the relative price :

$$\frac{\varphi^{\frac{1}{\phi}} \left(C^H\right)^{\frac{-1}{\phi}} (C)^{\frac{1}{\phi}}}{(1-\varphi)^{\frac{1}{\phi}} \left(C^F(t)\right)^{\frac{-1}{\phi}} (C)^{\frac{1}{\phi}}} = \frac{P^H}{P^F}.$$

Using this equation, we find that

$$\frac{C^F}{C^H} = \left(\frac{1-\varphi}{\varphi}\right) .(P)^{\phi}, \tag{3.33}$$

where $P = P^H/P^F$. Plugging (3.196) into $P_C\,.C = P\,.C^H + C^F$ leads to optimal consumption in domestic and foreign goods :

$$C^H = \frac{\varphi\,.P_C C}{(P)^{\phi}\left[\varphi\,.(P)^{1-\phi} + (1-\varphi)\right]}, \tag{3.34a}$$

$$C^F = \frac{(1-\varphi)\,.P_C C}{\left[\varphi\,.(P)^{1-\phi} + (1-\varphi)\right]}. \tag{3.34b}$$

Using the fact that the consumption price index is a weighted average of the price of domestic goods, i.e., P^H, and the price of imported goods normalized to one, i.e., $P^F = 1$, and denoting by α_C the domestic content of consumption expenditure, a rise by 1% in the relative price of domestic goods P^H/P^F raises the consumption price index by α_C %. The consumption price index is given by (substitute (3.197) into (3.195) and set $C = 1$ so that $P_C C|_{C=1} = P_C$) :

$$P_C = \left[\varphi\,.(P)^{1-\phi} + (1-\varphi)\right]^{\frac{1}{1-\phi}}, \tag{3.35}$$

which allows us to reduce the expressions of consumption in domestically produced and imported goods :

$$C^H = \varphi\,.\left(\frac{P}{P_C}\right)^{-\phi} .C. \tag{3.36a}$$

$$C^F = (1-\varphi)\,.\left(\frac{1}{P_C}\right)^{-\phi} .C. \tag{3.36b}$$

The domestic content of consumption expenditure is given by :

$$\alpha_C = \frac{P\,.C^H}{P_C\,.C} = \varphi\,.\left(\frac{P}{P_C}\right)^{1-\phi}.$$

Denoting by β the discount rate, and $\sigma_C > 0$ the intertemporal elasticity of substitution, Households maximize the following objective function :

$$U = \int_0^{\infty} \left\{ \frac{C(t)^{1-\frac{1}{\sigma_C}}}{1-\frac{1}{\sigma_C}} \right\} \exp\left(-\beta t\right) \mathrm{d}t, \tag{3.37}$$

subject to the flow budget constraint

$$\dot{B}(t) = r^{\star} B(t) + P(t)\,.Y(t) - P_C\,.C(t). \tag{3.38}$$

The first order condition implies $C^{-\frac{1}{\sigma_C}} = \lambda\,.P_C$ where the consumption price index is given by (3.35). Differentiating $C^{-\frac{1}{\sigma_C}} = \lambda\,.P_C$ w.r.t P implies that the change of consumption in imported goods following a rise in the relative price by 1% is given by :

$$\begin{aligned} \hat{C}^F &= \phi\,.\hat{P}_C + \hat{C}, \\ &= \phi\,.\alpha_C\,.\hat{P} - \sigma_C\,.\alpha_C\,.\hat{P}, \\ &= \alpha_C\,.\left(\phi - \sigma_C\right)\,.\hat{P}. \end{aligned} \tag{3.39}$$

A rise in P raises the consumption price index P_C by α_C % and thus induces agents to cut consumption (second term on the RHS of (3.39) ; moreover, imported goods are less expensive which encourage agents to buy more foreign goods (first term on the RHS of (3.39).

We aim at evaluating the effect of a change in $P = P^H/P^F$ on $TB(t)$. We assume that exports are increasing with the P, i.e., $EX = P^{-\eta_X}$. Totally differentiating $P(t)\,.EX(t) - C^F(t)$ leads to the Marshall-Lerner condition :

$$\begin{aligned} \frac{\partial TB}{\partial P} &= EX + P\,.\frac{\partial EX}{\partial P} - \frac{C^F}{P}\,.\frac{\partial C^F}{\partial P}\,.\frac{\partial P}{\partial C^F} \\ &= EX\,.\left\{1 - \eta_X - \alpha_C\,.\left(\phi - \sigma_C\right)\right\}, \end{aligned} \tag{3.40}$$

where we substituted (3.39) to get the last line and used the fact that initially $C^F/P = EX$, i.e., imports are equal to exports. We set :

$$\Theta \equiv EX\,.\left\{\eta_X + \alpha_C\,.\left(\phi - \sigma_C\right) - 1\right\} > 0, \tag{3.41}$$

with $\nu_X \equiv \frac{\partial X}{\partial P}\frac{P}{X}$ is the price-elasticity of exports and $\alpha_C\,.\left(\phi - \sigma_C\right) = \frac{\partial C^F}{\partial P}\,.\frac{P}{C^F}$ is the price-elasticity of imports with ϕ the elasticity of substitution between C^F and C^H and σ_C the intertemporal elasticity of substitution for consumption. Plugging (3.41) into (**??**) yields :

$$\frac{\partial TB}{\partial P} = -\Theta. \tag{3.42}$$

To give a sense of the magnitude of the trade balance effect that a change in the terms of trade P might generate, we divide the left-hand side and the right-hand side terms by GDP $P\,.Y$:

$$\begin{aligned} \frac{dTB}{P\,.Y} &= -\frac{EX}{Y}\,.\left\{\eta_X + \alpha_C\,.\left(\phi - \sigma_C\right) - 1\right\}\,.\frac{dP}{P}, \\ &= -0.28\,.\left\{0.8 + 0.8\,.\left(1.5 - 0.5\right) - 1\right\}\,.1\%, \\ &= -0.28\,.\left\{1.6 - 1\right\}\,.1\%, \\ &= -0.168\,.1\%. \end{aligned} \tag{3.43}$$

An appreciation in the relative price of domestically produced goods by 1% reduces the trade balance by 0.17 percentage point of GDP when calibrating the model to French data.

At the steady-state, the stock of foreign bonds is constant and thus the accumulation of foreign bonds must cease. Hence, the net investment income $r^\star \tilde{B}$ must be the offset by the opposite of the trade balance :

$$\dot{B}(t) = 0, \Rightarrow \quad r^\star \tilde{B} = -\tilde{TB}. \tag{3.44}$$

Due to the Marshall-Lerner condition, the trade balance in the long-run is negatively related with the real exchange rate :

$$d\tilde{TB} = -\Theta \ .d\tilde{P}, \tag{3.45}$$

where $\Theta > 0$ is given by (3.41).

Eq. (3.44) just states that a country must run a steady-state trade surplus equal to the net investment income on its net foreign debt position. Eq. (3.45) says that, all else equal, the real exchange rate will be more depreciated, the bigger the steady-state trade surplus. Combining (3.44) with (3.45), we find that the real exchange rate is increasing in the net foreign asset position of the country :

$$d\tilde{P} = \frac{r^\star}{\Theta} \ .d\tilde{B}. \tag{3.46}$$

This is the type of equation typically estimated in the empirical literature on the long-run relation between net foreign assets and real exchange rates. However, this approach is potentially restrictive for two reasons. First, rates of return vary across countries, over time and between different categories of assets and liabilities. Second, in a nonzero growth environment, the intrinsic dynamics of the net foreign asset position depends on the output growth rate as well as rates of return.

Changes in the net foreign asset position are due to current account imbalances and to capital gains and losses. Assume initially that external assets and liabilities earn the same rate of return. In this case, the dynamics of net foreign assets can be written as the sum of the trade balance $TB(t)$ plus the receipts from trade bonds holding including interest receipts at the rate $i^\star$ plus capital gains $kg(t)$. Denoting the rate of return as the sum of the foreign interest rate plus capital gains, $R = i^\star + kg$, we have :

$$\Delta B_t = TB_t + (R-1) \ .B_{t-1}. \tag{3.47}$$

Because $\Delta B(t) = B(t) - B(t-1)$,

$$B_t = TB_t + R \ .B_{t-1}.$$

By dividing both sides by GDP and using the fact that $\frac{Y_t}{Y_{t-1}} = (1+g)$:

$$\frac{B_t}{Y_t} = \frac{TB_t}{Y_t} + \frac{R}{1+g} \ .\frac{B_{t-1}}{Y_{t-1}}. \tag{3.48}$$

Subtracting the ratio of foreign bonds to GDP at time $t-1$ b_{t-1} on both sides :

$$\Delta b_t = tb_t + \left(\frac{r-g}{1+g}\right) \ .b_{t-1} = tb_t + \Psi_t. \tag{3.49}$$

allows to obtain a negative relationship between the initial foreign asset position and the trade balance :

$$tb_t = -\Psi_t + \Delta b_t, \quad \Psi_t \equiv \left(\frac{r-g}{1+g}\right) .b_{t-1}, \tag{3.50}$$

where tb_t and b_{t-1} are the ratios of the trade balance and net foreign assets to GDP ; r is the nominal rate of return and g is the nominal GDP growth rate, both in US dollars. Eq. (3.51) gives the trade surplus in % of GDP which is required to stabilize the debt $-b_{t-1}$, taking into account the return on foreign assets and domestic liabilities held by foreigners, and given a rate of growth g which mechanically reduces the debt in % of GDP.

Lane and Milesi-Ferretti (2002) label Ψ_t the 'adjusted returns' variable : it determines the size of the trade imbalance - as a function of outstanding external wealth, investment returns, output growth - that is consistent with a unchanging ratio of net foreign assets to GDP. In the long run, we should observe an inverse relation between the net foreign asset position and the trade balance if the rate of return exceeds the growth rate : when the country is a net creditor, i.e., $b_{t-1} > 0$, the country can run a trade deficit.

Lane and Milesi-Ferretti (2002) examine the relation between the balance on goods and services tb_t, the net external position, its composition, and the 'adjusted returns' term Ψ_t. The sample spans the period 1970-1998 and includes 20 OECD countries. The data on the trade balance come from the IMF's Balance of Payments Statistics and refer to the balance of goods, services and transfers. The ratio of nominal investment returns to GDP is calculated as the sum of net investment income and net capital gains on outstanding external assets and liabilities measured in US dollars, divided by GDP in US dollars ($r_t \,.\frac{B_{t-1}}{Y_t}$). The 'adjusted returns' term is calculated as the difference between the ratio of nominal returns to GDP ($r_t \,.\frac{B_{t-1}}{Y_t}$) and the impact of GDP growth on the ratio of outstanding net foreign assets to GDP ($g_t \,.\frac{B_{t-1}}{Y_t}$).

Lane and Milesi-Ferretti (2002) examine the cross-sectional dimension in Table 3.1. The dependent variable is the trade balance averaged over the period 1974-1998 (columns (1), (3), (5) where the NIIP is obtained by calculating adjusted cumulative current accounts and 1983-1998 (columns (2), (4) and (6) where the NIIP is obtained by using the NIIP provided by the IMF :

$$\frac{1}{T}\sum_{t=1983}^{1998} tb_t = \beta \,.\frac{1}{T}\sum_{t=1983}^{1998} \Psi_t + \gamma \,.b_{1982}$$

From columns (1) and (2) of Table 3.1, it is clear that there is no cross-sectional relation between the initial net foreign asset position and the subsequent average trade balance. However, the relation between the average trade balance and the adjusted returns variable is close to one-to-one : countries with positive adjusted returns run trade deficits, while countries with negative adjusted returns run trade surpluses.

The cross-section relation between the average adjusted returns and the average trade balance is also illustrated graphically in Figure 3.2. Over the 1974-1998 period, countries that enjoyed positive adjusted returns (such as the US, UK and Greece) ran average trade deficits ; conversely, those countries that on net were paying out adjusted returns (such as the Netherlands, Canada, Denmark and Finland) ran average trade surpluses. Figure 3.3 shows instead the relation (or lack thereof) between the average trade balance over the period 1983-1998 and the stock of net foreign assets at the end of 1982 : differences in rates of return and growth rates means that the cross-section relation between net foreign assets and the trade balance is weaker than the relation between adjusted returns and the trade balance.

When breaking down the adjusted returns element into its underlying components in columns (4) and (5) of Table 3.1, we find that both the real return and the growth component are highly significant and have a coefficient that is statistically not different from minus one.

Table 3.4 gives the trade balance and provides a decomposition of the adjusted return term for different countries. Column (1) gives the average NIIP over the period 1983-1998, column (2) the initial NIIP (1983), column (3) the average trade balance (1983-1998), column (4) the adjusted returns term Ψ_t. Columns (5)-(9) provides information about the elements composing the adjusted returns term, Ψ_t. Column (5) gives the real returns as a ratio of GDP which are calculated as follows :

$$\begin{aligned} \frac{r_t \,. B_{t-1}}{Y_t} &= r_t^A \,. \frac{A_{t-1}}{Y_t} - r_t^L \,. \frac{L_{t-1}}{Y_t}, \\ &= \frac{r_t^A}{1+g_t} \,. a_{t-1} - \frac{r_t^L}{1+g} \,. l_{t-1}, \\ &= \frac{r_t^A}{1+g_t} \,. b_{t-1} + \frac{\left(r_t^A - r_t^L\right)}{1+g} \,. l_{t-1}, \end{aligned} \tag{3.51}$$

where A_{t-1} is the stock of foreign assets owned by the home country while L_{t-1} is the stock of liabilities. One of the most striking stylized facts emerging from the table is the high median real rate of return on external assets and liabilities (nominal dollar returns minus US inflation) for most countries (mean and median are above 6 percent). The main factor behind this result is the impact of capital gains on FDI and equity holdings - indeed, the mean and median values of real yields (not reported) are around 4 percent. Columns (6) and (7) give r^A and r^L, respectively.

Debtor countries such as Canada, Finland, New Zealand, Spain and Sweden have negative average adjusted returns (column (4)) and negative average net foreign assets (column (1)), suggesting a positive $r_t - g_t$. Conversely, the adjusted returns term is positive for debtor countries such as Greece, Portugal and the US and negative for creditor countries such as Germany and the Netherlands, suggesting that the term $\frac{r_t \,. B_{t-1}}{Y_t} - \frac{g_t \,. B_{t-1}}{Y_t}$ is negative because $r^L > r^A$ (see columns (6) and (7)) : what is at work here is a rate of return differential between external assets and liabilities (positive for the US, negative for Japan and the Netherlands) which makes the average ratio of real returns to GDP (and the adjusted returns term) positive for a debtor country like the US and negative for creditor countries such as Germany, Japan and the Netherlands. But what factors account for the high measured rates of return on US assets and liabilities ? Capital gains on FDI and equity holdings are the main factor.

Having established a link between the net foreign asset position and the trade balance, the objective of the second part of our empirical exercise is to capture the long-run relation between the trade balance and the real exchange rate.

Suppose the price level, P, is constructed as some average of the price of tradables and nontradables. We can then write

$$P = \phi\left(P_T, P_N\right). \tag{3.52}$$

where ϕ is increasing in P_T and P_N and homogeneous of degree one. For instance, if P is a geometric average of P_T and P_N, then $\phi\left(P_T, P_N\right) = \left(P_T\right)^{1-\alpha_N} \,. \left(P_N\right)^{\alpha_N}$. The assumption that $\phi(.,.)$ is homogeneous of degree one ensures that, if all individual prices increase by, say, 5%, then P also increases by 5%. Assume that the price level in the foreign country is also constructed as some average of the prices of tradables and nontradables, that is

$$P^{\star} = \phi\left(P_T^{\star}, P_N^{\star}\right). \tag{3.53}$$

We can then write the real exchange rate, E, as

$$\begin{aligned} \text{RER} &= \frac{P}{E\,.P^{\star}}, \\ &= \frac{\phi\left(P_T, P_N\right)}{E\,.\phi\left(P_T^{\star}, P_N^{\star}\right)}, \\ &= \frac{P_T\,.\phi\left(1, \frac{P_N}{P_T}\right)}{E\,.P_T^{\star}\,.\phi\left(1, \frac{P_N^{\star}}{P_T^{\star}}\right)}, \\ &= TOT\,.\frac{\phi\left(1, \frac{P_N}{P_T}\right)}{\phi\left(1, \frac{P_N^{\star}}{P_T^{\star}}\right)}, \end{aligned} \tag{3.54}$$

where $TOT = \frac{P^T}{E\,.P^{T,\star}} = \frac{P^H}{P^F} = P$ with $P^F = E\,.P^{T,\star}$. Totally differentiating (3.54) yields :

$$\ln \text{RER} = \ln \text{TOT} + \alpha_N\,.\left(p^N - p^{N,\star}\right), \tag{3.55}$$

where $p^N = \ln(P^N/P^T)$. According to (**??**), there exists an inverse relationship between TB and TOT :

$$d\frac{TB}{\text{TOT}\,.Y} = -\Theta\,.d\ln \text{TOT} \tag{3.56}$$

Initially, the terms of trade are assumed to be one and net exports are nil so that :

$$d\ln \text{TOT} \simeq \ln TOT, \quad d\frac{TB}{P\,.Y} \simeq \frac{TB}{Y} = tb. \tag{3.57}$$

Hence, the equation (3.55) that the authors explore empirically can be rewritten as follows :

$$\ln \text{RER} = -\frac{Y}{\Theta}\,.tb + \alpha_N\,.\left(p^N - p^{N,\star}\right). \tag{3.58}$$

The results for the real exchange rate equation are shown in Table 3.5. In columns (1) and (2), the full panel is employed. The sample is then split between the non-G3 and G3 countries in columns (3) and (4) and (5) and (6), respectively. It is natural to expect a difference in the sensitivity of the real exchange rate to various fundamentals between large and small countries : for instance, the relative size of the nontraded sector typically varies directly with the size of the country. In all cases, country fixed effects are employed : these are necessary since the real exchange rate data are index measures and therefore not comparable across countries. Results are reported both with and without time fixed effects. The trade balance enters significantly in all specifications in Table 3.5. Taking the specification that includes time dummies, the trade balance coefficient for the full panel is -0.72. However, the split between the non-G3 and G3 sub-samples reveals a large difference in magnitude. For the non-G3 countries, a 3 percentage point increase in the trade surplus as a ratio to GDP is associated with only a 1 percent real depreciation. For the G3 countries, by contrast, the same improvement in the trade balance is associated with a 19.3 percent real depreciation. A similar story applies for the role played by relative output per capita : in all specifications, its relation with the real exchange rate is significantly positive but the point coefficient is

Trade balance, net foreign assets and adjusted returns: cross-sectional regressions, 1974–1998 and 1983–1998

	(1) Trade balance 1974–1998	(2) Trade balance 1983–1998	(3) Trade balance 1974–1998	(4) Trade balance 1983–1998	(5) Trade balance 1974–1998	(6) Trade balance 1983–1998
Initial net foreign assets (ratio of GDP)	0.008 (0.40)	0.002 (0.15)	0.017 (1.16)	0.006 (0.52)		
Adjusted returns/GDP			−0.940 (4.05)***	−0.677 (3.16)***		
Real returns/GDP					−0.939 (4.46)***	−0.678 (3.22)***
Growth term/GDP					−1.790 (3.95)***	−0.920 (2.60)**
Observations	20	20	20	20	20	20
Adjusted R^2	−0.05	−0.05	0.44	0.32	0.53	0.30
F-test Adjusted returns = −1 (p-val in par.)			0.07 (0.80)	2.28 (0.15)		
Joint F test compon. adj. returns = −1 (p-val in par.)					2.58 (0.11)	2.34 (0.14)

Note: The trade balance, adjusted returns, returnsand growth effects are averages over the periods, 1974–1998 (columns (1), (3), (5)) and 1983–1998 (columns (2), (4), (6)). Net foreign assetsare the outstanding stocks at the beginning of each period. Forthe 1974–1998 regressions, we use the CUMCA measure of net foreign assets. For the period 1983–1998 we use International Investment Position data for the following countries: Austria, Canada, Finland, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, United States. t-statistics in parentheses.
***,**,* denote significance at the 1, 5 and 10 percent levels, respectively.

FIG. 3.1 – Trade balance, net foreign assets and adjusted returns : cross-sectional regressions, 1974-1998 and 1983-1998. Source : Lane and Milesi-Ferretti (2002) External wealth, the trade balance, and the real exchange rate. *European Economic Review*, 46(6), pp. 1049-1071.

ten times larger for the G3 than for the non-G3 countries - a 10 percent increase in relative output per capita is associated with less than a 2 percent real appreciation for the non-G3 countries but a 19 percent real appreciation for the G3 countries.

Overall, the results in Table 3.5 provide broad support for an inverse long-run relation between the trade balance and the real exchange rate, holding fixed relative output per capita and the terms of trade. As is illustrated in Figure 3.6, a negative relation between country size and the magnitude of the trade balance coefficient is clearly evident in the data (the correlation is -0.46). The explanation is that the depreciation in the real exchange rate that is associated with a given improvement in the trade balance is directly related to the relative size of the nontraded sector in the economy. To see it more formally, divide both sides of (3.199) by Y_t :

$$dtb = -ex\ .\left\{\eta_X + \alpha_C\ .\left(\phi - \sigma_C\right) - 1\right\}\ .d\ln P, \tag{3.59}$$

where $tb = TB/Y$, $ex = EX/Y$ and $d\ln P = dP/P$. The term ex is smaller for large countries which have a lower trade openness. As a result, a given fall in P leads to a lower trade balance surplus or alternatively a given trade surplus requires a greater depreciation in countries of larger size which explains the reason why G3 countries have a higher elasticity of the RER to the trade balance.

FIG. 3.2 – Trade balance and adjusted returns. Source : Lane and Milesi-Ferretti (2002) External wealth, the trade balance, and the real exchange rate. *European Economic Review*, 46(6), pp. 1049-1071.

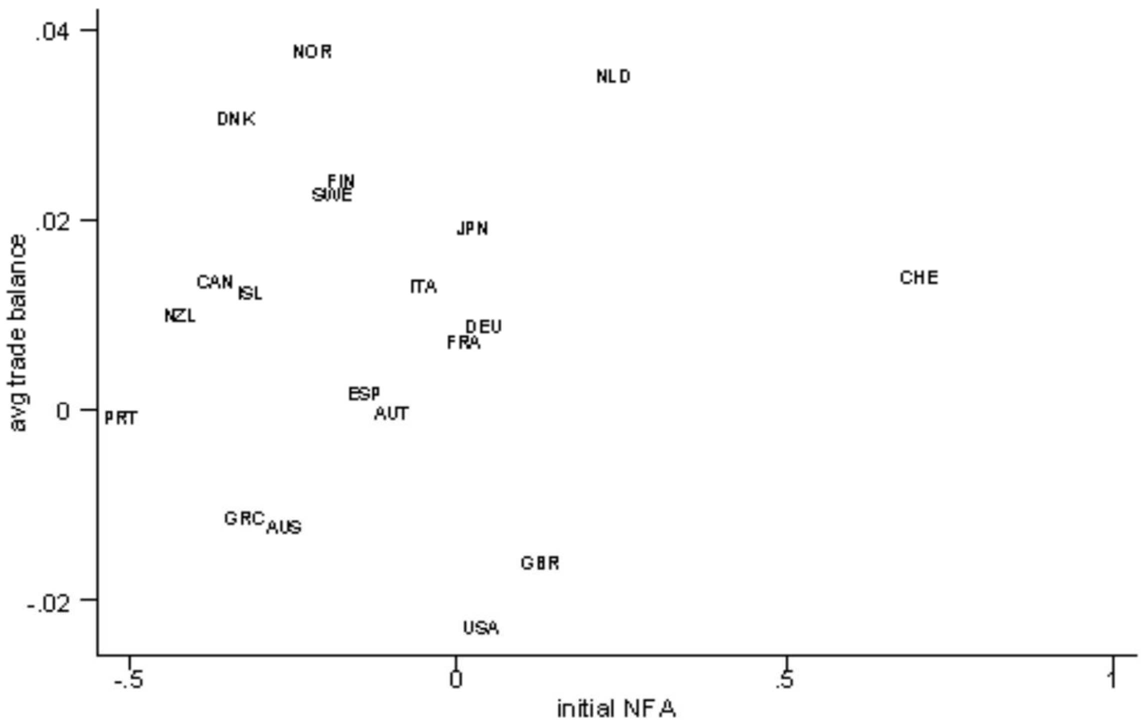


Fig. 2. Initial net foreign assets and average trade balance (1983–1998).

FIG. 3.3 – Initial net foreign assets and average trade balance (1983-1998). Source : Lane and Milesi-Ferretti (2002) External wealth, the trade balance, and the real exchange rate. *European Economic Review*, 46(6), pp. 1049-1071.

Data summary, 1983–1998

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Country	Avg NFA (ratio of GDP)	NFA in 1983 (ratio of GDP)	Trade balance (ratio of GDP)	Adj. returns (ratio of GDP)	Real returns (ratio of GDP)	Median real rate of return (Assets)	Median real rate of return (Liabilities)	Avg change in RER	Avg real growth rate
United States	−5.0	3.6	−2.4	1.0	0.9	11.3	8.1	0.0	2.2
United Kingdom	6.0	12.7	−1.7	−0.5	−0.1	6.6	7.1	1.3	2.2
Austria	−11.5	−10.1	−0.1	−0.5	−0.9	6.5	8.3	1.9	2.0
Denmark	−22.6	−34.0	3.0	−1.4	−2.6			2.0	2.2
France	−0.1	1.0	0.6	−0.6	−0.6	8.5	8.9	1.2	1.5
Germany	10.9	3.8	0.8	−1.0	−0.2	2.9	4.3	1.7	1.0
Italy	−7.0	−5.2	1.2	−0.5	−0.7	8.3	9.6	2.0	1.8
Netherlands	26.1	23.7	3.5	−2.9	−2.3	5.9	7.3	0.7	2.2
Norway	−3.2	−22.1	3.7	−1.0	−1.2			−0.6	2.7
Sweden	−30.0	−19.1	2.2	−2.8	−3.1	11.7	11.0	0.6	1.3
Switzerland	97.2	70.2	1.3	−0.1	3.7	6.9	8.7	1.7	0.8
Canada	−38.9	−36.8	1.3	−1.8	−2.2	3.6	4.2	−1.4	1.5
Japan	13.2	2.4	1.8	−1.9	−1.7	7.0	9.6	2.3	2.3
Finland	−34.6	−17.8	2.3	−5.4	−5.7	0.4	7.2	0.6	1.9
Greece	−42.2	−32.2	−1.2	0.5	−1.0			1.2	1.4
Iceland	−34.4	−31.6	1.2	−1.4	−2.3	6.0	3.3	0.7	1.4
Portugal	−30.4	−51.4	−0.1	1.0	−0.6			3.1	2.9
Spain	−14.9	−14.1	0.1	−0.7	−0.9	6.1	6.9	1.5	2.5
Australia	−45.3	−26.3	−1.3	−1.0	−1.6	6.1	3.5	−1.8	2.3
New Zealand	−64.0	−42.4	0.9	−3.1	−4.5			0.2	0.7

Note: The NFA data is the International Investment Position (IIP) data for the following countries: Austria, Canada, Finland, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, United States. For the remaining countries, we use our own estimate of NFA. The median real rates of return can only be calculated for those countries for which IIP data are available. Among those, we excluded those countries for which IIP data are available for less than 10 years.

Fig. 3.4 – Data summary (1983-1998). Source : Lane and Milesi-Ferretti (2002) External wealth, the trade balance, and the real exchange rate. *European Economic Review*, 46(6), pp. 1049-1071.

	(1) Full sample	(2) Full sample	(3) Non-G3	(4) Non-G3	(5) G3	(6) G3
tb	−0.51	−0.72	−0.38	−0.33	−5.64	−6.44
	(2.83)***	(3.33)***	(2.28)**	(1.64)*	(5.15)***	(3.79)***
yd	0.4	0.39	0.18	0.19	1.61	1.89
	(3.8)***	(5.66)***	(2.02)**	(2.83)**	(5.82)***	(5.32)***
tt	0.44	0.55	0.48	0.52	0.19	0.05
	(7.5)***	(11.68)***	(10.46)***	(10.95)***	(1.33)	(.25)
Adjusted R^2	0.51	0.55	0.54	0.57	0.65	0.62
Number of observ.	519	519	442	442	77	77
Number of countries	21	21	18	18	3	3
Time dummies?	No	Yes	No	Yes	No	Yes

Note: The sample comprises all countries in columns (1) and (2); Germany, Japan, and United States (G3) are excluded from the regressions in columns (3) and (4); the sample comprises Germany, Japan and United States only in columns (5) and (6). Estimation is by DOLS; *t*-statistics in parentheses.
***,**,* denote significance at the 1, 5 and 10 percent levels, respectively.

FIG. 3.5 – Real exchange rate equation : panel results. Note : The sample comprises all countries in columns (1) and (2) ; Germany, Japan, and United States (G3) are excluded from the regressions in columns (3) and (4) ; the sample comprises Germany, Japan and United States only in columns (5) and (6). Estimation is by DOLS ; t-statistics in parentheses. $\star\star\star$, $\star\star$, $\star$ denote significance at the 1, 5 and 10 percent levels, respectively. Source : Lane and Milesi-Ferretti (2002) External wealth, the trade balance, and the real exchange rate. *European Economic Review*, 46(6), pp. 1049-1071.

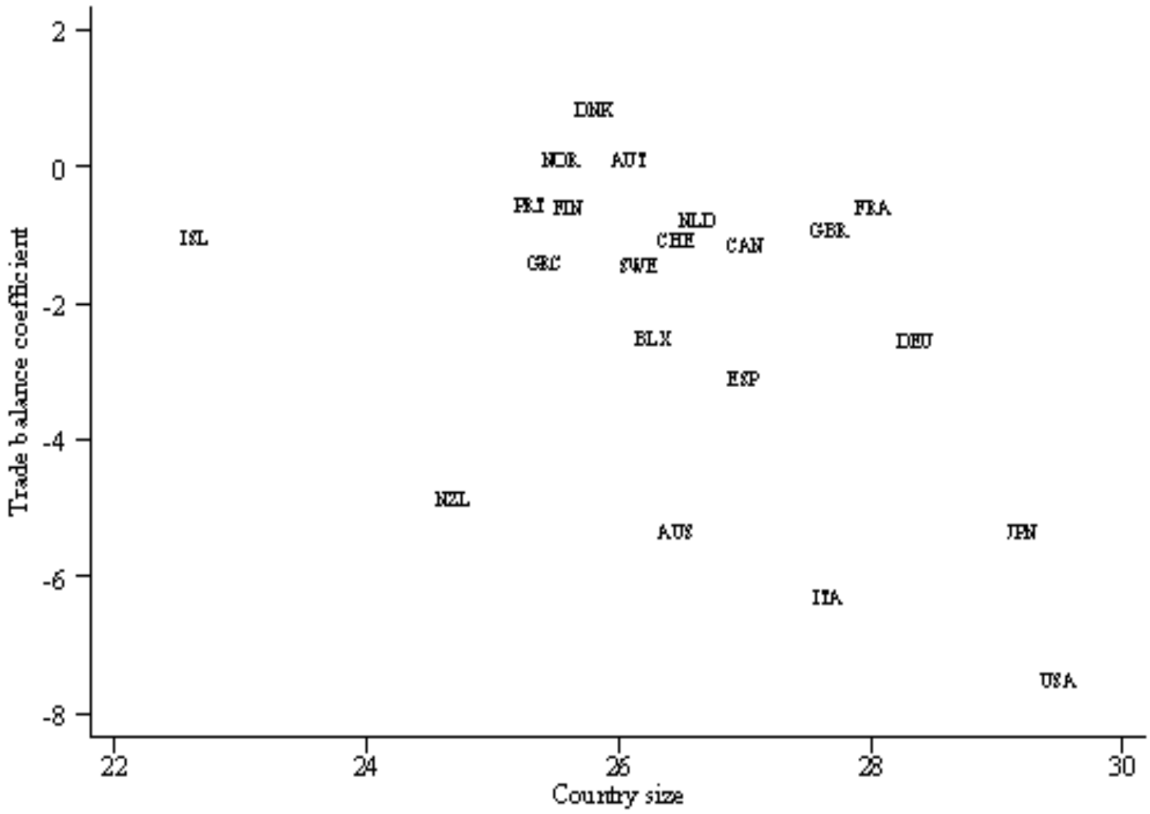


FIG. 3.6 – The trade balance and the real exchange rate : country size. Country size is the log of GDP in 1990, measured in constant US dollars. The trade balance coefficient is the coefficient on the trade balance in country-by-country regressions of the real exchange rate on the trade balance, terms of trade and relative GDP per capita. Source : Lane and Milesi-Ferretti (2002) External wealth, the trade balance, and the real exchange rate. *European Economic Review*, 46(6), pp. 1049-1071.

3.3 Current Account Determination in a Two-Period Production Economy

In this section, we study the determination of the current account in an economy with investment in physical capital. In this economy, output is not given exogenously, but is instead produced by firms. In the closed economy, access to international financial markets is precluded. At the same time, the increase in the interest rate has a negative effect on investment in physical capital. If period 1 output is low, the country must sacrifice consumption while a high interest rate reduces investment opportunities. In the open economy, households will smooth consumption by borrowing in the international capital market at a constant interest rate, thus running a current account deficit in period 1 while raising investment compared with the closed economy.

3.3.1 Firms

Consider an economy in which output is produced with physical capital. Specifically, let K_i denote the capital stock at the beginning of period i, and assume that output is an increasing function of capital with decreasing returns to scale

$$Y_i = F\left(K_i\right), \quad F' > 0, F'' < 0. \tag{3.60}$$

The marginal product of capital is the amount by which output increases when the capital stock is increased by one unit and is given by the derivative of the production function with respect to capital :

$$\frac{\Delta Y_i}{\Delta K_i} = F' > 0, \tag{3.61}$$

Finally, we assume that the marginal product of capital is decreasing in K, that is, $F'' < 0$, which implies that the production function is concave.

Figure 3.7(a) displays output as a function of the capital stock. The marginal product of capital at $K^\star$, $F'\left(K^\star\right)$, is given by the slope of $F(K)$ at $K = K^\star$. Figure 3.7(a) displays the marginal product of capital as a function of K.

Output is produced by firms. Because we abstract from labor supply and the capital K_1 is predetermined at period 1, the production is exogenous in period 1, and thus so is the profit in period 1 :

$$\Pi_1 = F\left(K_1\right) - \left(r_1 + \delta\right)\ .K_1. \tag{3.62}$$

To produce in period 2 firms must borrow capital in period 1 at the interest rate r_1. Physical capital depreciates at the rate δ between periods 1 and 2. Therefore, the total cost of borrowing one unit of capital in period 1 is $r_2 + \delta$. Profits in period 2, Π_2, are then given by the difference between output and the rental cost of capital, that is

$$\Pi_2 = F\left(K_2\right) - \left(r_2 + \delta\right)\ .K_2. \tag{3.63}$$

Firms choose K_2 so as to maximize profits, taking as given the interest rate r_2. Figure 3.8 displays the level of capital that maximizes profits. For values of K below K_2, the marginal product of capital exceeds the rental cost $r_2 + \delta$, thus, the firm can increase profits by

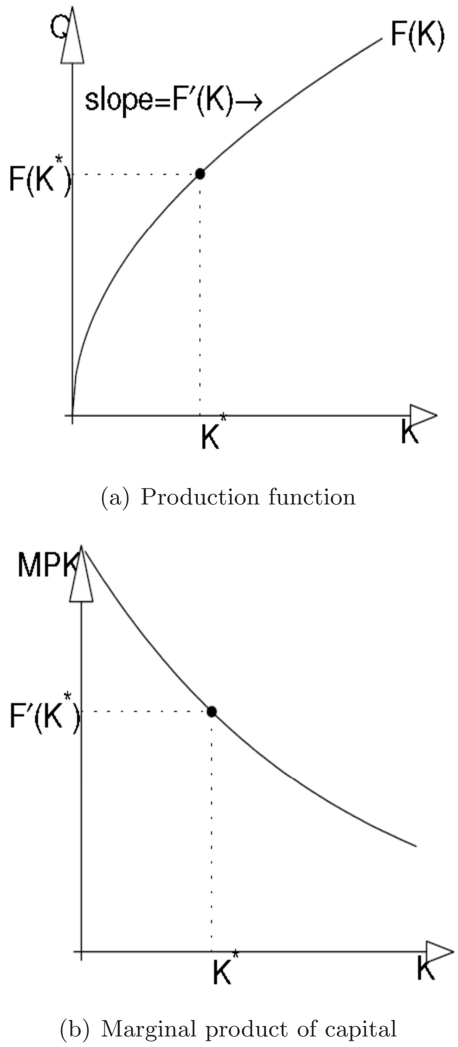


Fig. 3.7 – The production function and the marginal product of capital - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

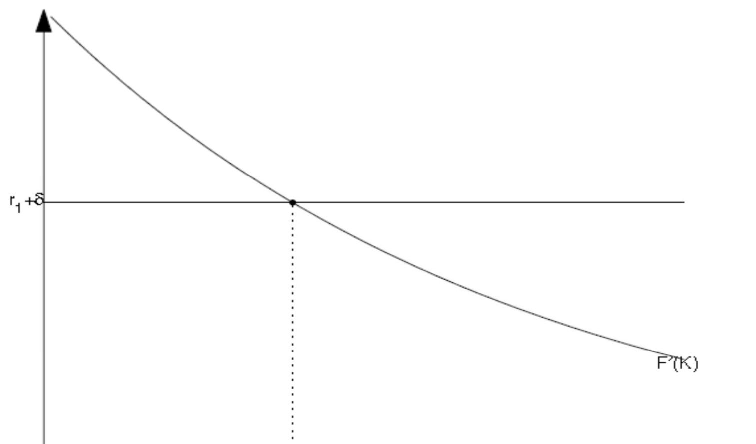


Fig. 3.8 – Optimal capital decision, $K^{\star}$ - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

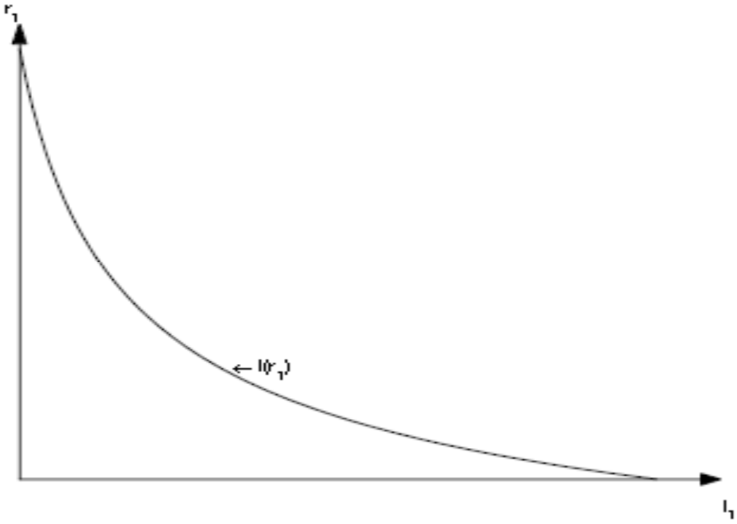


FIG. 3.9 – Investment as a decreasing function of the capital capital - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

renting an additional unit of capital. Because the profit function is hump-shaped, the profit is maximum when the slope of the tangent of the profit function is horizontal. At this point, the optimal level of capital, is the one at which the marginal product of capital equals the rental cost of capital, that is :

$$F'(K_2) = r_2 + \delta. \tag{3.64}$$

In order to bring the capital stock from K_1 to K_2, i.e., to bring the initial predetermined capital stock to its level consistent with the equality (3.64), firms must invest an amount I_1 by taking into account that a share δ of existing capital will be lost due to depreciation and thus must be replaced :

$$I_1 = K_2 - (1 - \delta) \, .K_1. \tag{3.65}$$

Because the optimal capital stock K_2 falls as the interest rate r_2 rises, investment is a decreasing function of the interest rate as illustrated in Figure 3.9 :

$$I_1 = I(r_2), \quad I_r < 0. \tag{3.66}$$

3.3.2 Households

Consider now the behavior of households. At the beginning of period 1, the household is endowed with A_0 units of interest bearing wealth. The rate of return on wealth is given by r_1. Thus, interest income is given by $r_1 \, .A_0$. In addition, the household is the owner of the firm and thus receives the firm's profits, Π_1. Therefore, total household income in period 1 equals $r_1 \, .A_0 + \Pi_1$. As in the endowment economy, the household uses its income for consumption and financial wealth accumulation. The budget constraints of the household in periods 1 and 2 are then given by

$$A_1 = (1 + r_1) \, .A_0 + \Pi_1 - C_1, \tag{3.67a}$$
$$A_2 = (1 + r_2) \, .A_1 + \Pi_2 - C_2 = 0, \tag{3.67b}$$

where we impose the transversality condition $A_2 = 0$; because period 2 is the last period of life, the household will not want to hold any positive amount of assets maturing after that

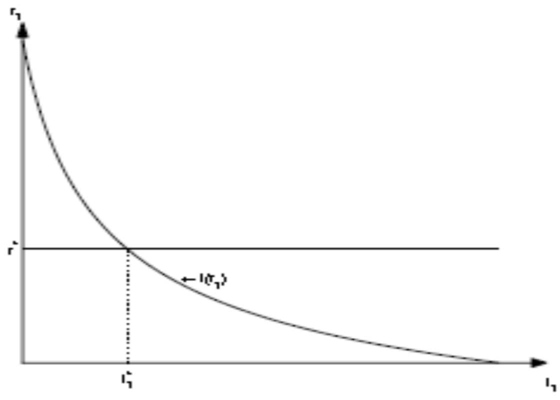


FIG. 3.10 – Investment Function - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

period, so that $A_2 \leq 0$; at the same time, the household is not allowed to end period 2 with unpaid debts (the no-Ponzi-game condition), so that $A_2 \geq 0$. By combining the two budget constraints (3.67a) et (3.67b), we obtain the intertemporal budget constraint :

$$C_1 + \frac{C_2}{1+r_1} = (1+r_2) \ .A_0 + \Pi_1 + \frac{\Pi_2}{1+r_2} \equiv \Omega. \tag{3.68}$$

The household chooses C_1 and C_2 so as to maximize the utility function subject to the intertemporal budget constraint (3.68) taking as given Π_1, pi_2, $(1+r)\,A_0$, and r_1. The intertemporal utility is a measure of overall welfare which gives the present discounted value of utility flows :

$$\Lambda = U\left(C_1\right) + \frac{1}{1+\rho}.U\left(C_2\right), \tag{3.69}$$

where the parameter ρ corresponds to the time preference rate which is equal to the subjective time discount rate. Eliminating C_2 from (3.67a) by using (3.67b), i.e., $C_2 = (1+r_2)\ .(\Omega - C_1)$, differentiating the intertemporal utility w.r.t. C_1 and equating the partial derivative to zero, we get the optimal temporal path for consumption which is obtained by equating the intertemporal marginal rate os substitution which corresponds to the slope of the indifference curve and the relative price of present consumption at period 1 $(1+r_2)$ which corresponds to the slope of the budget constraint in the (C_1, C_2)-space :

$$\frac{U'\left(C_1\right)}{U'\left(C_2\right)}\left(1+\rho\right) = 1 + r_2. \tag{3.70}$$

3.3.3 General equilibrium in a Closed Economy and Adjustment to a Temporary Negative Output Shock

Before studying the determination of the current account, it is instructive to analyze a closed economy, that is, an economy in which agents do not have access to international financial markets, so that the current account is always zero.

In a closed economy, agents do not have access to the world capital market. As a consequence, the household's wealth must be held in the form of claims to domestic capital, that is

$$A_0 = K_1, \quad A_1 = K_2. \tag{3.71}$$

Replacing profit Π_1 in period 1 given by (3.62), the budget constraint (3.67a) can be rewritten as an identity which equalizes GDP to total expenditure which consists of consumption and investment :

$$Y_1 = C_1 + K_2 - (1-\delta)\ .K_1. \tag{3.72}$$

Then replacing profit Π_2 in period 2 given by (3.63), the budget constraint (3.67b) can be rewritten as an identity which equalizes GDP to total expenditure :

$$Y_2 = C_2 - (1-\delta) K_2. \tag{3.73}$$

Note that because the world ends after period 2, in that period the household chooses to consume the entire undepreciated stock of capital, $(1-\delta) K_2$, so that investment is negative and equal to $I_2 = -(1-\delta) K_2$.

Eliminating K_2 from (3.73) by using (3.72), i.e., $K_2 = Y_1 - C_1 + (1-\delta)\ .K_1$, and using the fact that $Y_2 = F(K_2)$, we obtain the resource constraint of the economy labelled the **Production possibilities frontier** (PPF) :

$$C_2 = F[Y_1 + (1-\delta)\ .K_1 - C_1] + (1-\delta)\ .[Y_1 + (1-\delta) K_1 - C_1]. \tag{3.74}$$

The PPF simplifies when the depreciation rate is assumed to be 100 percent (set $\delta = 1$ into (3.74). In this case we have that consumption is equal to output in period 2 ;

$$C_2 = F(Y_1 - C_1), \tag{3.75}$$

where output in period 1 $Y_1 = F(K_1)$ is exogenous because K_1 is a predetermined variable.

Figure 3.11 depicts the production possibility frontier (3.75) in the space (C_1, C_2). Because the production function is increasing and concave, the PPF is downward sloping and concave toward the origin. If in period 1 the household allocates the entire output to consumption and thus does not save, we have $C_1 = Y_1$, then output in period 2 is nil (point A in Figure 3.11). The maximum possible consumption in period 2 can be obtained by setting consumption equal to zero in period 1 ($C_1 = 0$) so that the household saves the entire output which is devoted to capital accumulation (point B in the figure). The slope of the PPF is negative and measured by the marginal product of capital : $dC_2/dC_1 = -F'$. As consumption in period 1, C_1, falls, the economy saves more and thus invest a larger amount which raises output in period 2 and thus increases consumption in this period. Because these are decreasing returns to scale w.r.t. capital, the production rises but a decreasing rate which explains the reason the PPF is concave.

The slope of the PPF indicates the quantity of consumption C_2 that the agent must give up in order to obtain one additional unit of consumption in period 1, C_1. The quantity that the agent must give up is measured by the marginal product of capital F' because it measures the quantity Y_2 that would have been produced with this unit of C_1.

In order to understand the choice along the PPF, it is simpler to adopt a line of reasoning in terms of goods market equilibrium, keeping in mind that the agent exchanges units of consumption across time through the capital market. The demand curve is given by the intertemporal marginal rate of substitution (MRS) which simplifies by using a logarithmic utility function $\ln(C_i)$:

$$-\frac{dC_2}{dC_1}\Big|_{\Lambda=\bar{\Lambda}} = \text{MRS} = \frac{C_2 \, . \, (1+\rho)}{C_1}. \tag{3.76}$$

According to (3.76), the agent consumes more in the present, the maximum price that he is willing to pay to obtain more units in period 1 decreases as a result of declining marginal utility. The demand curve is thus decreasing in the ($\frac{C_1}{C_2}$, MRS)-space : it means that consuming more units of good 1 gives a smaller additional utility so that the maximum price that the agent is willing to pay gets lower.

The supply curve is measured by the marginal rate of transformation (MRT) which is the slope of the PPF :

$$-\frac{dC_2}{dC_1}\Big|_{Y_1} = \text{MRT} = F'(Y_1 - C_1). \tag{3.77}$$

According to (3.77), as the agent consumes more in the present, he saves less which in turn reduce investment and thus capital stock in period 2, K_2, which results in a lower ouput $F(K_2)$. Because there are decreasing returns to scale in capital accumulation, the marginal product of capital F' increases. Hence, the supply curve is increasing in the ($\frac{C_1}{C_2}$, MRT)-space : it means that producing more units of good 1 costs a larger amount so that its relative price increases.

Since the MRS implies a trade off between present and future consumption and thus determines private savings, eq. (3.76) can be rewritten so that the supply of capital shows up :

$$\text{TMS} = \frac{F(K_2)(1+\rho)}{Y_1 - K_2},$$

where we used the fact that $Y_2 = C_2 = F(K_2)$, $C_1 = Y_1 - K_2$. The combination of capital supply described by the intertemporal MRS which is upward sloping in the (K_2, R_2)-space and the MRT which downward sloping in the (K_2, R_2)-space, the intercept gives the equilibrium value of K_2,

$$\frac{Y_2(1+\rho)}{Y_1 - K_2} = F'(K_2). \tag{3.78}$$

This capital stock maximizes intertemporal utility and profits and guarantees that the capital market clears. The value of K_2 determines the equilibrium value of $R_2 = \delta + r_2 = 1 + r_2$ (recall that we set $\delta = 1$).

Figure 3.12 represents the general equilibrium at point C ; at this point, the intertemporal MRS which represents the slope of the indifference curve is equal to the MRT qui represents the slope of the PPF. Since we have imposed a capital depreciation rate of 100%, the capital cost is measured by $1 + r_2$ which is represented graphically by the line which is tangent to the MRT. In equilibrium at point C, the slope of PPF and the marginal cost of capital $1 + r_2$ equalize so that firms maximize their profits and produce Y_1 and Y_2. These quantities guarantee the goods market equilibrium in both periods because when the MRT is equal to the MRS, relative supply is equal to relative demand of good 1. Because the capital cost $1+r_2$ represents the relative price of present consumption and corresponds to the slope of the intertemporal budget constraint, the agent gets the maximum intertemporal utility because the budget constraint is tangent to the indifference curve.

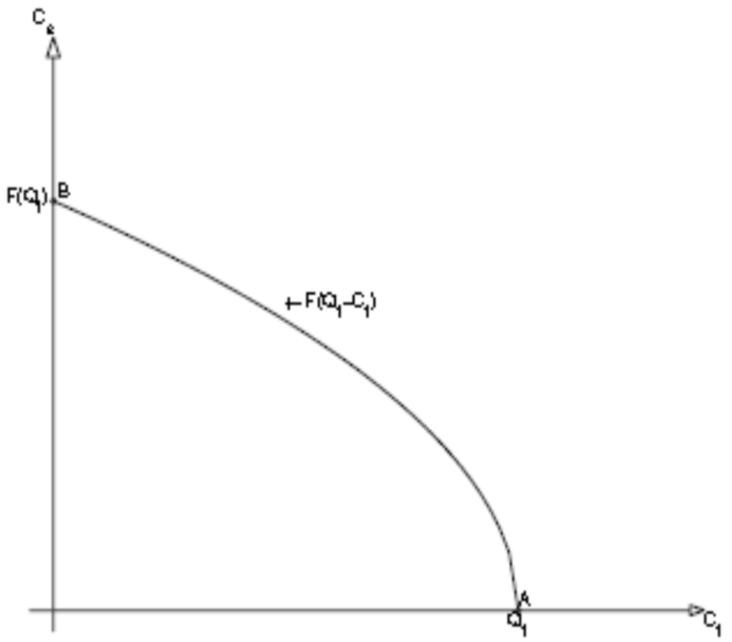


Fig. 3.11 – Production possibilities frontier (PPF) - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

In a closed economy, investment must be financed by national savings. When abstracting from government spending and thus taxes, savings reduce the households'savings.[1] The equality between households' savings $S_1 = Y_1 - C_1$ and investment is obtained by using the goods market equilibrium and by noting that investment $I_1 = K_2 - (1-\delta) K_1$:

$$S_1 = Y_1 - C_1 = K_2 - (1-\delta) K_1 = I_1. \tag{3.79}$$

The current account is equal to the difference between savings and investment (see equation (2.11)). Therefore, in a closed economy the current account is always equal to zero.

We now investigate the effects of a negative transitory shock (such as a negative productivity shock) that lowers output in period 1, which is captured in the model by an exogenous fall in Y_1. One simple way to analyze the effect of the shock graphically is to keep in mind that assuming full capital depreciation $\delta = 1$ implies that the PPF reduces to : $C_2 = F(Y_1 - C_1)$. The effect of a temporary shock can be analyzed in three steps :

- As illustrated in Figure 3.10, the exogenous fall in Y_1 shifts the PPF toward the origin because the agent must reduce consumption in both periods : the fall in savings in period 1 lower investment, and thus output and consumption in period 2. Yet, the shift is not uniform and biased toward consumption in period 1 because decreasing returns in capital accumulation moderates the fall in consumption in period 2. Put otherwise, savings S_1 fall and thus the capital stock K_2 which reduces Y_2. Yet, at the same time, the capital reduction makes the utilization of capital goods more productive which in turn moderates the fall in Y_2.
- Because consumptions C_1 et C_2 in both periods are normal goods : the agent consumes less of both goods as the revenue declines ; hence, consumption in period 1, C_1, falls

[1] We assume that households own firms. In reality, they rent the funds to firms and obtain a capital return $(r_1 + \delta) K_1$; savings is equal to $(r_1 + \delta) K_1 + \Pi_1 - C_1$ Firms get a profit $\Pi_1 = Y_1 - (r_1 + \delta) K_1$ which corresponds to firms'savings. Because houeholds own firms, we aggregate both types of savings : $S_1 = Y_1 - C_1$.

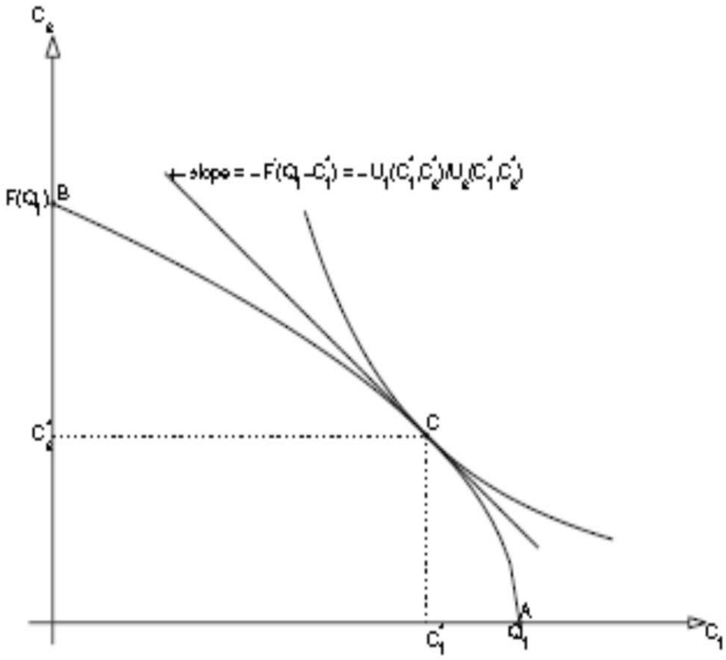


FIG. 3.12 – General equilibrium in a closed economy in a two-period model - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

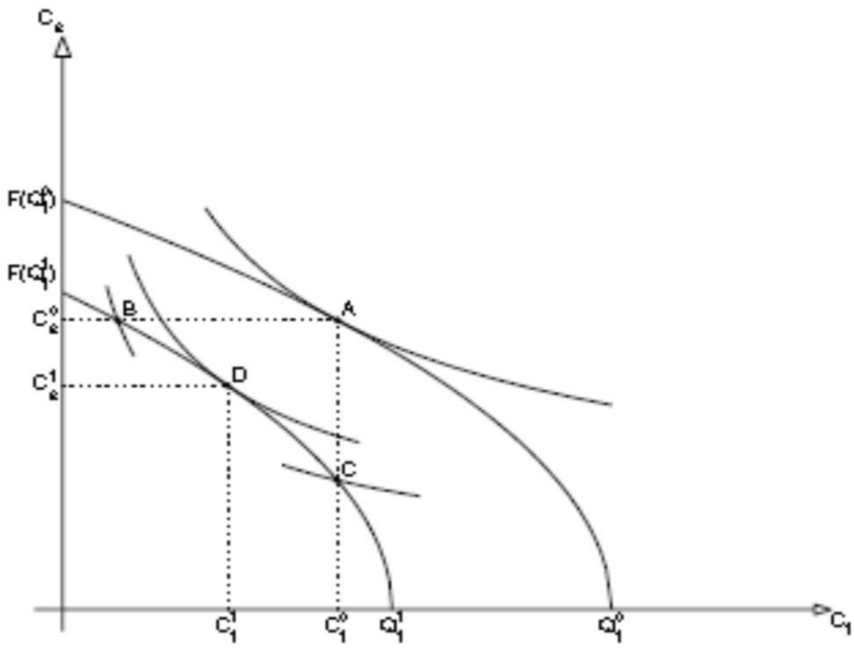


FIG. 3.13 – The effect of a negative output shock in a closed economy - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

less than output in period 1 Y_1, because the fall in expenditure is spread across both periods. Thus savings decline, which reduce K_2 which implies that the capital marginal product $F'(K_2)$ increases which raises the interest rate $r_2 = F'(K_2) - 1$. Graphically, the budget constraint line become steeper : the slope $1 + r_1$ is larger.

– The indifference curve shifts toward the south-west and is tangent with the new PPF which is below to the initial PPF. Because consumption across both periods fall, welfare Λ is lower. The final equilibrium is at point D of Figure 3.10.

Summing up, the effects of a decline in output in period 1 in the closed production economy are : i) a decline in consumption in period 1 that is less than the decline in output ; ii) a decline in savings that is matched by a decline in investment of equal magnitude ; and ii) an increase in the interest rate. We turn next to the analysis of current account determination in a production economy that has access to the world capital market.

3.3.4 Analytical derivation of the general equilibrium by using the capital market

To determine analytically consumption in both periods, C_1, C_2, investment K_2, and interest rate r_2, which are endogenous variables, we make assumptions : the rate of time preference is nil, $\rho = 0$, capital fully depreciates, $\delta = 1$, and the production functions takes a power form :

$$Y_2 = (K_2)^{\alpha}, \quad 0 < \alpha < 1. \tag{3.80}$$

The system comprises fours equations which jointly determine the four variables

$$C_2 = (1 + r_1)\, C_1, \tag{3.81a}$$

$$F\prime\,(K_2) = 1 + r_2, \tag{3.81b}$$

$$Y_1 = C_1 + K_2, \tag{3.81c}$$

$$Y_2 = C_2. \tag{3.81d}$$

Note that $Y_1 = F(K_1)$ is exogenous. The combination of (3.81a), (3.81c), and (3.81d) leads to capital supply (or savings $S_1 = K_2^S$) :

$$\frac{Y_2}{Y_1 - K_2^S} = (1 + r_2), \tag{3.82}$$

which is a decrasing function of r_2. The demand for capital $K_2^D$is desribed by (3.81b). Using (3.80), since $\alpha < 1$, the demand of capital decreases as r_2 rises. The combination of (3.81b) and (3.82), together with $F' = \alpha\,.\frac{Y_2}{K_2}$, leads to the capital market equlibrium :

$$\frac{Y_2}{Y_1 - K_2} = \alpha \frac{Y_2}{K_2}, \quad \Rightarrow \quad K_2 = \frac{\alpha}{1 + \alpha} \times Y_1. \tag{3.83}$$

Goods market equilibrium implies : $C_1 = Y_1 - K_2$. Plugging (3.83), one obtains period 1 (optimal) consumption :

$$C_1 = Y_1 - \frac{\alpha}{1 + \alpha} \times Y_1 = \frac{1}{1 + \alpha} \times Y_1. \tag{3.84}$$

The equlibrium interest rate is obtained by plugging (3.83) into (3.81b) :

$$r_2 = \alpha \left[\frac{\alpha}{1 + \alpha} \times Y_1 \right]^{\alpha - 1} - 1. \tag{3.85}$$

Period 2 consumption is determined by substituting (3.83) into (3.81d) :

$$C_2 = F(K_2) = \left[\frac{\alpha}{1+\alpha} \times Y_1\right]^{\alpha}. \tag{3.86}$$

A fall in Y_1 lower consumption in both periods, C_1, C_2, and investment K_2 (since savings fall), and raise interest rate.

3.3.5 Equilibrium in a Small Open Economy

In a small open economy households and firms can borrow and lend at an exogenously given world interest rate, which we denote by $r^\star$. Therefore, the interest rate prevailing in the small open economy has to be equal to the world interest rate, that is,

$$r_1 = r^\star. \tag{3.87}$$

The underlying assumption is that the economy is small on the world capital market. Also, in an open economy, households are not constrained to hold their wealth in the form of domestic capital. In addition to domestic capital, households can hold foreign assets, which are denoted by B. Thus, the stock of financial wealth A_i consists of foreign bonds B_i and capital claims K :

$$A_0 = K_1 + B_0, \quad A_1 = K_2 + B_1. \tag{3.88}$$

Because the interest rate is exogenous, the capital cost remains fixed as long as the world interest rate $r^\star$ is unchanged. As previously, the demand for capital is determined by equating the marginal product of capital and the marginal cost of capital :

$$F'(K_2^\star) = r^\star + \delta, \tag{3.89}$$

where $K_2^\star$ is the equilibrium capital stock in period 2. As the capital cost $r^\star + \delta$ increases, the equilibrium capital stock $K_2^\star$ falls. Because investment corresponds to capital accumulation in order to bring the initial capital stock to the equilibrium capital stock, i.e. $I_1 = K_2 - (1-\delta) K_1$, where the initial capital stock K_1 is predetermined, a rise in $r^\star$ lowers $K_2^\star$, and thus investment because the distance between the initial capital stock K_1 and the optimal capital stock $K_2^\star$ gets smaller ; hence, investment is a decreasing function of the world interest rate as illustrated in Figure 3.23(a) :

$$I_1 = I(r^\star), \quad I_r < 0. \tag{3.90}$$

By using the stocks of wealth (3.88), assuming a full capital depreciation $\delta = 1$, and supposing that the initial foreign asset position is null, cad $B_0 = 0$, by substituting profits in both periods, i.e., $\Pi_1 = Y_1 - (1+r^\star) K_1$ and $\Pi_2 = Y_2 - (1+r^\star) K_2$, the period 1 budget constraint (3.67a) reduces to :

$$\begin{aligned} A_1 &= B_1 + K_2 \\ &= (1+r^\star) A_0 + \Pi_1 - C_1, \\ &= (1+r^\star) K_1 + \Pi_1 - C_1, \\ &= Y_1 - C_1. \end{aligned}$$

Substituting $\Pi_2 = Y_2 - (1 + r^\star) K_2$ dans (3.67b) and using the fact that $A_1 = B_1 + K_2$, on obtient :

$$\begin{aligned} A_2 &= (1 + r_2) A_1 + \Pi_2 - C_2, \\ &= (1 + r_2) \ . B_1 + Y_2 - C_2 = 0. \end{aligned}$$

Hence, budget constraints (3.67) are modified as follows :

$$B_1 = Y_1 - K_2 - C_1, \tag{3.91a}$$

$$C_2 = (1 + r^\star) B_1 + Y_2. \tag{3.91b}$$

Eliminating B_1 from (3.91b) by using (3.91a), and using the fact that $K_2 = K_2^\star$ is determined by the world interest rate which is exogenous, we can determine the PPF in an open economy :

$$C_2 = (1 + r^\star) \ . (Y_1 - K_2^\star - C_1) + F(K_2^\star). \tag{3.92}$$

This resource constraint (3.92) says that the economy can consume the output, cad $Y_2 = F(K_2^\star)$, plus the principal and the interest from traded bonds holding, i.e. $(1 + r^\star) \ . B_1$.

The striking feature of an open economy compared with a closed economy is that in a closed economy, as shown in (3.92), the substitution of consumption across time occurs through the production function : when the agent consumes more in the present, savings declines which lower output in period 2. In an open economy, output in period 2 is fixed and the substitution of consumption across time occurs through traded bonds. Because the world interest rate is fixed, the relative price of present consumption does not change so that the consumption possibilities frontier (CPF) expands.

While the PPF is not affected, the resource constraint is now described by a linear relationship between C_2 et C_1 where the substitution between the two goods is determined by the world interest rate :

$$\frac{dC_2}{dC_1}\Big|^{OPEN} = -(1 + r^\star) < 0. \tag{3.93}$$

Since the CPF is linear, when savings is two times larger, future consumption doubles. When present consumption rises, the relative price of present consumption remains fixed so that agents may consume more when they borrow abroad at a fixed interest rate.

In order to determine the investment point graphically, we proceed as follows. Investment is determined by equating the marginal product $F'(K_2)$ with the capital cost $1 + r^\star$. Since the marginal product of capital is equal to the slope of FPP, and since the slope of the CFP is equal to the capital cost $1 + r^\star$, the tangency point between the CFP and the PPF. This point determines the equilibrium value of the capital stock $K_2^\star$ in a small open economy :

$$\begin{aligned} -\frac{dC_2}{dC_1}\Big|^{\text{FERM}} &= (1 + r^\star), \\ &= F'(K_2^\star) = F'(Y_1 - C_1). \end{aligned}$$

This point corresponds to the production choice in an open economy since the firms choose to use a capital stock $K_2^\star$ by equating the slope of the PPF and the line with a slope equal to $1 + r^\star$. The vertical line at point B would give a consumption $C_1 = Y_1 - K_2^\star$ which is associated with $B_1 = 0$ (since along the PPF, the capital account is closed). The horizontal line at point B would give a consumption in period 2 equal to $C_2 = F(K_2^\star)$. To summarized, at point B, we have :

$$B_1 = 0, \quad \Rightarrow \quad K_2^\star = K_2 = Y_1 - C_1.$$

We have to determine the consumption choice in an open economy. To do so, we write out optimal profile of intertemporal consumption :

$$C_2 = \frac{1+r^\star}{1+\rho}.C_1. \tag{3.94}$$

The time preference rate is defined as the marginal rate of substitution along a path where consumption is equal, i.e., $C_1 = C_2$. The Figure 3.15 shows that the agent chooses point A : it means that the time preference rate is higher than $r^\star$ since along the 45° degree line, the slope of the indifference curve $1+\rho$ is larger than the slope of the budget of the budget constraint $1+r^\star$ (since the the indifference curve crosses the 45° degree line with a slope equal to $1+\rho > 1+r^\star$).

Because consumption is higher in period 1 after opening the capital account than that in a closed economy, savings S_1 are smaller. The current account balance CA_1 is defined as the difference between savings and investment. In a closed economy, savings is equal to investment. In an open economy, the country may sustain a higher investment rate without sacrificing consumption, so that the differential is financed by a capital inflow :

$$CA_1 = S_1 - I_1 = (Y_1 - C_1) - K_2^\star < 0. \tag{3.95}$$

Because both consumption and investment rise, the trade balance deficit leads to a current account deficit. The current account deteriorates by the same amount as the trade balance because we have assumed that the initial foreign asset position was zero, $B_0 = 0$,

$$CA_1 = TB_1 < 0. \tag{3.96}$$

When the initial net foreign asset position B_0 is zero, (3.8) implies that trade balance deficit in period 1 must be offset by a trade balance surplus in period 2 :

$$TB_2 = -(1+r^\star)\ .TB_1 > 0. \tag{3.97}$$

The term $(1+r^\star)$ captures the fact that the economy must pay back both the principal and interests by running a trade balance surplus TB_2 in period 2.

Figure 3.15 depicts the general equilibrium in the closed economy which allows us to discuss the effects of opening to the world capital markets. The Figure shows the PPF together with the representative household's indifference curve that is tangent to the PPF. The point of tangency (point A in the Figure) represents the equilibrium allocation. At point B, the slope of the indifference curve is equal to the slope of the PPF. In terms of Figure 3.15, consumption, investment, and savings are determined as follows :

- Consumption $C_1^\star$ is determined by the point of tangency of the new budget constraint and the higher indifference curve.
- Investment is determined by the horizontal line located between Y_1 (period 1 endowment) and $Y_1 - K_2^\star$ (consumption when borrowing abroad is nil).
- Since $Y_1 - C_1^\star = S_1$ is savings and the horizontal line between Y_1 and $Y_1 - K_2^\star$ corresponds to investment, and since the current account is the difference between savings and investment, external borrowing $B_1 < 0$ corresponds to the horizontal line between $C_1^\star$ and $Y_1 - K_2^\star$.

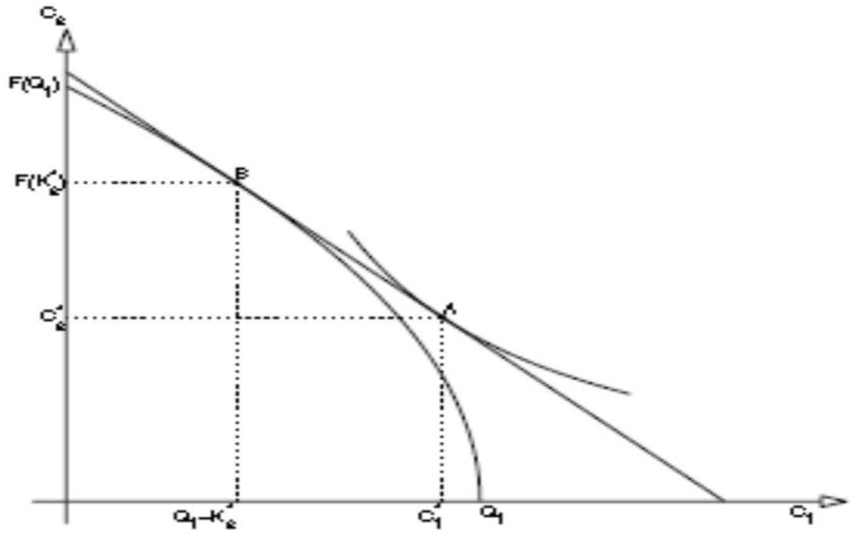


FIG. 3.14 – The determination of investment in a small open economy - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

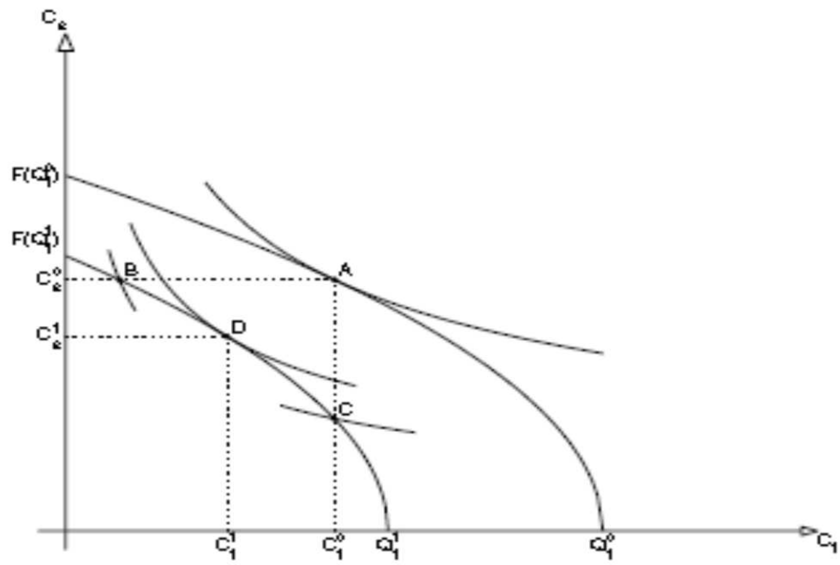


FIG. 3.15 – General equilibrium in a small open economy in a two-period model - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

- If the economy decides to close its capital account, the indifference passing though point B would be located in the southwest of the indifference curve passing through point A. Hence, opening the capital account leads to welfare gains.

3.3.6 Analytical derivation of the general equilibrium in a small open economy

In a small open economy, the world interest rate is exogenous, $r = r^{\star}$. We have to determine four endogenous variables, $C_1^{\star}$, $C_2^{\star}$, investment, $K_2^{\star}$, and the net investment income

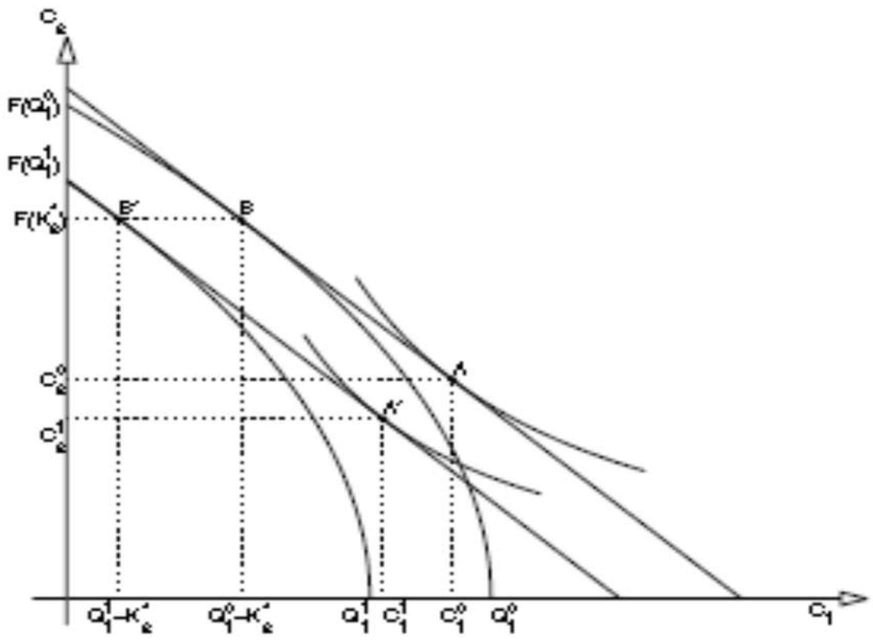


FIG. 3.16 – The effect of a negative output shock in an open economy - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 5

position, $B_1^\star$. The system comprises four equations which jointly determine four variables :

$$C_2 = (1 + r_1) C_1, \tag{3.98a}$$

$$F_\prime (K_2^\star) = 1 + r^\star, \tag{3.98b}$$

$$Y_1 = C_1 + K_2^\star + B_1, \tag{3.98c}$$

$$C_2 = F [K_2^\star] + (1 + r^\star) B_1. \tag{3.98d}$$

The demand for capital K_2^D is determined by (3.98b). By using the production function, we have :

$$K_2^\star = \left(\frac{\alpha}{1 + r^\star} \right)^{\frac{1}{1-\alpha}}. \tag{3.99}$$

The combination of (3.98a), (3.98c), et (3.98d), leads to the equilibrium value of the NIIP :

$$\frac{F (K_2)^\star + (1 + r^\star) B_1}{Y_1 - K_2^\star - B_1} = (1 + r^\star). \tag{3.100}$$

Solving yielfs :

$$B_1^\star = \frac{1}{2} \left[Y_1 - K_2^\star - \frac{F (K_2^\star)}{1 + r^\star} \right]. \tag{3.101}$$

Substituting (3.111) into (3.98c), leads to period 1 consumption :

$$C_1^\star = \frac{1}{2} \left[(Y_1 - K_2^\star) + \frac{F (K_2^\star)}{1 + r^\star} \right]. \tag{3.102}$$

Inserting (3.111) into (3.98a), leads to period 2 consumption :

$$C_2^\star = \frac{1}{2} \left[(1 + r^\star) (Y_1 - K_2^\star) + F (K_2^\star) \right]. \tag{3.103}$$

In a small open economy, a fall in Y_1 lowers consumption in both periods, but leave investment $K_2^\star$ unaffected as the world interest rate is exogenous and thus does not change.

3.3.7 A Negative Temporary Output Shock

We consider a negative output shock captured by a fall in Y_1. In a closed eocnomy, consumption in both periods falls, savings and investment decline while the interest rate increases. In an open economy :

- Investment is unaffected, i.e. $I_1 = I(r^\star)$, since it is determined by an exogneous world interest rate. As a result, the period 2 output is unchanged since $Y_2 = F(K_2^\star)$.
- Because the agent is poorer, he/she cuts consumption expenditure in both periods. However, because the agent can borrow abroad, C_1 falls but less than that in a closed economy. While the nation must satisfy its intertemporal solvency condition, the world interest rate is fixed so that the rise in consumption in period 1 does not raise the interest rate.
- Since investment $K_2^\star$ is unchanged while savings fall, the current account deficit gets larger (i.e., $CA_1 = B_1$ is more negative).
- The period 2 consumption C_2 may decline more than in a closed economy because the agent must repay its debts ; however, since investment and thus period 2 output does not fall, it may be possible, if the world interest rate is not too high, that period 2 consumption falls less in an open eocnomy than in a closed economy.

Figure 2.24 shows the effect of a temporary shock graphically :

- Point B correponds to the investment decision (determined graphically by the horizontal line between Y_1^0 and $Y_1^0 - K_2^\star$) and thus period 2 output $F(K_2^\star)$; point B is the tangency point between the budget constraint and the PPF ;
- point A corresponds to the consumption decision and is determined by the tangency between the budget constraint and the indifference curve ;
- Following the fall in Y_1, the PPF shifts toward the origin but not in an uniform way ; since the capital cost is unchanged, investment $K_2^\star$ remains unaffected and so does period 2 output $Y_2 = F(K_2^\star)$; graphically, points B and B' are along the same horizontal line ;
- the indifference curve shifts toward the origin so that consumptions in both periods are lower ; since the agent spreads the negative shock over consumption in both periods while only Y_1 falls, it implies that savings decline ;
- since savings fall while investment is unchanged, the current account deficit becomes larger : the horizontal line increases a length between C_1^0 and $Y_1^0 - K_2^\star$ to a length between C_1^1 and $Y_1^1 - K_2^\star$;
- the reduction of consumption C_2 in period 1 allows to produce a trade surplus and thus to pay back the debt, including interest payments : $F(K_2^\star) - C_2^1$ which is equal to $-(1 + r^\star) .B_1^1$

3.3.8 The Fall in Interest Rates in 1990's and Current Account Imbalances in the Euro Area in 2000's

The model of a small open economy is well suited to analyze the emergence of large current account deficits experienced by catching-up economies in the euro-area. As shown in 3.17 and 3.18, the fall in interest rates triggered by the financial and monteray integration has caused large current account deficits in Greecen, Ireland, Portugal, and Spain.

In terms of the model, it is tedious to show that a fall in interest rates leads to a current account deficit by raising investment and lowering private savings. Assuming a general form for utility, i.e., $U(C_i) = \frac{C_i^{1-\frac{1}{\sigma_C}}}{1-\frac{1}{\sigma_C}}$ $(i = 1, 2)$ with σ_C the intertemporal elasticity of substitution, the macroeconomic equilibrium reduces to the first-order conditions for consumption and investment, plus the intertemporal budget constraint :

$$C_2 = (1 + r^\star)^{\sigma_C} \; .C_1, \tag{3.104a}$$

$$K_2^\star = \left(\frac{\alpha}{1 + r^\star}\right)^{\frac{1}{1-\alpha}}, \tag{3.104b}$$

$$C_1 + K_2^\star + \frac{C_2}{1 + r^\star} = Y_1 + \frac{Y_2}{1 + r^\star}. \tag{3.104c}$$

Totally differentiating (3.104a) yields :

$$\hat{C}_2 = \sigma_C \; .(1 \hat{+} r^\star) + \hat{C}_1, \tag{3.105}$$

where $\hat{X} = dX/X \simeq d \ln X$. Totally differentiating (3.104c) gives :

$$\begin{aligned} C_1 \; .\hat{C}_1 \quad = \quad & -K_2 \; .\hat{K}_2 - \frac{C_2}{1 + r^\star} \; .\hat{C}_2 + \frac{C_2}{1 + r^\star} \; .(1 \hat{+} r^\star) \\ & + \frac{Y_2}{1 + r^\star} \; .\hat{Y}_2 - \left(\frac{Y_2}{1 + r^\star}\right) \; .(1 \hat{+} r^\star). \end{aligned} \tag{3.106}$$

Plugging (3.105) into (3.106), using the fact that $B_1 = \frac{C_2 - Y_2}{1+r^\star}$ and $\hat{Y}_2 = \alpha \; .\hat{K}_2$ (recall that $Y_2 = (K_2)^\alpha$), eq. (3.106) can be rewritten :

$$\begin{aligned} \left(C_1 + \frac{C_2}{1 + r^\star}\right) \; .\hat{C}_1 \quad = \quad & \left(\alpha \; .\frac{Y_2}{1 + r^\star} - K_2\right) \; .\hat{K}_2 \\ + \quad & \left(B_1 - \frac{C_2}{1 + r^\star} \; .\sigma_C\right) \; .(1 \hat{+} r^\star). \end{aligned} \tag{3.107}$$

Using the fact that $\alpha \; .\frac{Y_2}{1+r^\star} - K_2 = \frac{\alpha}{1+r^\star} \cdot \left(\frac{\alpha}{1+r^\star}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha}{1+r^\star}\right)^{\frac{1}{1-\alpha}} = 0$, eq. (3.107) finally reduces to :

$$\left(C_1 + \frac{C_2}{1 + r^\star}\right) \; .\hat{C}_1 = \left(B_1 - \frac{C_2}{1 + r^\star} \; .\sigma_C\right) \; .(1 \hat{+} r^\star). \tag{3.108}$$

When $\sigma_C = 1$, we have $B_1 - \frac{C_2}{1+r^\star} \; .\sigma_C = B_1 - \frac{C_2}{1+r^\star} = -\frac{Y_2}{1+r^\star} < 0$. In this case, a fall in interest rate $r^\star$ raises unambiguously C_1. As a result, private savings fall ; intuitively, a fall in $r^\star$ lowers the relative price of present consumption. At the same time, $K_2^\star$ rises because the capital cost $r^\star$ declines. Since the current account is equal to $B_1 = Y_1 - C_1 - K_2^\star$, the country borrows abroad in period 1 and thus runs a current account deficit.

Current account balances in Greece, Ireland, Italy, and Spain worsened significantly during the first decade of European Monetary Union, while Portugal's deficit remained at the very high levels it had reached early in the decade (see Table 3.19). As a result of the increasing recourse to external financing, net external liabilities of these countries rose sharply, reaching levels close to or above 100 percent of GDP by the end of 2010 in Greece, Ireland, Portugal, and Spain (Figure 3.20)). During this period, Germany and a number of other smaller countries in Northern Europe progressively built large current account surpluses, with the current account for the euro area as a whole remaining in broad balance throughout the period.

While current account trends were broadly similar across debtor countries, there were significant differences in the underlying evolution of saving and investment. In Ireland and

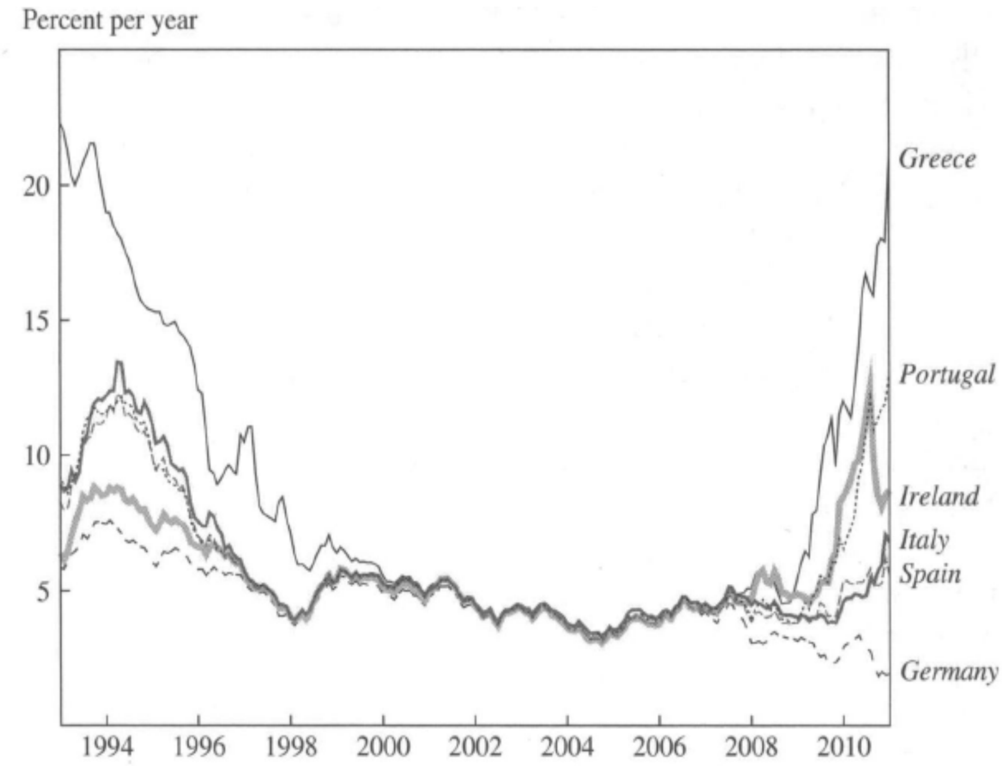


Fig. 3.17 – Interest Rates on Government Debt, 1993-2011. Period average - Source : Shambaugh, Reis, and Rey (2012) What caused the Asian currency and financial crisis. *Brookings Papers on Economic Activity*, Spring 2012, pp. 157-231

Spain, investment rates were boosted by construction booms, and growth rates were considerably above the average for the euro area, also thanks to rising labor forces. In Greece, growth was also stronger than in the rest of the euro area, with the widening current account deficit mostly explained by a large decline in saving. In contrast, growth was very modest in Portugal, with declines of both investment and household saving. Italy also experienced relatively weak growth and some decline in saving, although the current account deficit in percent of GDP remained much more contained than in other countries.

The sectoral destination of capital inflows reflected a combination of purchases of government bonds (in all countries, but particularly in Greece and Portugal) and purchases of bank bonds and lending to domestic banks (particularly in Spain, Portugal, Ireland) with Italy standing out as having the largest accumulation of assets overseas, reflecting capital outflows by the nonbank private sector (see Figure 3.21). In sum, the net position of the general government and the financial sector account for the lion share of the increase in net external liabilities for the debtor countries. This helps explain why concerns about government finances and the health of bank balance sheets took center stage during the crisis starting in early 2010.

However, a parallel analysis of domestic financial balance sheets reveals a more complex picture : the worsening external position of debtor countries is to a significant extent associated with a worsening in the financial balance sheet of the private sector, specifically households (Table 3.22). In turn, this worsening of the financial balance sheet of households is mostly explained by an increase in purchases of nonfinancial assets (primarily housing). The net position of the general government (as of end-2008) was still stronger than early in the decade (the exception being Portugal) but the domestic private sector reduced substantially

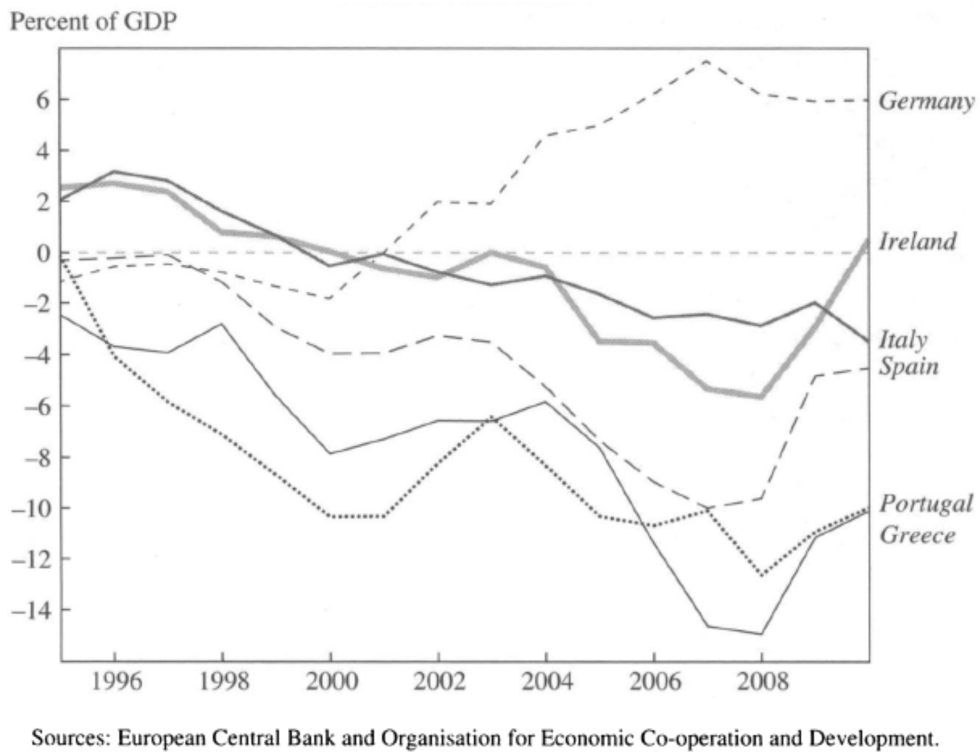


FIG. 3.18 – Current Account Balances, 1995-2010 - Source : Shambaugh, Reis, and Rey (2012) What caused the Asian currency and financial crisis. *Brookings Papers on Economic Activity*, Spring 2012, pp. 157-231

its holdings of domestic government debt and increased its indebtedness vis-à-vis the domestic financial system, which in turn increased its reliance on external funding. What changed therefore was the pattern of ownership of domestic public debt, rather than its overall size - worsening private sector balance sheets were the driving force behind increased external imbalances.

3.4 Uncertainty and the Current Account in a Simple Two-Period Economy

Thus far, we have studied the response of the current account to changes in fundamentals that are known with certainty. The real world, however, is an uncertain place. A natural question, therefore, is how the overall level of uncertainty affects the macroeconomy, and, in particular, the external accounts. This section is devoted to addressing this question. It begins by documenting a period of remarkable stability in the United states, known as the Great Moderation. It then shows that this period coincided with the emergence of large current account deficits. Finally, the chapter expands the small open economy model to allow for uncertainty by abstracting from physical capital.

		1999-2001	2007-2008	Change 1999-01 to 2007-08
Greece [1/]	**Current Account**	**-6.8%**	**-14.5%**	**-7.7%**
	Investment	22.9%	21.6%	-1.4%
	Savings	16.2%	7.1%	-9.1%
	Public Savings	-0.7%	-3.0%	-2.3%
	Private Savings	16.9%	10.1%	-6.7%
	Household Savings	2.0%	0.3%	-1.7%
	Corporate Savings	13.8%	9.8%	-4.0%
Ireland	**Current Account**	**-0.3%**	**-5.3%**	**-5.0%**
	Investment	23.4%	23.9%	0.5%
	Savings	23.2%	18.7%	-4.5%
	Public Savings	8.3%	-0.8%	-9.1%
	Private Savings	14.8%	19.4%	4.6%
	Household Savings	...	...	...
	Corporate Savings	...	...	...
Italy	**Current Account**	**0.0%**	**-2.9%**	**-3.0%**
	Investment	20.4%	21.5%	1.0%
	Savings	20.5%	18.5%	-1.9%
	Public Savings	1.3%	1.5%	0.2%
	Private Savings	19.2%	17.0%	-2.2%
	Household Savings	10.8%	10.2%	-0.5%
	Corporate Savings	8.4%	6.8%	-1.6%
Portugal	**Current Account**	**-9.5%**	**-10.8%**	**-1.2%**
	Investment	27.5%	22.3%	-5.3%
	Savings	18.0%	11.5%	-6.5%
	Public Savings	0.9%	-0.5%	-1.3%
	Private Savings	17.1%	11.9%	-5.2%
	Household Savings	7.3%	4.4%	-2.9%
	Corporate Savings	9.8%	7.5%	-2.2%
Spain [1/]	**Current Account**	**-3.6%**	**-9.8%**	**-6.2%**
	Investment	25.9%	30.1%	4.2%
	Savings	22.3%	20.3%	-2.0%
	Public Savings	2.3%	2.9%	0.6%
	Private Savings	20.1%	17.5%	-2.6%
	Household Savings	7.4%	7.7%	0.2%
	Corporate Savings	12.5%	9.8%	-2.7%
France	**Current Account**	**2.2%**	**-1.6%**	**-3.9%**
	Investment	19.9%	22.2%	2.3%
	Savings	22.0%	20.6%	-1.5%
	Public Savings	2.1%	5.1%	3.0%
	Private Savings	19.9%	15.4%	-4.5%
	Household Savings	10.0%	10.2%	0.3%
	Corporate Savings	9.9%	5.2%	-4.8%
Germany	**Current Account**	**-1.0%**	**7.2%**	**8.1%**
	Investment	20.9%	18.8%	-2.2%
	Savings	19.9%	25.9%	6.0%
	Public Savings	0.9%	2.5%	1.5%
	Private Savings	19.0%	23.5%	4.4%
	Household Savings	10.6%	11.6%	1.0%
	Corporate Savings	8.4%	11.8%	3.4%

Sources: Eurostat, IFS, and Staff Calculations

1/: households and corporate savings data start in 2000.

Fig. 3.19 – Saving-Investment Balance (In percent of GDP), 1999-2001 and 2007-2008 - Source : Chen, Milesi-Ferretti, Tressel (2012) External Imbalances in the Euro Area. *IMF Working Paper*.

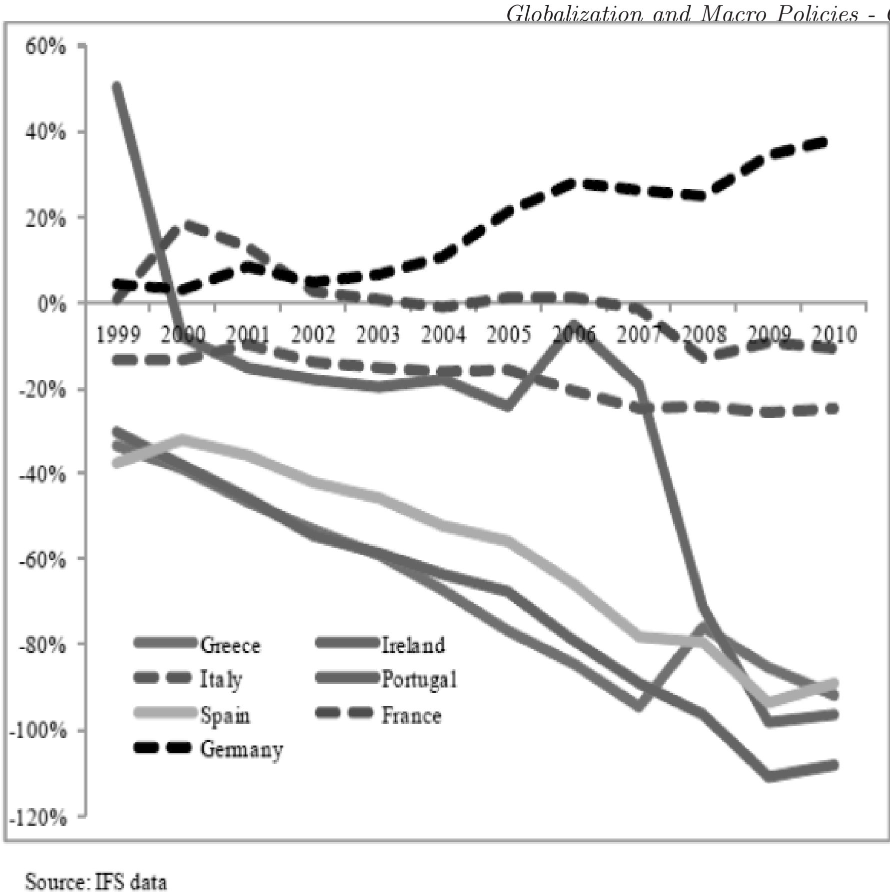


FIG. 3.20 – Net Foreign Asset Positions 1999-2010, in Percent of GDP - Source : Chen, Milesi-Ferretti, Tressel (2012) External Imbalances in the Euro Area. *IMF Working Paper.*

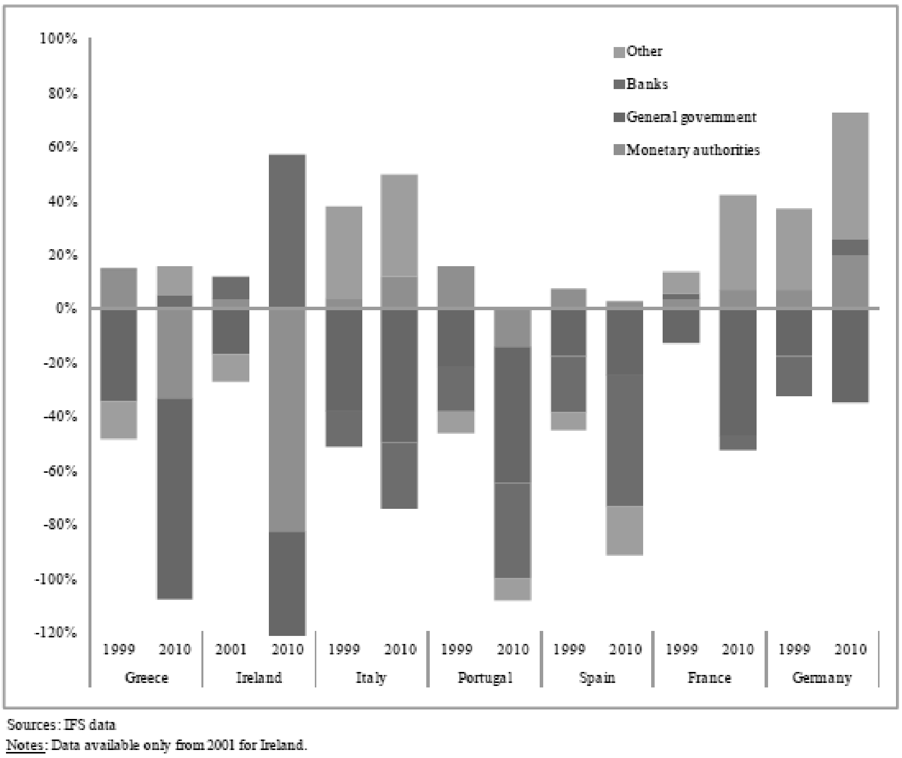


FIG. 3.21 – Sectoral Net Foreign Asset Positions (In percent of GDP) - Source : Chen, Milesi-Ferretti, Tressel (2012) External Imbalances in the Euro Area. *IMF Working Paper.*

	Sector	2001	2009	Change
Greece	Households	131	59	-71
	Government	-93	-87	6
	Financial Sector	-9	-6	4
	Non-financial Sector	-71	-69	2
	Total	-42	-102	-60
Ireland	Households	103	65	-38
	Government	-13	-28	-15
	Financial Sector	-2	1	3
	Non-financial Sector	-103	-105	-2
	Total	-15	-67	-52
Italy	Households	202	186	-16
	Government	-96	-103	-7
	Financial Sector	2	19	17
	Non-financial Sector	-99	-117	-18
	Total	9	-16	-25
Portugal	Households	140	127	-13
	Government	-30	-57	-27
	Financial Sector	-10	-1	9
	Non-financial Sector	-148	-174	-26
	Total	-48	-106	-58
Spain	Households	107	76	-31
	Government	-42	-34	7
	Financial Sector	3	11	7
	Non-financial Sector	-103	-143	-40
	Total	-34	-90	-56
France	Households	118	131	14
	Government	-37	-51	-14
	Financial Sector	11	19	8
	Non-financial Sector	-77	-102	-25
	Total	15	-2	-17
Germany	Households	98	130	32
	Government	-36	-48	-12
	Financial Sector	0	7	7
	Non-financial Sector	-58	-59	-1
	Total	4	30	26

Source: Eurostat statistics, OECD statistics

Fig. 3.22 – Net Financial Assets by Sector (In percent of GDP, 2001-09) - Source : Chen, Milesi-Ferretti, Tressel (2012) External Imbalances in the Euro Area. *IMF Working Paper*.

3.4.1 The Great Moderation

3.4.1.1 The Great Moderation and the Reduction of Volatility of Output

A number of researchers have documented that the volatility of U.S. output declined significantly starting in the early 1980s. This phenomenon has become known as the Great Moderation. The most commonly used measure of volatility in macroeconomic data is the standard deviation. According to this statistic, U.S. output growth became half as volatile in the past quarter century. Specifically, the standard deviation of quarter-to-quarter output growth was 1.2 percent over the period 1948 to 1983, but only 0.5 percent over the period 1984 to 2006. Figure 3.23(a) depicts the quarterly growth rate of U.S. output from 1948 :Q1 to 2009 :Q3. It also shows with a vertical line the beginning of the Great Moderation in 1984. It is evident from the figure that the time series of output growth in the United States is much smoother in the post 1984 subsample than it is in the pre-1984 subsample.

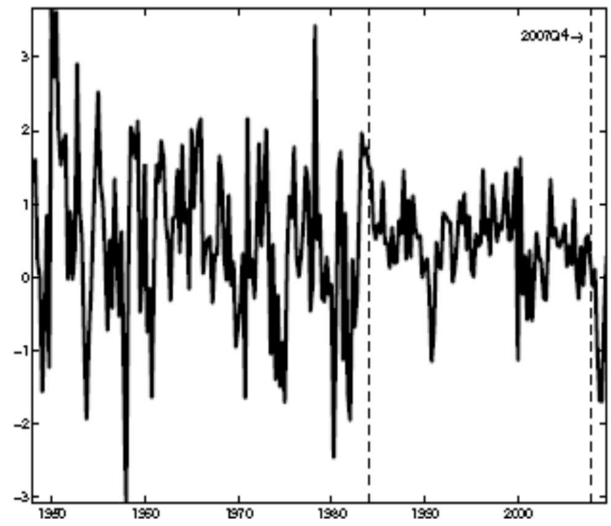
Researchers have put forward three alternative explanations of the Great Moderation : good luck and good policy. The good luck hypothesis states that by chance, starting in the early 1980s the U.S. economy has been blessed with smaller shocks. The good policy hypothesis maintains the monetary policy conducted by Paul Volker and Alan Greespan has brought to an end the high inflation of the 1970s and has brought macroeconomic stability. In particular, since the Kydland and Prescott's (1977) article which emphasizes the inflation bias, central banks became independent and pursued the objective of inflation stability by raising the interest rate during an economic boom while lowering the interest rate during a slump.

3.4.1.2 The Great Moderation and the U.S. Trade Imbalances

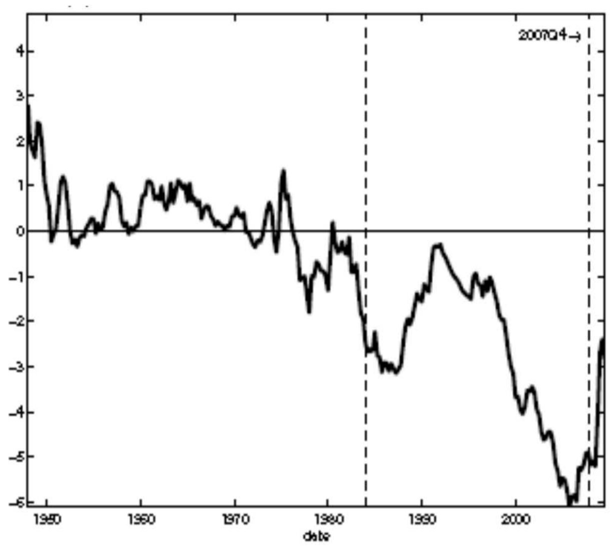
Our interest is in possible connections between the Great Moderation and the significant trade balance deterioration observed in the U.S. over the period 1984 to 2006. Figure 3.23(b) depicts the ratio of the trade balance to GDP in the United States over the period 1948-2009. During the period 1948-1984 the United States experienced on average positive trade balances of about 0.2 percent of GDP. Starting in the early 1980s, however, the economy was subject to a string of large trade deficits averaging 2.6 percent of GDP. Is the timing of the Great Moderation and the emergence of protracted trade deficits pure coincidence, or is there a causal connection between the two ? To address this issue, we will explore the effects of changes in output uncertainty on the trade balance and the current account in the context of our theoretical framework of current account determination.

3.4.2 A Model with Uncertainty

In the economy studied in section 3.3, the endowments Y_1 and Y_2 are known with certainty. What would be the effect of making the future endowment, Y_2, uncertain ? That is, how would households adjust their consumption and savings decisions in period 1 if they knew that the endowment in period 2 could be either high or low with some probability ? Intuitively, we should expect the emergence of precautionary savings in period 1. That is, an increase in savings in period 1 to hedge against a bad income realization in period 2. The desired



(a) Per Capita U.S. GDP Growth 1948-2009



(b) U.S. Trade Balance to GDP ratio 1948-2009

Fig. 3.23 – The Great Moderation in the U.S. - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 4

increase in savings in period 1 must be brought about by a reduction in consumption in that period. With period-1 endowment unchanged and consumption lower, the trade balance must improve. We therefore have that an increase in uncertainty brings about an improvement in the trade balance. By the same token, a decline in income uncertainty, such as the one observed in the United States since the early 1980s, should be associated with a deterioration in the trade balance.

3.4.3 The Solution in a Model without Uncertainty

To formalize these ideas, consider an economy in which initially, the stream of output is known with certainty and constant over time. Specifically suppose that $Y_1 = Y_2 = Y$. Assume further that preferences are of the form

$$\Lambda = \ln C_1 + \ln C_2. \tag{3.109}$$

To simplify the analysis, assume that initial asset holdings are nil, that is, $B_0 = 0$, and that the world interest rate is nil, or $r^\star = 0$. In this case, the intertemporal budget constraint of the representative household reduces to :

$$C_2 + C_1 = 2 \times Y. \tag{3.110}$$

Using this expression to eliminate C_2 from the utility function (3.109), we have that the household's utility maximization problem consists in choosing C_1 so as to maximize

$$\ln C_1 + \ln\left(2 \times Y - C_1\right). \tag{3.111}$$

The first order condition implies that the agent chooses a temporal path for consumption so that marginal utilities equalize across periods :

$$\frac{1}{C_1} = \frac{1}{C_2}. \tag{3.112}$$

Substituting $C_1 = C_2$ into the intertemporal budget constraint (3.110), the solution to this problem with certainty reduces to

$$C_1 = C_2 = Y. \tag{3.113}$$

It follows that the trade balance in period 1, given by $TB_1 = Y_1 - C_1$, is zero :

$$TB_1 = Y_1 - C_1 = 0. \tag{3.114}$$

In this economy households do not need to save or dissave in order to smooth consumption over time because the endowment stream is already perfectly smooth.

3.4.4 Extending the Model to Uncertainty

Consider now a situation in which Y_2 is not known with certainty in period 1. Specifically, assume that with probability 1/2 output in period 2 equal is raised by an amount equal to σ (positive shock), and that with equal probability the output in period 2 is reduced by an amount equal to $-\sigma$ (negative shock), which can be summarized as follows :

$$Y_2 = \begin{cases} Y_H = Y + \sigma & \text{with probability } 1/2 \\ Y_L = Y - \sigma & \text{with probability } 1/2. \end{cases} \tag{3.115}$$

We continue to assume that $Y_1 = Y$. Note that the realization if output is not known with certainty but the expected value of the endowment in period 2 is equal to output in period 2 in the economy without uncertainty :

$$\frac{1}{2} \times (Y + \sigma) + \frac{1}{2} \times (Y - \sigma) = Y. \tag{3.116}$$

In this case, the uncertainty in period 2 income is said to be mean-preserving.

The standard deviation of the endowment in period 2 is given by σ. To see this, recall that the standard deviation is the square root of the variance and that, in turn, the variance is the expected value of the deviation of output from its mean (equal to Y, see (3.116)). The deviation of output from its mean is

$$\begin{aligned} \text{Variance} &= \frac{1}{2}\sum_{i=H}^{L}(Y_i - Y)^2 = \frac{1}{2}(Y_H - Y)^2 + \frac{1}{2}(Y_L - Y)^2, \\ &= \frac{1}{2}(Y + \sigma - Y)^2 + \frac{1}{2}(Y - \sigma - Y)^2, \\ &= \frac{1}{2}(\sigma)^2 + \frac{1}{2}(-\sigma)^2 = \sigma^2. \end{aligned}$$

The standard deviation of output inn period 2 is then given by

$$\text{Standard-deviation} = \sqrt{\text{Variance}} = \sigma.$$

It follows that the larger is σ, the more volatile is the endowment in period 2.

We must specify how households value uncertain consumption bundles. We will assume that households care about the expected value of utility. Specifically, preferences under uncertainty are given by

$$\ln C_1 + E \ln C_2, \tag{3.117}$$

where E is the expectation operator. Note that this preference formulation encompasses the preference specification we used in the absence of uncertainty. This is because when C2 is known with certainty, then $E \ln C2 = \ln C2$. The budget constraint of the household in period 2 is given by

$$C_2 = \begin{cases} Y + Y_H - C_1 = 2 \times Y + \sigma - C_1 & \text{with probability } 1/2 \\ Y + Y_L - C_1 = 2 \times Y - \sigma - C_1 & \text{with probability } 1/2. \end{cases} \tag{3.118}$$

the good state of the world (first line) and in the bad state of the world (second line). Therefore, expected lifetime utility, $lnC_1 + ElnC_2$, is given by :

$$\ln C_1 + \frac{1}{2} \times \ln(2 \times Y + \sigma - C_1) + \frac{1}{2} \times \ln(2 \times Y - \sigma - C_1).$$

Differentiating with respect to C_1 and setting the partial derivative to zero, we have that the marginal utility of consumption in period 1 must equalize the expected value of the marginal utility in period 2 :

$$\frac{1}{C_1} = \frac{1}{2} \times \left[\frac{1}{(2 \times Y + \sigma - C_1)}\right] + \frac{1}{2} \times \left[\frac{1}{(2 \times Y - \sigma - C_1)}\right]. \tag{3.119}$$

To get further insight into the implications of uncertainty, we analyze if the solution with certainty (3.113) is a solution in a model with uncertainty :

$$\begin{aligned}
\frac{1}{Y} &= \frac{1}{2} \times \left[\frac{1}{(2 \times Y + \sigma - Y)}\right] + \frac{1}{2} \times \left[\frac{1}{(2 \times Y - \sigma - Y)}\right], \\
&= \frac{1}{2} \times \left[\frac{1}{(Y + \sigma)}\right] + \frac{1}{2} \times \left[\frac{1}{(Y - \sigma)}\right], \\
&= \frac{Y}{Y^2 - \sigma^2}, \\
&= \frac{1}{Y - \frac{\sigma^2}{Y}},
\end{aligned}$$

which is impossible as long as $\sigma > 0$. Hence, if the agent chooses a flat time profile for consumption with uncertainty, the first order condition does not holds because the marginal utility of consumption in period 1 is lower than the expected value of the marginal utility of consumption in period 2. Since the marginal utility is decreasing with consumption, this outcome implies that consumption in period 1 is excessive and consumption in period 2 is not sufficient. It follows that the household would be better off consuming less in period 1 and more in period 2. Formally, because the left side of optimality condition (3.119) is decreasing in C_1 whereas the right side is increasing in C_1, it must be the case that the optimal level of consumption in period 1 satisfies :

$$\tilde{C}_1 < Y. \tag{3.120}$$

It then follows that **in the economy with uncertainty the trade balance is positive in period 1**, or

$$Y - \tilde{C}_1 = \tilde{TB}_1 > 0. \tag{3.121}$$

Households use the trade balance as a vehicle to save in period 1. In this way, they avoid having to cut consumption by too much in the bad state of period 2.

By means of Figure 3.24, we can build intuition graphically. We put revenue on the horizontal axis and utility $u(Y)$ on the vertical axis. Utility is concave : as consumption increases, utility increases at a smaller rate. The concave form of the utility function reflects the relative risk aversion of the individual. To see it, let us consider two scenarios, in line with the model set out above. We first assume that there is no savings and thus the individual consumes his/her whole revenue, i.e., $C_1 = Y_1$. In the first scenario, i.e., in period 1, the agent gets a certain revenu of $Y_1 = Y$ which gives a utility of $u(Y)$. In the second scenario, i.e., in period 2, the revenue can be low at $Y_L = Y - \sigma$ with a probability of 1/2 or can be high at $Y_H = Y + \sigma$ with a probability of 1/2. The excpected revenu in period 2 is $E(Y_2) = Y$ (see (3.116)). To determine the expected utility, we have to draw a line between point A and point B. As illustrated in Figure 3.24, at point A, the agent gets a revenue Y_L which leads to utility $u(Y_L)$. At point B, the agent gets a revenue Y_H which leads to utility $u(Y_H)$. Denoting by p the probability that revenue is low, expected utility can be written as follows :

$$E(u(Y)) = p \,.u(Y_L) + (1-p) \,.u(Y_H).$$

Expected utility varies between point A and point B : ecpected utility increases as the probability of the good state, $1-p$, rises. When assuming that $p = 1-p$ so that $p = 1/2$, expected income $E(Y_2)$ in period 2 is equal to the certain income in period 1, Y_1 ; however, expected utility is lower that the utility from a certain income :

$$\frac{1}{2}\,.u(Y_L) + \frac{1}{2}\,.u(Y_H) < u(Y_1).$$

Because the utility is concave, the utility gain (when the revenue rises from Y to $Y+\sigma$) is lower than the utility loss (when the revenue falls from Y to $Y-\sigma$). When setting period 1 consumption at $C_1 = Y$ while period 2 consumption varies between $C_2 = Y_L$ and $C_2 = Y_H$, we find that the marginal utility from period 1 consumption $u'(Y)$ is lower than the marginal expected utility (we use a logarithmic utility) :

$$\frac{1}{2}\cdot\frac{1}{Y-\sigma}+\frac{1}{2}\cdot\frac{1}{Y+\sigma}>\frac{1}{Y}.$$

or

$$\frac{Y}{Y^2-\sigma^2}>\frac{1}{Y},$$

which unambigously holds as long as $\sigma > 0$. However, the optimal profil for consumption requires that the period 1 marginal utility to equalize with period 2 expected marginal utility (see eq. (3.119)). In other words, the agent consumes too much in period 1 which results in a too low marginal utility. Because the marginal utility in each period must be equal, agent must consume the same level, $C_1 = C_2$, so that the marginal utility is given by the slope of the utility at point E. Consumption that gives the same utility than that prevailaing when the revenue is uncertain corresponds to the 'certain equivalent' which is lower than the expected income $E(Y) = Y$. The difference between the expected income and the equivalent certain measures the risk premium, i.e., the revenue that the agent is willing to give up in order to have the same utility whether the revenue is certain or uncertain.[2]

Le revenu auquel doit renoncer l'individu pour être sûr de maintenir ce niveau de consommation aux deux périodes est égal à $Y-\tilde{C}$: c'est la prime de risque qui mesure ce que l'agent est prêt à payer pour échapper au risque. On note PR la prime de risque : elle est définie comme la différence entre le gain espéré $E(Y)$, et l'équivalent certain (ou monétaire) de la loterie. Calculons d'abord l'équivalent certain qui mesure la consommation aboutissant à une utilité identique à celle obtenue lorsque le revenu est incertain :

$$\frac{1}{2}\ln(Y+\sigma)+\frac{1}{2}\ln(Y-\sigma)=\ln\tilde{C}.$$

ce qui donne en résolvant :

$$\begin{aligned}\ln\tilde{C} &= \frac{1}{2}\cdot[\ln(Y-\sigma)\ .(Y+\sigma)] \\ &= \frac{1}{2}\cdot\ln\left(Y^2-\sigma^2\right),\end{aligned}$$

ou encore en utilisant le fait que $x^a = \exp(a\ln x) = \exp[(\ln x)^a]$, l'expression ci-dessus peut être réécrite de la façon suivante :

$$\begin{aligned}\tilde{C} &= \exp\left[\frac{1}{2}\ln\left(Y^2-\sigma^2\right)\right], \\ &= \exp\left[\ln\left(Y^2-\sigma^2\right)^{\frac{1}{2}}\right], \\ &= \left(Y^2-\sigma^2\right)^{\frac{1}{2}}.\end{aligned}$$

[2] On observe que la consommation 'certaine' est plus faible que le revenu espéré de l'individu $E(Y)$. Cette consommation certaine est appelée **equivalent certain** : c'est le montant de revenu qui lui procurerait la même utilité que celle obtenue par sa participation à cette loterie. Mais pour obtenir ce revenu avec certitude, l'individu va devoir recourir à un système d'assurance qui consiste à constituer une épargne de précaution. Cette épargne de précaution, $E(Y)-\tilde{C}$ correspond au montant que l'individu est prêt à payer pour échapper au risque.

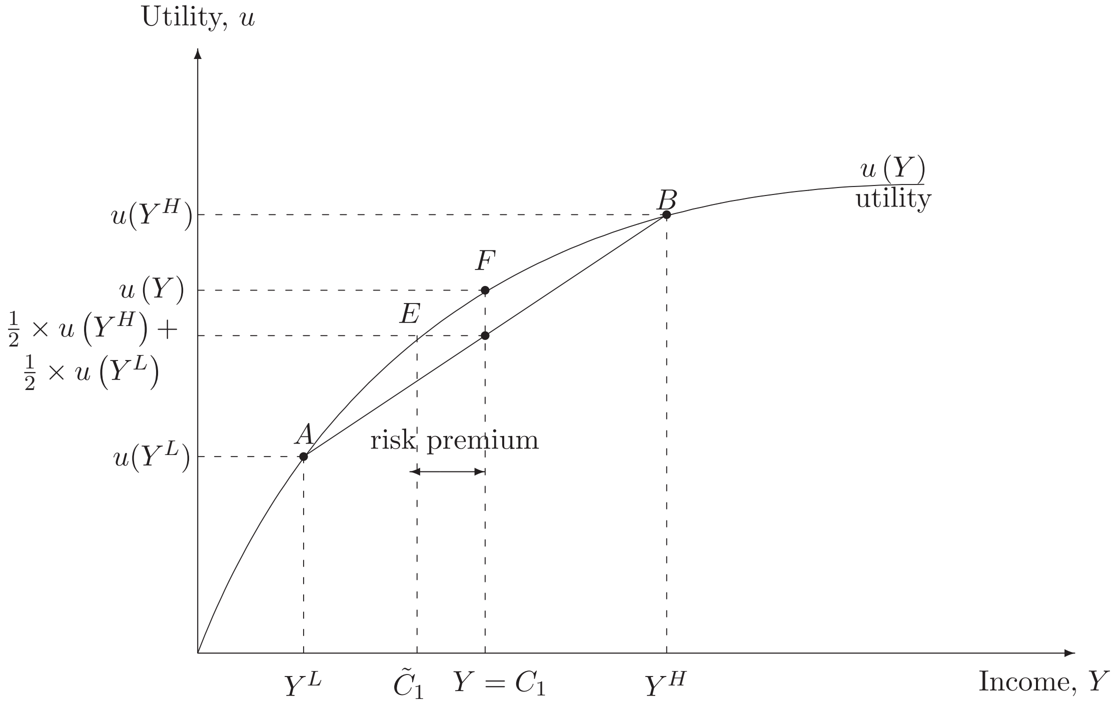


FIG. 3.24 – Uncertainty, risk aversion, certainty equivalent, and risk premium : $\tilde{C}_1 < Y$

Par conséquent, la prime de risque est égale à :

$$\begin{aligned} PR &= E(Y) - \tilde{C}, \\ &= Y - \left(Y^2 - \sigma^2\right)^{\frac{1}{2}} \end{aligned} \tag{3.122}$$

In summary, Because at the optimum, today's marginal utility must equal next period's in expected value, and because current marginal utility is decreasing in current consumption, the adjustment to a mean-preserving increase in uncertainty about next period's endowment takes the form of a reduction in current consumption.

3.4.5 The Return of Uncertainty : The Great Contraction and The Current Account

The model presented in this chapter captures qualitatively the joint occurrence of diminished output uncertainty and trade deficits observed during the Great Moderation (1984-2006). A natural test of the model is whether the elevated level of aggregate uncertainty the U.S. economy has been experiencing since the onset of the Great Contraction of 2007 has been accompanied by an improvement in the trade balance. It turns out that this is indeed the case. Look at Figure 3.25. It displays the U.S. current account balance as a percentage of GDP

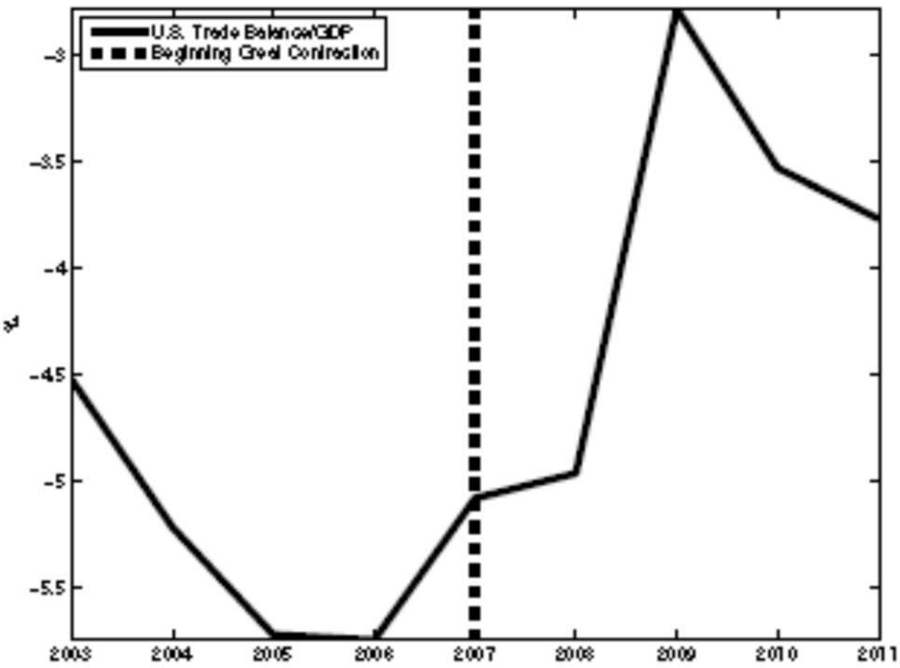


FIG. 3.25 – The Great Contraction and The Trade Balance - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 4

between 2003 and 2011. Over the four years preceding the Great Contraction, 2003-2006, the average U.S. trade balance deficit was about 5.4 percent of GDP. Over the four years since the onset of the Great Contraction, 2007-2011, the trade balance deficit was on average 3.4 percent of GDP. This means that the crisis era was associated with an improvement in the trade balance of 2 percent of output.

3.5 External Adjustment in Small and Large Economies

In section 3.3, we provide the microfundations for savings and investment behavior. This chapter takes stock of those results by condensing them in a convenient, user-friendly, synthetic apparatus. The resulting framework provides a simple graphical toolkit to study the determination of savings, investment, and the current account at the aggregate level.

3.5.1 The Current Account Schedule

3.5.1.1 The Savings and Investment Schedules

Figure 3.26 summarizes the results obtained thus far in section 3.3. Panel (a) plots the investment and saving schedules. The investment schedule, $I(r_1)$, is the same as the one shown in Figure 3.14. It describes a negative relation between the level of investment and the interest rate resulting from the profit-maximizing investment choice of firms (see equation (3.47)) :

$$F'(K_2) = 1 + r. \tag{3.123}$$

The schedule is downward sloping because an increase in the interest rate raises the rental cost of capital thus inducing a decline in the demand for equipment, structures, and the like.

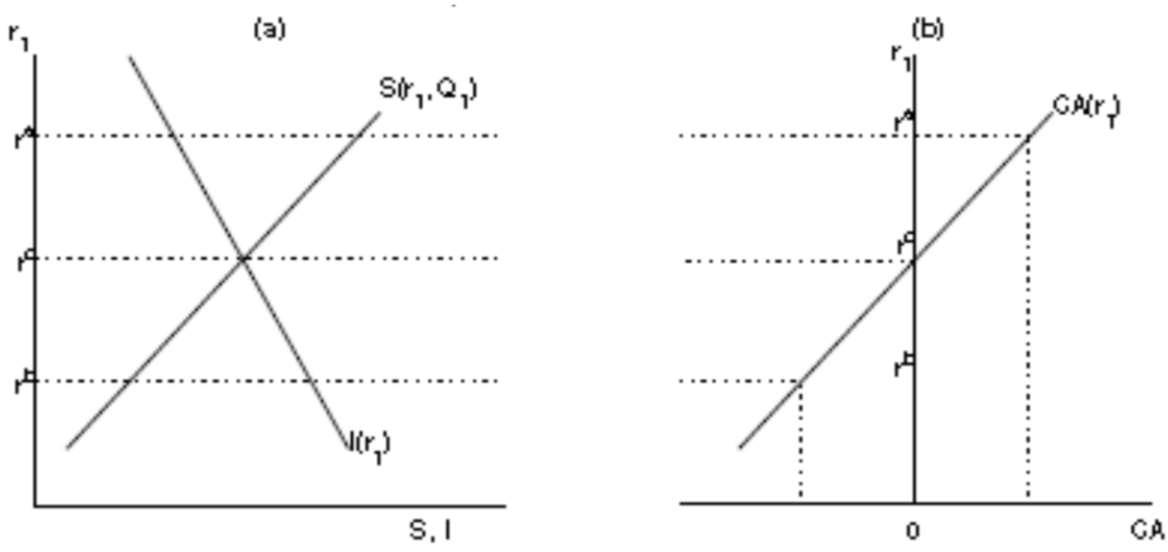


FIG. 3.26 – Savings, investment and the current account - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

The saving schedule, $S(r_1, Y_1)$, relates savings to the interest rate and output in period 1. Savings are increasing in both the interest rate and output. An increase in the interest rate affects savings through three channels : first, it induces an increase in savings as agents substitute future for current consumption. This is called the substitution effect. Second, an increase in the interest rate affects savings through an income effect. If the country is a net foreign debtor, an increase in the interest rate makes its residents poorer and induces them to cut consumption. In this case, the income effect reinforces the substitution effect. However, if the country is a net creditor, then the increase in the interest rate makes households richer, allowing them to consume more and save less. In this case the income effect goes against the substitution effect. Third, an increase in the interest rate has a positive effect on savings because it lowers the present discounted value of income from profit in period 2 ($\frac{\Pi_2}{1+r}$). We will assume that the first and third effects combined are stronger than the second one, so that savings is an increasing function of the interest rate. In sections **??** and 3.3.5 we analyzed the effects of temporary output shocks in the context of a two-period economy and derived the result that savings are increasing in period 1's output, Y_1. This result arises because an increase in Y_1 represents, holding other things constant, a temporary increase in income, which induces households to increase consumption in both periods. Thus, households save more in period 1 in order to consume more in period 2 as well.

3.5.1.2 Analytical Derivation of the Savings Schedule

Pour décomposer les trois effets de manière analytique, nous procédons en deux étapes. D'abord, il est nécessaire de différentier totalement l'égalité entre le TMS intertemporel et le taux d'intérêt $1 + r^\star$:

$$\frac{dC_2}{C_2} = \sigma_C \times d\ln(1+r) + \frac{dC_1}{C_1}. \tag{3.124}$$

Puis dans une deuxième étape, on différentie totalement la contrainte budgétaire intertemporelle $C_1 = \Omega - \frac{C_2}{1+r}$:

$$dC_1 = d\Omega - \frac{C_2}{1+r}\frac{dC_2}{C_2} + \frac{C_2}{1+r}\frac{d(1+r)}{1+r}, \tag{3.125}$$

où $\frac{d(1+r)}{1+r} = d\ln(1+r)$. En substituant (3.124) dans (3.125), on obtient la variation de la consommation à la période 1 :

$$dC_1 = d\Omega - \frac{C_2}{1+r}\frac{dC_1}{C_1} + \frac{C_2}{1+r}\left(1-\sigma_C\right)\frac{d(1+r)}{1+r}.$$

En réarrangeant les termes et en utilisant le fait que $C_1 + \frac{C_2}{1+r} = \Omega$, on obtient la variation de la consommation à la période 1 en fonction de la variation de la richesse et du taux d'intérêt :

$$\frac{dC_1}{C_1} = \frac{d\Omega}{\Omega} + \frac{\Omega - C_1}{\Omega}\left(1-\sigma_C\right)\frac{d(1+r)}{1+r} \tag{3.126}$$

où on a utilisé le fait que $\frac{C_2}{1+r} = \Omega - C_1$.

En posant $A_0 = 0$, l'épargne est simplement égale à la part du revenu Y_1 qui n'est pas consommée C_1. En différentiant $S = Y_1 - C_1$, on obtient :

$$dS = dY_1 - C_1\frac{d\Omega}{\Omega} - C_1\frac{\Omega - C_1}{\Omega}\left(1-\sigma_C\right)\frac{d(1+r)}{1+r}. \tag{3.127}$$

L'épargne est composé de trois termes : i) la variation du revenu à la première période, ii) la variation du revenu permanent, iii) la variation du taux d'intérêt.

Comme l'indique le troisième terme terme, l'effet revenu l'emporte sur l'effet-substitution lorsque $\sigma_C < 1$; dans ce cas, l'individu est peu enclin à substituer la consommation future à la consommation présente. Lorsque l'on considère des préférences logarithmiques, $\sigma_C = 1$ de telle sorte que le troisième terme de (**??**) disparaît. Pourtant, nous avons vu qu'une hausse du taux d'intérêt diminuait l'épargne. La raison est que la variation du revenu permanent se décompose lui-même en trois termes :

$$d\Omega = dY_1 + \frac{d\Pi_2}{1+r} - \frac{\Pi_2}{1+r}\frac{d(1+r)}{1+r}, \tag{3.128}$$

où $d\Pi_1 = dY_1$ car nous avons supposé que $A_0 = 0$ ce qui implique $K_1 = 0$ et donc $\Pi_1 = Y_1$. Le troisième terme montre qu'une hausse du taux d'intérêt réduit également la valeur actualisée des revenus ce qui en retour conduit l'individu à réduire sa consommation à la période 1 comme il est moins riche en valeur présente. Puisque le revenu à la période 1 est inchangé, il est conduit à épargner davantage : S augmente.

Dans la section 3.3.4, nous avons déterminé les valeurs d'équilibre en imposant plusieurs hypothèses qui permettent une résolution analytique : une fonction d'utilité logaritmique de telle sorte que $\sigma_C = 1$, le taux de préférence pour le présent est nul, $\rho = 0$, le taux de dépréciation du capital est égal à 1, $\delta = 1$. Dans ce cas, en utilisant le fait que $C_2 = (1+r)\,C_1$ et en utilisant cette relation pour éliminer C_2 de la CBI, $C_1 + \frac{C_2}{1+r} = \Omega$, la consommation à la période 1 est égale à

$$C_1 = \frac{1}{2} \times \Omega. \tag{3.129}$$

En utilisant le fait que $S_1 = Y_1 - C_1$, on obtient l'épargne :

$$S_1 = \frac{1}{2}Y_1 - \frac{1}{2} \times \frac{\Pi_2}{1+r}. \tag{3.130}$$

La relation (3.130) montre bien que l'épargne est une fonction croissante du taux d'intérêt r.

3.5.1.3 Derivation of the Current Account Schedule

Having established the way in which the interest rate and current output affect savings and investment, it is easy to determine the relationship between these two variables and the current account. This is because the current account is given by the difference between savings and investment ($CA_1 = S_1 - I_1$). Panel (b) of figure 3.26 illustrates this relationship. Suppose that the interest rate is r^a. Then savings exceed investment, which implies that the current account is in surplus. If the interest rate is equal to r^c, then investment equals savings and the current account is zero. Note that $r^c = r_1$ is the interest rate that would prevail in a closed economy, that is, in an economy that does not have access to international capital markets. For interest rates below r^c, such as r^b, investment is larger than savings so that the country runs a current account deficit. Therefore, as shown in panel (b), the current account is an increasing function of the interest rate.

Formally, assuming the initial foreign asset position is nil, $B_0 = 0$, capital fully depreciates, $\delta = 1$, and the time preference rate is set to zero, $\rho = 0$, the current account is equal to the net foreign asset position in period 1

$$B_1 = CA_1 = S_1 - I_1 = \frac{1}{2}Y_1 - \frac{1}{2} \times \frac{\Pi_2}{1+r} - \left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}. \tag{3.131}$$

where $I_1 = K_2$ and we used (3.80), i.e., $Y_2 = (K_2)^\alpha$ with $0 < \alpha < 1$. A rise in r increases savings while reducing investment which in turn improves the net foreign asset position $B_1 = CA_1$. Using (3.85), we thus impose

$$r^\star < r_1 = \alpha \left[\frac{\alpha}{1+\alpha} \times Y_1\right]^{\alpha-1} - 1. \tag{3.132}$$

3.5.2 The Current Account Schedule

In a small open economy with free capital mobility, in equilibrium the domestic interest rate must equal the world interest rate, $r^\star$, that is :

$$r_1 = r^\star. \tag{3.133}$$

Depending on whether the equilibrium interest rate is higher or smaller than the world interest rate, the economy will be a net debtor ($B_1 < 0$) or a net creditor ($B_1 > 0$). Graphically he equilibrium level of the current account is obtained by evaluating the current account schedule at $r = r^\star$. Figure 3.27 shows the equilibrium level of the current account, $CA(r^\star)$. Because the world interest rate is lower than the equilibrium interest rate in a closed economy, the country runs a country deficit as investment rises (because the capital cost is lower) and savings falls (as the present value of profits declines).

3.5.2.1 Interest Rate Shock

We begin by revisiting the effects of world interest rate shocks. Suppose a small open economy that initially faces the world interest rate $r^{0,\star}$ as shown in Figure 3.28. At that interest rate, the country runs a current account deficit equal to CA^0. Suppose now that the world interest rate rises to $r^{1,\star}$. The change in the world interest rate does not shift the current

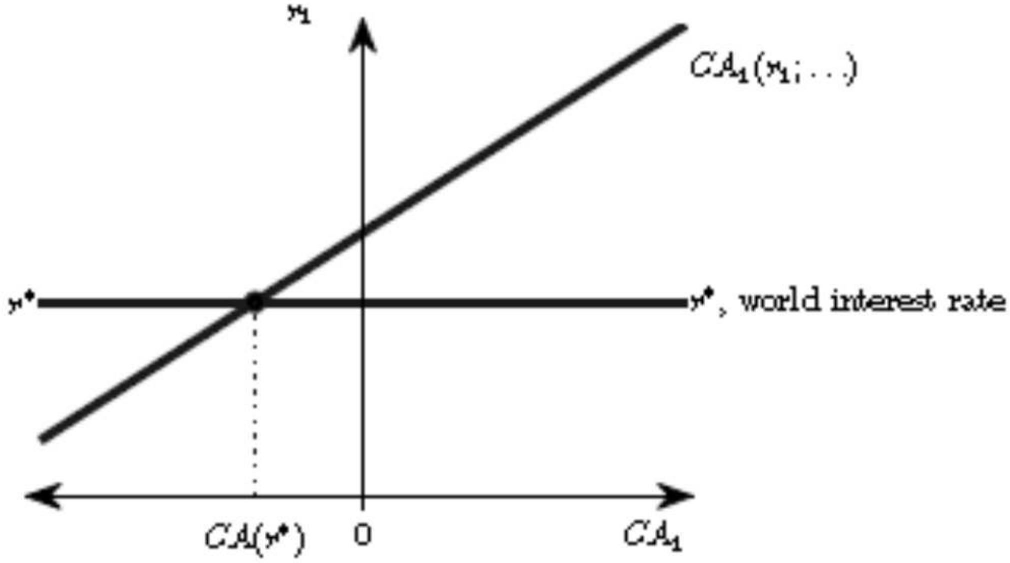


FIG. 3.27 – Current Account Determination in a Small Open Economy - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

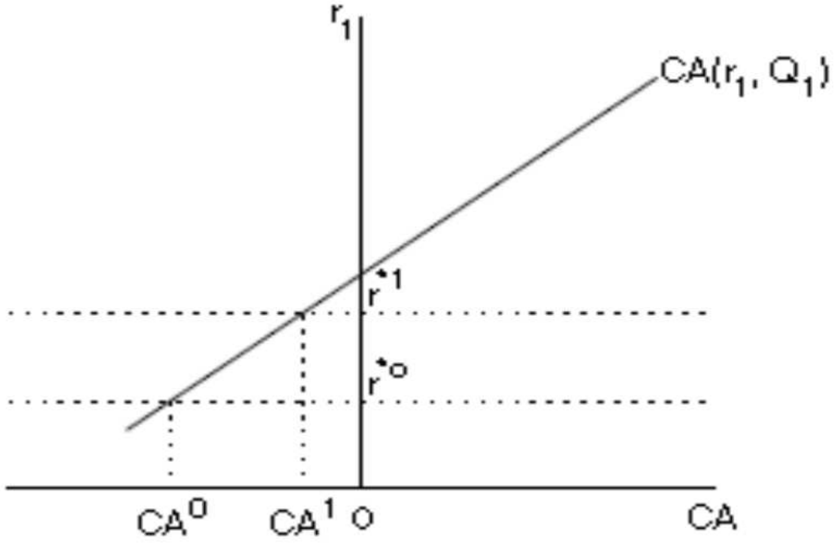


FIG. 3.28 – Current account adjustment to an increase in the world interest rate - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

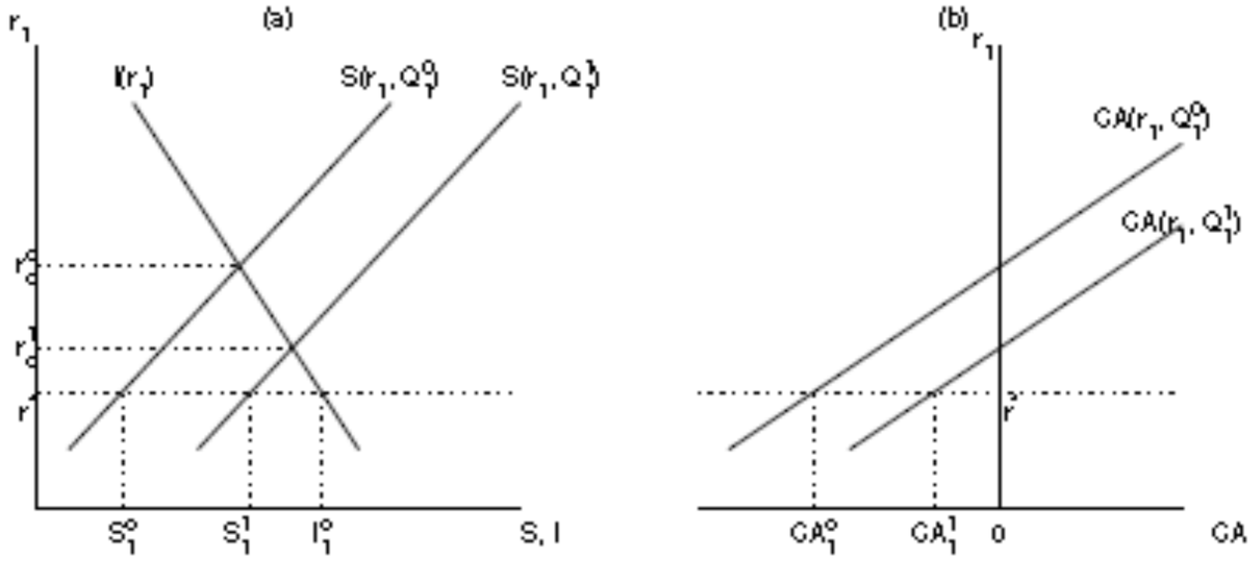


FIG. 3.29 – Current account adjustment to a temporary increase in output - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

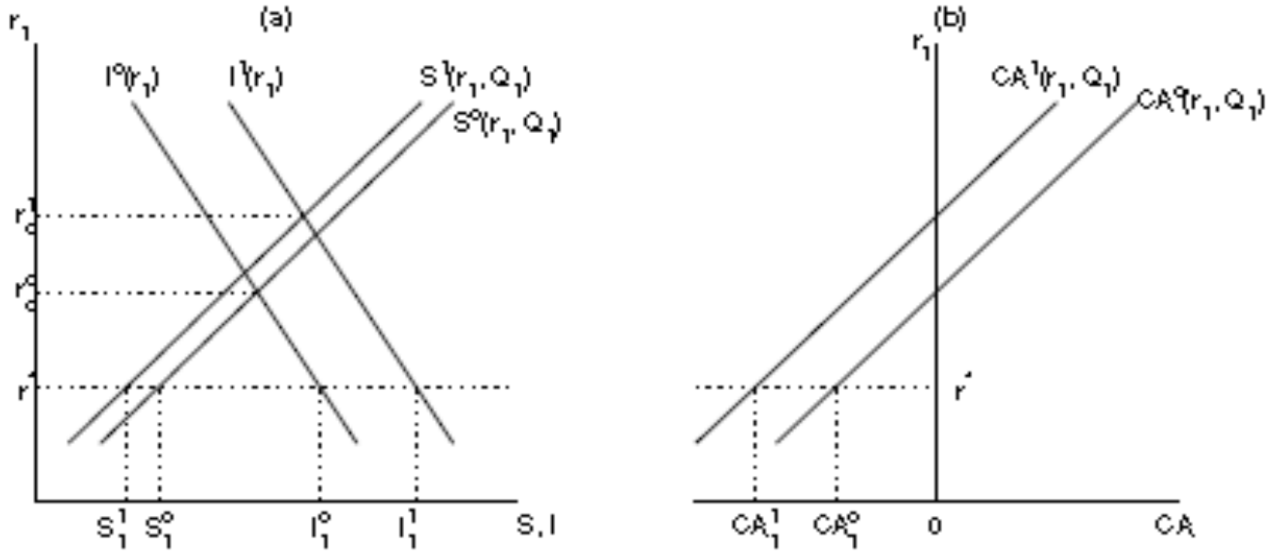


FIG. 3.30 – An Investment Surge - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

account schedule. Hence the equilibrium value of the current account is given by the point where the (unchanged) current account schedule intersects the new higher world interest rate level. The higher world interest rate encourages domestic saving and forces firms to reduce investment in physical capital. As a result, in equilibrium the current account deficit declines from CA^0 to CA^1.

3.5.2.2 Temporary Output Shock

Consider next the effects of a temporary positive income shock, that is, an increase in Y_1. We illustrate the effects of this shock in Figure 3.29. Suppose that output in period 1 is initially equal to Y_1^0. At the world interest rate $r^\star$, savings are equal to S_1^0, investment is equal to I_1^0, and the current account is $CA^0 = S_1^0 - I_1^0$. Suppose now that Y_1 increases to $Y_1^1 > Y_1^0$. As shown in section 3.3.6, this increase in Y_1 :

- raises consumption C_1 and C_2 ; in order to reach higher consumption in period 2, the agent must save a fraction of the output increases ; hence, savings rise ; as a result, as depicted in Panel (a) of Figure 3.29, the rise in Y_1 shifts the saving schedule to the right because households in an effort to smooth consumption over time, save part of the increase in income.
- The rise in saving lowers the interest rate prevailing in the closed economy r_1. Yet, in an open economy, the interest rate is fixed ; as a result, the investment schedule does not move because investment is not affected by current income.
- The current account is the difference between savings and investment, i.e., $CA_1^1 = S_1^1 - I_1^0$. The difference between savings and investment is larger than before the increase in income. As a result, the current account schedule shifts to the right which reflects an improvement in the current account balance.

3.5.2.3 An Investment Surge

Suppose that in period 1 agents learn that in period 2 the productivity of capital will increase. For example, suppose that the production function in period 2 was initially given by $F(K_2) = \sqrt{K_2}$ (setting $\alpha = 1/2$ into the production function (3.80)) and that due to a technological advancement it changes to $F(K_2) = 2 \times \sqrt{K_2}$. Another example of an investment surge is given by an expected increase in the price of exports. In Norway, for instance, the oil price increase of 1973 unleashed an investment boom of around 10% of GDP. This scenario is illustrated in Figure 3.30. As shown in section 3.3.6, this increase in Y_1 :

- increases investment $I_1 = K_2$ in period 1 car the marginal product of capital rises ; the news of the future productivity increase shifts the investment schedule to the right to I^1,
- raises consumption C_1 and C_2 because agents are richer ; savings decline in period 1 S_1 since consumption C_1 rises while output in period 1 is unchanged ; the savings schedule shifts to the left to S^1 ;
- because investment increases while savings fall, the current account CA_1 deteriorates ; the current account schedule shifts to the left from CA^0 to CA^1.

3.5.3 External Adjustment in a Large Open Economy

3.5.3.1 The Current Account Schedule : Graphical Apparatus

Thus far, we have considered current account determination in a small open economy. We now turn to the determination of the current account in a large open economy like the United States. Let's divide the world into two regions, the United States (US) and the rest of the world (RW). Because a U.S. current account deficit represents the current account surplus of the rest of the world and conversely, a U.S. current account surplus is a current account deficit of the rest of the world, it follows that the world current account must always be equal to zero ; that is,

$$CA^{US} + CA^{RW} = 0, \tag{3.134}$$

where CA^{US} and CA^{RW} denote, respectively, the current account balances of the United States and the rest of the world. Figure 3.31 shows the current account schedules of the U.S. and the rest of the world. The innovation in the graph is that the current account of the rest of the world is measured from right to left, so that to the left of the vertical axis, the rest of the world has a CA surplus and the U.S. a CA deficit, whereas to the right of the vertical axis, the U.S. runs a CA surplus and the rest of the world a CA deficit. Equilibrium in the world capital markets is given by the intersection of the CA^{US} and CA^{RW} schedules. In the Figure 3.31, the equilibrium is given by point A, at which the U.S. runs a current account deficit and the rest of the world a current account surplus. Consider now an investment surge in the U.S. that shifts the CA^{US} schedule to the left to $CA^{US,\prime}$. The new equilibrium is given by point B, where the schedule $CA^{US,\prime}$ and the schedule CA^{RW} intersect. At point B, the world interest rate is higher, the US runs a larger CA deficit, and the rest of the world runs a larger CA surplus. Note that because the U.S. is a large open economy, the investment surge produces a large increase in the demand for loans, which drives world interest rates up. As a result, the deterioration in the U.S. current account is not as pronounced as the one that would have resulted if the interest rate had remained unchanged (point C in Figure 3.31). Note further in a closed economy, the current account is nil because savings equalize investment. An investment surges produces a rise in the interest rate in a closed economy as a result of higher demand for capital. In a two-country world, the increase in the U.S. interest rate is smaller than the one that would have occurred if the US economy was closed (given by the distance between D' and D in Figure 3.31) because savings in the rest of the world rises which moderates the interest rate increase.

3.5.3.2 The Current Account Schedule : A Two-Country Model

The Framework

Consider a two-period model with two large open endowment economies, US for the United States and CH for China, and a single traded good. In both countries household preferences over period-1 consumption, C_1, and period-2 consumption, C_2, are given by the following time-separable utility function

$$U(C_1, C_2) = \ln C_1 + \ln C_2. \tag{3.135}$$

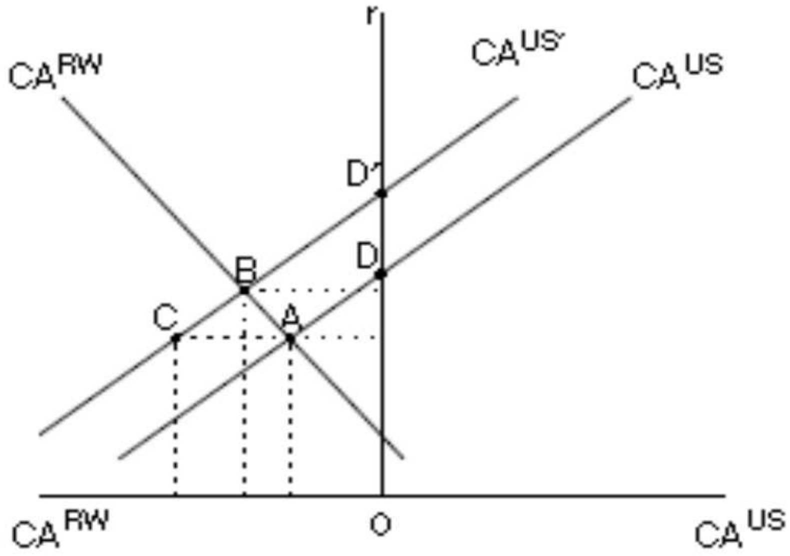


FIG. 3.31 – Current account determination in a large open economy - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

The endowment in country US is constant over time and equal to Y, that is,

$$Y_1^{US} = Y, \quad Y_2^{US} = Y. \tag{3.136}$$

By contrast, the endowment in country CH is growing over time. Let Y_1^{CH} denote the endowment in country CH in period 1, and QC2 its endowment in period 2. Assume that

$$Y_1^{CH} = \frac{Y}{2}, \quad Y_2^{CH} = Y. \tag{3.137}$$

Further assume that the net foreign asset position at the beginning of period 1 is zero in both countries :

$$B_0^{US} = B_0^{CH} = 0. \tag{3.138}$$

Optimal Current Account

The problem of households in country US consist in choosing consumption in period 1, C_1^{US}, consumption in period 2, C_2^{US}, and net foreign assets at the end of period 1, B_1^{US} so as to maximize lifetime utility, which is given by (3.135). The budget constraints of households in country US in period 1 and in period 2 are given by :

$$B_1^{US} = Y_1^{US} - C_1^{US}, \tag{3.139a}$$
$$C_2^{US} = (1 + r_1^s)\ .B_1^{US} + Y_2^{US}. \tag{3.139b}$$

Each country must have $B_2^{US} = B_2^{CH} = 0$ for the intertemporal solvency condition do hold. Use the budget constraint in period 2 (3.139b) to eliminate B_1^{US} from the period 1 budget constraint (3.139a) :

$$C_1^{US} + \frac{C_2^{US}}{1 + r_1^s} = Y_1^{US} + \frac{Y_2^{US}}{1 + r_1^s} \equiv \Omega^{US}. \tag{3.140}$$

Eliminating C_2^{US} from the utility function (3.135) :

$$\ln C_1^{US} + \ln (1 + r_1^s) \left(\Omega^{US} - C_1^{US}\right)$$

where $\Omega^{US} \equiv Y_1^{US} + \frac{Y_2^{US}}{1+r_1^s}$. Differentiating w.r.t. C_1^{US}, we get the equality between the intertemporal MRS and the gross return on net foreign assets (equal to the slope of the intertemporal budget constraint)

$$\frac{C_2^{US}}{C_1^{US}} = 1 + r_1^s. \tag{3.141}$$

Next use the intertemporal budget constraint, equation (3.140), to find the optimal level of period 1 consumption as a function of the interest rate :

$$C_1^{US} = \frac{1}{2}\,.\Omega^{US}. \tag{3.142}$$

The combination of (3.139a) and (3.142) gives :

$$CA_1^{US} = Y - C_1^{US} = \frac{1}{2}\,.Y\frac{r_1^s}{1+r_1^s}. \tag{3.143}$$

Notice that the current account schedule is upward sloping in the (CA, r)-space, that is, the higher the interest rate, the higher the current account.

Applying the same logic to country CH, we find the optimal level of period 1 consumption :

$$C_1^{US} = \frac{1}{2}\,.\Omega^{CH} = \frac{1}{2}\,.\left(\frac{Y}{2} + \frac{Y}{1+r_1^c}\right). \tag{3.144}$$

Using the period 1 budget constraint according to which $CA_1^{CH} = B_1^{CH} = \frac{Y}{2} - C_1^{CH}$ leads to the current account schedule of country CH as a function of the interest rate : current account schedule of country C as a function of the interest rate,

$$\begin{aligned} CA_1^{CH} &= \frac{Y}{2} - C_1^{CH}, \\ &= \frac{Y}{2} - \frac{1}{2}\,.\left(\frac{Y}{2} + \frac{Y}{1+r_1^c}\right), \\ &= \frac{Y}{4} - \frac{1}{2}\,.\frac{Y}{1+r_1^c}. \end{aligned} \tag{3.145}$$

The current account schedule is upward sloping in the (CA, r)-space. More importantly, as long as $r_1^c < 1$ (which is an assumption mostly reasonable), country CH will run a current account deficit in period 1. The reason is that the country has much higher income in period 2 than in period 1. To smooth consumption over time country CH thus has to borrow in period 1 and repay in period 2. This example cannot explain why the country that is expected to grow faster runs a surplus (China) against the slower growing country (the U.S.).

Market Clearing in World Capital Markets

In equilibrium the world current account balance has to be zero, that is,

$$CA_1^{US} + CA_1^{CH} = 0, \tag{3.146}$$

which implies $CA_1^{CH} + CA_1^{US} = \frac{Y}{2} - C_1^{CH} + Y - C_1^{US} = 0$

$$C_1^{US} + C_1^{CH} = \frac{3}{2}\,.Y, \tag{3.147a}$$

$$C_2^{US} + C_2^{CH} = 2\,.Y, \tag{3.147b}$$

where the second equation comes from the fact that $B_1^{US} = CA_1^{US}$ and $C_2^{US} + C_2^{CH} = Y + (1+r_1)CA_1^{US} + Y + (1+r_1)CA_1^{CH} = 2\,.Y$.

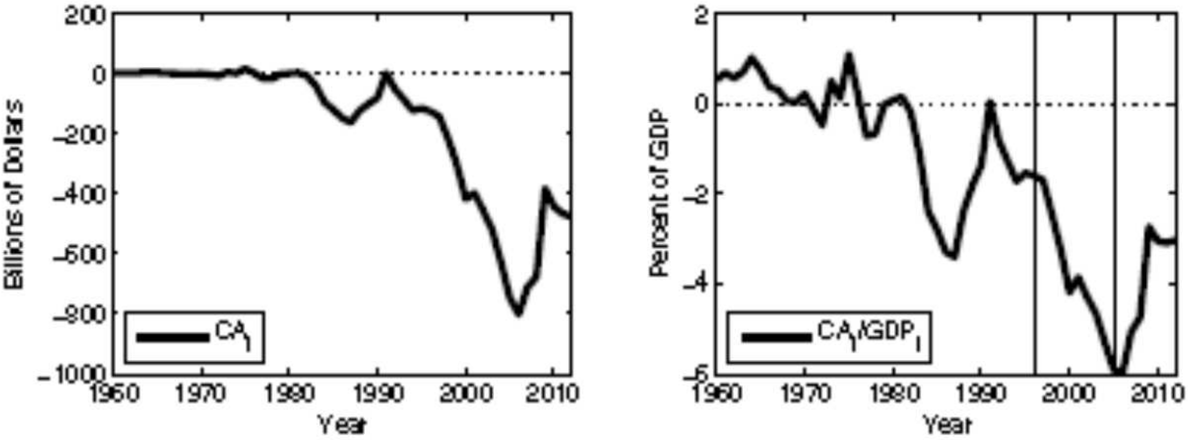


FIG. 3.32 – The U.S. Current Account Balance : 1960-2012 - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

Market Clearing in World Capital Markets

We assume that there is free capital mobility in both countries. Then in equilibrium the interest rate must be the same in both countries, that is,

$$r_1 = r_1^s = r_1^c. \tag{3.148}$$

Substituing CA_1^{US} (3.143) et CA_1^{CH} (3.145) into (3.146) gives :

$$\frac{1}{2}\left(\frac{r_1}{1+r_1}\right).Y + \frac{1}{4}\left(\frac{r_1-1}{1+r_1}\right).Y = 0.$$

Solving yields :

$$r_1 = \frac{1}{3}. \tag{3.149}$$

3.5.4 What factors are responsible for the U.S. current account deficit

As shown in left panel of Figure 3.32, between 1995 and 2005, the U.S. current account deficit experienced a dramatic increase from 125$ to 623$ billion dollars. This 500 billion dollar increase brought the deficit from a relatively modest level of 1.5 percent of GDP in 1995 to close to 6 percent of GDP in 2005. With the onset of the great recession of 2007, the ballooning of the current account deficits came to an abrupt stop. By 2009, the current account deficit had shrunk back to 3 percent of GDP, as illustrated in the left panel of Figure 3.32. An important question is what factors are responsible for these large swings in the U.S. current account. In particular, we wish to know whether the recent rise and fall in the current account deficit were driven by domestic or external factors.

3.5.4.1 The Period 1996-2006 : The Global Savings Glut Hypothesis : Caballero, Farhi and Gourinchas (2008)

In 2005 Ben Bernanke, then a governor of the Federal Reserve, gave a speech in which he argued that the deterioration in the U.S. current account deficits between 1996 and 2004 were caused by external factors. He raises the 'global saving glut' hypothesis to provide an

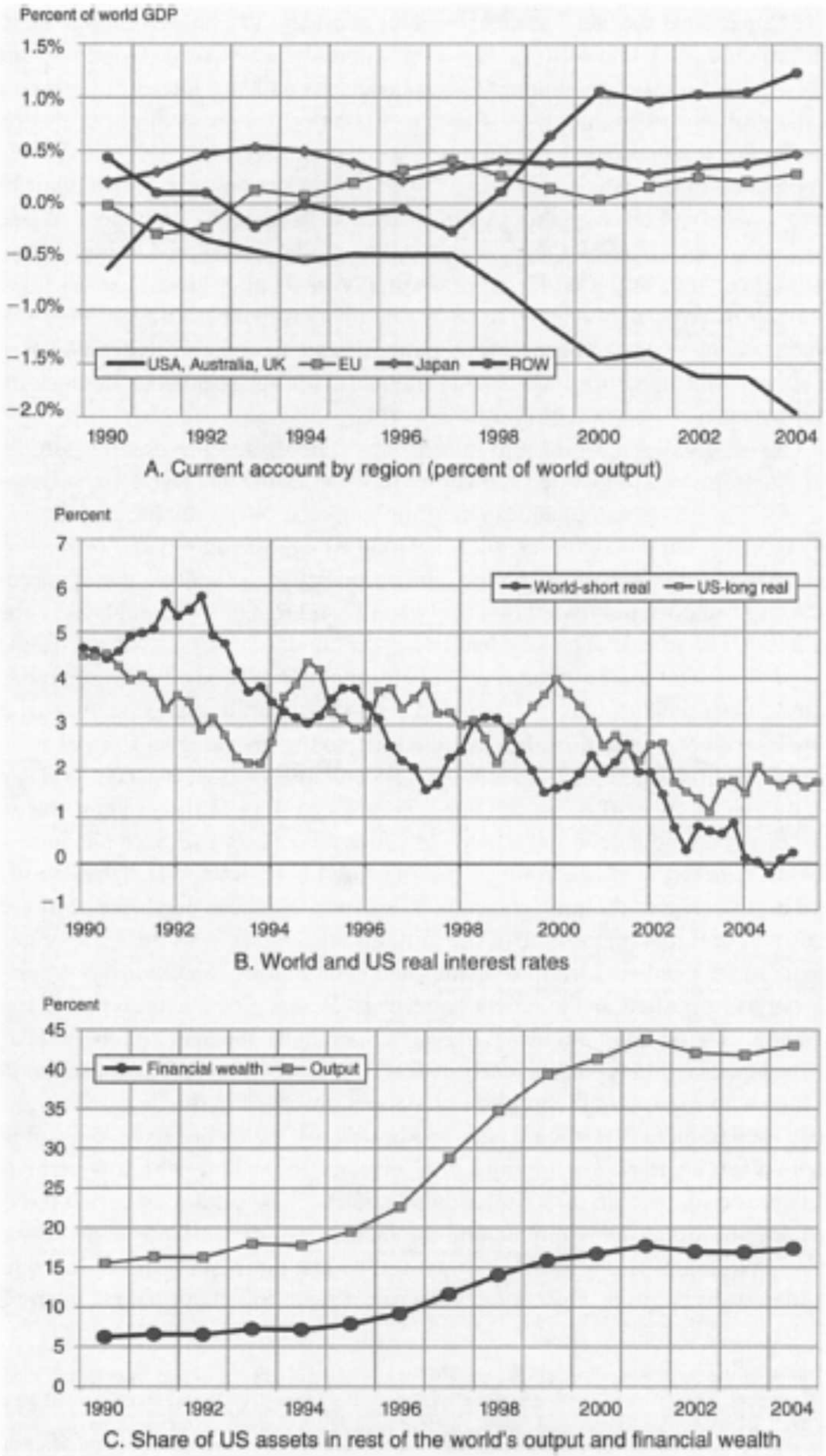


Fig. 3.33 – Three Stylized Facts - Source : Caballero, Farhi, et Gourinchas (2008) An equilibrium model of global imbalances and low interest rates. *American Economic Review*, 98(1), pp. 358-393

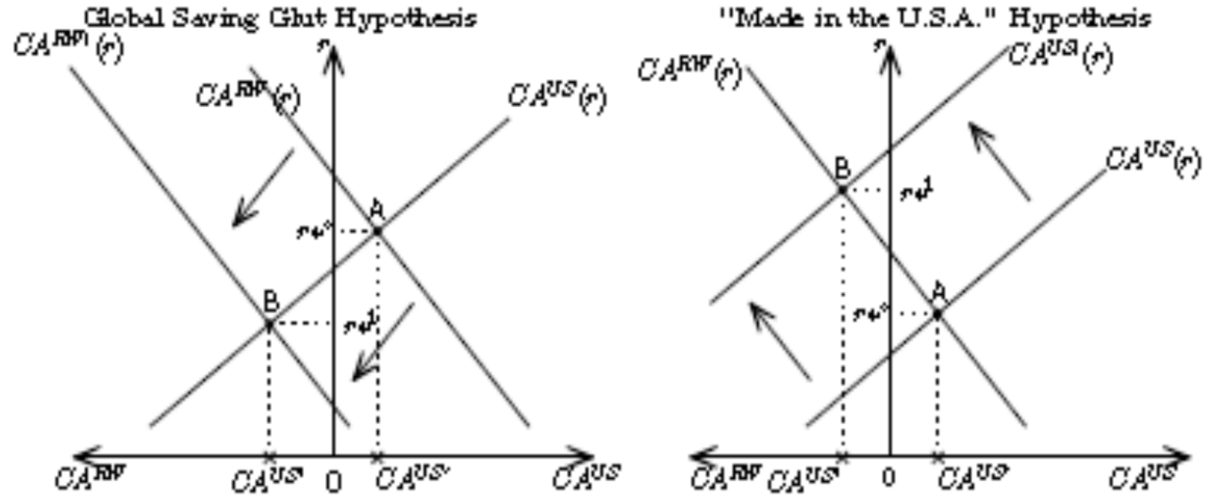


FIG. 3.34 – U.S. Current Account Deterioration : Global Saving Glut or internal factors - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

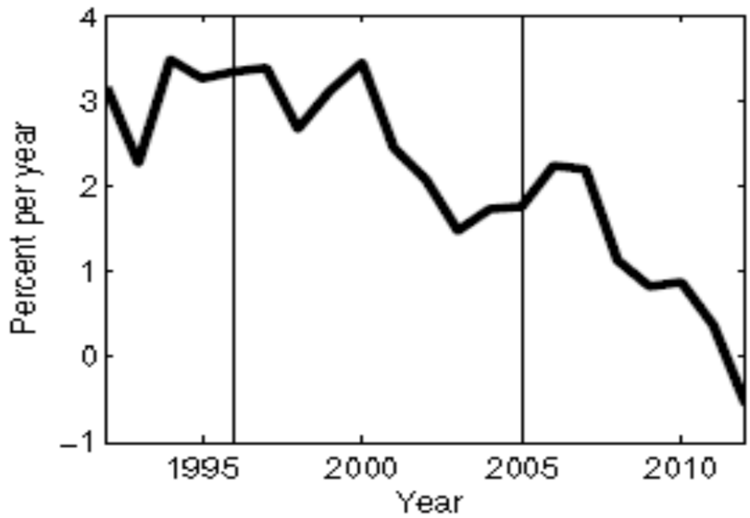


FIG. 3.35 – The World Interest Rate : 1992-2012 - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 6

explanation of the US current account deficit. In particular, Bernanke argued that the rest of the world experienced a heightened desire to save but did not have incentives to increase domestic capital formation in a commensurate way. As a result, the current account surpluses of the rest of the world had to be absorbed by current account deficits in the United States. Much of the increase in the desired current account surpluses in the rest of the world during this period originated in higher desired savings in emerging market economies. In particular, Bernanke attributes the increase in the desire to save to two factors : (1) Increased foreign reserve accumulation to avoid or be better prepared to face future external crises of the type that had afflicted emerging countries in the 1990s. And (2) Currency depreciations aimed at promoting export-led growth.

In a research paper published in the *American Economic Review* in 2008, Caballero, Farhi and Gourinchas formalize this idea by using a two-country world model. To test the 'global savings glut' hypothesis, the authors aim at replicating three stylized facts which are illustrated in Figure 3.33 :

1. Fact 1 : The United States has run a persistent current account deficit since the early 1990s, which has accelerated dramatically since the late 1990s. By 2004, it exceeded US$600 billion a year. The solid dark line in Figure 3.33A illustrates this path, as a ratio of world GDP (this line also includes the deficits of the United Kingdom and Australia, for reasons that will be apparent below, but it is overwhelmingly dominated by the US pattern). The counterpart of these deficits has been driven by the surpluses in Japan and Continental Europe throughout the period and, starting at the end of the 1990s, by the large surpluses in Asia minus Japan, commodity producers, and the turnaround of the current account deficits in most non-European emerging market economies.
2. Fact 2 : The long-run real interest rate has been steadily declining over the last decade, despite efforts from central banks to raise interest rates, as shown in Figure 3.33B.
3. Fact 3 : The importance of US assets in global portfolios has increased throughout the period, and by 2004 it amounted to over 17 percent of the rest of the world's financial wealth, which is equivalent to 43 percent of the annual output of the rest of the world (see Figure 3.33C).

To present the idea of the author, we can use the graphical tools developed above. In Figure 3.33A, the world is divided into four groups. The Figure shows that the most important interaction is between the US and the RW. Thus, our analysis is about global equilibrium in a US-RW.

'Global Savings Glut' Hypothesis

The left panel of Figure 3.34 illustrates the effect of a desired increase in savings in the rest of the world. The initial position of the economy, point A, is at the intersection of the CA^{US} and CA^{RW} schedules. In the initial equilibrium, the U.S. current account equals CA_0^{US} and the world interest rate equals $r^{\star,0}$. The increase in the desired savings of the rest of the world shifts the current account schedule of the rest of the world down and to the left as depicted by the schedule $CA^{RW,\prime}$. The new equilibrium at point B in the left panel of Figure 3.34, features a deterioration in the current account deficit of the U.S. from CA_0^{US} to CA_1^{US} and a fall in the world interest rate from $r^{\star,0}$ to $r^{\star,1}$. Intuitively, the United States will borrow more from the rest of the world only if it becomes cheaper to do so, that is, only if the interest rate falls. This prediction of the model implies that if the global saving glut hypothesis is valid, then we should have observed a decline in the interest rate.

'Made in the U.S.A.' Hypothesis

The 'Made in the U.S.A.' hypothesis is illustrated in the right hand panel of Figure 3.34. Again, in the initial equilibrium at point A, the U.S. current account equals CA_0^{US} and the world interest rate equals $r^{\star,0}$. Under this view, the current account schedule of the rest of the world is unchanged and instead the current account schedule of the United States shifts to the left as depicted by the schedule CAUS0. The new equilibrium at point B in the right hand panel of Figure 3.34, features a deterioration in the current account deficit of the U.S. from CA_0^{US} to CA_1^{US} and a rise in the world interest rate from $r^{\star,0}$ to $r^{\star,1}$.

Both hypotheses can explain a deterioration in the U.S. current account. However, the 'global saving glut' hypothesis implies that the CA deterioration should have been accompanied by a decline in world interest rates, whereas the 'Made in the U.S.A.' hypothesis implies that world interest rates should have gone up. Hence we can use data on the behavior of interest rates to find out which hypothesis is right.

Figure 3.35 plots the world interest rate. It shows that over the period in question, 1996 to 2005, interest rates fell, validating the 'global saving glut' hypothesis and rejecting the 'Made in the U.S.A.' hypothesis.

3.5.4.2 The Period 2006 to 2012

Can the global saving glut hypothesis also explain changes in U.S. current account dynamics after 2005 ? Figure 3.32 shows that at its peak in 2006 the U.S. current account deficit had reached 6 percent of GDP. Over the following 3 years, the deficit was reduced to half, or 3 percent of GDP. Under the global saving glut hypothesis, this reduction in the current account deficit would be attributed to a decline in desired savings in the rest of the world.

Again we can use the graphical tools developed earlier in this chapter to evaluate the plausibility of this view. Consider the left panel of Figure 3.34. Assume that the initial equilibrium is at point B, where the world interest rate is equal to $r^{\star,1}$ and the U.S. current account deficit is equal to CA_1^{US}. We can represent a decline in desired savings in the rest of the world as a shift up and to the right in the current account schedule of the rest of the world. For simplicity, assume that this adjustment is shown as a return of the current account schedule of the rest of the world back to its original position given by CA^{RW} so that the new equilibrium is given by point A. This shift in the current account schedule of the rest of the world causes the U.S. current account to improve from CA_1^{US} to CA_0^{US} and the interest rate to rise from $r^{\star,1}$ to $r^{\star,0}$. It follows that under the global saving glut hypothesis, the V-shape of the U.S. current account balance observed between 1996 and 2009 (see figure 3.32), should have been accompanied by a V-shaped pattern of the interest rate. However, Figure 3.32 shows that the interest rate does not display a V-shaped pattern as predicted by the global saving glut hypothesis. In fact, since 2005 the interest rate has declined further rejecting the global saving glut hypothesis as an explanation of U.S. current account dynamics since 2005. We conclude that the global saving glut hypothesis presents a plausible explanation for the observed developments in the U.S. current account deficit over the period 1996-2005. At the same time, the empirical evidence, in particular, the behavior of interest rates, suggests that the dynamics of the U.S. current account since 2005 were not primarily driven by external factors, but instead by domestic disturbances.

In particular, a negative productivity shock lowers investment by reducing the marginal product of capital and produces a rise in savings by inducing agents to cut consumption. As a result, the current account schedule shifts to the right. In terms of the right panel of Figure 3.34, the U.S. current account improves from $CA^{US,\prime}$ to CA^{US} and the interest rate falls from $r^{\star,1}$ to $r^{\star,0}$: the economy moves from point B to point A. The rise in the current account is thus accompanied by a decline in the world interest rate, in line with the evidence shown in Figures 3.32 and 3.35, respectively.

3.5.5 Analytical Exploration of the Savings Glut

Households

At every instant, households face an instantaneous probability of dying θ. Since θ is common to all households, it represents the fraction of the population that dies every instant. A fraction θ of the population is also born every instant, so that total population remains constant, normalized to 1. Agents consume an amount C_t, hold financial assets W_t, and non financial wealth H_t. Total wealth is equal to $W_t + H_t$. Agents have a (subjective) rate of time preference equal to ρ. At each instant of time, agents consume a fraction $\rho + \theta$ of their total wealth (theory of permanent income in a model with a probability of death equal of θ) :

$$C_t = (\rho + \theta) \ . \left[W_t + H_t\right], \tag{3.150}$$

while the accumulation of financial wealth (i.e., savings) is equal to interest receipts from bonds holding (more on this later) plus labor income Z_t less consumption expenditure C_t :

$$\dot{W}_t = r_t \ .W_t + Z_t - C_t, \tag{3.151}$$

where H_t represents the present discounted value of non financial income of all currently alive cohorts :

$$H_t \equiv \int_t^{\infty} Z_\tau e^{-R(t,\tau)} \ .e^{-(\theta+\phi) \ .(\tau - t)} d\tau, \tag{3.152}$$

with $R(t,\tau)$ the annuity factor between time t and time τ (allowing us express future income in present value terms) :

$$R\left(t,\tau\right) = \int_t^{\tau} r_v dv. \tag{3.153}$$

More specifically, H_t represents the present discounted value of labor income accruing in the future to people currently alive, discounted at rate r_t or equivalently, it is the present discounted value of total future labor income discounted at the rate $r_t + \theta + \phi$ because θ represents the rate at which the population born at a certain date falls. The parameter ϕ takes into account the time profile of labor income, i.e., the fact that labor income decreases with age (due to retirement) :

$$z\left(s,t\right) = \frac{\phi + \theta}{\theta} \ .Z_t \ .e^{-\phi \ .(t-s)}, \quad \phi \geq 0. \tag{3.154}$$

Equation (3.154) states that, at any given time t, older workers (lower s) receive lower income with a slope controlled by ϕ. In the limit of $\phi \to \infty$, all non financial income is received by the newborn generation : $z(t,t) = Z_t$, and $z(s,t) = 0$ for $s < t$, so that $H_t = 0$ (because time is discounted at an infinite rate). As shall be clear later, this assumption simplifies the calculus.

Firms

We close the model by specifying the market structure and technology available to the household. Suppose that output is produced with the aggregate production function

$$Y_t = K_t^{\alpha} \, . \, (\xi_t \, .N_t)^{1-\alpha} \, , \tag{3.155}$$

where K_t is physical capital, $N_t = 1$ labor, and ξ_t labor augmenting productivity which rises at a (constant) rate :

$$\frac{\dot{\xi}_t}{\xi_t} = g. \tag{3.156}$$

Under financial autarky, physical capital K is the only asset available, so

$$W_t = K_t. \tag{3.157}$$

We make two simplifying assumptions. First, we assume that there is no depreciation of capital : $\delta_k = 0$. Firms choose aggregate capital stock by equating the marginal product of capital to the capital cost r. We assume that the allocation fo capital across sectors is inefficient (due to capital market frictions, see Antraàs and Caballero (2009)) which is captured by a wedge $1 - \tau$ between the social and private returns of capital :

$$(1-\tau) \; .\alpha \, .\frac{Y}{K} = r. \tag{3.158}$$

Setting $(1-\tau) \; .\alpha = \delta$, eq. (3.158) can be rewritten

$$\frac{K}{Y} = \frac{\delta}{r}, \quad \frac{r \, .K}{Y} = \delta < \alpha. \tag{3.159}$$

The second equality of equation (3.159) shows that the capital intensity of production δ (measured by the share of capital income in GDP) is lower that if capital market imperfections were absent. The reason is that inefficient allocation of capital across sectors reduces the private returns of capital which in turn reduces investment K/Y. Using the Euler theorem which implies that $Y = r \, .K + Z$, substituting (3.159) gives labor income

$$Z = Y - r \, .K = (1-\delta) \; .Y > (1-\alpha) \, . \tag{3.160}$$

Eq. (3.160) shows that the economy is more labor intensive due to capital market imperfections.

Capital Market Equilibrium in Autarchy

In autarchy, financial wealth consists only of claims on capital stock :

$$W_t = K_t. \tag{3.161}$$

Additionally, in the long-run, the interest rate is fixed at r_{ss}. The demand for capital (3.159) implies that GDP, Y_t, must rise at the same speed as the capital stock, K_t, i.e.,

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t}. \tag{3.162}$$

Applying logarithm and differentiating the production function (3.155) and substituting (3.162) implies that the capital stock rises at the same speed as labor-augmenting productivity (see (3.156)) :

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{\xi}_t}{\xi_t} = g. \tag{3.163}$$

Because $W_t = K_t$ (see (3.161)), both variables increase at the same rate, i.e.,

$$\frac{\dot{W}_t}{W_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{\xi}_t}{\xi_t} = g. \tag{3.164}$$

Using (3.151), we have :

$$\frac{\dot{W}_t}{W_t} = r_{ss} + \frac{Z_t}{K_t} - \frac{C_t}{K_t}, \tag{3.165}$$

where we used the fact that

$$\frac{C_t}{K_t} = (\rho + \theta)\ .\left(1 + \frac{H_t}{K_t}\right), \tag{3.166}$$

and labor income along a constant growh path is given by the present discounted value over an infinite time of labor income (keeping in mind that Z rises at rate g) :

$$H_t = \frac{Z_t}{r_{ss} + \theta + \phi - g}. \tag{3.167}$$

To determine (3.167), we proceed as follows. First, from (3.160), we know that Z_t rises at the same speed as Y_t. Hence, labor income at time τ when starting from time t is : $Z_\tau = Z_t\ .e^{g\ .\tau}$. Second, substituting $Z_t\ .e^{g\ .\tau}$ into (3.152) ans solving yields (we use the fact that Z_t is now independent from time τ) :

$$\begin{aligned} H_t &= Z_t\ .\int_t^\infty e^{-(\rho+\theta+r_{ss}-g)\ .(\tau-t)}d\tau, \\ &= \frac{Z_t}{r_{ss} + \theta + \phi - g}, \end{aligned} \tag{3.168}$$

where we used the fact that $\int_t^\infty e^{-(\rho+\theta+r_{ss}-g)\ .(\tau-t)}d\tau = \frac{1}{-(r_{ss}+\theta+\phi-g)}\ .(-1) = \frac{1}{r_{ss}+\theta+\phi-g}$.

Dividing H_t given by (3.167) by the capital stock K_t and using (3.159)-(3.160) gives

$$\begin{aligned} \frac{H_t}{K_t} &= \frac{Z_t}{K_t\ .(r_{ss} + \theta + \phi - g)}, \\ &= \frac{(1-\delta)\ .Y_t}{\frac{\delta}{r_{ss}}\ .Y_t\ .(r_{ss} + \theta + \phi - g)}, \\ &= \frac{(1-\delta)\ .r_{ss}}{\delta\ .(r_{ss} + \theta + \phi - g)}. \end{aligned} \tag{3.169}$$

Substituting consumption-to-capital ratio (3.166) and the non financial wealth-to-capital ratio (3.170) into the rate of growth of the stock of financial wealth (3.164), one obtains :

$$\begin{aligned} \frac{\dot{W}_t}{W_t} &= r_{ss} + \frac{Z_t}{K_t} - \frac{C_t}{K_t}, \\ &= r_{ss} + \frac{(1-\delta)\ .r_{ss}}{\delta} - (\rho+\theta)\ .\left[1 + \frac{(1-\delta)\ .r_{ss}}{\delta\ .(r_{ss} + \theta + \phi - g)}\right], \end{aligned} \tag{3.170}$$

where we used the fact that $\frac{Z_t}{K_t} = \frac{(1-\delta)\ .Y_t}{\frac{\delta}{r_{ss}}\ .Y_t} = \frac{(1-\delta)\ .r_{ss}}{\delta}$. Using the fact that in the long-run, $\frac{\dot{W}_t}{W_t} = g$, eq. (3.170) can be rewritten :

$$\begin{aligned} &\frac{\delta\ .r_{ss} + (1-\delta)\ .r_{ss}}{\delta} \\ - &\ (\rho+\theta)\ .\left[\frac{\delta\ .(r_{ss} + \theta + \phi - g) + (1-\delta)\ .r_{ss}}{\delta\ .(r_{ss} + \theta + \phi - g)}\right] \\ = &\ g. \end{aligned}$$

Rearranging terms,

$$\begin{aligned} & r_{ss} \cdot (r_{ss} + \theta + \phi - g) - (\rho + \theta) \ .\delta \cdot (r_{ss} + \theta + \phi - g) \\ & - (\rho + \theta) \ .(1 - \delta) \ .r_{ss} = g \ .\delta \cdot (r_{ss} + \theta + \phi - g) \, , \end{aligned}$$

the autarchy interest rate satisfies :

$$\begin{aligned} & [r_{ss} + \phi + \theta - g] \ .[r_{ss} - \delta \cdot (g + \rho + \theta)] \\ = \ & (\rho + \theta) \ .(1 - \delta) \ .r_{ss}. \end{aligned} \tag{3.171}$$

Two cases emerge :

- When $\phi = \theta = 0$ (which corresponds to the infinite horizon model), the autarchy interest rate reduces to $r^a_{ss} = g + \rho$.
- When $\phi \to \infty$, $H_t = 0$ (see (3.168). In this case, we have :

$$\begin{aligned} \frac{\dot{W}_t}{W_t} &= r_{ss} + \frac{Z_t}{K_t} - \frac{C_t}{K_t}, \\ &= r_{ss} + \frac{(1 - \delta) \ .r_{ss}}{\delta} - (\rho + \theta) = g. \end{aligned} \tag{3.172}$$

  Rearranging terms, the autarchy interest rate is found to be decreasing with the extent of capital market frictions δ (the smaller δ, the larger the investment wedge, the lower the capital-to-GDP ratio and the more labor intensive the economy) :

$$r^a_{ss} = \delta \cdot (g + \rho + \theta) \, . \tag{3.173}$$

  Compared to the neoclassical model, two parameters influence the autarky rate. First, the interest rate increases because the mortality risk θ makes agents more impatient, which reduces saving. Second, the interest rate decreases because only a share $\delta \leq 1$ of income is paid out as financial income. This second effect is due to the scarcity of stores of value in the non-Ricardian economy. When $\delta < \frac{g+\rho}{g+\rho+\theta}$, this second effect dominates and the interest rate falls below the autarky rate of the benchmark model. Economies with distorted domestic capital markets (low δ or high τ) are more likely to have lower autarky interest rate.

The main implication of the model is that low levels of financial development, associated with sufficiently low δ, can depress autarky interest rates. It is then possible for a country to have a low autarky rate, despite a high growth rate of productivity g. When $\phi \to \infty$, the marginal product of capital remains constant and equal to :

$$\begin{aligned} MP_k &= \delta \, .\frac{Y}{K} = r^a_{ss} = \delta \cdot (g + \rho + \theta) \, , \\ &= \frac{Y}{K} = (g + \rho + \theta) \, , \\ &= \alpha \, .\frac{Y}{K} = \alpha \cdot (g + \rho + \theta) \, , \end{aligned} \tag{3.174}$$

regardless of δ. In that case, we obtain the opposite result from the neoclassical benchmark model : variations in τ (or δ) are fully reflected in r^a_{ss}, and not in the marginal product of capital or the capital-output ratio. The explanation relies upon the asynchronicity between income and consumption decisions :

- In the neoclassical model, there is one generation which lives forever. In the steady state of the neoclassical model, this capital wedge does not affect the private rate of return to capital, still equal to $\rho + g$; the effect of the financial friction τ falls entirely on the marginal product of capital : $MP_k = \frac{\rho+g}{1-\tau}$.

– Instead, Caballero, Fahri, and Gourinchas (2008) emphasize that financial frictions also influence the autarky interest rate. In the overlapping generations model, when assuming ϕ is large, the newborn receives all labor income ; as a result, older people must build up savings in order to maintain consumption. If $\phi \to \infty$, this case maximizes the asynchronicity between income and consumption decisions since all income is received at birth, but consumption decisions need to be sequenced over a random lifetime. Due to high savings, the autarchy interest rate is low. In this special case, the fall in the autarchy interest rate exactly offsets the investment wedge so that the marginal product of capital MP_k is unaffacted.
– When ϕ takes intermediate values, the autarchy interest rate is low due to the asynchronicity between income and consumption decisions while the (social) marginal product of capital MP_k is larger than its private return. In this case, the fall in the autarchy interest rate due to higher savings does not compensate for the decline in the private return of capital. The model provides simultaneously a rationale for high marginal product of capital and low autarky rates in countries with low levels of financial development.

Open Economy and the Direction of Capital Flows

Following the steps described in the previous section, consider now the case of a small open economy facing a constant real interest rate r. For simplicity, we limit ourselves to the case where $\phi \to \infty$. We can determine the stock of financial wealth as a share of GDP. When $\phi \to \infty$, $H_t = 0$ (see (3.168). In this case, we have :

$$\begin{aligned} \frac{\dot{W}_t}{Y_t} &= r \, . \frac{W_t}{Y_t} + \frac{Z_t}{Y_t} - \frac{C_t}{Y_t}, \\ &= r \, . \frac{W_t}{Y_t} + (1-\delta) - (\rho+\theta) \, . \frac{W_t}{Y_t} = g. \end{aligned} \tag{3.175}$$

Rearranging terms, the financial wealth-GDP ratio is an increasing function of the world interest rate :

$$\frac{W_t}{Y_t} = \frac{1-\delta}{g+\rho+\theta-r}. \tag{3.176}$$

This equation expresses domestic wealth, i.e. the domestic demand for stores of value per unit of output, W_t/Y_t, as a function of the world interest rate. A higher interest rate increases the demand for stores of value since wealth accumulates at a higher rate.

As determined previously, the capital-to-GDP ratio is decreasing with the extent of capital market frictions and the interest rate :

$$\frac{K_t}{Y_t} = \frac{\delta}{r}. \tag{3.177}$$

This equation expresses the domestic supply of stores of value (here capital) as a function of the interest rate. A higher interest rate depresses the present discounted value of the payments to capital $\delta \, .Y$, which lowers the equilibrium capital-output ratio.

The difference between W_t and K_t represents the net foreign asset position of the country, B. Combining (3.176) with (3.177), we can express the net foreign asset position as a function of the autarky and world interest rates,

$$\begin{aligned}
\frac{B_t}{Y_t} &= \frac{W_t - K_t}{Y_t}, \\
&= \frac{1-\delta}{g+\rho+\theta-r} - \frac{\delta}{r}, \\
&= \frac{(1-\delta)\ .\delta\ .r - \delta\ .\left(r^a_{ss} - \delta\ .r\right)}{\left(r^a_{ss} - \delta\ .r\right)\ .r}, \\
&= \frac{\delta\ .\left(r - r^a_{ss}\right)}{\left(r^a_{ss} - \delta\ .r\right)\ .r},
\end{aligned} \tag{3.178}$$

where we used the fact that $g + \rho + \theta = \frac{r^a_{ss}}{\delta}$ when $\phi \to \infty$ (see eq. (3.173)). To determine the current account CA_t which corresponds to the change in the net foreign asset position $\dot{B}_t = \dot{W}_t - \dot{K}_t$, we first determine an expression of capital accumulation

$$\frac{\dot{K}_t}{Y_t} = \frac{K_t}{Y_t}\ .g = \frac{r}{\delta}\ .g. \tag{3.179}$$

Then we determine an expression of the rate of accumulation of financial wealth as ratio of GDP :

$$\begin{aligned}
\frac{\dot{W}_t}{Y_t} &= r\ .\frac{W_t}{Y_t} + \frac{Z_t}{Y_t} - \frac{C_t}{Y_t}, \\
&= r\ .\frac{W_t}{Y_t} + (1-\delta) - (\rho+\theta)\ .\frac{W_t}{Y_t}, \\
&= (1-\delta) + [r - (\rho+\theta)]\ .\left(\frac{1-\delta}{g+\rho+\theta-r}\right), \\
&= \left(\frac{1-\delta}{g+\rho+\theta-r}\right)\ .g, \\
&= g\ .\frac{W_t}{Y_t}.
\end{aligned} \tag{3.180}$$

Substituting (3.180) and (3.179) into $CA_t = \dot{B}_t = \dot{W}_t - \dot{K}_t$, the current account can be written as follows :

$$\begin{aligned}
\frac{CA_t}{Y_t} &= g\ .\left(\frac{W_t}{Y_t} - \frac{K_t}{Y_t}\right) = g\ .\frac{B_t}{Y_t}, \\
&= g\ .\frac{\delta\ .\left(r - r^a_{ss}\right)}{\left(r^a_{ss} - \delta\ .r\right)\ .r}.
\end{aligned} \tag{3.181}$$

This expression makes clear that the net foreign asset position is positive (resp. negative) depending on whether the world interest rate is higher (resp. lower) than the autarky interest rate. From the previous discussion, we infer that it is now possible for capital to flow out of emerging countries, provided that they have a sufficiently low autarky interest rate, i.e. a sufficiently low supply of stores of value.

Asymptotic Metzler Diagram

The vertical axis in Figure 3.36 reports the real interest rate while the horizontal axis reports either the long run domestic financial wealth W or the value of domestic assets K, scaled by output Y. By construction, the difference between domestic financial wealth and the value of domestic assets equals the country's long run net foreign asset position : $B = W - K$. From the previous discussion, the value of domestic assets decreases with the real interest rate, while the value of domestic wealth increases with the real interest rate. Financial autarky corresponds to the situation where $W = K$. This pins down the autarky real interest rate

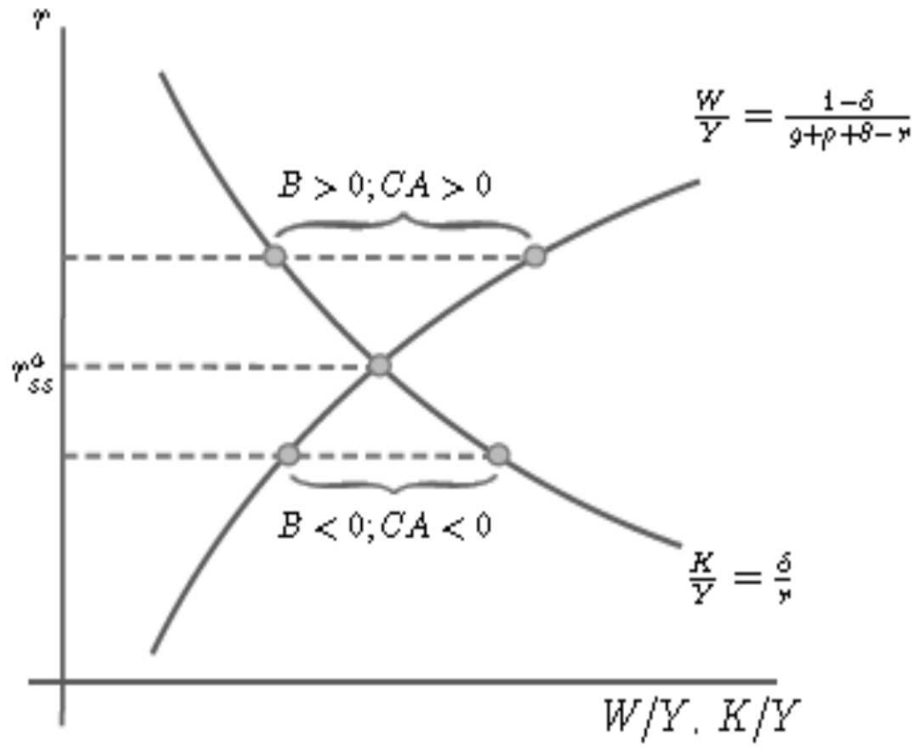


FIG. 3.36 – The Metzler Diagram - Source : Gourinchas and Rey (2014) External Adjustment, Global Imbalances, Valuation Effects. Handbook of International Economics, vol IV.

r^a_{ss}. When $r > r^a_{ss}$, the small open economy runs an asymptotic current account surplus and is a net foreign creditor. Conversely, when $r < r^a_{ss}$ the country runs an asymptotic current account deficit and is a net foreign borrower.

Consider now a world economy composed of two countries, a and b. The two countries are identical, except in terms of their level of financial development, captured by δ : Assume that $\delta^a > \delta^b$. It follows that country a will have a higher autarky interest rate than country b : Each country satisfies equations (18) and (21). Combining these equations, and denoting $\omega^a = \frac{Y^a}{Y^a+Y^b}$ the share of country a in global output, the steady state world interest rate r^a_{ss} is a weighted average of the autarky interest rate in both countries :

$$r^a_{ss} = \omega^a \ .r^{a,a}_{ss} + (1 - \omega_{ss}) \ .r^{a,b}_{ss} = \bar{\delta} \ . (g + \rho + \theta) . \tag{3.182}$$

Since $r^{a,b}_{ss} < r^a_{ss} < r^{a,a}_{ss}$, following a financial liberalization, capital will flow from b to a, and a will run an asymptotic negative net foreign asset position.

According to the model, a simultaneous decline in world interest rates and the emergence of global imbalances (stylized fact 1) can be the result of the integration of countries with low financial development (low δ) into the world economy (e.g. China after 1980), or the decline in the market perception financial development in some countries (e.g. emerging Asia after the Asian financial crisis of 1997). We can think of a variety of reasons why countries may be unable to pledge a high share of future output. Government, managers or insiders can dilute and divert a substantial share of profits. The parameter δ can thus capture a number of capital market frictions, from explicit taxation, lack of enforcement of property rights, corruption or rent-seeking etc... Many of these features tend to be associated with developing economies, as measured by indicators of social infrastructure.

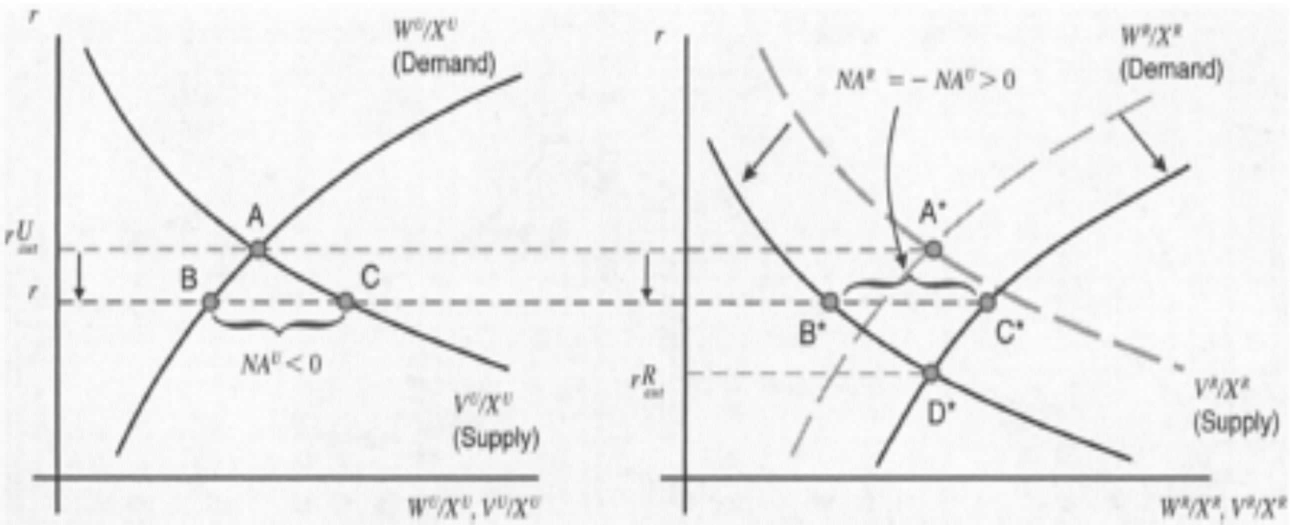


FIG. 3.37 – The Metzler Diagram for a Permanent Drop in δ - Source : Caballero, Farhi, et Gourinchas (2008) An equilibrium model of global imbalances and low interest rates. *American Economic Review*, 98(1), pp. 358-393

3.5.6 Testable Implications of the Twin Deficit Hypothesis

The fact that there seems to be no systematic relationship between large changes in government savings and changes in the current account does not necessarily invalidate the twin-deficit hypothesis. In reality, economies are hit simultaneously by a multitude of shocks of different nature. As a result, it is difficult to infer from raw data, like that presented in Figure 2.12, the effect of an increase in the fiscal deficit on the current account.

What then led some economists to conclude that the Reagan fiscal deficits were the cause of the current account deficits ? To answer this question, we need to look at the implications that the twin-deficit hypothesis has for the behavior of variables other than the current account and the fiscal deficit and then compare those predictions to actual data.

In the early 1980s not all economic observers attributed the emergence of current accounts deficits to the fiscal stance. There were two prevailing theoretical views on the source of current account deficits.

One view was that in those years the rest of the world wanted to send their savings to the U.S., so the U.S. had to run a current account deficit. This view is illustrated in Figure 3.38. The increase in the rest of the world's demand for U.S. assets is reflected in a shift to the left of the current account schedule of the rest of the world. As a result, in the new equilibrium position, the current account in the U.S. deteriorates from CA_0^{US} to CA_1^{US} and the world interest rate falls from $r^{\star,0}$ to $r^{\star,1}$.

What could have triggered such an increase in the desire of the rest of the world to redirect savings to the U.S. ? A number of explanations have been offered. First, in the early 1980s, the U.S. was perceived as a 'safe heaven', that is, as a safer place to invest. This perception, that we label the first view, triggered an increase in the supply of foreign lending. For example, it has been argued that international investors were increasingly willing to hold U.S. assets due to instability in Latin America ; in the jargon of that time, the U.S. was the recipient of the 'capital flight' from Latin America. Second, as a consequence of the debt crisis of the early 1980s, international credit dried up, forcing developing countries, particularly in Latin America, to reduce current account deficits. Third, financial deregulation in several countries made it easier for foreign investors to hold U.S. assets. An example is Japan in the late 1980s.

A second view of what caused the U.S. current account deficit is that in the 1980s the U.S. wanted to save less and spend more at any level of the interest rate. As a result, the American economy had to draw savings from the rest of the world. Thus, U.S. foreign borrowing went up and the current account deteriorated. Figure 3.39 illustrates this view. As a result of the increase in desired spending relative to income in the U.S., the CA schedule for the U.S. shifts to the left, causing a deterioration in the U.S. current account from CA_0^{US} to CA_1^{US} and an increase in the world interest rate from $r^{\star,0}$ to $r^{\star,1}$. Under view 2, the deterioration of the U.S. current account is the consequence of a decline in U.S. national savings or an increase in U.S. investment or a combination of the two.

How could we tell views 1 and 2 apart ? One strategy is to look for an economic variable about which the two views have different predictions. Once we have identified such a variable, we could look at actual data to see which view its behavior supports. Comparing Figures 3.38 and 3.39, it is clear that a good candidate for testing the two views is the real interest rate. The two views have different implications for the behavior of the interest rate in the U.S. Under view 1, the interest rate falls as the foreign supply of savings increases, whereas under view 2 the interest rate rises as the U.S. demand for funds goes up. What does the data show ? In the early 1980s, the U.S. experienced a large increase in real interest rates (see Figure 3.40). This evidence seems to vindicate view 2. We will therefore explore this view further.

As already mentioned, view 2 requires that either the U.S. saving schedule shifts to the left, or that the U.S. investment schedule shifts to the right or both (see Figure 3.41).

Before looking at actual data on U.S. savings and investment a comment about national savings is in order. National savings is the sum of private sector savings, which we will denote by S^P, and government savings, which we will denote by S^G. Letting S denote national savings, we have :

$$S = S^P + S^G. \tag{3.183}$$

Thus far we have analyzed a model economy without a public sector. In an economy without a government, national savings is simply equal to private savings, that is, $S = S^P$. However, in actual economies government savings accounts for a non-negligible fraction of national savings. To understand what happened to U.S. savings in the 1980s the distinction between private savings and government savings is important. With this comment in mind, let us now turn to the data.

Figure 3.42 displays with a solid line private savings, S^P, with a broken line national savings, S, and with a circled line investment, I. The difference between the solid and the broken lines represents government savings, Sg. The figure shows that national savings and private savings begin to diverge in 1980, with national savings falling consistently below private savings. This gap reflects the fiscal deficits created by the Reagan fiscal expansion. Specifically, the increase in the fiscal deficit in the early 1980s arose due to, among other factors, a tax reform, which reduced tax revenues, and an increase in defense spending.

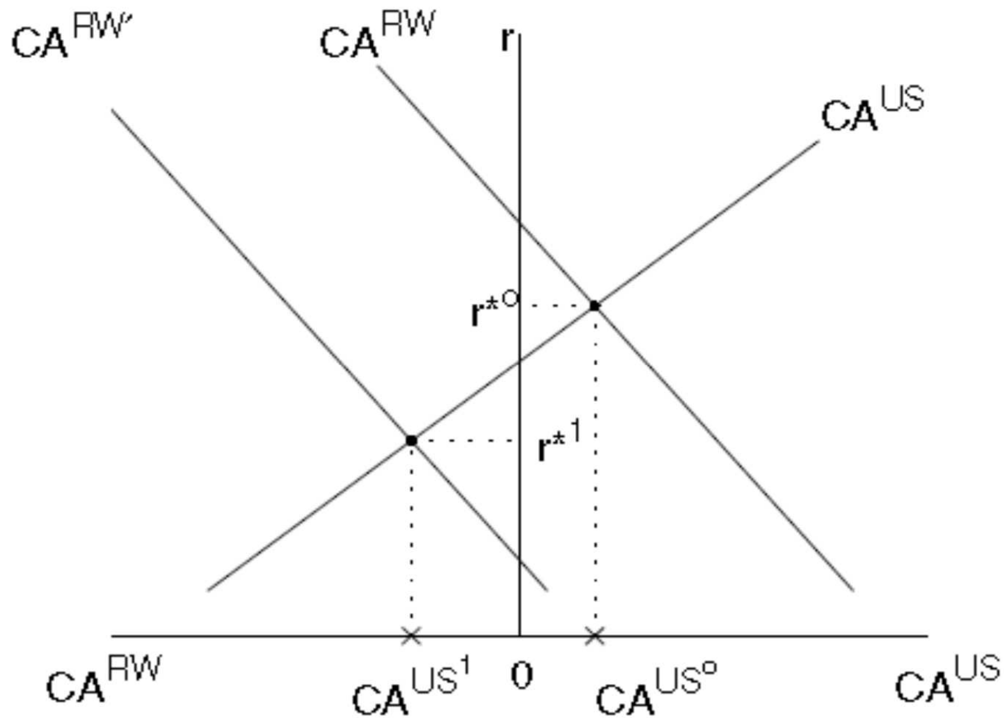


Fig. 3.38 – The U.S. current account in the 1980s : View 1 - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 7

Advocates of the twin-deficit hypothesis emphasize the fact that the decline in the current account balance, given by $S - I$ (the difference between the broken line and the circled line in Figure 3.42), is roughly equal to the decline in government savings (given by the difference between the solid and the broken lines). They therefore conclude that the increase in the fiscal deficit caused the decline in the current account. However, this causal direction, which implies that that the increase in the government deficit, that is, a decline in government savings, shifted the U.S. savings schedule to the left is not necessarily correct. The reason is that changes in fiscal policy that cause the fiscal deficit to increase may also induce offsetting increases in private savings, leaving total savings - and thus the current account - unchanged.

We conclude that if the current account deficit of the 1980s is to be explained by the fiscal imbalances of the Reagan administration, then this explanation will have to rely on a combination of an increase in government expenditure and multiple factors leading to the failure of Ricardian equivalence.

3.6 International Financial Adjustment : Gourinchas and Rey (2007)

In this section, we try to answer to the following question : How large current account deficits can be sustainable ? As emphasized in the last section of chapter 2, the cumulated current account deficits do not coincide with the NIIP due to valuation effect. This section highlights the quantitative importance of valuation effects and the financial channel of external adjustment. To do so, we have to explore the implications of the external solvency constraint. Derivations are presented in discrete time for two reasons. First, it allows for an

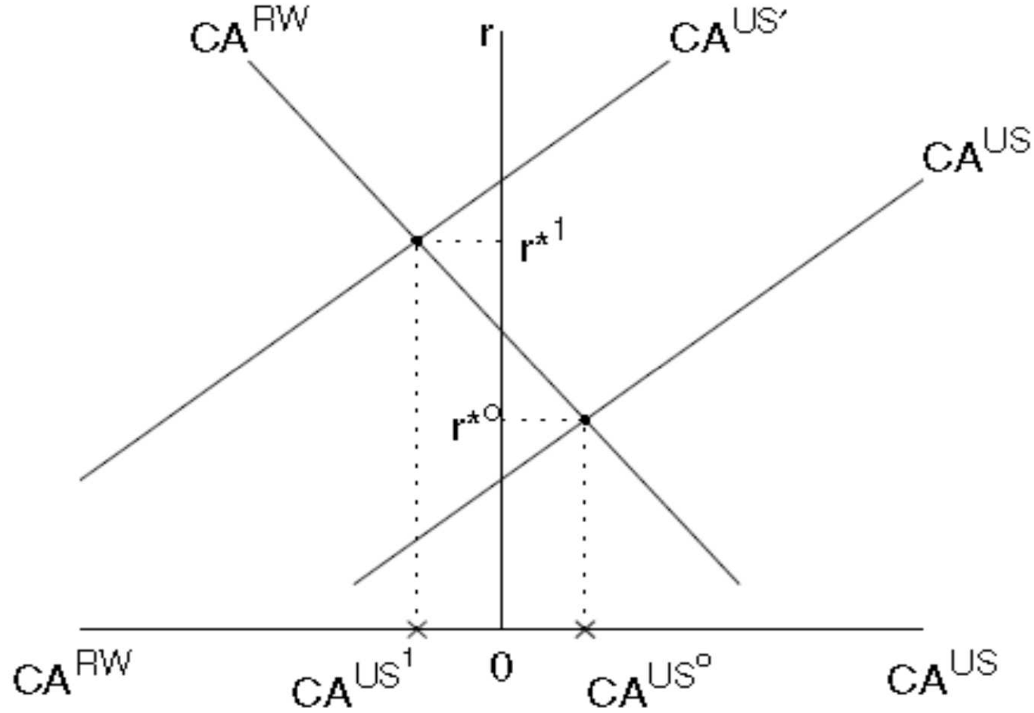
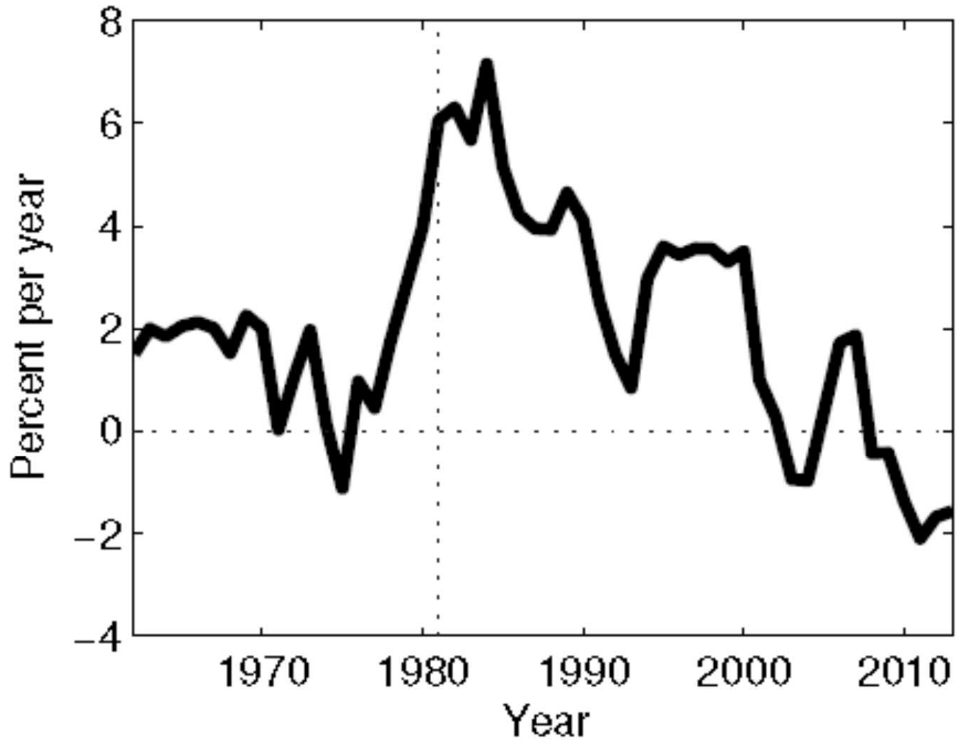


FIG. 3.39 – The U.S. current account in the 1980s : View 2 - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 7



Note: The real interest rate is measured as the difference between the 1-year constant maturity Treasury rate and one-year expected inflation.

FIG. 3.40 – Real interest rates in the United States 1962-2013 - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 7

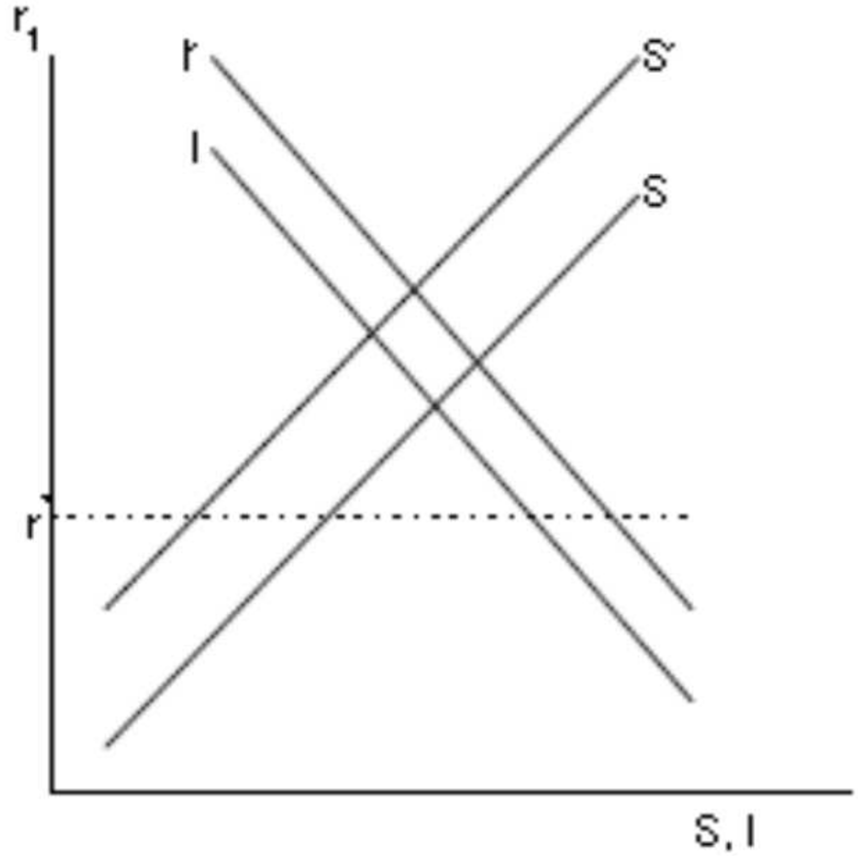


FIG. 3.41 – View 2 requires shifts in the U.S. savings or investment schedules - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 7

Source: Norman S. Fieleke, "The USA in Debt," *New England Economic Review*, September-October 1990, pages 34-54, table 11.

Fig. 3.42 – U.S. Saving and Investment in Percent of GNP - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 7

easier mapping between the theoretical objects of analysis and their empirical counterpart. Second, many of the issues discussed in this section have a business cycle dimension, for which a discrete time set-up is better adapted. Third, the approach followed by Gourinchas and Rey (2007) is to analyze the international adjustment around the trends and thus the authors do not investigate the slow moving trends, i.e., the structural change.

3.6.1 External Budget Constraint

We define a measure of external imbalances and show that current imbalances must be offset by future improvements in trade surpluses, or excess returns on the net foreign portfolio, or both. We start with the accumulation identity for net foreign assets between periods t and $t+1$. If we define the stock of net foreign assets NA'_t at the end of period t instead of the beginning,

$$NA'_{t+1} - NA'_t = r_{t+1} NA'_t + NX'_{t+1}, \tag{3.184}$$

where NA'_{t+1} is net foreign assets at the end of period $t+1$. If instead the net foreign asset position NA_t is defined at the beginning of period t, and NX_t denotes net exports over period t. If we define the stock of foreign assets at the beginning of the period t, the change in the net foreign asset position between t and $t+1$ is equal to interest receipts on net foreign assets at the beginning of period t plus the change in foreign assets due to net exports, plus trade balance :

$$NA_{t+1} - NA_t = r_{t+1}\left(NA_t + NX_t\right) + NX_t.$$

Defining the gross return on net foreign assets $1 + r_{t+1} = R_{t+1}$ as one plus the interest rate, the stock of net foreign assets at the beginning of period 1 is equal to :

$$NA_{t+1} \equiv A_{t+1} - L_{t+1} = R_{t+1} \times \left(NA_t + NX_t\right), \tag{3.185}$$

NX represents net exports, defined as the difference between t exports X and imports M of goods and services ; NA_t represents net foreign assets foreign assets, defined as the difference between gross external assets A and gross external liabilities L measured in the domestic currency ; R_{t+1} denotes the (gross) return on the net foreign asset portfolio which is a combination of the (gross) return on assets R^a_{t+1} and the (gross) return on liabilities R^l_{t+1}. Equation (3.184) states that the net foreign position improves with positive net exports and with the return on the net foreign asset portfolio.

3.6.2 Log-Linearization of the External Constraint

By using the fact that net foreign asset position NA_t is the difference between assets and liabilities and divided both sides of (3.185) by the stock of financial wealth W_{t+1} (which contains both domestic and foreign assets), and denoting by lower-case letters the ratio of X_{t+1} to W_{t+1}, the identity can be rewritten as follows :

$$\begin{aligned} \frac{A_{t+1} - L_{t+1}}{W_{t+1}} &= \frac{W_t}{W_{t+1}} R_{t+1} \left(\frac{A_t - L_t + X_t - M_t}{A_t} \right), \\ a_{t+1} - l_{t+1} &= \frac{R_{t+1}}{\Gamma_{t+1}} \left(a_t - l_t + x_t - m_t\right), \end{aligned} \tag{3.186}$$

where $\Gamma_{t+1} = W_{t+1}/W_t$ correspond to the growth rate of wealth.

We assume that a variable $X_t = A_t, L_t, X_t, M_t$ displays a deterministic trend and denote with a bar $\bar{X}_t$ the trend component of this variable. The accumulation equation (3.186) where all variable are at their trend reads :

$$\bar{a}_{t+1} - \bar{l}_{t+1} = \frac{\bar{R}_{t+1}}{\bar{\Gamma}_{t+1}} \left(\bar{a}_t - \bar{l}_t + \bar{x}_t - \bar{m}_t \right). \tag{3.187}$$

The deviation of the variable from trend at time t is equal to $x_{t+1} - \bar{x}_{t+1} = \Delta x_{t+1}$; we express the deviation from trend in percentage of the trend : $\frac{x_{t+1} - \bar{x}_{t+1}}{\bar{x}_{t+1}} = \frac{\Delta x_{t+1}}{\bar{x}_{t+1}}$; we denote this deviation from trend in percentage by $\epsilon^x_{t+1} = \frac{\Delta x_{t+1}}{\bar{x}_{t+1}}$. Using this definition, the current value of the variable x_{t+1} can be rewritten as $\bar{x}_{t+1}\left(1 + \epsilon^x_{t+1}\right)$.

We linearize equation (3.186) around trend and express the resulting expression as a percentage of trend ; we define several ratios for assets, liabilities, exports, and imports which correspond to trend shares :

$$\mu^a_t = \frac{\bar{a}_t}{\bar{a}_t - \bar{l}_t}, \quad \mu^l_t = \frac{\bar{l}_t}{\bar{a}_t - \bar{l}_t}, \tag{3.188a}$$

$$\mu^x_t = \frac{\bar{x}_t}{\bar{x}_t - \bar{m}_t}, \quad \mu^m_t = \frac{\bar{m}_t}{\bar{x}_t - \bar{l}_t}, \tag{3.188b}$$

$$\frac{1}{\rho_t} = \frac{\bar{a}_t - \bar{l}_t}{\bar{a}_t - \bar{l}_t + \bar{x}_t - \bar{m}_t}, \quad 1 - \frac{1}{\rho_t} = \frac{\bar{x}_t - \bar{m}_t}{\bar{a}_t - \bar{l}_t + \bar{x}_t - \bar{m}_t}. \tag{3.188c}$$

We also define the deviation from trend in percentage of net exports and net foreign asset position as follows. The net foreign asset position and net exports can be rewritten as follows :

$$\begin{aligned}
a_t - l_t &= \bar{a}_t\left(1 + \epsilon^a_t\right) - \bar{l}_t\left(1 + \epsilon^l_t\right), \\
&= \left(\bar{a}_t - \bar{l}_t\right)\left(1 + \frac{\bar{a}_t\epsilon^a_t - \bar{l}_t\epsilon^l_t}{\bar{a}_t - \bar{l}_t}\right), \\
x_t - m_t &= \bar{x}_t\left(1 + \epsilon^x_t\right) - \bar{m}_t\left(1 + \epsilon^m_t\right), \\
&= \left(\bar{x}_t - \bar{m}_t\right)\left(1 + \frac{\bar{x}_t\epsilon^x_t - \bar{m}_t\epsilon^m_t}{\bar{x}_t - \bar{m}_t}\right).
\end{aligned}$$

Dividing both sides by $\bar{a}_t - \bar{l}_t$ and $\bar{x}_t - \bar{m}_t$, respectively, and denoting by $na_t = \frac{a_t - l_t}{\bar{a}_t - \bar{l}_t} - 1$ and $nx_t = \frac{x_t - m_t}{\bar{x}_t - \bar{m}_t} - 1$ the detrended component of the net foreign asset position and net exports respectively, these cyclical components can be written as follows :

$$na_t \equiv \mu^a_t\epsilon^a_t - \mu^l_t\epsilon^l_t, \tag{3.189a}$$

$$nx_t \equiv \mu^x_t\epsilon^x_t - \mu^m_t\epsilon^m_t. \tag{3.189b}$$

The term μ^x_t represents the (trend) share of exports in the trade balance. Similarly, μ^a_t denotes the (trend) share of assets in the net foreign assets. The variable nx is a linear combination of the stationary components of (log) exports and imports to wealth ratios, which we shall call 'detrended net exports'.

Using the definition of cyclical or detrended components of the net foreign asset na_t and net exports nx_t given, by (3.189), the log-linear version of (3.186) is

$$na_{t+1} = \frac{1}{\rho t} na_t + \left(r_{t+1} - \epsilon^w_{t+1}\right) + \left(1 - \frac{1}{\rho_t}\right) nx_t. \tag{3.190}$$

Equation (3.190) involves only the stationary component in percentage $\ln x_t - \ln \bar{x}_t = \epsilon^x_t$; everything is normalized by wealth so that the deviation are measured in percentage of wealth ; second, these stationary components are multiplied by time-varying weights μ^x_t that reflect the trends in the data; finally, everything is normalized by wealth; hence the rate of return r_{t+1} is adjusted for the cyclical component of the growth rate of wealth ϵ^w_{t+1}. Note that r_{t+1} measures the deviation from trend of the return on net foreign asset : when $r^{t+1} > 0$, it means that the return on net foreign asset increases. Eq. (3.190) states that the detrended component of the net foreign asset position at $t+1$ is a weighted average of the net foreign asset position at the previous period and net exports, and is positively affected by a temporary increase in the return on net foreign assets.

3.6.3 A Measure of External Imbalances

In order to have a measure of external imbalances, we assume that the trend shares are constants and define the following variables

$$nxa_t \equiv na_t - nx_t = |\mu^a|\epsilon^a_t - |\mu^l|\epsilon^l_t + |\mu^x|\epsilon^x_t - |\mu^m|\epsilon^m_t, \tag{3.191a}$$

$$\Delta nx_{t+1} \equiv |\mu^x|\Delta\epsilon^x_{t+1} - |\mu^m|\Delta\epsilon^m_{t+1}. \tag{3.191b}$$

When defining these the measure of cyclical external imbalances, nxa_t, we assume that assets are higher that liabilities, i.e., $\bar{A} > \bar{L}$, so that $\mu^a > 0$ and exports are lower than imports, $\bar{X} < \bar{M}$, so that $\mu^x < 0$ and $\mu^m < 0$; in order to have positive shares, we define a new measure of imbalances by subtracting net exports nx_t from net foreign asset position na_t in order to have positive shares in absolute terms, i.e., $|\mu^x| > 0$ and $|\mu^m| > 0$.

Assuming that the shares are constant over time and using the definitions (3.191), we get a measure of global imbalances :

$$nxa_{t+1} = \frac{1}{\rho} nxa_t + r_{t+1} + \Delta nx_{t+1}, \tag{3.192}$$

where

$$\rho = 1 - \frac{\bar{X} - \bar{M}}{\bar{A} - \bar{L}}. \tag{3.193}$$

The term nxa_t defined by (3.191a) combines linearly the stationary components of exports, imports, assets, and liabilities. It is a well-defined measure of cyclical external imbalances. Unlike the current account, it incorporates information from both the trade balance (the flow) and the foreign asset position (the stock). Since it is defined using the absolute values of the weights μ^x ($x = a, l, x, m$), the measure nxa always increases with assets and exports and decreases with imports and liabilities.

The term Δnx_{t+1} represents detrended net export growth between t and $t+1$. It increases with cyclical export growth $\Delta\epsilon^x_{t+1}$ and decreases with cyclical import growth $\Delta\epsilon^m_{t+1}$. Just like (3.186) and (3.190), equation (3.192) shows that a country can improve its net foreign asset position either through a trade surplus $\Delta nx_{t+1} > 0$ or through a high return on its net foreign asset portfolio $r_{t+1} > 0$.

3.6.4 The Intertemporal Solvency Condition

In order to build intuition about the term ρ given by eq. (3.193), we evaluate (3.186) at the steady-state. We denote with a prime the ratio of variable X_t to the stock of wealth W_t.

Expressing both sides of (3.185) as a share of wealth, we have :

$$\begin{aligned} W_{t+1}\frac{NA_{t+1}}{W_{t+1}} &= W_t R_{t+1}\frac{(NA_t + NX_t)}{W_t}, \\ NA'_{t+1} &= \frac{R_{t+1}}{\Gamma_t}\left(NA'_t + NX'_t\right). \end{aligned}$$

Assumption that the economy settles at the steady-state, and plugging $NA'_{t+1} = NA'_t = \bar{NA}'$ and $NX'_{t+1} = NX'_t = \bar{NX}'$ into $NA'_{t+1} = \frac{R}{\Gamma}\left(NA'_t + NX'_t\right)$, we obtain

$$\left(\frac{\Gamma}{R} - 1\right)\bar{NA}' = \bar{NX}'.$$

This equality can be rewritten in order to determine the ratio of net exports to the net foreign asset position :

$$\frac{\bar{NX}'}{\bar{NA}'} = \frac{\bar{X} - \bar{M}}{\bar{A} - \bar{L}} = \frac{\Gamma}{R} - 1 < 0. \tag{3.194}$$

where we made use of (3.193), i.e., $\rho - 1 = \frac{\bar{NX}}{\bar{NA}}$. Note that it is assumed that the steady-state rate of return R is higher than the long-term growth rate of the economy Γ. According to (3.194), the steady-state ratio of net exports to net foreign asset position must be smaller than one in the long-run. In words, countries with long-run creditor positions ($\bar{NA} > 0$), should run trade deficits ($\bar{NX} < 0$), whereas countries with steady-state debtor positions ($\bar{NA} < 0$) should run trade surpluses ($\bar{NX} > 0$). Put otherwise, in the long-run, the net foreign asset position remains constant at $NA_{t+1} = NA_t = \bar{NA}$ so that interest payments $(R-1)\,\bar{NA}$ if the net foreign asset position is negative must be offset by a trade balance surplus.

Eq. (3.192) can be solved forward by imposing first the no-Ponzi condition that the detrended measure of external imbalances nxa_t cannot grow faster than the interest rate adjusted with the steady-state growth rate, i.e., $\frac{R}{\Gamma} = \rho - 1$, in the long-run :

$$\lim_{t\to\infty} nxa_{t+j}\left(\frac{R}{\Gamma}\right)^{-j} = \lim_{t\to\infty} nxa_{t+j}\rho^j = 0. \tag{3.195}$$

Solving eq. (3.192) forward, we get :

$$\begin{aligned} nxa_t &= \rho nxa_{t+1} - \rho\left(r_{t+1} + \Delta nx_{t+1}\right), \\ &= \rho\left[\rho nxa_{t+2} - \rho\left(r_{t+2} + \Delta nx_{t+2}\right)\right] - \rho\left(r_{t+1} + \Delta nx_{t+1}\right), \\ &= \rho^2 nxa_{t+2} - \rho\left(r_{t+1} + \Delta nx_{t+1}\right) - \rho^2\left(r_{t+2} + \Delta nx_{t+2}\right), \\ &= \rho^j nxa_{t+j} - \rho\left(r_{t+1} + \Delta nx_{t+1}\right) - ... - \rho^j\left(r_{t+j} + \Delta nx_{t+j}\right). \end{aligned}$$

The solution for the measure of external imbalance is thus given by :

$$nxa_t = \rho^j nxa_{t+j} - \sum_{j=1}^{\infty}\rho^j\left(r_{t+j} + \Delta nx_{t+j}\right).$$

Imposing the no-ponzi game (3.195), the first term on the RHS of the equation above vanishes ; we get the intertemporal solvency condition of the open economy :

$$nxa_t = -\sum_{j=1}^{\infty}\rho^j\left(r_{t+j} + \Delta nx_{t+j}\right). \tag{3.196}$$

Equation (3.196) plays a key role in the analysis. It shows that the initial external imbalance of the country is negatively correlated with future net exports and returns changes.

– Consider the case of a country with a negative value for nxa, because of either a deficit in the cyclical component of the trade balance or a cyclical net debt position, or both. Suppose first that returns on net foreign assets are expected to be constant. In that case, equation (3.196) reveals that any adjustment must come through future increases in net exports : Δnx_{t+j}. This is the standard implication of the intertemporal approach to the current account. We call this channel the 'trade channel'.
– When the open economy experiences a negative external asset position, The adjustment may also come from high net foreign portfolio returns : $r_{t+j} > 0$. We call this channel the 'valuation channel'. Importantly, such higher returns can occur via a depreciation of the domestic currency. While such depreciation may also help to improve future net exports, the important point is that it operates through an entirely different channel : a predictable wealth transfer from foreigners to domestic residents.

To summarize, if the home country experiences a large initial external imbalance, it must run at some date in the future an improvement in the trade balance and/or a rise in the return on the asset position. Such an adjustment is achieved through an exchange rate depreciation which stimulates exports and depressed imports and thus raises net exports. When liabilities are labelled in the domestic currency and assets are labelled in foreign currencies, as for the USA (or the euro area), the interest rate parity condition according to which the return on domestic assets and foreign assets must equalize when expressed in the same currency :

$$r^a_{t+1} = r^{\star,a}_{t+1} + \Delta e_{t+1} \tag{3.197}$$

where $\Delta e_{t+1} = \frac{E(e_{t+1})-e_t}{e_t}$ corresponds to a dollar depreciation which raises the return on foreign assets r^a while the interest payments on liabilities r^l are not modified since they are labelled in the domestic currency. Using (3.197), the real return is defined as the difference between the return on foreign assets and the interest payment on liabilities, both denominated in the domestic currency and weighted by that the share of assets and liability (in absolute terms) in the net foreign asset position :

$$r_{t+1} = |\mu^a| \left(r^{\star,a}_{t+1} + \Delta e_t\right) - |\mu^l| r^l_t. \tag{3.198}$$

Hence, Gourinchas and Rey (2007) argue that when the exchange rate depreciation reduces the external imbalance by i) increasing net exports through a 'trade channel', and ii) raises interest receipts denominated in dollars through a 'valuation channel'.

3.6.5 The Financial and Trade Channels of External Adjustment

The term nxa in eq. (3.196) is a theoretically well-defined measure of cyclical external imbalances. Gourinchas and Rey (2007) decompose it into a return and a net export component :

$$nxa_t \equiv nxa^r_t + nxa^{\Delta nx}_t, \tag{3.199}$$

where nxa^r corresponds to the future reduction of cyclical external imbalances through the valuation channel and $nxa^{\Delta nx}_t$ represents the reduction of external imbalances through the trade channel. Put otherwise, if the country is initially a net debtor, i.e., $nxa_t < 0$, then the country must run in the future a sequence of trade surplus or a rise in the real return on foreign assets : these two channels reduce the size of external imbalances and thus allow the country to satisfy its intertemporal solvency condition.

		Discount Factor ρ		
Line	Percent	.96	.95	.94
1	$\beta_{\Delta nx}$	71.77	63.96	57.05
2	β_r of which:	23.76	26.99	28.85
3	β_{ra}	19.91	20.78	20.65
4	β_{rl}	3.87	6.22	8.21
5	Total (lines 1+2)	95.53	90.95	85.89
6	μ^a	6.72	8.49	10.08

Note.—$\beta_{\Delta nx}$ (β_r) represents the share of the unconditional variance of *nxa* explained by future net export growth (future excess returns); β_{ra} (β_{rl}) represents the share of the unconditional variance of nxa^r explained by future returns on gross external assets (liabilities). The sum of coefficients $\beta_{ra} + \beta_{rl}$ is not exactly equal to β_r because of numerical rounding in the VAR estimation. The sample is 1952:1–2004:1.

Fig. 3.43 – Unconditional Variance Decomposition of nxa_t. Note : $\beta_{\Delta nx}$ (β_r) represents the share of the unconditional variance of nxa explained by future net export Δnx (future excess returns) ; β_{r^a} (β_{r^l}) represents the share of the unconditional variance of nxa^r explained by future returns on gross external assets (liabilities). The sum of coefficients $\beta^{r^a} + \beta_{r^l}$ is not exactly equal to β_r because of numerical rounding in the VAR estimation. The sample is 1952 :1-2004 :1. Source : Gourinchas and Rey (2007) International Financial Adjustment. *Journal of Political Economy*, 115(4), pp. 665-703

They estimate the cyclical component of nxa_t by computing $|\mu^a|\epsilon_t^a - |\mu^l|\epsilon_t^l + |\mu^x|\epsilon_t^x - |\mu^m|\epsilon_t^m$ when μ^z is the trend share in absolute terms and ϵ^z is the cyclical component of assets, liabilities, exports and imports. To breakdown the measure of external imbalances into two components, they look at the coefficients from regressing independently nxa^r and $nxa^{\Delta nx}$ on nxa. The resulting regression coefficients, β_r and $\beta_{\Delta nx}$, represent the share of the unconditional variance of nxa explained by future returns or future net export growth :

$$\begin{aligned} 1 &= \frac{\text{Cov}\,(nxa, nxa)}{\text{Var}\,(nxa)} \\ &= \frac{\text{Cov}\,(nxa^r, nxa)}{\text{Var}\,(nxa)} + \frac{\text{Cov}\left(nxa^{\Delta nx}, nxa\right)}{\text{Var}\,(nxa)}. \end{aligned}$$

Table 3.43 reports the decomposition for different values of ρ between 0.94 and 0.96.

For the benchmark value of $\rho = 0.95$, we get a breakdown of 64 percent (net exports) and 27 percent (portfolio returns), accounting for 91 percent of the variance in nxa. The results are sensitive to the assumed discount factor. These findings indicate that valuation effects play a major role but do not replace the need for an ultimate adjustment in net exports via a rise in exports and a fall in imports. What our estimates indicate, however, is that valuation effects profoundly transform the nature of the external adjustment process. By absorbing 25-30 percent of the cyclical external imbalances, valuation effects substantially relax the external budget constraint of the United States. With the same methodology, lines 3 and 4 of table 3.43 further decompose the variance of nxar into the contributions of returns on gross assets and liabilities. For the standard specification, we obtain a breakdown of roughly 21 percent (β_{r^a}) and 6 percent (β_{r^l}). These findings confirm that gross asset returns account for the bulk of the variance, whereas returns on gross liabilities, which are all in dollars, are much less responsive.

3.6.6 Forecasting short-run and long-run returns : The Role of Valuation Effects

Because the returns and net export changes are expected in an uncertain world, we introduce an expectation operator into equation (3.196) :

$$nxa_t = -\sum_{j=1}^{\infty} \rho^j E_t \left(r_{t+j} + \Delta nx_{t+j} \right). \tag{3.200}$$

It shows that movements in the detrended trade balance and net foreign asset position must forecast either future portfolio returns or future net export growth, or both. Hence, equation (3.200) indicates that nxa should help predict either future returns on the net foreign asset portfolio r or future net export growth Δnx, or both. This subsection looks specifically at the predictive power of nxa for future returns on the net foreign asset portfolio r_t^j at the quarterly horizon. Table 3.44 reports a series of results using nxa as a predictive variable. Each column of the table reports a regression of the form :

$$y_{t+1} = \alpha + \beta \times nxa_t + \delta \times z_t + \epsilon_{t+1}, \tag{3.201}$$

where y denotes a quarterly return between t and $t+1$, z denotes additional controls shown elsewhere in the literature to contain predictive power for asset returns or exchange rates, and ϵ_{t+1} is a residual.

Looking first at panel A of table 3.44, we see that nxa has significant forecasting power for the net portfolio return one quarter ahead (col. 1). The R^2 of the regressions is 0.10, and the negative and significant coefficient indicates that a positive deviation from trend predicts a decline in net portfolio return that is qualitatively consistent with equation (3.200). Col. 2 and Col. 3 of panel A of Table 3.44 show that the inclusion of lagged values of the net portfolio return or the difference between domestic and foreign dividend-price ratios or the deviation from trend of net exports does not improve the forecasting power.

A natural question is whether the predictive power of the measure of external imbalances increases with the forecasting horizon. According to equation (3.200), nxa could forecast any combination of r and Δnx at long horizons. Table 3.45 reports the results for forecasting horizons ranging between one and 24 quarters. Table 3 3.45 indicates that the in-sample predictability increases up to an impressive 0.26 (0.38 with separate regressors) for net foreign portfolio returns at a four-quarter horizon and then declines to 0.02 or 0.16 at 24 quarters. These results suggest that the financial adjustment channel operates at short to medium horizons, between one quarter and two years. It then declines significantly and disappears in the long run. As shown in the previous subsection, its overall contribution to external adjustment amounts to roughly 27 percent.

The picture is very different when we look at net export growth. We find that nxa predicts a substantial fraction of future net export growth in the long run : the R^2 is 0.58 at 24 quarters (0.79 with three regressors). A large positive external imbalance predicts low future net export growth, which restores equilibrium. The classic channel of trade adjustment is therefore also at work, especially at longer horizons (eight quarters and more).

Looking at exchange rates, we find a similarly strong long-run predictive power on the rate of depreciation of the dollar. The R^2 increases up to 0.41 (0.55 with three regressors) at 12 quarters. There is significant predictive power at short, medium, and long horizons.

	Total Real Return (r_{t+1})				Real Equity Differential (Δr^{e}_{t+1})			
		r_t	$(d_t/p_t) - (d^*_t/p^*_t)$	xm_t		Δr^e_t	$(d_t/p_t) - (d^*_t/p^*_t)$	xm_t
z_t	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\hat{\beta}$	**−.36**	**−.33**	**−.46**	**−.37**	**−.13**	**−.14**	**−.17**	−.07
	(.07)	(.07)	(.08)	(.16)	(.03)	(.03)	(.03)	(−.06)
$\hat{\delta}$		.09	−1.43	.01		−.07	−.63	−.09
		(.07)	(1.60)	(.19)		(.07)	(.61)	(.07)
$\bar{R}^2$	.10	.10	.15	.10	.07	.07	.12	.07
Observations	208	207	136	208	208	207	136	208

B. Depreciation Rates

	FDI-Weighted (Δe_{t+1})				Trade-Weighted (Δe^T_{t+1})			
		Δe_t	xm_t	$i_t - i^*_t$		Δe^T_t	xm_{t-1}	$i_t - i^*_t$
z_t	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\hat{\beta}$	**−.08**	**−.09**	**−.10**	**−.09**	**−.09**	**−.09**	**−.08**	**−.08**
	(.02)	(.02)	(.04)	(.02)	(.02)	(.02)	(.03)	(.02)
$\hat{\delta}$		−.04	.02	.32		.02	−.01	−.67
		(.07)	(.05)	(.32)		(.07)	(.05)	(.34)
$\bar{R}^2$	.09	.08	.08	.08	.11	.10	.10	.13
Observations	125	124	125	125	124	123	124	124

Note.—Regressions of the form $y_{t+1} = \alpha + \beta nxa_t + \delta z_t + \epsilon_{t+1}$, where y_{t+1} is the total real return (r_{t+1}), the equity return differential ($\Delta r^e_{t+1} = r^{e*}_{t+1} - r^e_{t+1}$) (panel A), the FDI-weighted depreciation rate (Δe_{t+1}), or the trade-weighted depreciation rate (Δe^T_{t+1}) (panel B). $(d_t/p_t) - (d^*_t/p^*_t)$ is the relative dividend-price ratio (available since 1970:1); $i_t - i^*_t$ is the short-term interest rate differential; xm_t is the stationary component from the trade balance, defined as $\epsilon^x_t - \epsilon^m_t$. The sample is 1952:1–2004:1 for total returns and 1973:1–2004:1 for depreciation rates. Robust standard errors are in parentheses. Boldface entries are significant at the 5 percent level.

Fig. 3.44 – Forecasting Quarterly Returns. Source : Gourinchas and Rey (2007) International Financial Adjustment. *Journal of Political Economy*, 115(4), pp. 665-703

Taken together, these findings indicate that two dynamics are at play. At horizons smaller than two years, the dynamics of the portfolio returns seem to dominate, and exchange rate adjustments create valuation effects that have an immediate impact on cyclical external imbalances. At horizons longer than two years, there is little predictability of asset returns. But there is still substantial exchange rate predictability, which goes hand in hand with a corrective adjustment in future net exports.

3.7 International Capital Market Integration

Figure 3.47 displays the stock of cross-border capital as a percentage of world GDP. The figure highlights two phases of increased capital market integration.

3.7.1 Measuring the degree of capital mobility : Saving-Investment correlations

In 1980 Feldstein and Horioka wrote a provoking paper in which they showed that changes in countries' rates of national savings had a very large effect on their rates of investment. Feldstein and Horioka examined data on average investment-to-GDP and saving-to-GDP

	Forecast Horizon (Quarters)							
	1	2	3	4	8	12	16	24
	A. Real Total Net Portfolio Return $r_{t,k}$							
nxa	**−.36**	**−.35**	**−.35**	**−.33**	**−.22**	**−.14**	**−.09**	**−.04**
	(.07)	(.05)	(.04)	(.04)	(.03)	(.03)	(.02)	(.02)
$\bar{R}^2(1)$	[.11]	[.18]	[.24]	[.26]	[.21]	[.13]	[.09]	[.02]
$\bar{R}^2(2)$	[.14]	[.25]	[.34]	[.38]	[.35]	[.24]	[.19]	[.16]
	B. Real Total Excess Equity Return $r^{eq}_{t,k} - r^{b}_{t,k}$							
nxa	**−.14**	**−.13**	**−.12**	**−.11**	**−.06**	**−.03**	−.02	.01
	(.03)	(.02)	(.02)	(.02)	(.01)	(.01)	(.01)	(.01)
$\bar{R}^2(1)$	[.07]	[.13]	[.17]	[.18]	[.10]	[.03]	[.01]	[.00]
$\bar{R}^2(2)$	[.11]	[.20]	[.28]	[.31]	[.26]	[.15]	[.10]	[.17]
	C. Net Export Growth $\Delta nx_{t,k}$							
nxa	**−.08**	**−.08**	**−.07**	**−.07**	**−.07**	**−.06**	**−.06**	**−.04**
	(.02)	(.02)	(.01)	(.01)	(.01)	(.01)	(.01)	(.01)
$\bar{R}^2(1)$	[.05]	[.10]	[.13]	[.17]	[.31]	[.44]	[.53]	[.58]
$\bar{R}^2(2)$	[.04]	[.08]	[.12]	[.17]	[.38]	[.55]	[.66]	[.79]
	D. FDI-Weighted Effective Nominal Rate of Depreciation $\Delta e_{t,k}$							
nxa	**−.08**	**−.08**	**−.08**	**−.08**	**−.07**	**−.06**	**−.04**	**−.02**
	(.02)	(.02)	(.01)	(.01)	(.01)	(.01)	(.01)	(.01)
$\bar{R}^2(1)$	[.09]	[.16]	[.28]	[.31]	[.41]	[.41]	[.33]	[.12]
$\bar{R}^2(2)$	[.10]	[.21]	[.35]	[.40]	[.52]	[.55]	[.55]	[.38]

Note.—Regressions of the form $y_{t,k} = \alpha + \beta nxa_t + \epsilon_{t+k}$, where $y_{t,k}$ is the k-period real total net portfolio return ($r_{t,k}$), total excess equity return ($r^{eq}_{t,k} - r^{b}_{t,k}$), net export growth ($\Delta nx_{t,k}$), or the FDI-weighted depreciation rate ($\Delta e_{t,k}$). Newey-West robust standard errors are in parentheses with a $k-1$ Bartlett window. Adjusted R^2's are in brackets. $\bar{R}(1)$ reports the adjusted R^2 of the regression on nxa_t; $\bar{R}(2)$ reports the adjusted R^2 of the regression on ϵ^{x}_t, ϵ^{m}_t, ϵ^{a}_t, and ϵ^{l}_t. The sample is 1952:1–2004:1 (1973:1–2004:1 for the exchange rate). Boldface entries are significant at the 5 percent level.

Fig. 3.45 – Long-Horizon Regressions. Source : Gourinchas and Rey (2007) International Financial Adjustment. *Journal of Political Economy*, 115(4), pp. 665-703

	Horizon (Quarters)						
	1	2	3	4	8	12	16
	A. FDI-Weighted Depreciation Rate						
MSE_u/MSE_r	.960	.920	.858	.841	.804	.818	.903
ΔMSE-adjusted	1.48	1.53	1.61	1.51	1.20	.74	.35
	(.68)	(.60)	(.57)	(.53)	(.37)	(.24)	(.23)
p-value	[.01]	[.01]	[<.01]	[<.01]	[<.01]	[<.01]	[.06]
	B. Trade-Weighted Depreciation Rate						
MSE_u/MSE_r	.949	.900	.830	.788	.733	.929	.961
ΔMSE-adjusted	2.76	3.03	2.94	2.78	1.91	.67	.29
	(1.03)	(1.03)	(1.02)	(.98)	(.69)	(.38)	(.24)
p-value	[<.01]	[<.01]	[<.01]	[<.01]	[<.01]	[.03]	[.11]

Note.—ΔMSE-adjusted = MSE_r − MSE_u-adjusted is the Clark-West (2006) test statistic based on the difference between the out-of-sample MSE of the driftless random walk model and the out-of-sample MSE of a model that regresses the rate of depreciation Δe_{t+1} against nxa_t. Rolling regressions are used with a sample size of 105. *t*-statistics are in parentheses. The *p*-value of the one-sided test using critical values from a standard normal distribution is in brackets. Under the null, the random walk encompasses the unrestricted model. The sample is 1952:1–2004:1. The cutoff is 1978:1.

Fig. 3.46 – Out-of-Sample Tests for Exchange Rate Depreciation against the Martingale Hypothesis. Source : Gourinchas and Rey (2007) International Financial Adjustment. *Journal of Political Economy*, 115(4), pp. 665-703

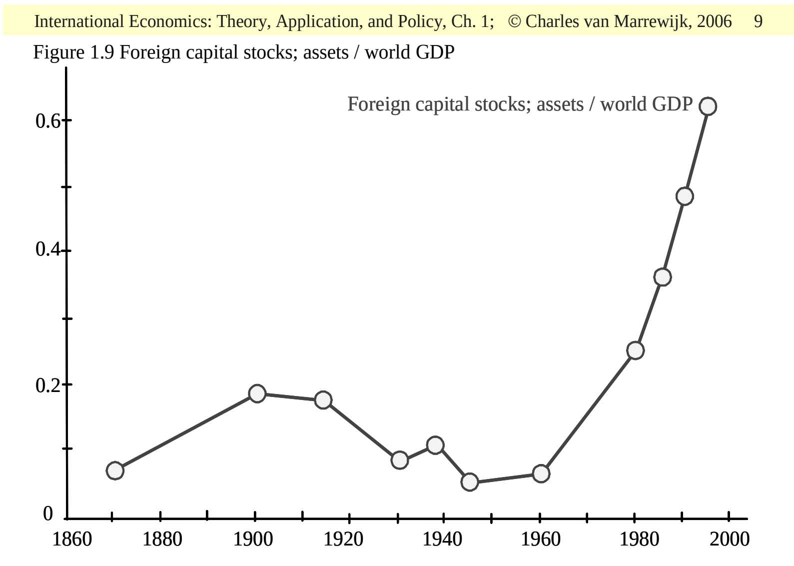
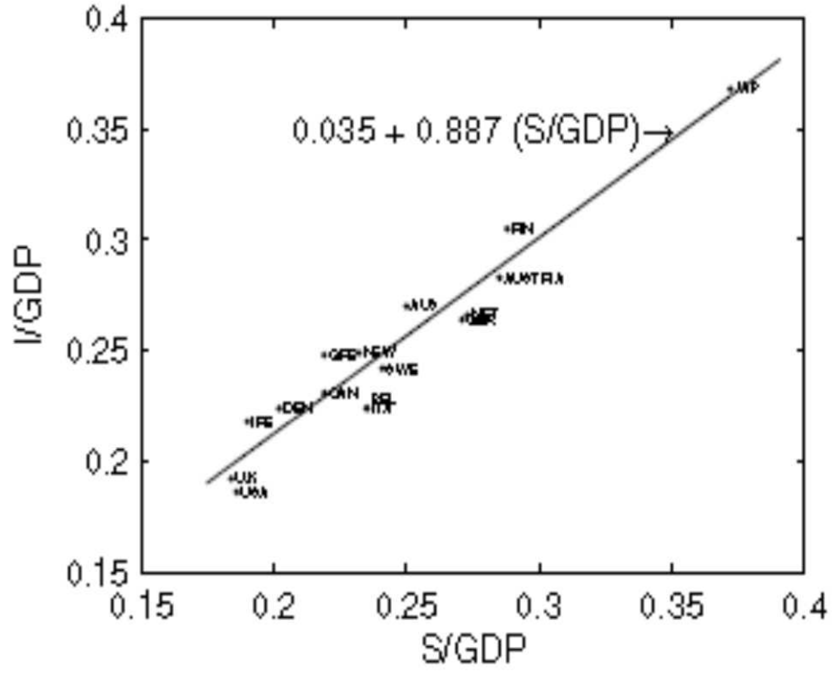


Fig. 3.47 – Stock of Cross-Border Capital as a Percentage of World GDP



Source: M. Feldstein and C. Horioka, "Domestic Saving and International Capital Flows," *Economic Journal* 90, June 1980, 314-29.

FIG. 3.48 – Saving and Investment Rates for 16 Industrialized Countries, 1960-1974 Averages - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 8

ratios from 16 industrial countries over the period 1960-74. The data used in their study is plotted in Figure 3.48.

Feldstein and Horioka argued that if capital was highly mobile across countries, then the correlation between savings and investment should be close to zero, and therefore interpreted their findings as evidence of low capital mobility. The reason why Feldstein and Horioka arrived at this conclusion can be seen by considering the identity :

$$CA = S - I, \tag{3.202}$$

where CA denotes the current account balance, S denotes national savings, and I denotes investment. In a closed economy - i.e., in an economy without capital mobility - the current account is always zero, so that $S = I$ and changes in national savings are perfectly correlated with changes in investment. On the other hand, in a small open economy with perfect capital mobility, the interest rate is exogenously given by the world interest rate, so that, if the savings and investment schedules are affected by independent factors, then the correlation between savings and investment should be zero. For instance, events that change only the savings schedule will result in changes in the equilibrium level of savings but will not affect the equilibrium level of investment (Figure 3.49a). Similarly, events that affect only the investment schedule will result in changes in the equilibrium level of investment but will not affect the equilibrium level of national savings (Figure 3.49b).

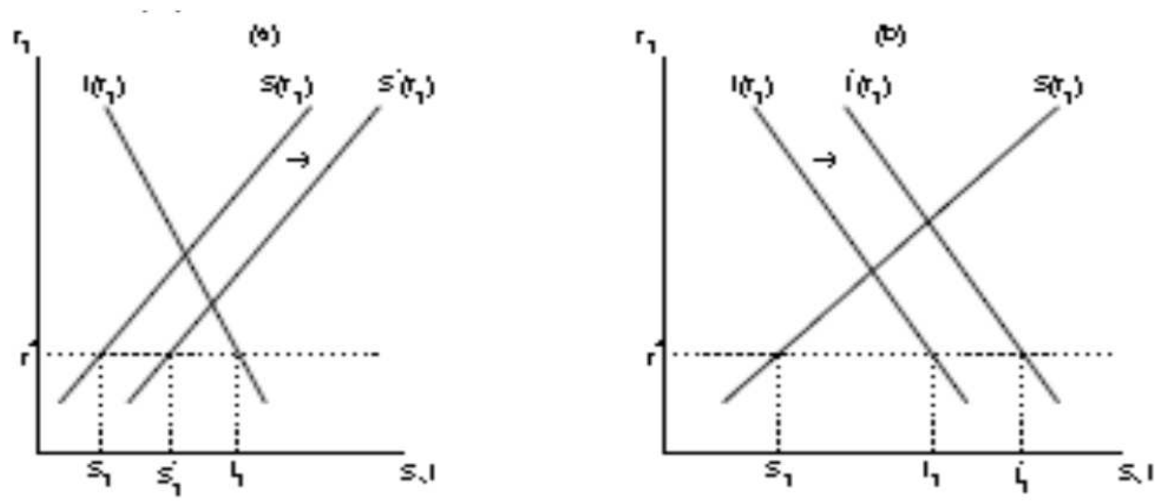


FIG. 3.49 – Response of S and I to independent shifts in (a) the savings schedule and (b) the investment schedule - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 8

Feldstein and Horioka fit the following line through the cloud of points shown in Figure 3.48 :

$$\left(\frac{I}{Q}\right)_i = 0.035 + 0.887 \,.\, \left(\frac{S}{Q}\right)_i + \nu_i; \quad R^2 = 0.91. \tag{3.203}$$

where $(I/Q)_i$ and $(S/Q)_i$ denote, respectively, the average investment-to-GDP and savings-to-GDP ratios in country i over the period 1960-74. Figure 3.48 shows the fitted relationship as a solid line. Feldstein and Horioka used data on 16 OECD countries, so that their regression was based on 16 observations. The high value of the coefficient on S/Q of 0.887 means that there is almost a one-to-one positive association between savings and investment rates. The reported R^2 statistic of 0.91 means that the estimated equation fits the data quite well, as 91 percent of the variation in I/Q is explained by variations in S/Q.

The Feldstein-Horioka regression uses cross-country data. A positive relationship between savings and investment rates is also observed within countries over time (i.e., in time series data). Specifically, for OECD countries, the average correlation between savings and investment rates over the period 1974-90 is 0.495. The savings-investment correlation has been weakening overtime. Figure 3.50 shows the U.S. savings and investment rates from 1955 to 1987. Until the late 1970s savings and investment were moving closely together whereas after 1980 they drifted apart. As we saw earlier (see Figure 3.42), in the first half of the 1980s the U.S. economy experienced a large decline in national savings. A number of researchers have attributed the origin of these deficits to large fiscal deficits. Investment rates, on the other hand, remained about unchanged. As a result, the country experienced a string of unprecedented current account deficits. The fading association between savings and investment is reflected in lower values of the coefficient on S/Q in Feldstein-Horioka style regressions. Specifically, Frankel (1993) estimates the relationship between savings and investment rates using time series data from the U.S. economy and finds that for the period 1955-1979 the coefficient on S/Y is 1.05 and statistically indistinguishable from unity. He then extends the sample to include data until 1987, and finds that the coefficient drops to 0.03 and becomes statistically indistinguishable from zero. In the interpretation of Feldstein and Horioka, these regression results show that in the 1980 the U.S. economy moved from a situation of very limited capital mobility to one of near perfect capital mobility.

But do the Feldstein-Horioka findings of high savings-investment correlations really imply imperfect capital mobility? Feldstein and Horioka's interpretation has been criticized on at least two grounds. First, even under perfect capital mobility, a positive association between savings and investment may arise because the same events might shift the savings and investment schedules. For example, suppose that, in a small open economy, the production functions in periods 1 and 2 are given by $Q_1 = A_1F(K_1)$ and $Q2 = A_2F(K_2)$, respectively. Here Q_1 and Q_2 denote output in periods 1 and 2, K_1 and K_2 denote the stocks of physical capital (such as plant and equipment) in periods 1 and 2, $F(.)$ is an increasing and concave production function stating that the higher is the capital input the higher is output, and A_1 and A_2 are positive parameters reflecting factors such as the state of technology, the effects of weather on the productivity of capital, and so forth.

1st criticism of the Feldstein-Horioka's puzzle

Consider a persistent productivity shock. Specifically, assume that A_1 and A_2 increase and that A_1 increases by more than A_2. This situation is illustrated in Figure 3.51, where the initial situation is one in which the savings schedule is given by $S(r)$ and the investment schedule by $I(r)$. At the world interest rate $r^\star$, the equilibrium levels of savings and investment are given by S and I. In response to the expected increase in A_2, firms are induced to increase next period's capital stock, K_2, to take advantage of the expected rise in productivity. In order to increase K_2, firms must invest more in period 1. Thus, I_1 goes up for every level of the interest rate. This implies that in response to the increase in A_2, the investment schedule

shifts to the right to $I^1(r)$. At the same time, the increase in A_2 produces a positive wealth effect which induces households to increase consumption and reduce savings in period 1. As a result, the increase in A_2 shift the savings schedule to the left. Now consider the effect of the increase in A_1. This should have no effect on desired investment because the capital stock in period 1 is predetermined. However, the increase in A_1 produces an increase in output in period 1 ($\Delta Q_1 > 0$). Consumption-smoothing households will want to save part of the increase in Q_1. Therefore, the effect of an increase in A_1 is a rightward shift in the savings schedule. Because we assumed that A_1 increases by more than A_2, on net the savings schedule is likely to shift to the right. In the figure, the new savings schedule is given by $S^1(r)$. Because the economy is small, the interest rate is unaffected by the changes in A_1 and A_2. Thus, both savings and investment increase to S1 and I1, respectively.

2nd criticism of the Feldstein-Horioka's puzzle

A second reason why savings and investment may be positively correlated in spite of perfect capital mobility is the presence of large country effects. Consider, for example, an event that affects only the savings schedule in a large open economy like the one represented in Figure 3.52. In response to a shock that shifts the savings schedule to the right from $S(r)$ to $S'(r)$ the current account schedule also shifts to the right from $CA(r)$ to $CA'(r)$. As a result, the world interest rate falls from $r^\star$ to $r^{\star,\prime}$. The fall in the interest rate leads to an increase in investment from I to I'. Thus, in a large open economy, a shock that affects only the savings schedule results in positive comovement between savings and investment.

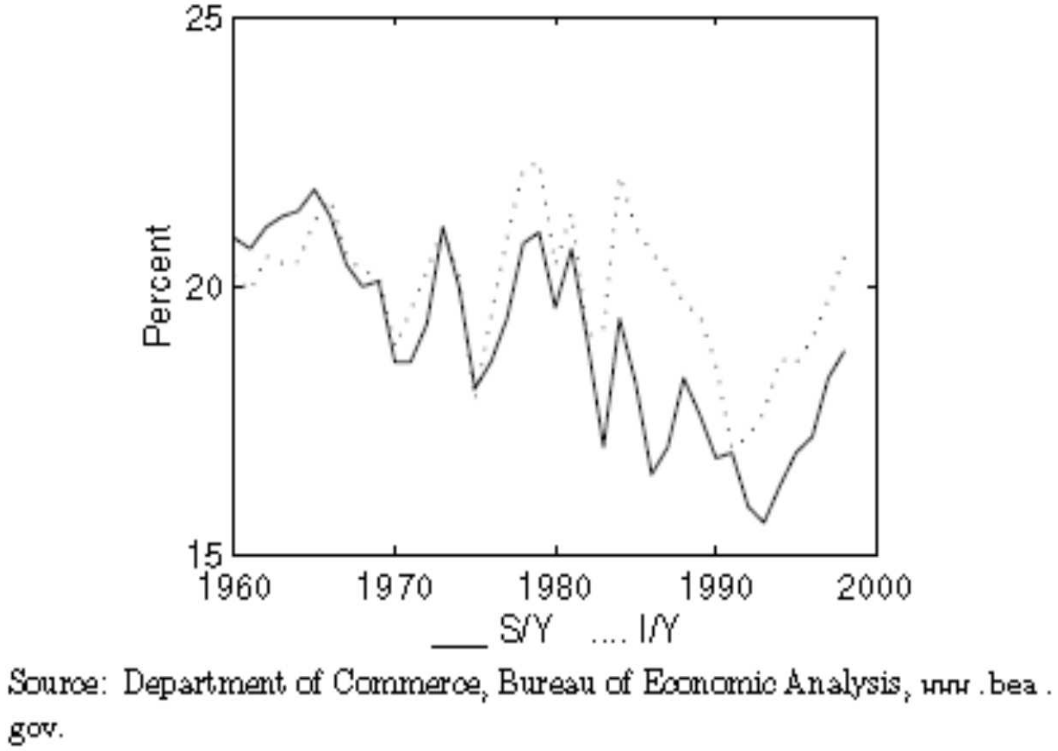


FIG. 3.50 – U.S. National Saving, Investment, and the Current Account as a Fraction of GNP, 1960-1998 - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 8

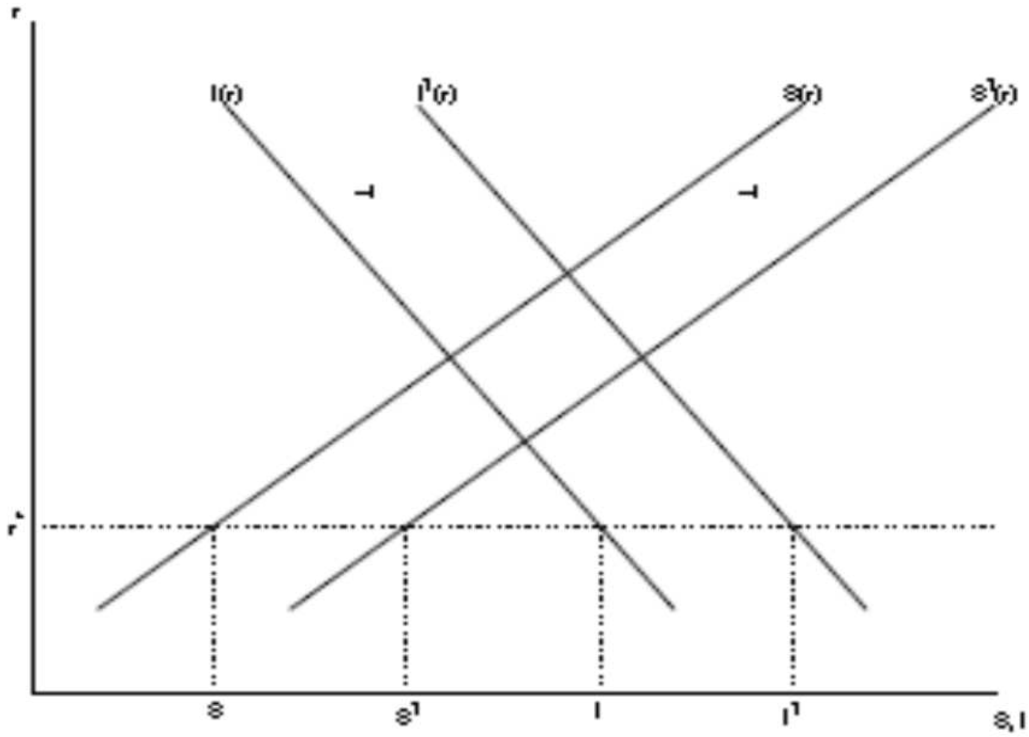


FIG. 3.51 – Response of S and I to a persistent productivity shock - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 8

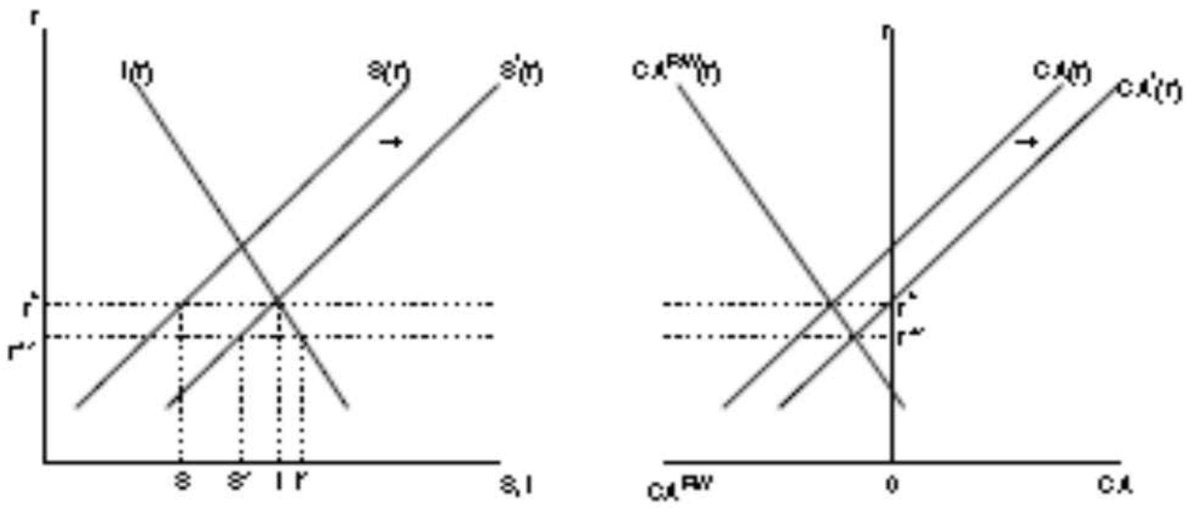


FIG. 3.52 – Large open economy : response of S and I to a shift in the savings schedule - Source : Schmitt-Grohé, Stephanie et Martin, Uribe (2014) International Macroeconomics, Chapter 8

3.7.2 Current Account Deficits in the Euro Area : Blanchard and Giavazzi (2002)

At the time of euro accession, Greece and Portugal's current account deficits were already large. In 2000-01 the current account deficit of Portugal reached 10 percent of its GDP, up from 2-3 percent at the start of the 1990s. These deficits have continued and reach almost 11% of GDP in 2007-2008. Greece is not far behind. Its current account deficit in 2000-01 was equal to 6-7 percent of GDP, and up to almost 15% in 2007-2008. Spain had a moderate current account deficit, while Italy and Ireland had a balanced current account.

As shown in Table 3.53, current account balances in Greece, Ireland, Italy, and Spain worsened significantly during the first decade of European Monetary Union, while Portugal's deficit remained at the very high levels it had reached early in the decade. As a result of the increasing recourse to external financing, net external liabilities of these countries rose sharply, reaching levels close to or above 100 percent of GDP by the end of 2010 in Greece, Ireland, Portugal, and Spain, as shown in Figure 3.54.

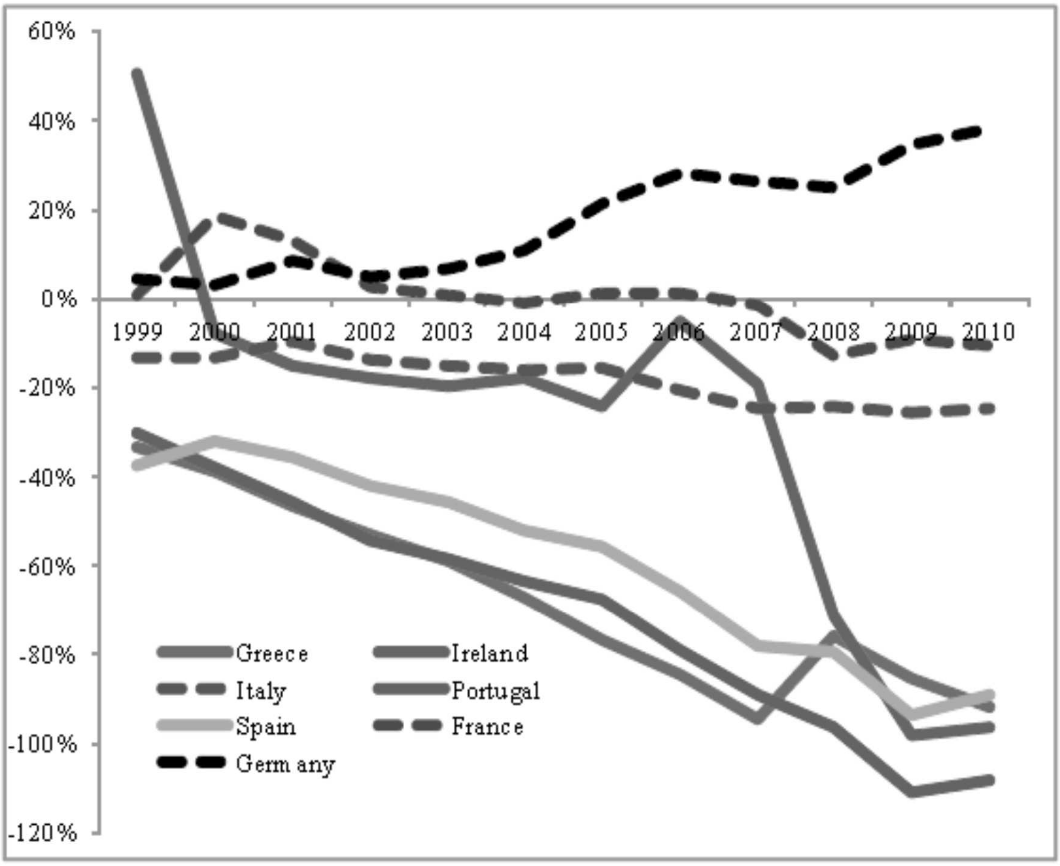
According to the standard modern growth theory, these deficits are the symptom of a catching-up process. More precisely, the speed at which the country is catching-up depends on the distance between actual and potential output levels, where potential output depends on total factor productivity, savings and population growth, as well as on policies. Financial integration and lower interest rates due to the elimination of the exchange-rate premium eases or fastens the catching-up process by removing obstacles to capital flows. The most striking evidence of the extent of financial integration of debtor countries with the rest of the euro area is the well-known convergence in bond yields that occurred between the mid-1990s and the onset of the global financial crisis. Figure 3.55 shows the spreads between 10-year government bonds and German bonds, illustrating the process of financial integration and convergence of interest rates that started in the mid-1990s. With the exception of Greece, most of the reduction of bond spreads in Southern Europe took place in the run–up to EMU, and spreads remained stable and low until the onset of the crisis, suggesting that government

		1999-2001	2007-2008	Change 1999-01 to 2007-08
Greece 1/	**Current Account**	**-6.8%**	**-14.5%**	-7.7%
	Investment	22.9%	21.6%	-1.4%
	Savings	16.2%	7.1%	-9.1%
	Public Savings	-0.7%	-3.0%	-2.3%
	Private Savings	16.9%	10.1%	-6.7%
	Household Savings	2.0%	0.3%	-1.7%
	Corporate Savings	13.8%	9.8%	-4.0%
Ireland	**Current Account**	**-0.3%**	-5.3%	-5.0%
	Investment	23.4%	23.9%	0.5%
	Savings	23.2%	18.7%	-4.5%
	Public Savings	8.3%	-0.8%	-9.1%
	Private Savings	14.8%	19.4%	4.6%
	Household Savings	...	...	...
	Corporate Savings	...	...	...
Italy	**Current Account**	**0.0%**	**-2.9%**	**-3.0%**
	Investment	20.4%	21.5%	1.0%
	Savings	20.5%	18.5%	-1.9%
	Public Savings	1.3%	1.5%	0.2%
	Private Savings	19.2%	17.0%	-2.2%
	Household Savings	10.8%	10.2%	-0.5%
	Corporate Savings	8.4%	6.8%	-1.6%
Portugal	**Current Account**	-9.5%	**-10.8%**	**-1.2%**
	Investment	27.5%	22.3%	-5.3%
	Savings	18.0%	11.5%	-6.5%
	Public Savings	0.9%	-0.5%	-1.3%
	Private Savings	17.1%	11.9%	-5.2%
	Household Savings	7.3%	4.4%	-2.9%
	Corporate Savings	9.8%	7.5%	-2.2%
Spain 1/	**Current Account**	**-3.6%**	**-9.8%**	**-6.2%**
	Investment	25.9%	30.1%	4.2%
	Savings	22.3%	20.3%	-2.0%
	Public Savings	2.3%	2.9%	0.6%
	Private Savings	20.1%	17.5%	-2.6%
	Household Savings	7.4%	7.7%	0.2%
	Corporate Savings	12.5%	9.8%	-2.7%
France	**Current Account**	**2.2%**	**-1.6%**	**-3.9%**
	Investment	19.9%	22.2%	2.3%
	Savings	22.0%	20.6%	-1.5%
	Public Savings	2.1%	5.1%	3.0%
	Private Savings	19.9%	15.4%	-4.5%
	Household Savings	10.0%	10.2%	0.3%
	Corporate Savings	9.9%	5.2%	-4.8%
Germany	**Current Account**	**-1.0%**	7.2%	**8.1%**
	Investment	20.9%	18.8%	-2.2%
	Savings	19.9%	25.9%	6.0%
	Public Savings	0.9%	2.5%	1.5%
	Private Savings	19.0%	23.5%	4.4%
	Household Savings	10.6%	11.6%	1.0%
	Corporate Savings	8.4%	11.8%	3.4%

Sources: Eurostat, IFS, and Staff Calculations

1/ households and corporate savings data start in 2000.

FIG. 3.53 – Saving-Investment Balance (In percent of GDP) - Source : Chen, Milesi-Ferretti and Tressel (2012) External Imbalances in the Euro Area. *IMF Working Paper* n° 236.



Source: IFS data

FIG. 3.54 – Net Foreign Asset Positions 1999-2010, in Percent of GDP - Source : Chen, Milesi-Ferretti and Tressel (2012) External Imbalances in the Euro Area. *IMF Working Paper n°* 236.

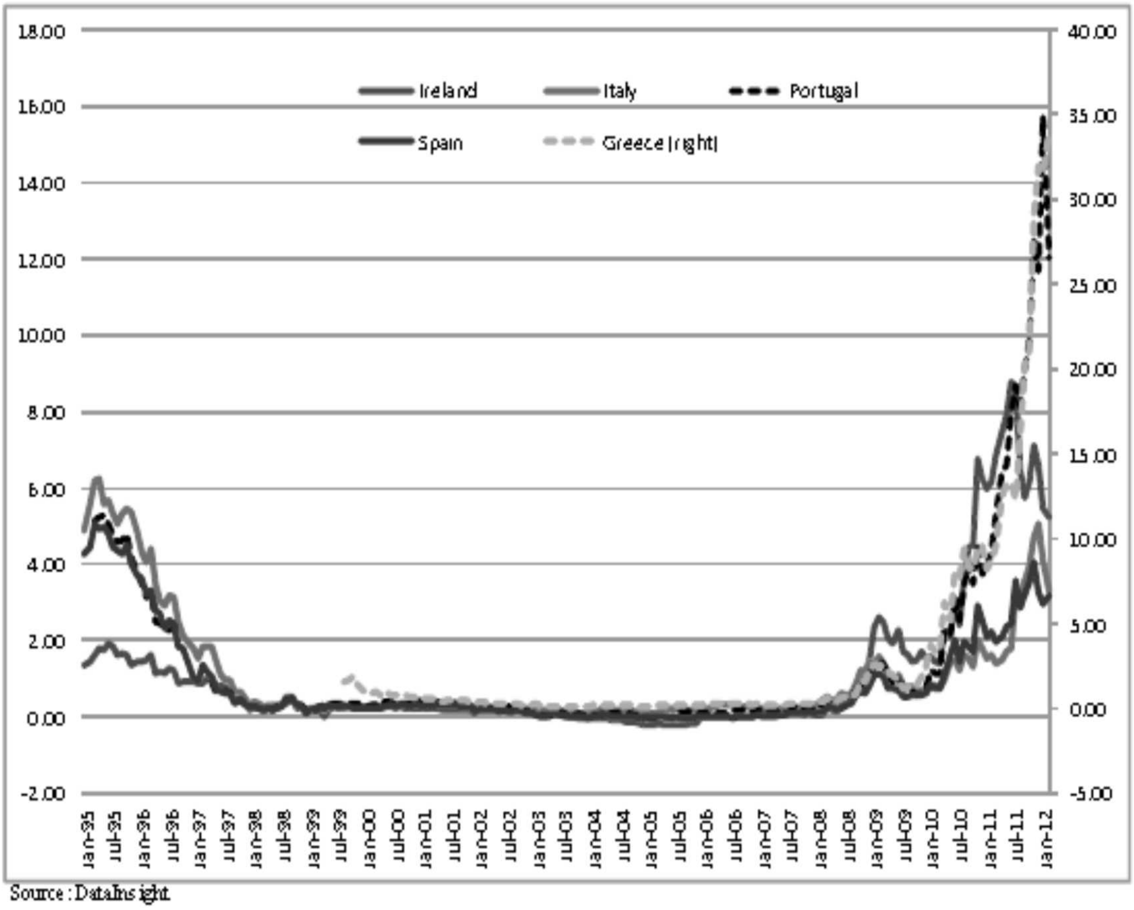


FIG. 3.55 – Ten-Year Government Bond Spreads Against German Bunds - Source : Chen, Milesi-Ferretti and Tressel (2012) External Imbalances in the Euro Area. *IMF Working Paper* n° 236.

bonds of euro area debtor countries became close substitutes to German bonds for marginal investors.

The fact that both Portugal and Greece are members of both the European Union and the euro area (the group of countries that use the euro as their common currency), and the fact that they are the two poorest members of both groups, suggest a natural explanation for 1999-2008 current account deficits. To the extent that they are the countries with higher expected rates of return, poor countries should see an increase in investment. And to the extent that they are the countries with better growth prospects, they should also see a decrease in saving. Thus, on both counts, poorer countries should run larger current account deficits, and, symmetrically, richer countries should run larger current account surpluses. In line with the theory, during the catching-up period, Germany and a number of other smaller countries in Northern Europe progressively built large current account surpluses, with the current account for the euro area as a whole remaining in broad balance throughout the period.

Portugal and Greece should exactly be what theory suggests can and should happen when countries become more closely linked in goods and financial markets. A country that wants to borrow from the rest of the world must take into account two things : the interest rate it faces, and the price cuts it will need to make to generate sufficient export revenue to repay the debt.

- Increased financial integration, which brings about a lower or a flatter cost of borrowing, clearly makes it more attractive to borrow.
- Increased goods market integration, which leads to a more elastic demand for the country's goods, decreases the price cuts required in the future and so has a similar effect.

Thus, in response to increased integration, borrower countries will want to borrow more. And, by a symmetric argument, lender countries will want to lend more. Thus, the distribution of current account balances will widen.

In this section, we investigate whether optimal borrowing triggered by catching-up process fits the facts. We conclude that it does, and that saving rather than investment is the main channel through which integration affects current account balances.

We proceed in four steps :

- First, using a workhorse open-economy model, we show how, for poorer countries, goods and financial market integration are likely to lead to both a decrease in saving and an increase in investment, and so to a larger current account deficit. We also discuss how other, less direct implications of the process of integration, such as domestic financial liberalization, are likely to reinforce that outcome.
- Second, we look at panel data evidence from the countries of the Organization for Economic Cooperation and Development (OECD) since 1975.
- Third, we return to the cases of Portugal and Greece. We conclude that the recent history of these two countries is largely consistent with the findings of the panel data regressions. Lower private saving-due to both internal and external financial market liberalization but also to better future growth prospects - and, to a lesser extent, higher investment appear to be the main drivers of the larger current account deficits.
- We end by taking up two issues raised by our findings. First, we relate our results to the large body of research triggered by what has been called the Feldstein-Horioka puzzle :

the finding of a high cross-country correlation between saving and investment. We show that, consistent with our findings, this correlation has substantially declined over time in this sample of countries, especially within the euro area. At least for this last group, the Feldstein-Horioka phenomenon appears to have largely disappeared.

3.7.2.1 Current Account Balances and Economic Integration : An Open Economy Model

We consider a group of J countries trading goods and assets among themselves. Each country produces its own good, but households in each country consume the same composite good. We denote by P_t^C the consumption price index and P_t' the output price. To keep things simple, the rate of time preference ρ is set to zero. Households live for two periods and maximize intertemporal utility :

$$\Lambda \equiv \ln\left(C_t^j\right) + \ln\left(C_{t+1}^j\right), \tag{3.204}$$

where consumption in each period is an aggregate of n varieties

$$C^j \equiv \left(\sum_{i=1}^{N} (C_i^j)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}; \tag{3.205}$$

the parameter σ is the elasticity of substitution among varieties. The demand for each variety is a decreasing function of its own price (P_i'), an increasing function of the average price (P^C) and of the number of varieties, N :

$$C_i^j = \left(\frac{P_i'}{P^C}\right)^{-\sigma} .C^j, \tag{3.206}$$

où $C^j = X^j/P^C$ and the consumption price index is a weighted average of varieties' prices :

$$P^C = \left(\sum_{i=1}^{N} (P_i')^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{3.207}$$

Countries may borrow or lend at rate $1+r = R$. The intertemporal budget constraint states that present discounted flow of consumption expenditure cannot exceed the present discounted flow of revenue :

$$P_t^C .C_t + \frac{P_{t+1}^C .C_{t+1}}{R} = P_t' .Y_t + \frac{P_{t+1}' .Y_{t+1}}{R}. \tag{3.208}$$

Dividing both sides by the consumption price index at time t, i.e., P_t^C, and denoting by $P_t \equiv \frac{P'}{P_t^C}$ the price of the good produced by the country in terms of consumption, the intertemporal budget constraint (3.208) can be rewritten as follows :

$$\begin{aligned} & C_t + \frac{P_{t+1}^C}{P_t^C} .\frac{C_{t+1}}{R} \\ = & \frac{P_t'}{P_t^C} .Y_t + \frac{P_{t+1}'}{P_{t+1}^C} .\frac{P_{t+1}^C}{P_t^C} .\frac{Y_{t+1}}{R}, \\ = & P_t .Y_t + \frac{P_{t+1}^C}{P_t^C} .\frac{P_{t+1} .Y_{t+1}}{R}. \end{aligned} \tag{3.209}$$

We denote by $R^C = 1 + r^C$ the consumption-based real interest rate :

$$R^C = \frac{R .P_t^C}{P_{t+1}^C} = R .(1+x). \tag{3.210}$$

The interest rate R^C gives the real return following a loan of one unit of C_t. More precisely, when selling one unit of C_t, the agent gets an mount equal to P_t^C ; lending this amount on capital markets gives $R\ .P_t^C$ at time $t+1$ which allows the agent to but $\frac{R\ .P_t^C}{P_{t+1}^C}$ units of C_{t+1}. In other words, R^C is the price of present consumption in terms of future real consumption. The price of present consumption increases when the interest rate R rises or when the price of future consumption falls relative to present consumption, i.e., if $x > 0$. Using (3.210) which implies that $\frac{P_t^C}{P_{t+1}^C} = 1 + x$, the intertemporal budget constraint (3.209) can be rewritten as follows :

$$C_t + \frac{C_{t+1}}{(1+x)\ .R} = P_t\ .Y_t + \frac{P_{t+1}\ .Y_{t+1}}{(1+x)\ .R} \equiv \Omega, \tag{3.211}$$

where $(1+x)\ .R \equiv R^C$ is the consumption-based real interest rate.

To solve the intertemporal maximization problem, we eliminate future consumption C_{t+1} from the intertemporal utility (3.204) by using the intertemporal budget constraint (3.211), i.e., $C_{t+1} = (1+x)\ .R\,(\Omega - C_t)$:

$$\Lambda \equiv \ln(C_t) + \ln\left[(1+x)\ .R\,(\Omega - C_t)\right]. \tag{3.212}$$

Differentiating (3.212) w.r.t. C_t and setting the partial derivative to zero, one obtains :

$$\frac{1}{C_t} - \frac{(1+x)\ .R}{C_{t+1}} = 0.$$

Rearranging terms, we get the standard equality between the intertemporal MRT and the relative price of present consumption measured by R^C :

$$\frac{C_{t+1}}{C_t} = (1+x)\ .R \equiv R^C. \tag{3.213}$$

Inserting eq. (3.213), i.e. $\frac{C_{t+1}}{(1+x)\ .R}$ into the intertemporal budget constraint (3.211) leads to :

$$C_t = \frac{1}{2}\ .\Omega, \tag{3.214}$$

where $\Omega \equiv P_t\ .Y_t + \frac{P_{t+1}\ .Y_{t+1}}{(1+x)\ .R}$.

To determine the current account at time t, we use the first period budget constraint. We denote by B_t the net foreign asset position at the end of period t; we further assume that the economy starts with a net foreign position equal to zero, i.e., $B_{t-1} = 0$ so that the budget constraint reads as :

$$P_t'\ .B_t = P_t'\ .Y_t - P_t^C\ .C_t. \tag{3.215}$$

Dividing both sides by P_t^C and denoting by $P_t = \frac{P_t'}{P_t^C}$ the price of the good in terms of consumption, denoting by $CA_t = B_t - B_{t-1}$ the current account at time t, using the fact that $B_{t-1} = 0$ by assumption, the current account can be written as follows :

$$P_t\ .B_t = P_t\ .CA_t = P_t\ .Y_t - C_t. \tag{3.216}$$

Defining $ca_t = \frac{P_t \, .CA_t}{P_t \, .Y_t}$ as the ratio of the current account balance to GDP (at current prices), and inserting optimal consumption C_t (3.214), eq. (3.216) reads as :

$$\begin{aligned}
\frac{P_t \, .CA_t}{P_t \, .Y_t} &= 1 - \frac{C_t}{P_t \, .Y_t}, \\
&= 1 - \frac{1}{2} \cdot \frac{\Omega}{P_t \, .Y_t}, \\
&= 1 - \frac{1}{2} \left[\frac{P_t \, .Y_t}{P_t \, .Y_t} + \frac{P_{t+1} \, .Y_{t+1}}{P_t \, .Y_t \, . (1+x) \, .R} \right], \\
&= \frac{1}{2} - \frac{1}{2} \cdot \frac{Y_{t+1}}{Y_t} \cdot \frac{1}{(1+x) \, .R} \cdot \frac{P_{t+1}}{P_t}.
\end{aligned} \tag{3.217}$$

The three terms in the expression in brackets on the RHS of eq. (3.217) give the determinants of the current account balance :

- The growth of domestic output : $\frac{Y_{t+1}}{Y_t} = 1 + g$. A rise in GDP implies that the country raises both future, C_{t+1}, and present consumption, C_t. Because the current income is unchanged, the country borrows abroad to finance increased present consumption C_t.
- The interest rate faced by the country : $(1+x) \, .R$. The higher the consumption interest rate, the larger the relative price of present consumption, the more expensive it is to borrow abroad, and thus the smaller the current account deficit.
- The change in the terms of trade : $\frac{P_{t+1}}{P_t}$. The terms of trade are defined as the ratio of the price of exports to the price of imports. The price of exports corresponds to the price of the good produced by the country and the price of imports is defined as the consumption price index. The larger the fall in the price of the domestic good required next period to sell enough domestic goods to pay down the debt, the more expensive it is to borrow, and thus the smaller the current account deficit.

To simplify expression (3.217), we use the fact that the world demand for one variety is obtained by summing over the N countries $\sum_{j=1}^{N} C_i^j$; the world demand for one variety must be equal to output in one country producing this variety :

$$\sum_{j=1}^{N} C_i^j = Y_i = Y, \tag{3.218}$$

where Y is the output of one variety which determines GDP in one given country. The demand for all goods must be equal to world output

$$\sum_{j=1}^{N} C^j = Y^{\star}. \tag{3.219}$$

Aggregating the demand for one variety over the N countries by using eq. (3.206), we get

$$\begin{aligned}
\sum_{j=1}^{N} C_i^j &= \left(\frac{P_i'}{P^C} \right)^{-\sigma} . \sum_{j=1}^{N} C^j, \\
Y_i &= (P_i)^{-\sigma} \, .Y^{\star}, \\
Y &= (P)^{-\sigma} \, . \sum_{j=1}^{N} Y^{\star}.
\end{aligned} \tag{3.220}$$

Adding the time subscript t, one obtains the demand for the good produced in a given country :

$$P_t = \left(\frac{Y_t}{Y_t^{\star}} \right)^{-\sigma}. \tag{3.221}$$

Assuming that the world consumption-based interest rate is determined by the rate of growth of world output, i.e.

$$\frac{Y^\star_{t+1}}{Y^\star_t} = 1 + g^\star = R \,.\, (1+x)\,, \tag{3.222}$$

the current account can be expressed as follows :

$$\begin{aligned} ca_t &= \frac{1}{2} - \frac{1}{2} \,.\, \frac{1+g}{1+g^\star} \,.\, \left(\frac{Y_{t+1}/Y_t}{Y^\star_{t+1}/Y^\star_t}\right)^{-\frac{1}{\sigma}}, \\ &= \frac{1}{2} - \frac{1}{2} \,.\, \frac{1+g}{1+g^\star} \,.\, \left(\frac{1+g}{1+g^\star}\right)^{-\frac{1}{\sigma}}, \\ &= \frac{1}{2} - \frac{1}{2} \,.\, \left(\frac{1+g}{1+g^\star}\right)^{\frac{\sigma-1}{\sigma}}. \end{aligned} \tag{3.223}$$

Assuming that poorer countries face a risk premium θ on their borrowing, the current account balance as a percentage of GDP reads as :

$$\frac{1}{2} - \frac{1}{2} \,.\, \left(\frac{1+g}{(1+\theta)\ .\,(1+g^\star)}\right)^{\frac{\sigma-1}{\sigma}}. \tag{3.224}$$

The authors assume that $\sigma > 1$ for the Marshall-Lerner condition to hold (which implies that a fall in the terms of trade improves the trade balance because export and import demands are elastic enough to relative price changes). According to (3.224) :

- If output growth g exceeds the output growth of its trading partners $g^\star$, and the borrowing wedge θ is not too large, the country will run a current account deficit.
- By Financial integration amplifies the current account deficit by reducing the risk premium (as captured by a fall in θ) and thus the cost of borrowing.
- Monetary union has led to a further decrease in θ within the euro area, by eliminating currency risk.
- By eliminating tariffs, setting up a stricter enforcement of competition rules across the European Union, and raising the number of products due to the fall in the cost of entry (in particular in 'Transport and Communication'), goods market integration (captured by a rise in σ) also magnifies the the current account deficit because the price cuts which are required to raise exports in order to repay debt are smaller as demand becomes more elastic.

So far we have focused only on saving. Allowing production to depend on capital, i.e., $Y_t = A_t \,.\, (K_t)^\alpha$, and using the fact that $I_t = K_{t+1} - K_t + \delta \,.K_t$, the optimal decision for the stock of capital K_{t+1} is :

$$\begin{aligned} K_{t+1} &= \left[\frac{\alpha \,.A_{t+1}}{\frac{P'_t}{P'_{t+1}} \,.R \,.\, (1+\theta) - (1-\delta)}\right]^{\frac{1}{1-\alpha}}, \\ &= \left[\frac{\alpha \,.A_{t+1}}{\frac{P_t}{P_{t+1}} \,.\, (1+x)\ .R \,.\, (1+\theta) - (1-\delta)}\right]^{\frac{1}{1-\alpha}}, \end{aligned} \tag{3.225}$$

where we used the fact that $\frac{P'_t}{P'_{t+1}} = \frac{P'_t}{P^C_t} \,.\, \frac{P^C_t}{P^C_{t+1}} \,.\, \frac{P^C_{t+1}}{P'_{t+1}} = \frac{P_t}{P_{t+1}} \,.\, (1+x)$. If we consider a country that is poorer in the sense of having less capital than the others in the group, the initial capital stock K_t is much lower than the next period capital stock which in turn requires higher investment.

How much investment takes place will depend both on the cost of borrowing and on the future terms of trade : the lower the relative price of domestic goods in the future, the less attractive it is to invest in the production of domestic goods.

- The extent that financial integration leads to a lower cost of finance, investment will increase.
- It will also increase to the extent that goods market integration leads to an increase in the elasticity of demand for domestic goods : the higher the elasticity of demand, the smaller the decrease in price needed to sell the additional output in the future, and so the more attractive investment is this period.

3.7.2.2 Current Account Balances and Economic Integration : An Open Economy Model

We have to explain why countries such as Greece or Portugal have been running larger deficits and why other countries have been running larger surpluses. According to the open-economy model presented in the previous section, these deficits should be associated with rising domestic investment-to the extent that the marginal product of capital is higher than in richer countries - and/or by a decrease in savings - that would be the consequence of stronger growth prospects and declining borrowing constraints for firms and households following financial liberalization.

To test the predictions of the model, Blanchard and Giavazzi (2002) examine the following specification :

$$\left(\frac{CA}{Y}\right)_{it} = \alpha_t + \beta_t \cdot \left[\frac{(Y/N)_{it}}{(Y/N)_t}\right] + \gamma \cdot X_{it} + \epsilon_{it}. \tag{3.226}$$

In this specification, the ratio of the current account balance to output in year t for country i depends on a common time effect, a_t, on the level of income per capita in year t for country i, $(Y/N)_{it}$, relative to the average level of income per capita in year t for the group of countries under consideration $(Y/N)_t$, and on other control variables included in the vector X_{it} (dependency ratio, constructed as the ratio of population to the labor force, and rate of growth of output from year $t-1$ to t, included to capture cyclical effects of movements in output on the current account). Finally, Blanchard and Giavazzi allow the effect of relative income per capita to vary from year to year (estimated coefficient β_t varies across time). The period of estimation runs from 1975 to 2001.

The simplest way to present our results is by plotting the set of estimated coefficients β_t against time. Figure 3.56 suggests that, for the European Union, the widening of current account positions can be largely accounted for by an increased dependence of the current account balance on income per capita. The effect seems weaker, if present at all, for the OECD. And there is no strong evidence of an additional euro effect.

We turn to the question of whether the increased dependence of current account balances on income per capita reflects an increased dependence of saving or an increased dependence of investment. To address this question, Blanchard and Giavazzi simply reran the basic specification, replacing the ratio of the current account to GDP first with the ratio of saving to GDP and then with the ratio of investment to GDP. Figure 3.57 shows the results of the saving regressions. We draw two conclusions :

- First, for the OECD as a whole, the coefficient tends to be close to zero : there is not much evidence of a significant effect of income per capita on saving.

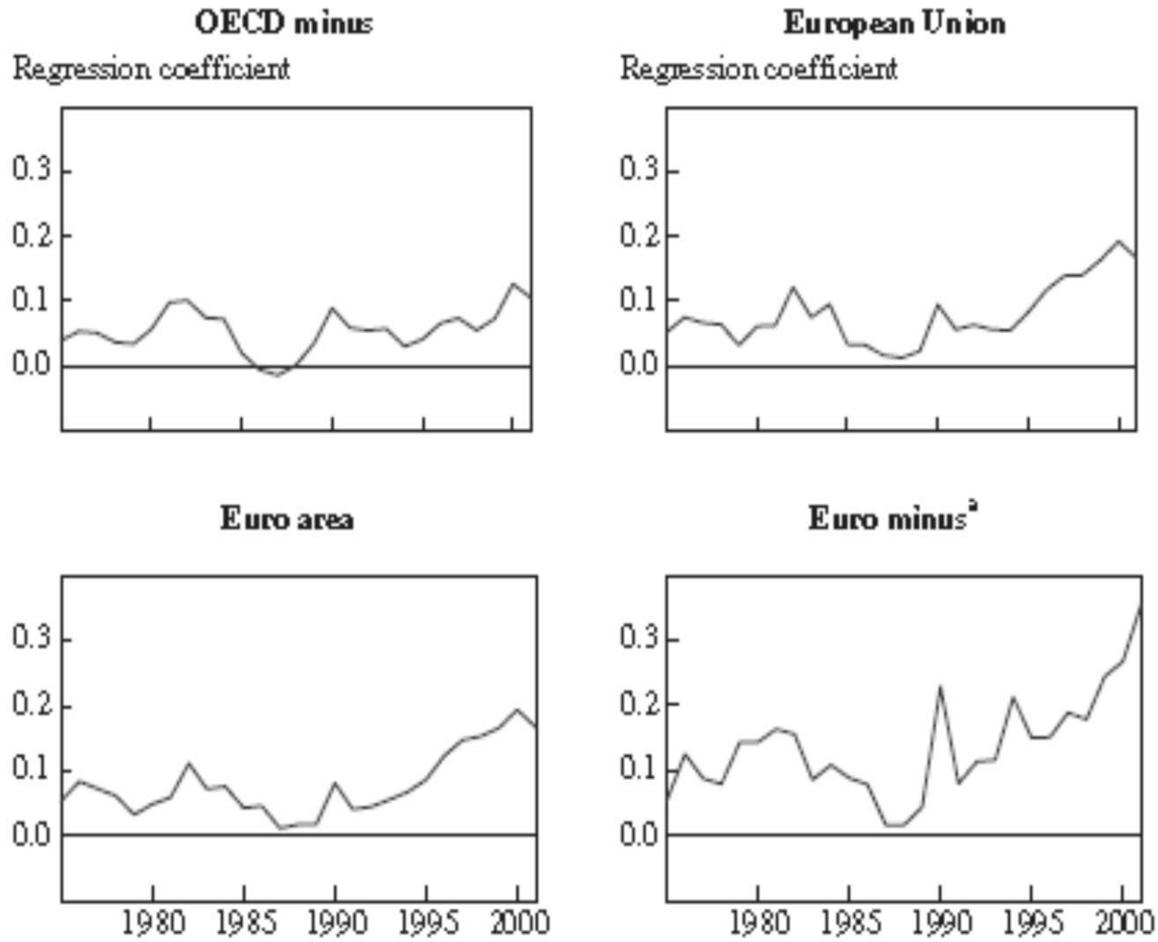


FIG. 3.56 – Yearly Coefficients of Current Account Balances on Output per Capita from Panel Regressions, 1975-2001 - Source : Blanchard and Giavazzi (2002) Current Account Deficits in the Euro Area : The End of the Feldstein-Horioka Puzzle ? *Brookings Papers on Economic Activity*, 33(2), pp. 147-210.

– Second, for both the European Union and the euro area, there is much clearer evidence of a trend. At the start of the sample, saving is negatively related to income per capita - the opposite of what the standard open-economy growth model predicts. The relationship turns positive in the late 1980s, in line with the prediction of the model, and the coefficient becomes larger in the euro area than in the European Union, which can be explained by financial integration.

Figure 3.58 shows the results of the investment regressions. We draw two conclusions :

– First, the coefficient is typically negative : a lower income per capita is associated with higher investment, as predicted by the standard model
– Second, there is no evidence of a trend toward a more negative effect of income per capita on investment over time.

In short, the **increased dependence of current account balances on income per capita reflects, for the most part, an effect through saving rather than an effect through investment**.

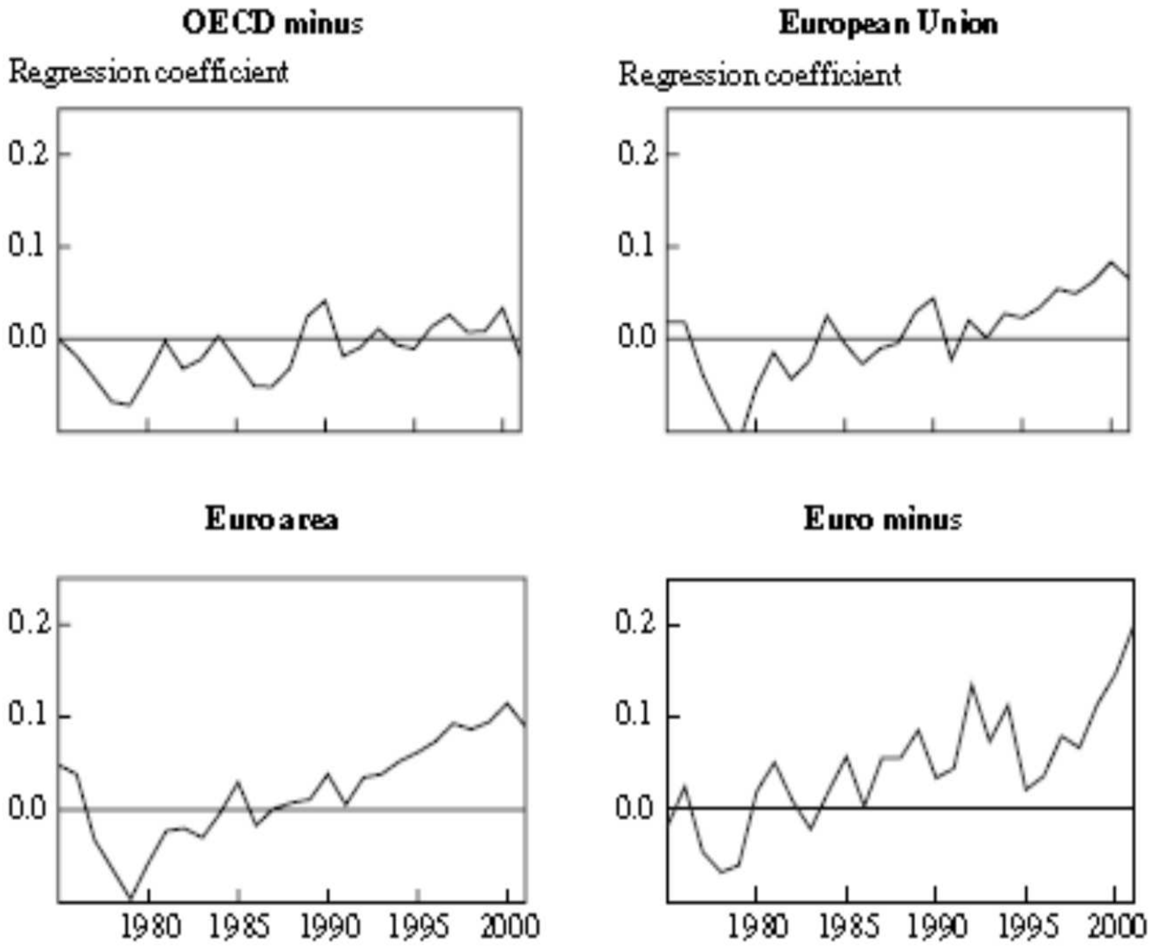


FIG. 3.57 – Yearly Coefficients of Saving on Income per Capita from Panel Regressions, 1975-2001 - Source : Blanchard and Giavazzi (2002) Current Account Deficits in the Euro Area : The End of the Feldstein-Horioka Puzzle ? *Brookings Papers on Economic Activity*, 33(2), pp. 147-210.

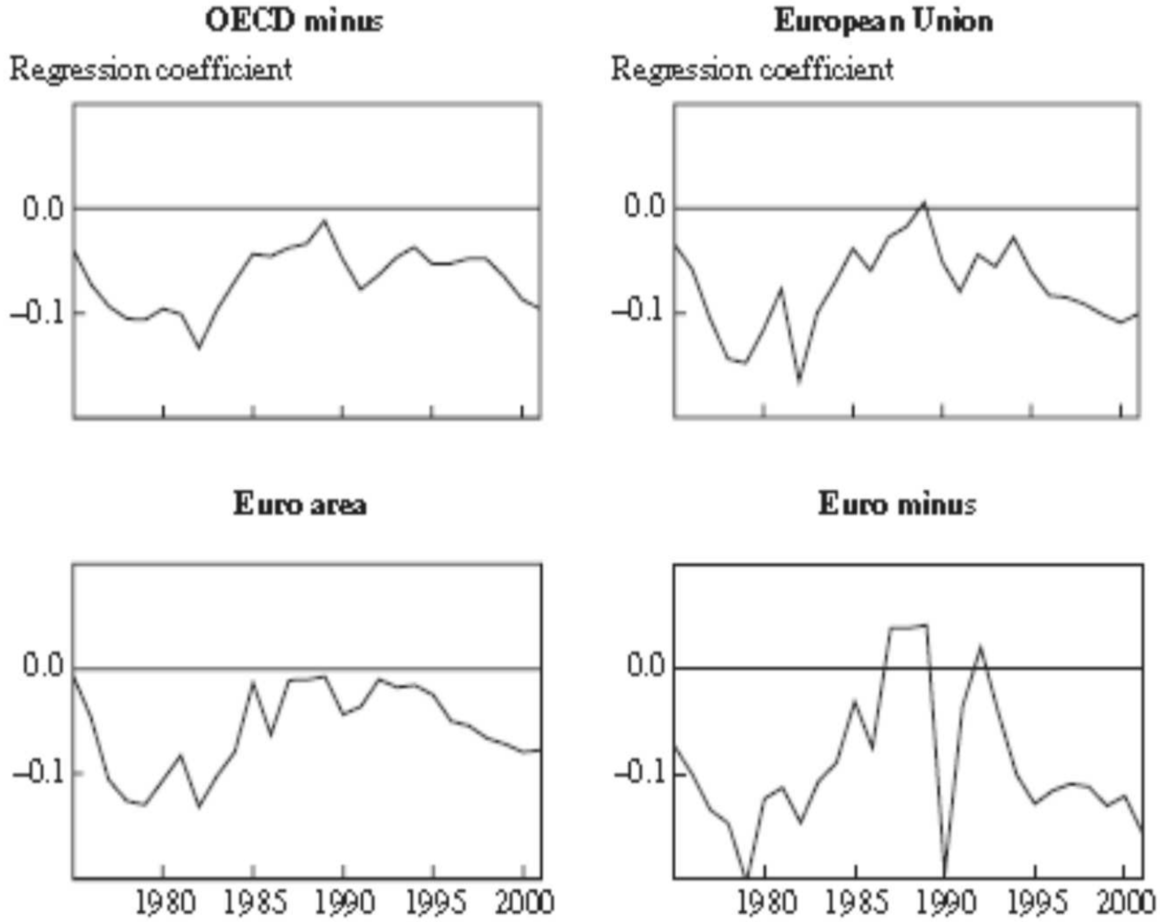


Fig. 3.58 – Yearly Coefficients of Investment on Income per Capita from Panel Regressions, 1975-2001 - Source : Blanchard and Giavazzi (2002) Current Account Deficits in the Euro Area : The End of the Feldstein-Horioka Puzzle ? *Brookings Papers on Economic Activity*, 33(2), pp. 147-210.

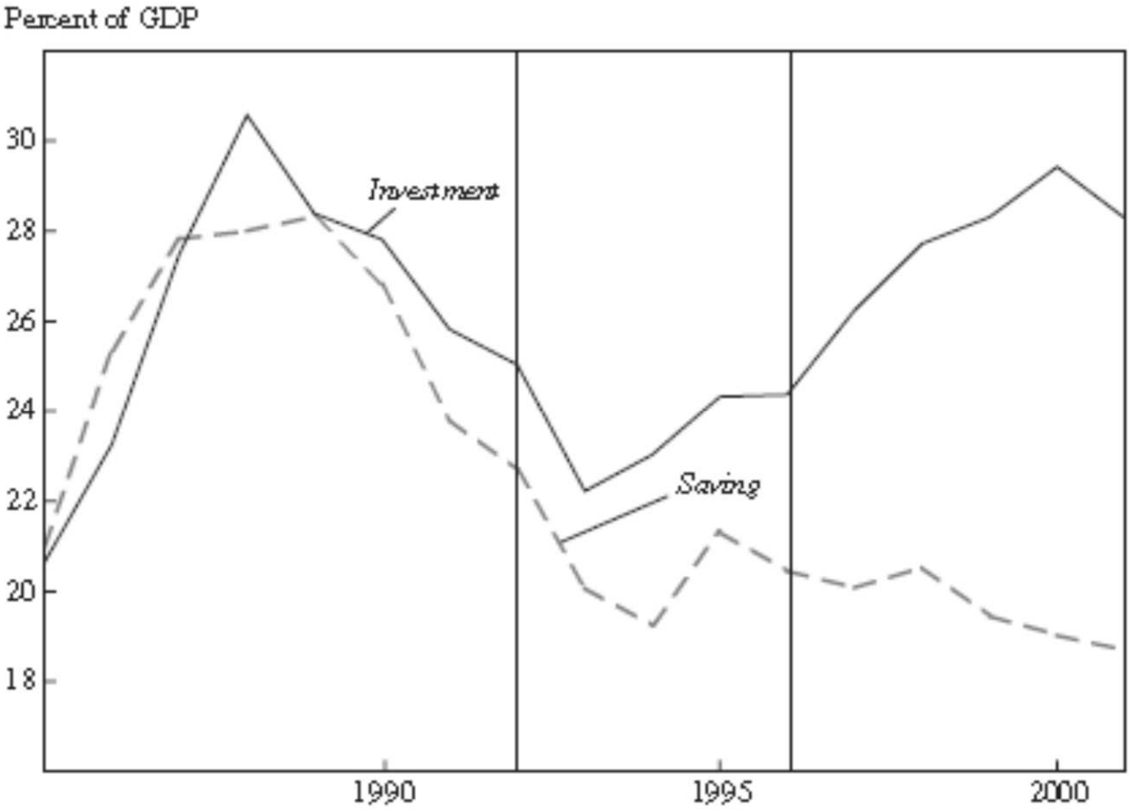


Fig. 3.59 – Portugal : Investment and Saving, 1985-2001 - Source : Blanchard and Giavazzi (2002) Current Account Deficits in the Euro Area : The End of the Feldstein-Horioka Puzzle ? *Brookings Papers on Economic Activity*, 33(2), pp. 147-210.

Percent of GDP

Item	*1985–91*	*1992–95*	*1996–2001*	*2000–2001*	*Change, 1985–91 to 2000–01*
Current account	0.6	–2.0	–7.0	–10.0	–10.6
Investment	25.3	22.8	26.6	28.1	2.8
Saving	25.9	20.8	19.5	18.1	–7.8
Public	4.6	2.3	2.6	2.4	–2.2
Private	21.3	18.5	16.9	15.7	–5.6
Household	9.2	8.3	5.7	5.4	–3.8
Corporate	12.1	10.2	11.2	10.3	–1.8

Fig. 3.60 – Portugal : Current Account Balance, Investment, and Saving, 1985-2001 - Source : Blanchard and Giavazzi (2002) Current Account Deficits in the Euro Area : The End of the Feldstein-Horioka Puzzle ? *Brookings Papers on Economic Activity*, 33(2), pp. 147-210.

3.7.2.3 Back to Portugal

Figure 3.59 shows Portuguese investment and saving, as ratios to GDP, from 1985 to 2001. It clearly shows the steadily increasing divergence between the two, and the resulting steady increase in the current account deficit, starting in the 1980s. In trying to assess how much of the change in the current account deficit is due to a change in saving or to a change in investment, Table 3.60 divides the data into three subperiods. While the fist (1985-1991) and the second sub-period (1992-1995) were periods of slow growth, the third (1996-2001) is a period of sustained growth averaging 3.5 percent a year. Figure 3.59 and Table 3.60 suggest four conclusions :

- The increase in the current account deficit dates back to the late 1980s but accelerated in the second half of the 1990s. When 1985-91 is used as the base period, the current account deficit has increased by 10.6 percent of GDP.
- Less than one-third of the increase in the current account deficit is due to an increase in investment. The ratio of investment to GDP has increased by 2.8 percentage points relative to 1985-91.
- More than two-thirds of the increase in the current account deficit is due to a decrease in saving. The ratio of saving to GDP has decreased by about 7.8 percent of GDP relative to its 1985-91 value.
- The decrease in saving reflects primarily a decrease in private saving. Public saving has decreased by 2.2 percent of GDP relative to 1985-91 ; private saving has decreased by 5.6 percent of GDP. The decrease in private saving reflects primarily a decrease in household saving. The ratio of household saving to GDP has decreased by 3.8 percentage points, and the ratio of corporate saving by 1.8 percentage points.

The decline in private savings can be explained by the fall ijn interest rates triggered by financial integration :

- Why has there been such an increase in household debt ? The decrease in interest rates must be a central part of the story : short-term nominal interest rates have decreased sharply, from 16 percent a year in 1992 to around 4 percent in 2001 (for the euro area as a whole the numbers are 11 percent and 4 percent). Real short-term interest rates (nominal interest rates minus realized inflation, measured using the GDP deflator) fell from 6 percent in 1992 to roughly zero in 2001.
- Why the low interest rates ? Apart from factors common to the OECD, much of the decline is clearly traceable to financial integration. Adoption of the euro has eliminated country risk. And it has opened the euro interbank loan market to Portuguese banks. The net foreign debt position of Portuguese banks has increased from 610 billion in 1999 to 624 billion in 2001. In 2000 the increase in net indebtedness of resident Portuguese banks was equal to 10.7 percent of GDP - hence exceeding the current account deficit in that year.[3]

In conclusion, financial integration (eliminating country risk) and financial liberalization (reducing the borrowing cost) have made it easier to borrow, and easier to borrow abroad. Hence, financial integration has led to lower saving and, to a lesser extent, to higher investment. Together these have led to larger current account deficits.

[3]Note that net FDI, which had been an important source of capital inflows following Portugal's entry into the European Union in 1986, turned negative in 1995 and has remained negative since then.

3.7.3 Back to the Feldstein-Horioka Puzzle

Blanchard and Giavazzi's (2002) findings are obviously closely related to the Feldstein-Horioka puzzle. More precisely, they find an increasing positive dependence of saving on income per capita and a negative dependence of investment on income per capita which raise the possibility that the correlation between national saving and national investment has decreased through time. To explore the relation between investment and saving across countries and time, they run conventional Feldstein-Horioka regressions of investment on saving, over different periods :

$$\left(\frac{I}{Y}\right)_{it} = \alpha + \beta_t \cdot \left(\frac{S}{Y}\right)_{it} + \epsilon_{it}, \tag{3.227}$$

where $\left(\frac{I}{Y}\right)_{it}$ and $\left(\frac{S}{Y}\right)_{it}$ are ratios of investment and saving to GDP, respectively, in country i and year t. Table 3.61 shows the estimated values for β, first from estimation over the whole period 1975-2001, and then over two subperiods, 1975-90 and 1991-2001, for each of our four groups of countries. Table 3.61 suggests two main conclusions :

- The coefficient in the original Feldstein-Horioka regression, run on a sample of sixteen OECD countries over the period 1960-74, was 0.89. When Obstfeld and Rogoff ran the same regression on a sample of twenty-two OECD countries over the period 1982-91, they obtained a coefficient of 0.62. The results document by Blanchard and Giavazzi for the OECD as a whole give a coefficient of 0.58, with no evidence of a decline in the coefficient over time.
- As we move from the OECD to the European Union and to the euro area, however, the coefficient steadily declines, suggesting steadily higher degrees of integration. It also declines over time, reaching much lower values in the 1990s. The coefficient for the euro area for 1991-2001 is only 0.14.

Figure 3.62 plots the time series for estimated β_t for the European Union and the Euro Area. the two panels confirm and amplify the results from Table 3.61 :

- The coefficients for the European Union and the euro area show an inverse-U shape, with the coefficient initially increasing from a value close to zero in 1975, and then steadily declining from the late 1980s on.
- For the European Union and the euro area, the estimated coefficient is close to zero or even negative at the end of the 1990s. Previous results suggest a natural interpretation : to the extent that investment and saving depend with opposite signs on income per capita, and to the extent that integration reinforces these two effects, the estimated coefficient from a regression of investment on saving may well be negative, and this may be what we are observing at the end of the period.

In short, for the countries of the European Union, and even more so for the countries of the euro area, there no longer appears to be a Feldstein-Horioka puzzle. In highly integrated regions, investment and saving appear increasingly uncorrelated.

Period	*OECD*	*OECD minus*	*European Union*	*Euro area*	*Euro minus*
1975–2001	0.58	0.51	0.47	0.35	0.39
1975–1990	0.56	0.55	0.50	0.41	0.49
1991–2001	0.57	0.38	0.36	0.14	0.26

Fig. 3.61 – Estimated Feldstein-Horioka Coefficients, 1975-2001 - Source : Blanchard and Giavazzi (2002) Current Account Deficits in the Euro Area : The End of the Feldstein-Horioka Puzzle ? *Brookings Papers on Economic Activity*, 33(2), pp. 147-210.

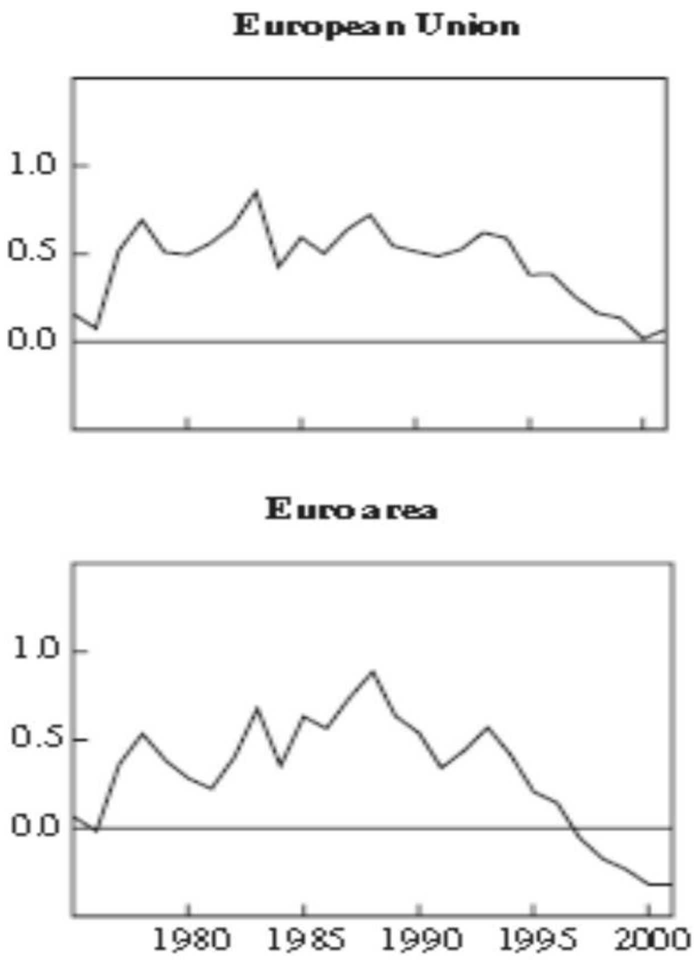


Fig. 3.62 – Yearly Coefficients of Investment on Saving from Panel Regressions, 1975-2000 - Source : Blanchard and Giavazzi (2002) Current Account Deficits in the Euro Area : The End of the Feldstein-Horioka Puzzle ? *Brookings Papers on Economic Activity*, 33(2), pp. 147-210.