RELATIVE PRODUCTIVITY AND SEARCH UNEMPLOYMENT IN AN OPEN ECONOMY

TECHNICAL APPENDIX

NOT MEANT FOR PUBLICATION

Luisito BERTINELLI, Olivier CARDI, and Romain RESTOUT

- Section A presents the source and construction of the data used in the empirical and quantitative analysis, and provides summary statistics as well.
- Section B starts with a quick look at the data and provides additional empirical results related to panel unit root and cointegration tests and shows alternative cointegration estimates as well. The section also gives more details and results on the split-sample analysis.
- Sections C-E give more details on the model. Section C develops an open economy version of the neoclassical model with search frictions and sectoral endogenous labor supply, and derives first-order conditions. Section D presents the matching process and derives the Nash bargaining wage. Section E sets out the approach taken to solve the model, analyzes equilibrium dynamics, and provides formal solutions.
- Section F provides the main steps leading to equations in the main text of section 2.1.
- Section G characterizes the initial steady-state and the transitional paths by using phase diagrams.
- In section H, we derive analytically the steady-state effects of higher productivity in tradables relative to non tradables and investigate the adjustment along the stable path by using phase diagrams whose constructions are presented in section G. In section I, we describe the graphical framework which allows us to characterize initial steady-state values for the relative wage and the relative price.
- In section J, we decompose analytically the steady-state changes in the relative wage and the relative price following higher relative productivity of tradables. In section L, we break down the change in the unemployment rate differential into labor market frictions and labor accumulation effects.
- In section K, we analyze graphically the long-term adjustment in the relative price, the relative wage, and the unemployment differential following a productivity shock biased toward the traded sector and investigate the implications of labor market regulation.
- In section M, we explore the case of total immobility and perfect mobility as well in order to highlight the role of the elasticity of the labor supply at the extensive margin.
- Section N gives more details about the calibration of the model to data.

A Data Description

A.1 Data for Empirical Analysis: Source and Construction

Country Coverage: Our sample consists of a panel of 18 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Germany (DEU), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), the United Kingdom (GBR), Ireland (IRL), Italy (ITA), Japan (JPN), Korea (KOR), the Netherlands (NLD), Norway (NOR), Sweden (SWE), and the United States (USA). These countries have the most extensive coverage of variables of our interest.

Period Coverage: The period is running from 1970 to 2007, except for Japan (1974-2007).

Sources: We use the EU KLEMS [2011] database (the March 2011 data release) for all countries of our sample with the exceptions of Canada and Norway. For these two countries, sectoral data are taken from the Structural Analysis (STAN) database provided by the OECD [2011]. Both the EU KLEMS and STAN databases provide annual data at the ISIC-rev.3 1-digit level for eleven industries.

The eleven 1-digit ISIC-rev.3 industries are split into tradables and non tradables sectors. To do so, we adopt the classification proposed by De Gregorio et al. [1994] who treat an industry as traded when it exports at least 10% of its output. Following Jensen and Kletzer [2006], we have updated the classification suggested by De Gregorio et al. [1994] by treating "Financial Intermediation" as a traded industry. Jensen and Kletzer [2006] use the geographic concentration of service activities within the United States to identify which service activities are traded domestically. The authors classify activities that are traded domestically as potentially traded internationally. The idea is that when a good or a service is traded, the production of the activity is concentrated in a particular region to take advantage of economies of scale in production.

Jensen and Kletzer [2006] use the two-digit NAICS (North American Industrial Classification System) to identify tradable and non tradable sectors. We map their classification into the NACE-ISIC-rev.3 used by the EU KLEMS database. The mapping was clear for all sectors except for "Real Estate, Renting and Business Services". According to the EU KLEMS classification, the industry labelled "Real Estate, Renting and Business Services" is an aggregate of five sub-industries: "Real estate activities" (NACE code: 70), "Renting of Machinery and Equipment" (71), "Computer and Related Activities" (72), "Research and Development" (73) and "Other Business Activities" (74). While Jensen and Kletzer [2006] find that industries 70 and 71 can be classified as tradable, they do not provide information for industries 72, 73 and 74. We decided to classify "Real Estate, Renting and Business Services" as non tradable.

Traded Sector comprises the following industries: Agriculture, Hunting, Forestry and Fishing; Mining and Quarrying; Total Manufacturing; Transport, Storage and Communication; and Financial Intermediation.

Non Traded Sector comprises the following industries: Electricity, Gas and Water Supply; Construction; Wholesale and Retail Trade; Hotels and Restaurants; Real Estate, Renting and Business Services; and Community Social and Personal Services.

Relevant to our work, the EU KLEMS and STAN database provides series, for each industry and year, on value added at current and constant prices, permitting the derivation of sectoral deflators of value added, as well as details on labor compensation and employment data, allowing the construction of sectoral wage rates. We describe below the construction for the data employed in Section 2 (mnemonics are given in parentheses):

- Sectoral value-added deflator P_t^j for $j = T, N$: value added at current prices (VA) over value added at constant prices (VA_QI) in sector j. Source: EU KLEMS database. The relative price of non tradables P_t corresponds to the ratio of the value added deflator of non traded goods to the value added deflator of traded goods: $P_t = P_t^N/P_t^T$.
- Sectoral labor L_t^j for $j = T, N$: total hours worked by persons engaged (H_EMP) in sector j. Source: EU KLEMS database.
- Sectoral nominal wage W_t^j for $j = T, N$: labor compensation in sector j (LAB) over total hours worked by persons engaged (H_EMP) in that sector. Source: EU KLEMS database. The relative wage, Ω_t is calculated as the ratio of the nominal wage in the non traded sector W^N to the nominal wage in the traded sector: $\Omega_t = W_t^N/W_t^T$.

- The construction of sectoral unemployment rates is detailed below in section A.2.

Because data source and construction are heterogenous across variables as a result of different nomenclatures, Table 11 provides a summary of the classification adopted to split value added and its demand components as well into traded and non traded goods.

Summary statistics of the data used in the empirical analysis are displayed in Table 12. As shown in the first three columns, all countries of our sample experience higher productivity gains in tradables relative to non tradables, an appreciation in the relative price of non tradables (except for Norway) and a decline in the ratio of the non traded wage relative to the traded wage.

Table 11: Construction of Variables and Data Sources

Variable	Countries covered	Period	Construction and aggregation	Database
Value added Y^T & Y^N (constant prices)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation N: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services	EU KLEMS
Value added P^TY^T & P^NY^N (current prices)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation N: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services	EU KLEMS
Labor L^T & L^N (total hours worked by persons engaged)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation N: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services	EU KLEMS
Labor compensation LAB^T & LAB^N (current prices)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation N: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services	EU KLEMS
Consumption C^T & C^N (constant prices)	BEL (95-07), DEU (91-07), DNK, ESP (95-07), FIN (75-07), FRA, ITA, GBR (90-07), IRL (96-07) JPN (80-07), KOR, NLD (80-07), SWE (93-07), USA	1990-2007	T: Food, Beverages, Clothing, Furnishings, Transport, Recreation, Other N: Housing, Health, Communication, Education, Restaurants, Recreation (Recreation is defined as 50% tradable and 50% non tradable)	COICOP
Government spending P^TG^T & P^NG^N (current prices)	BEL, DEU (91-07), DNK, ESP (95-07), FIN, FRA (95-07), GBR, IRL, ITA, JPN, KOR (00-07), NLD (95-07), SWE (95-07), USA	1990-2007	T: Energy, Agriculture, Manufacturing, Transport N: Public Services, Defense, Safety, Education, Health, Welfare, Housing, Environment, Recreation	OECD-FMI
Trade balance NX (constant prices)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, SWE, USA	1970-2007	External balance of goods and services at current prices (source: OCDE) over price of traded goods (P^T)	authors' calculations
Price P^T & P^N (value added deflator)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	Value added at current prices (P^jY^j) over value added at constant prices (Y^j)	authors' calculations
Relative Price P (index 1995=100)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	Value added deflator of non traded goods (P^N) over value added deflator of traded goods (P^T)	authors' calculations
Wage W^T & W^N (nominal and per hour)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	Labor compensation (LAB^j) over total hours worked by hired persons (L^j)	authors' calculations
Relative Wage Ω (index 1995=100)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	Nominal wage in non tradables (W^N) over nominal wage in tradables (W^T)	authors' calculations
Sectoral Productivity A^T & A^N (index 1995=100)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN (74-07), KOR, NLD, SWE, USA	1970-2007	Measured by labor productivity $A^j = Y^j/L^j$	authors' calculations
Relative Productivity (index 1995 =100)	BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, JPN (74-07), KOR, NLD, ESP, SWE, USA	1970-2007	Computed as the ratio A^T/A^N	authors' calculations

To empirically assess the role of labor market institutions in the determination of the relative wage response to higher productivity in tradables relative to non tradables, we use three indicators aimed at capturing the stringency of labor market regulation. We detail below the construction and the source of these three indicators:

- The strictness of legal protection against dismissals for permanent workers is measured by the **employment protection legislation** index, $\text{EPL}_{i,t}$ in country i at time t, provided by OECD. Source for $\text{EPL}_{i,t}$: OECD Labour Market Statistics database. Data coverage: 1985-2007 (1990-2007 for KOR). This index can be misleading since regulation was eased for temporary contracts (in Spain) while the regulation for workers with permanent contracts hardly changed. To have a more accurate measure of legal protection against dismissals, we construct a new index denoted by $\text{EPLadj}_{i,t}$ in country i at time t by adjusting $\text{EPL}_{i,t}$ for regular workers with the share share $\text{perm}_{i,t}$ of permanent workers in the economy, i.e., $\text{EPLadj}_{i,t} = \text{EPL}_{i,t} \times \text{share perm}_{i,t}$. Source for share $\text{perm}_{i,t}$: OECD Labour Market Statistics database. Data coverage: 1985-2007 (1990-2007 for KOR).
- The generosity of the unemployment benefit scheme, $\varrho_{i,t}$ in country i at time t, is commonly captured by the **unemployment benefit replacement rate**. It is worthwhile noticing that the unemployment benefit rates are very similar across counties when considering short-term unemployment (less than one year) but display considerable heterogeneity for long-term unemployment. To have a more accurate measure of the generosity of the unemployment benefit scheme, we calculate ϱ as the average of the net unemployment benefit (including social assistance and housing benefit) replacement rates (for two earnings levels and three family situations) for three durations of unemployment (1 year, 2&3 years, 4&5 years). Source: OECD, Benefits and Wages Database. Data coverage: 2001-2007. In order to have longer time series, we calculated ϱ over the period running from 1970 to 2000, by using the growth rate of the historic OECD measure of benefit entitlements which is defined as the average of the gross unemployment benefit replacement rates for two earnings levels, three family situations and three durations of unemployment. Source: OECD, Benefits and Wages Database. Data coverage: 1970-2001 for all countries while data are unavailable for Korea.
- The worker bargaining power is measured by the collective **bargaining coverage**, $\text{BargCov}_{i,t}$, which corresponds to the employees covered by collective wage bargaining agreements as a proportion of all wage and salary earners in employment with the right to bargaining. Source: Data Base on Institutional Characteristics of Trade Unions, Wage Setting, State Intervention and Social Pacts, 1960-2010 (ICTWSS), version 3.0, Jelle Visser [2009]. Data coverage: 1970-2007 for AUS, AUT, CAN, DEU, DNK, FIN, GBR, IRL, ITA, JPN, SWE and USA, 1970-2005 for NLD and NOR, 1970-2002 for BEL and FRA, 1977-2004 for ESP and 2002-2006 for KOR.

Summary statistics of the labor market regulation indicators used in the empirical analysis are displayed in the three last columns Table 12.

A.2 Calibration of the Labor Market

To calibrate the labor market for the traded and the non traded sector, we need to estimate the sectoral unemployment rate, the job finding and the job destruction rate for each sector, and the sectoral labor market tightness. We provide below the source and construction of the data.

A.2.1 Source and Construction of Data

In this subsection, we first describe the data employed to calibrate some key features of OECD labor markets. Then, we present the dataset we use to estimate a set of sectoral search unemployment parameters. Summary statistics for the key indicators of the labor market are displayed in Table 13.

- **Sectoral unemployment rate**, u^j, is the number of unemployed workers U^j in sector $j = T, N$ as a share of the labor force $L^j + U^j$ in this sector. LABORSTA database from the International Labour Organization (ILO) provides annual data for unemployed and employed workers at the 1-digit ISIC-rev.3 level. To construct L^j and U^j for $j = T, N$, we map the classification used previously to compute series for sectoral wages, prices and real labor productivity indexes (see section A.1) into the 1-digit ISIC-rev.3 classification used by the LABORSTA database. The mapping was clear for all industries except for "Not classifiable by economic activity" (1-digit ISIC-Rev.3 code: X) when constructing L^j and U^j, and, "Unemployed seeking their first job" to identify U^j. These two categories have been split between tradables and non tradables according to the shares of total unemployment (excluding the two sectors) between tradables and non tradables by year and country. In a few rare

Table 12: Summary Statistics per Country

Countries	Variables					
	$\hat{p}$ (1)	$\hat{\omega}$ (2)	$\hat{a}^T - \hat{a}^N$ (3)	ϱ (4)	BargCov (5)	EPL$_{adj}$ (6)
AUS	0.91	-0.27	1.83	0.50	0.71	1.21
AUT	1.97	-0.72	2.89	0.50	0.97	2.48
BEL	2.26	-0.04	2.53	0.67	0.94	1.65
CAN	0.54	-0.42	1.55	0.54	0.36	0.81
DEU	0.85	-0.62	1.62	0.72	0.69	2.36
DNK	0.78	-0.91	2.21	0.61	0.82	1.93
ESP	2.62	-0.97	3.67	0.41	0.76	2.04
FIN	2.56	-0.78	4.22	0.59	0.86	2.02
FRA	2.14	-0.98	2.68	0.47	0.85	2.11
GBR	1.57	-0.50	2.31	0.63	0.45	1.02
IRL	2.55	-0.88	4.37	0.54	0.58	1.32
ITA	2.02	-0.92	3.05	0.08	0.83	2.53
JPN	2.60	-0.44	2.68	0.51	0.24	1.49
KOR	3.35	-2.15	6.49	0.38	0.11	1.98
NLD	1.86	-0.39	2.38	0.67	0.85	2.60
NOR	-0.37	-0.39	1.96	0.43	0.70	2.06
SWE	2.34	-0.11	2.76	0.48	0.89	2.31
USA	1.74	-0.23	2.64	0.26	0.20	0.24
Average	1.79	-0.65	2.88	0.50	0.66	1.79

Notes: $\hat{p}$ is the relative price of non tradables average growth rate, $\hat{\omega}$ is the relative wage of non tradables average growth rate and $(\hat{a}^T - \hat{a}^N)$ is the average growth rate of the labor productivity differential between tradables and non tradables. Data coverage for $\hat{p}$, $\hat{\omega}$ and $(\hat{a}^T - \hat{a}^N)$ is 1970-2007 (1974-2007 for Japan). ϱ is the unemployment benefit replacement rate. Data coverage: 1970-2007 (2001-2007 for KOR). BargCov is the collective bargaining coverage. Data coverage: 1970-2007 for AUS, AUT, CAN, DEU, DNK, FIN, GBR, IRL, ITA, JPN, SWE and USA, 1970-2005 for NLD and NOR, 1970-2002 for BEL and FRA, 1977-2004 for ESP and 2002-2006 for KOR. EPL$_{adj}$ is the employment protection legislation index adjusted with the share of permanent workers in the economy. Data coverage: 1985-2007 (1990-2007 for KOR).

cases, the sum of sectoral employment provided by ILO did not correspond to total unemployment. These differences were usually due to missing data for some industries in the sectoral databases. In these cases, we added these differences in level, keeping however the share of each sector constant. In Table 13 we provide a overview of the classifications used to construct traded and non traded sectors variables. Once industries have been classified as traded or non traded, series for unemployed and employed workers are constructed by adding unemployed and employed workers of all sub-industries k in sector $j = T, N$ in the form $U^j = \sum_{k \in j} U_k$ and $L^j = \sum_{k \in j} L_k$. Data coverage: AUS (1995-2007), AUT (1994-2007), BEL (2001-2007), CAN (1987-2007), DEU (1995-2007), DNK (1994-1998 and 2002-2004), ESP (1992-2007), FIN (1995-2007), GBR (1988-2007), IRL (1986-1997), ITA (1993-2007), JPN (2003-2007), KOR (1992-2007), SWE (1995-2007) and USA (2003-2007). Data for unemployed workers by economic activity are not available for FRA, NLD and NOR.

- **Labor market tightness**, θ^j for $j = T, N$, is calculated as the ratio of employment vacancies in sector j (V^j) to the number of unemployed workers in that sector (U^j). To construct the variables θ^j, we collect information on job vacancies and unemployed workers by economic activity. Sources for V^j: Job Openings and Labor Turnover Survey (JOLTS) provided by the Bureau of Labor Statistics (BLS) for USA and Eurostat database (NACE 1-digit) for a range of European Countries, Labour Market Statistics from the Office for National Statistics for the UK. Sources for U^j: Current Population Survey (CPS) published by the BLS for USA and LABORSTA (ILO) for European Countries.[58] As shown in Table 13, the level of detail in the definition of traded and non traded sectors differs across databases in two dimensions. First, the number of items to split disaggregated data varies across nomenclatures from a low eleven categories in the Eurostat database to a high of eighteen items in the LABORSTA

[58]The JOLTS and CPS databases provide (not seasonally adjusted) monthly data on vacancies and unemployed workers. We convert monthly data series into a annual data series by summing the twelve monthly data points.

database. Second, the definitions of items are not harmonized across the different sets of data. To generate sectoral variables in a consistent and uniform way, series on disaggregated data for vacancies and unemployed workers are added up to form traded and non traded sectors following, as close as possible, the classification we used for value added, hours worked and labor compensation. Once industries have been classified as traded or non traded, series for employment vacancies (unemployed workers resp.) are constructed by adding job openings (unemployed workers resp.) of all sub-industries k in sector $j = T, N$ in the form $V^j = \sum_{k \in j} V_k$ ($U^j = \sum_{k \in j} U_k$ resp.). Data coverage for V^j and U^j: AUT (2004-2005), DEU (2006-2007), FIN (2002-2007), GBR (2001-2007), SWE (2005-2007) and USA (2001-2007).

For reason of space, Table 13 does not provide the classification between tradables and non tradables for job vacancies for the United Kingdom. The classification is detailed below. The Office for National Statistics provides series for the UK that cover 19 sectors, according to SIC 2007 classification. Sectors have been aggregated into tradables (Financial and insurance activities; Information and communication; Manufacturing; Mining and quarrying; Transport and storage) and non tradables (Accomodation and food service activities; Administrative and support service activities; Arts, entertainment and recreation; Construction; Education; Electricity, gas, steam and air conditioning supply; Human health and social work activities; Other service activities; Public administration and defense; Compulsory social security; Real estate activities; Water supply, sewerage, waste and remediation activities; Wholesale and retail trade; repair of motor vehicles and motor cycles).

A.2.2 The Methodology

In this section, we present the approach we adopted to measure the job finding and employment exit rates by using readily accessible data. We apply the methodology developed by Shimer [2012] who assume that the labor force is fixed. Applying the same logic to our two-sector model, we need to impose that the labor force F^j is fixed at a sectoral level. The implication of such an assumption is twofold. First, we explicitly assume that there are no movements into and out of the labor force at an aggregate level. Second, we assume that there are no movements between the traded and the non traded sectors. Reassuringly, Shimer [2012] shows that a two-state model where workers simply transit between employment and unemployment does a good job of capturing unemployment fluctuations. Because the reallocation of labor across sectors is relatively low, the second assumption should not substantially affect the results. In particular, Shimer [2012] finds that the job finding rate to worker averaged 0.44 over the post-war period for the U.S., while our own estimates indicate that the job finding rate averages about 0.40 from 2003 to 2007.

The presentation below borrows heavily from Elsby, Hobijn, and Sahin [2013]. We assume that during period t, all unemployed workers find a job according to a Poisson process with arrival rate $m^j(t) = -\ln\left(1 - M^j(t)\right)$ and all employed workers lose their job according to a Poisson process with arrival rate $s^j(t) = -\ln\left(1 - S^j(t)\right)$. We refer to $m^j(t)$ and $s^j(t)$ as the job finding and job destruction rates in sector j and to $M^j(t)$ and $S^j(t)$ as the corresponding probabilities.

The evolution over time of the unemployed workers, which we denote by $U^j(t)$, can be written as:

$$\dot{U}^j(t) = s^j(t)L^j(t) - m^j(t)U^j(t), \tag{46}$$

where $L^j(t)$ is employment in sector j; the evolution over time of the unemployed workers can be written alternatively by using the fact that $L^j(t) = F^j - U^j(t)$:

$$\dot{U}^j(t) = s^j(t)\left(F^j - U^j(t)\right) - m^j(t)U^j(t), \tag{47}$$

where $s^j(t)$ is the monthly rate of inflow into unemployment, $m^j(t)$ is the monthly outflow rate from unemployment, and t indexes months.

Collecting terms, assuming that the job destruction rate and the job finding rate are constant within years and solving eq. (47), pre-multiplying by $e^{-(m+s)\tau}$, and integrating over the time interval $[t-12, t]$, leads to the temporal path for unemployed workers:

$$U^j(t) = \psi^j(t)\tilde{u}^j(t)F^j(t) + (1 - \psi(t))\, U^j(t-12), \tag{48}$$

where $\tilde{u}^j$ is the long-run unemployment rate in sector j:

$$\tilde{u}^j(t) = \frac{s^j(t)}{s^j(t) + m^j(t)}, \tag{49}$$

and ψ^j is the annual rate of convergence to the long-run sectoral unemployment rate:

$$\psi^j(t) = 1 - e^{-\left(s^j(t) + m^j(t)\right)12}. \tag{50}$$

Table 13: Summary of Sectoral Classifications

Sector	EU KLEMS/STAN	LABORSTA Employment	LABORSTA Unemployment	JOLTS (BLS)	CPS (BLS)	EUROSTAT
Tradables	Agriculture, Hunting, Forestry and Fishing (AtB)	Agriculture, Hunting, Forestry and Fishing (A-B)	Agriculture, Hunting, Forestry (A)		Agriculture	Agriculture and fishing
			Fishing (B)			
	Mining and Quarrying (C)	Mining and Quarrying (C)	Mining and Quarrying (C)	Mining and logging	Mining and quarrying	Mining and quarrying
	Total Manufacturing (D)	Manufacturing (D)	Manufacturing (D)	Manufacturing	Manufacturing	Manufacturing
	Transport and Storage and Communication (I)	Transport, Storage and Communications (I)	Transport, Storage and Communications (I)	Transportation	Transportation and utilities	Transport, storage and communication
				Information	Information	
	Financial Intermediation (J)	Financial Intermediation (J)	Financial Intermediation (J)	Finance and insurance	Financial activities	Financial intermediation
			Unemployed seeking their first job			
Non Tradables	Electricity, Gas and Water Supply (E)	Electricity, Gas and Water Supply (E)	Electricity, Gas and Water Supply (E)			Electricity, gas and water supply
	Construction (F)	Construction (F)	Construction (F)	Construction	Construction	Construction
	Wholesale and Retail Trade (G)	Wholesale and Retail Trade (G)	Wholesale and Retail Trade (G)	Wholesale trade	Wholesale and retail trade	Wholesale and retail trade
				Retail trade		
	Hotels and Restaurants (H)	Hotels and Restaurants (H)	Hotels and Restaurants (H)			Hotels and restaurants
	Real Estate, Renting and Business Activities (K)	Real Estate, Renting and Business Activities (K)	Real Estate, Renting and Business Activities (K)	Real estate and rental		Real estate, renting and business activities
				Business services	Business services	
	Community Social and Personal Services (LtQ)	Public Adm., Defence and Compulsory Social Security (L)	Public Adm., Defence and Compulsory Social Security (L)	Government	Government workers	Public adm. and community services
		Education (M)	Education (M)	Education and health	Education and health services	
		Health and Social Work (N)	Health and Social Work (N)			
		Other Community, Social and Personal Service Activities (O)	Other Community, Social and Personal Service Activities (O)	Leisure and hospitality	Leisure and hospitality	
		Households with Employed Persons (P)	Households with Employed Persons (P)			
		Extra-Territorial Organizations and Bodies (Q)	Extra-Territorial Organizations and Bodies (Q)			
		Not classifiable by economic activity (X)	Not classifiable by economic activity (X)	Other services	Other services	
			Unemployed seeking their first job			
Unclassified					Self-employed, unincorporated and unpaid family workers	

To infer the monthly outflow probability $M^j(t)$ and then the monthly job finding rate $m^j(t)$, we follow Shimer [2012] and write the dynamic equations of sectoral unemployment and sectoral short term unemployment, i.e.,

$$\dot{U}^j(t+d) = s^j(t)L^j(t) - m^j(t)U^j(t),, \tag{51a}$$

$$\dot{U}^{j,<d}(t+d) = s^j(t)L^j(t) - m^j(t)U^{j,<d}(t), \tag{51b}$$

where $U^{j,<d}(t+d)$ denotes short-term unemployment, i.e., the stock of unemployed workers who are employed at some time $\tau \in]t, t+d]$ but lose their job and thus are unemployed at time $t+d$; hence, by construction, $U^{j,<d}(t) = 0$ since all short-term unemployed workers were employed at time t. Combining (51a) and (51b) to eliminate $s^j(t)L^j(t)$ leads to a dynamic equation relating changes of unemployment to changes of short-term unemployment:

$$\dot{U}^j(t+d) = \dot{U}^{j,<d}(t+d) - m^j(t)\left(U^j(t) - U^{j,<d}(t)\right). \tag{52}$$

Solving eq. (52) above by integrating over $[t-d, t]$, and using the fact that at time t, short-term unemployment is such that $U^{j,<d}(t) = 0$, leads to:

$$U^j(t+d) = U^{j,<d}(t+d) + e^{-m^j(t)\,.d}U^j(t).$$

Inserting $e^{-m^j(t)\,.d} = \left(1 - M^{j,<d}(t)\right)$ where $M^{j,<d}$ is the probability that an unemployed worker exits unemployment within d months, one obtains:

$$U^j(t+d) - U^j(t) = U^{j,<d}(t+d) - M^{j,<d}(t)U^j(t). \tag{53}$$

Eq. (53) states that the change of unemployment in sector j is equal to the inflows into unemployment $U^{j,<d}(t+d)$ of workers who were employed at time t but are unemployed at time $t+d$ less the number of unemployed workers who find a job $M^{j,<d}(t)U^j(t)$. Solving (53) for $M^{j,<d}(t)$, it is possible to write the probability that an unemployed worker exits unemployment within d months as

$$M^{j,<d}(t) = 1 - \left[\frac{U^j(t+d) - U^{j,<d}(t+d)}{U^j(t)}\right]. \tag{54}$$

The probability of finding a job within d months given by eq. (54) can be mapped as the monthly job finding rate for unemployment duration $d = 1, 3, 6, 12$:

$$m^{j,<d}(t) = -\frac{1}{d}\ln\left(1 - M^{j,<d}(t)\right). \tag{55}$$

To estimate the monthly job finding rate, we use the duration of unemployment lower than one month. In this configuration, the probability of finding a job can be rewritten as follows:

$$M^{j,<1}(t) = 1 - \left[\frac{U^j(t) - U^{j,<1}(t)}{U^j(t-1)}\right]$$

or alternatively

$$1 - M^{j,<1}(t) = \frac{U^j(t) - U^{j,<1}(t)}{U^j(t-1)}. \tag{56}$$

Since $U^j(t-1)$ corresponds to monthly unemployment, we have to convert annual data on a monthly basis:

$$U^j(t-1) = \left(U^j(t-12)\right)^{1/12}\left(U^j(t)\right)^{11/12}. \tag{57}$$

Using (55) with $d = 1$, the monthly job finding rate is:

$$m^{j,<1}(t) = -\ln\left(U^j(t) - U^{j,<1}(t)\right) + \ln\left(U^j(t-1)\right), \tag{58}$$

where the construction of $U^j(t-1)$ is given by eq. (57) while the same logic applies to $U^j(t)$.

Since series for unemployment by duration are expressed in percentage, we define $\alpha^{j,<1}(t)$ the share of unemployment less than one month among total unemployment as follows:

$$\alpha^{j,<1}(t) = \frac{U^{j,<1}(t)}{U^j(t)}. \tag{59}$$

Because the share of short-term unemployment is not available by economic activity, we assume that $\alpha^{j,<1}(t)$ is identical across sectors:

$$\alpha^{j,<1}(t) = \alpha^{T,<1}(t) = \alpha^{N,<1}(t). \tag{60}$$

The job destruction rate can be estimated by solving this equation:

$$U^j(t) = \psi^j(t)\frac{s^j(t)}{s^j(t) + m^{j,<1}(t)}\left(U^j(t) + L^j(t)\right) + \left(1 - \psi^j(t)\right)U^j(t-1), \tag{61}$$

where ψ^j is the monthly rate of convergence to the long-run sectoral unemployment rate:

$$\psi^j(t) = 1 - e^{-\left(s^j(t) + m^{j,<1}(t)\right)}. \tag{62}$$

A.2.3 Computation of the job finding rate and the job separation rate at a sectoral level

To estimate the monthly job finding rate, $m^{j,<1}$, and the job destruction rate, s^j, for $j = T, N$, we proceed as follows:

- We estimate $\alpha^{<1}(t) = \alpha^{j,<1}(t) = \frac{U^{<1}(t)}{U(t)}$ where $U^{<1}(t)$ is unemployment of duration less than one month.
- Using the fact that $U^{j,<1}(t) = \alpha^{<1}(t)U^j(t)$, the probability of finding a job is

$$M^{j,<1}(t) = 1 - \left[\frac{\left(1-\alpha^{<1}(t)\right)U^j(t)}{U^j(t-1)}\right], \tag{63}$$

  where $U^j(t-1)$ corresponds to monthly unemployment which is calculated as follows $U^j(t-1) = \left(U^j(t-12)\right)^{1/12}\left(U^j(t)\right)^{11/12}$ by using annual data.
- The monthly job finding rate is:

$$m^{j,<1}(t) = -\ln\left(1 - M^{j,<1}(t)\right) \tag{64}$$

- The job destruction rate can be estimated by solving the following equation:

$$U^j(t) = \psi^j(t)\frac{s^j(t)}{s^j(t) + m^{j,<1}(t)}\left(U^j(t) + L^j(t)\right) + \left(1-\psi(t)\right)U^j(t-1), \tag{65}$$

  where ψ^j is the monthly rate of convergence to the long-run sectoral unemployment rate:

$$\psi^j(t) = 1 - e^{-\left(s^j(t)+m^j(t)\right)}. \tag{66}$$

To compute $m^{j,<1}$ and s^j, we need series for unemployment by economic activity in order to construct U^j, and unemployment less than 1 month in order to estimate $\alpha^{<1}(t)$. For unemployment at the sectoral level, data are taken from ILOSTAT database (ILO) while unemployment less than one month is provided by OECD which gives unemployment by duration. Data coverage: AUS (1995-2007), AUT (1994-2007), BEL (2001-2007), CAN (1987-2007), DEU (1995-2007), DNK (1994-1998 and 2002-2004), ESP (1992-2007), FIN (1991-2007), GBR (1988-2007), IRL (1986-1997), ITA (1993-2007), JPN (2003-2007), SWE (1995-2007) and USA (2003-2007). Because we calibrate the model so that the initial steady state is consistent with the empirical properties of each OECD economy while the series for the sectoral job separation rates are computed when the economy is out of the steady-state, we need to compute values for s^j which are consistent with the steady-state sectoral unemployment rate $\tilde{u}^j = \frac{s^j}{s^j+m^j}$ given the computed value for m^j. The two first columns in Table 14 show the actual values for the sectoral unemployment rates while columns 3 and 4 give the values for steady-state sectoral unemployment rates computed by using its long-run equilibrium $\tilde{u}^j = \frac{s^j}{m^j+s^j}$ where the job finding rate m^j is taken from columns 5 and 7 of Table 6 and the job destruction rate has been computed by solving eq. (65). The two last columns of Table 14 show the difference between the actual and the predicted value. Reassuringly, because computed values for m^j and s^j by using (64) and (65) are averaged over a long enough time horizon so that the unemployment rate should have reached its long-run value, actual and predicted values are close in most of the cases, except for Sweden, Australia and Italy (for u^T), and Ireland (for u^N). The values for sectoral job destruction rates shown in columns 6 and 8 of Table 6 are thus calculated by using the long-run equilibrium expression for the sectoral unemployment rate, i.e.,

$$s^j = \frac{m^j u^j}{1-u^j}, \tag{67}$$

where u^j is taken from columns 2 and 3 of Table 6 and m^j is taken from columns 5 and 7 of Table 6. Computed values for s^j using (67) are shown in columns 6 and 8 of Table 6.

For France, Korea, the Netherlands, and Norway, data are not available to compute the job finding and the job separation rate. We proceed as follows to get estimates of m and s when calibrating the model for each economy:

- Because data for unemployment by economic activity are not available for FRA, NLD, and NOR, estimates for the job finding rate $m = m^j$ are taken from Hobijn and Sahin [2009]. Note that estimates are not available at a sectoral level so that we have to assume that the job finding rate is identical across sectors, i.e., $m^j = m$. Building on estimates by Hobijn and Sahin [2009], we set $m = 6.7\%$ for France (1975-2004), $m = 4.7\%$ for the Netherlands (1983-2004), and $m = 30.5\%$ for Norway (1983-2004). To compute the job separation rate, we use

Table 14: Comparison of Actual Values with Calculated Values for the Sectoral Unemployment Rates

Country	Actual		Calculated		Error	
	u^T	u^N	$\tilde{u}^T$	$\tilde{u}^N$	$u^T - \tilde{u}^T$	$u^N - \tilde{u}^N$
	(1)	(2)	(3)	(4)	(5)	(6)
AUS	0.072	0.062	0.084	0.066	-0.012	-0.004
AUT	0.037	0.044	0.036	0.037	0.001	0.007
BEL	0.077	0.079	0.075	0.078	0.002	0.001
CAN	0.082	0.084	0.086	0.086	-0.004	-0.002
DEU	0.101	0.091	0.100	0.094	0.001	-0.003
DNK	0.064	0.061	0.067	0.060	-0.003	0.001
ESP	0.147	0.161	0.146	0.155	0.001	0.006
FIN	0.087	0.118	0.088	0.119	-0.001	-0.001
GBR	0.073	0.066	0.071	0.068	0.002	-0.002
IRL	0.130	0.154	0.132	0.144	-0.002	0.010
ITA	0.094	0.098	0.104	0.097	-0.010	0.001
JPN	0.033	0.033	0.024	0.025	0.009	0.008
SWE	0.056	0.060	0.043	0.045	0.013	0.015
USA	0.048	0.053	0.047	0.052	0.001	0.001

the steady-state expression for the unemployment rate $u = \frac{s}{s+m}$ where the unemployment rate is averaged over the appropriate period, i.e., 1975-2004 for France, 1983-2004 for the Netherlands and 1983-2004 for Norway. Series for harmonized unemployment rates are taken from Labor Force Survey, OECD.

- While we can construct series for unemployment by economic activity for Korea, series for unemployment by duration is not provided by the OECD for this economy. We thus average the job finding rates taken from Chang et al. [2004] over 1993-1994, i.e., $m = 26.2\%$ and compute the job destruction rate by using the steady-expression for the unemployment rate $u^j = \frac{s^j}{s^j+m}$ where u^j is the sectoral unemployment rate calculated by using the LABORSTA database from ILO.

A.3 Elasticity of substitution in consumption (ϕ): Empirical Strategy

When including physical capital investment and denoting recruiting costs by $F \equiv \kappa^T V^T + \kappa^N V^N$, according to the goods market equilibrium, we have:

$$\frac{Y^T - NX - I^T - G^T - F}{Y^N - I^N - G^N} = \frac{C^T}{C^N}, \tag{68}$$

where we used the fact that $\dot{B} - r^\star B = NX$ with B the net foreign asset position and NX net exports. Inserting the optimal rule for intra-temporal allocation of consumption (15), i.e., $\frac{C^T}{C^N} = \left(\frac{\varphi}{1-\varphi}\right) P^\phi$, into (68) leads to

$$\frac{Y^T - NX - I^T - G^T - F}{Y^N - I^N - G^N} = \left(\frac{\varphi}{1-\varphi}\right) P^\phi. \tag{69}$$

According to the market clearing condition, we could alternatively use data for consumption or for sectoral value added along with times series for its demand components to estimate ϕ. Unfortunately, classifications for valued added by industry and for consumption by items are different (because nomenclatures are different) and thus it is most likely that C^T differs from $Y^T - NX - G^T - I^T - F$, and C^N from $Y^N - G^N - I^N$ as well. Because time series for traded and non traded consumption display a short time horizon for half countries of our sample while data for sectoral value added and net exports are available for the 18 OECD countries of our sample over the period running from 1970 to 2007 (except for Japan: 1974-2007), we find appropriate to estimate ϕ by computing $Y^T - NX - E^T$ and $Y^N - E^N$ where $E^T \equiv G^T + I^T + F$ and $E^N \equiv G^N + I^N$. Yet, a difficulty shows up because the classification adopted to split government spending and investment expenditure into traded and non traded items is different from that adopted to break down value added into traded and non traded components. Moreover, the time horizon is short at a disaggregated level for most of the countries, especially for time series of G^j. To overcome these difficulties, we proceed as follows. Denoting the ratio of $E^T \equiv G^T + I^T + F$ to traded value added adjusted with net exports at current prices by $\upsilon_{E^T} = \frac{P^T E^T}{P^T Y^T - P^T NX}$, and denoting the ratio of $E^N \equiv G^N + I^N$ to non traded value added at current prices by $\upsilon_{E^N} = \frac{P^N E^N}{P^N Y^N}$, the goods market equilibrium (69) can be rewritten as follows:

$$\frac{\left(P^T Y^T - P^T NX\right)\left(1-\upsilon_{E^T}\right)}{P^N Y^N \left(1-\upsilon_{E^N}\right)} = \left(\frac{\varphi}{1-\varphi}\right) P^{\phi-1},$$

or alternatively

$$\frac{\left(Y^T - NX\right)\left(1-\upsilon_{E^T}\right)}{Y^N \left(1-\upsilon_{E^N}\right)} = \left(\frac{\varphi}{1-\varphi}\right) P^{\phi}. \tag{70}$$

Setting

$$\alpha \equiv \ln \frac{\left(1-\upsilon_{E^N}\right)}{\left(1-\upsilon_{E^T}\right)} + \ln\left(\frac{\varphi}{1-\varphi}\right), \tag{71}$$

and taking logarithm, eq. (70) can be rewritten as follows:

$$\ln\left(\frac{Y^T - NX}{Y^N}\right) = \alpha + \phi \ln P. \tag{72}$$

Indexing time by t and countries by i, and adding an error term μ, we estimate ϕ by exploring the following empirical relationship:

$$\ln\left(\frac{Y^T - NX}{Y^N}\right)_{i,t} = f_i + f_t + \alpha_i t + \phi_i \ln P_{i,t} + \mu_{i,t}, \tag{73}$$

where f_i captures the country fixed effects, f_t are time dummies, and $\mu_{i,t}$ are the i.i.d. error terms. Because the term (71) is composed of ratios which may display a trend over time, we add country-specific trends, as captured by $\alpha_i t$. **Eq. (73) corresponds to eq. (40) in the text**.

Instead of using time series for sectoral value added, we can alternatively make use of series for sectoral labor compensation. Multiplying both sides by $\frac{P^T}{P^N}$ and then by $\frac{\rho^T}{\rho^N}$ with $\rho^j = \frac{W^j L^j}{P^j Y^j}$ the sectoral labor income share, eq. (70) can be rewritten as follows

$$\ln\left(\frac{W^T L^T - \rho^T P^T NX}{W^N L^N}\right) = \eta + (\phi - 1)\ln P. \tag{74}$$

where

$$\eta \equiv \ln \frac{\left(1-\upsilon_{E^N}\right)}{\left(1-\upsilon_{E^T}\right)} + \ln\left(\frac{\varphi}{1-\varphi}\right) + \ln \frac{\rho^T}{\rho^N}. \tag{75}$$

Indexing time by t and countries by i, and adding an error term μ, we estimate ϕ by exploring the following empirical relationship:

$$\ln\left(\gamma^T/\gamma^N\right)_{i,t} = g_i + g_t + \eta_i t + \delta_i \ln P_{i,t} + \zeta_{i,t}, \tag{76}$$

where $\delta_i = (\phi_i - 1)$; g_t are time dummies which capture common macroeconomic shocks. Because η_i is composed of preference parameters (i.e., φ), and (logged) ratios which may display trend over time, we introduce country fixed effects g_i, and add country-specific trends, as captured by $\eta_i t$. Once we have estimated δ_i, we can compute $\hat{\phi}_i = \hat{\delta}_i + 1$ where a hat refers to point estimate in this context. **Eq. (76) corresponds to eq. (41) in the text**.

B Empirical results

B.1 A First Glance at the Data

We begin by examining the data for the 18 OECD economies over the period 1970-2007. Figure 5 plots the average relative price growth against the average relative wage growth which have been scaled (i.e., divided) by the average productivity growth differential between tradables and non tradables. Quantitatively, the BS model predicts that a productivity differential between tradables and non tradables of 1% leaves unaffected the relative wage of non tradables and appreciates the relative price of non tradables by 1%. Hence, according to the BS model, all countries should be positioned at point BS along the X-axis with coordinates (1,0). However, we find that all countries are positioned to the south-west of point BS. Quantitatively, we find that a productivity differential between tradables and non tradables by 1% is associated with a fall in the relative wage which varies between -0.02% for Belgium and -0.41% for Denmark. Regarding the relative price, we find that its appreciation varies between 0.34% for Canada to 0.97% for Japan while Norway experiences a fall in the relative price of non tradables due to the large increase of prices in traded industries such as 'Mining and Quarrying' (which accounts for about one fourth of GDP) over 1995-2007.

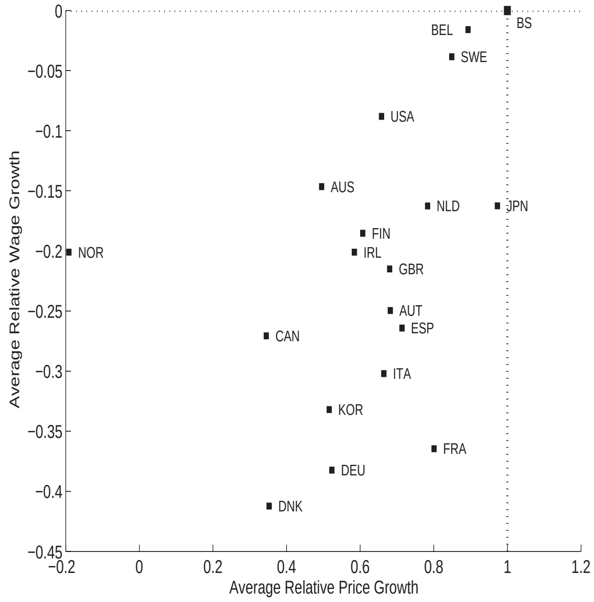


Figure 5: The Relative Price and the Relative Wage Growth. Notes: Figure 5 plots the annual average growth of the relative price of non tradables and the relative wage of non tradables, both scaled by the average productivity growth differential between tradables and non tradables, for each country of our sample over 1970-2007.

The data seem to challenge the conventional wisdom that labor mobility would gradually eliminate wage differences across sectors. If it were the case, the ratio of the non traded wage to the traded wage would remain unchanged. However, we observe that the relative wage tends to fall. Moreover, because non traded wages increase by a smaller amount that if labor were perfectly mobile, the relative price of non tradables appreciates by a smaller amount than suggested by the standard BS model. To confirm these findings, in the following, we have recourse to panel data unit root tests and cointegration methods.

B.2 Panel Unit Root Tests

We test for the presence of unit roots in the logged relative wage ω (i.e., $w^N - w^T$) and in the difference between the (log) relative price p (i.e., $p^N - p^T$) and the (log) relative productivities (i.e., $a^T - a^N$). If the wage equalization hypothesis was right, sectoral wages would increase at the same speed so that the relative wage of non tradables would be stationary. As a result, the non tradable unit labor cost would rise by the same amount as the productivity differential. Hence, the difference between the (logged) relative price and the (logged) relative productivity should be stationary as well.

We consider five panel unit root tests among those most commonly used in the literature: i) Levin, Lin and Chu's [2002] test based on a homogenous alternative assumption, ii) a t-ratio type test statistic by Breitung [2000] for testing a panel unit root based on alternative detrending methods, iii) Im, Pesaran and Shin's [2003] test that allows for a heterogeneous alternative, iv) Fisher type test by Maddala and Wu [1999], and v) Hadri [2000] who proposes a test of the null of stationarity against the alternative of a unit root in the panel data. Results are summarized in Table 15. We ran these five panel unit root tests for sectoral unemployment rates along with the unemployment rate differential.

As shown in the first column Table 15, all panel unit root tests, reveal that the relative wage variable is non-stationary at a 5% significance level. This finding suggests that labor market frictions prevent wage equalization across sectors in the long run. Regarding the relative price of non tradables and the productivity of tradables relative to productivity of non tradables, these variables are found to be non-stationary. As shown in the last column, the difference between the relative price of non tradables and the relative productivity is integrated of order one which implies that the productivity differential is not fully reflected in the non tradable unit labor cost and thus the relative price. As can be seen in The first two columns of Table 17, sectoral unemployment rates are stationary, except for Hadri's [2000] test.

The common feature of first generation tests is the restriction that all cross-sections are independent. We also consider some second generation unit root tests that allow cross-unit dependencies. We consider the tests developed by: i) Bai and Ng [2002] based on a dynamic factor model, ii) Choi [2001] based on an error-component model, iii) Pesaran [2007] based on a dynamic factor model

Table 15: Panel Unit Root Tests (p-values) for eqs. (7a)-(7b) involving the relative wage and the relative price

Test	Stat	Variables			
		ω	p	$a^T - a^N$	$p-(a^T-a^N)$
Levin et al. [2002]	t-stat	0.075	0.376	0.998	0.510
Breitung [2000]	t-stat	0.273	0.667	0.760	0.124
Im et al. [2003]	W-stat	0.558	1.000	1.000	0.999
Maddala and Wu [1999]	ADF	0.329	0.972	1.000	0.950
	PP	0.289	0.953	0.999	0.983
Hadri [2000]	Z_μ-stat	0.000	0.000	0.000	0.000

Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value $\geq$ 0.05 at a 5% significance level. For Hadri [2000], the null of stationarity is rejected if p-value $\leq$ 0.05 at a 5% significance level. ADF and PP are the Maddala and Wu's [1999] P test based on Augmented Dickey-Fuller and Phillips-Perron p-values respectively.

Table 16: Panel Unit Root Tests (second generation) for eqs. (7a)-(7b) involving the relative wage and the relative price

Test	Stat	Variables			
		ω	p	$a^T - a^N$	$p-(a^T-a^N)$
Bai and Ng [2002]	$Z^c_{\hat{e}}$	0.267	0.151	0.038	0.530
	$P^c_{\hat{e}}$	0.251	0.150	0.050	0.498
Choi [2001]	P_m	0.000	0.988	0.992	0.407
	Z	0.053	1.000	1.000	0.653
	$L^\star$	0.047	1.000	1.000	0.662
Pesaran [2007]	$CIPS$	0.010	0.320	0.450	0.015
	$CIPS^\star$	0.010	0.320	0.450	0.015
Chang [2002]	S_N	1.000	1.000	1.000	1.000

Notes: For all tests, the null of a unit root is not rejected if p-value $\geq$ 0.05 at a 5% significance level. $\hat{r}$ is the estimated number of common factors. For the idiosyncratic components, $P^c_{\hat{e}}$ is a Fisher's type statistic based on p-values of the individual ADF tests. Under H_0, $P^c_{\hat{e}}$ has a χ^2 distribution. $Z^c_{\hat{e}}$ is the standardized Choi's type statistic. Under H_0, $Z^c_{\hat{e}}$ has a $N(0,1)$ distribution. For the idiosyncratic components, the estimated number of independent stochastic trends in the common factors is reported. The first estimated value is derived from the filtered test MQ_c and the second one is derived from the corrected test MQ_f. The P_m test is a modified Fisher's inverse chi-square test. The Z test is an inverse normal test. The $L^\star$ test is a modified logit test. All these three statistics have a standard normal distribution under H_0. $CIPS$ is the mean of individual Cross sectionally ADF statistics (CADF). $CIPS^\star$ denotes the mean of truncated individual CADF statistics. The S_N statistic corresponds to the average of individual non-linear IV t-ratio statistics. It has a $N(0,1)$ distribution under H_0. Corresponding p-values are in parentheses.

and iv) Chang [2002] who proposes the instrumental variable nonlinear test. The results of second generation unit root tests are shown in Table 16.

In all cases, except for the Choi [2001] and Pesaran's [2007] tests applied to ω and $p-(a^T-a^N)$, we fail to reject the presence of a unit root in the relative price, the relative wage, the productivity differential, and the difference $p-\left(a^T-a^N\right)$, when cross-unit dependencies are taken into account.

B.3 Cointegration Tests and Alternative Cointegration Estimates

To begin with, we report the results of parametric and non parametric cointegration tests developed by Pedroni ([1999]), ([2004]). Cointegration tests are based on the estimated residuals of equations (5) and (6). Table 18 reports the tests of the null hypothesis of no cointegration. All Panel tests reject the null hypothesis of no cointegration between p and a^T-a^N at the 1% significance level while three Panel tests reject the null hypothesis of no cointegration between ω and a^T-a^N at the 5% significance level. Group-mean parametric t-test confirm cointegration between p and the labor productivity differential and between ω and a^T-a^N at 5% and 1% significance level, respectively, while group-mean non parametric t-tests are somewhat less pervasive. Pedroni [2004] explores finite sample performances of the seven statistics. The results reveal that group-mean parametric t-test is more powerful than other tests in finite samples. By and large, panel cointegration tests provide evidence in favor of cointegration between the relative price and relative productivity, and between the relative wage and relative productivity.

As robustness checks, we compare our group-mean FMOLS estimates and group-mean DOLS estimates with one lag ($q=1$), with alternative estimators. First, we consider the group-mean DOLS

Table 17: Panel Unit Root Tests (p-values) for eq. (8) involving sectoral unemployment rates

Test	Stat	Variables		
		du^T	du^N	$du^T - du^N$
Levin et al. [2002]	t-stat	0.000	0.000	0.000
Breitung [2000]	t-stat	0.049	0.045	0.000
Im et al. [2003]	W-stat	0.000	0.003	0.000
Maddala and Wu [1999]	ADF	0.000	0.003	0.000
	PP	0.000	0.000	0.000
Hadri [2000]	Z_μ-stat	0.074	0.051	0.013

Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value $\geq$ 0.05 at a 5% significance level. For Hadri [2000], the null of stationarity is rejected if p-value $\leq$ 0.05 at a 5% significance level. ADF and PP are the Maddala and Wu's [1999] P test based on Augmented Dickey-Fuller and Phillips-Perron p-values respectively.

Table 18: Panel cointegration tests results (p-values)

	wage equation	price equation
	eq. (5)	eq. (6)
Panel tests		
Non-parametric ν	0.000	0.000
Non-parametric ρ	0.012	0.003
Non-parametric t	0.004	0.002
Parametric t	0.046	0.000
Group-mean tests		
Non-parametric ρ	0.388	0.449
Non-parametric t	0.167	0.220
Parametric t	0.016	0.001

Notes: The null hypothesis of no cointegration is rejected if the p-value is below 0.05 (0.10 resp.) at 5% (10% resp.) significance level.

Table 19: Alternative Cointegration Estimates of β and γ

	Relative wage eq. (5)		Relative price eq. (6)	
	$\hat{\beta}$	$t(\beta = 0)$	$\hat{\gamma}$	$t(\gamma = 1)$
DOLS ($q = 2$)	-0.223^a (-27.69)	0.000	0.658^a (77.95)	0.000
DOLS ($q = 3$)	-0.220^a (-26.77)	0.000	0.673^a (79.22)	0.000
DOLS ($q = 4$)	-0.218^a (-26.51)	0.000	0.678^a (84.96)	0.000
DFE	-0.105^b (-2.51)	0.006	0.697^a (13.55)	0.000
MG	-0.145^a (-7.43)	0.000	0.608^a (17.25)	0.000
PMG	-0.164^a (-10.59)	0.000	0.668^a (31.03)	0.000
Panel DOLS ($q = 1$)	-0.214^a (-6.32)	0.000	0.621^a (22.39)	0.000
Panel DOLS ($q = 2$)	-0.216^a (-6.85)	0.000	0.620^a (22.62)	0.000
Panel DOLS ($q = 3$)	-0.213^a (-6.42)	0.000	0.624^a (23.88)	0.000

Notes: All regressions include country fixed effects. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. ^{a} denotes significance at 1% level. The columns $t(\beta) = 0$ and $t(\gamma) = 1$ report the p-value of the test of $H_0 : \beta = 0$ and $H_0 : \gamma = 1$ respectively.

estimator with 2 lags ($q = 2$) and 3 lags ($q = 3$). Second, we estimate cointegration relationships (7a) and (7b) using the panel DOLS estimator (Mark and Sul [2003]). We also use alternative econometric techniques to estimate cointegrating relationships (3): the dynamic fixed effects estimator (DFE), the mean group estimator (MG, Pesaran and Smith [1995]), the pooled mean group estimator (PMG, Pesaran et al. [1999]). All results are displayed in Table 19 and show that estimates of $\hat{\beta}$ and $\hat{\gamma}$ are close to those shown in Table 1 of the paper, except for the dynamic fixed effects estimator which suggests a fall in ω of 0.1% instead of 0.2%.

B.4 Split-Sample Analysis

In this subsection, we provide more details about the split-sample analysis we perform in the main text in order to differentiate the effects of a productivity differential according to the degree of labor market regulation.

B.4.1 Relative Wage and Relative Price Effects of Higher Relative Productivity of Tradables: Implications of Labor Market Regulation

To empirically explore the implications of labor market regulation for the effects of a productivity differential between tradables and non tradables, we apply cointegration techniques and perform a simple split-sample analysis. We consider three indicators which capture the extent of regulation on labor markets: the unemployment benefit replacement rate, the collective bargaining coverage, and the employment protection legislation index. We also we have recourse to a principal component analysis to construct an indicator that gives a more accurate measure of the degree of labor market regulation. Source and data construction are detailed in section A. We take the median to split the sample of 18 countries in 9 countries with high and 9 economies with low labor market regulation. Table 20 shows values of each labor market indicator for each country. For each indicator, countries are ranked in decreasing order.

We first compare the relative wage behavior of 9 countries with high and 9 economies with low labor market regulation by running the regression of the relative wage on relative productivity for each sub-sample:

$$\omega_{i,t} = \delta_i + \beta^c \left(a^T_{i,t} - a^N_{i,t}\right) + v_{i,t}, \quad c = H, L, \tag{77}$$

where β^H (β^L) captures the response of the relative wage to a productivity differential in countries with higher (lower) labor market regulation.

We adopt a similar approach for the relative price. Because the movements in the relative price of non tradables can be influenced by changes in the cost of entry in product market triggered by competition-oriented policies, we add country-specific linear time trends when we run the regression for each sub-sample in order to control for these effects:

$$p_{i,t} = \delta_i + \gamma^c \left(a^T_{i,t} - a^N_{i,t}\right) + u_{i,t}, \quad c = H, L, \tag{78}$$

where γ^H (γ^L) captures the response of the relative price to a productivity differential in countries where the index that captures the extent of labor market regulation is above (below) the median.

Building on our model's predictions, we expect the relative wage to decline more (i.e., $|\beta^H|$ is expected to take higher values) and the relative price to appreciate less (i.e., $|\gamma^H|$ is expected to take lower values) in countries where the unemployment benefit scheme is more generous (i.e., ϱ is higher) or the collective bargaining coverage is greater (i.e., $BargCov$ is higher). While we expect the relative wage to decline more in countries with strictness legislation against dismissals (i.e., EPL_{adj} takes higher values), the relative price should appreciate by a larger amount. While estimates summarized in Table 4 in the main text corroborate all of our conjectures related to the implications of labor market regulation for the relative wage and relative price effects of a productivity differential, Table 21 shows results when we base the split-sample analysis on sample mean for the three dimensions of labor market regulation. Reassuringly, all of our conclusions hold when we base the split of the sample of 18 OECD countries on sample mean. In a nutshell, our results are robust to the threshold used to perform the split-sample analysis.

Table 20: Split-Sample Analysis: Labor Market Indicators

Collective Bargaining Coverage		Unemployment Benefit Replacement Rate		Employment Protection Legislation		Labor Market Regulation	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
AUT	97.10	DNK	72.21	NLD	2.60	AUT	1.82
BEL	94.22	BEL	66.86	ITA	2.53	NLD	1.81
SWE	89.08	NLD	66.70	AUT	2.48	SWE	1.51
FIN	86.07	GBR	63.04	DEU	2.36	FRA	1.32
FRA	85.38	DEU	61.39	SWE	2.31	DNK	1.31
NLD	84.50	FIN	59.33	FRA	2.11	FIN	1.28
ITA	83.26	IRL	53.65	NOR	2.06	BEL	1.16
DNK	82.45	CAN	53.60	ESP	2.04	ESP	1.09
ESP	75.51	JPN	51.24	FIN	2.02	DEU	1.07
AUS	70.89	AUT	49.85	KOR	1.98	ITA	0.89
NOR	69.89	AUS	49.62	DNK	1.93	NOR	0.80
DEU	69.38	SWE	48.19	BEL	1.65	IRL	-0.17
IRL	57.58	FRA	47.18	JPN	1.49	AUS	-0.19
GBR	44.83	NOR	43.18	IRL	1.32	GBR	-0.86
CAN	35.75	ESP	41.34	AUS	1.21	JPN	-0.92
JPN	24.15	KOR	37.51	GBR	1.02	CAN	-1.18
USA	20.28	USA	25.72	CAN	0.81	USA	-2.47
KOR	10.50	ITA	7.68	USA	0.24	KOR	n.a.
Mean	65.60	Mean	49.91	Mean	1.79	Mean	0.40

Notes: Data coverage for Unemployment benefit replacement rate: 1970-2007 (2001-2007 for KOR). Data coverage for collective bargaining coverage: 1970-2007 for AUS, AUT, CAN, DEU, DNK, FIN, GBR, IRL, ITA, JPN, SWE and USA, 1970-2005 for NLD and NOR, 1970-2002 for BEL and FRA, 1977-2004 for ESP and 2002-2006 for KOR. Data coverage for the employment protection legislation index adjusted with the share of permanent workers in the economy: 1985-2007 (1990-2007 for KOR). The labor market regulation index is obtained by using a principal component analysis and thus the data coverage corresponds to the shortest period among the three indicators used.

B.4.2 Effect on Unemployment Rate Differential of Higher Relative Productivity of Tradables: Implications of Labor Market Regulation

One prediction of the two-sector model with search frictions developed in the paper is that a productivity differential between tradables and non tradables lowers the unemployment rate in both the traded and non traded sector, the decline of the former being larger than that of the latter. When we investigate the implications of labor market regulation, our model also predicts that the decline in the unemployment rate differential between tradables and non tradables following higher relative productivity of tradables is more pronounced in countries where labor markets are more regulated. To test these predictions, we proceed in two stages.

Firstly, indexing countries and time by i and t respectively, we explore the following relationship empirically:

$$du^T_{i,t} - du^N_{i,t} = \eta_i + \sigma \ . \left(\hat{a}^T_{i,t} - \hat{a}^N_{i,t}\right) + \lambda \ .LMR_{i,t} + z_{i,t}, \tag{79}$$

where η_i are the country fixed effects and $z_{i,t}$ are i.i.d. error terms. The dependent variable is the difference between the change in the unemployment rate in the traded sector and the change in the unemployment rate in the non traded sector (so that the unemployment rate differential is expressed

Table 21: Panel Cointegration Estimates of β and γ for Sub-Samples

LMR	ϱ		BargCov		EPL$_{adj}$		LMR	
	DOLS	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS	FMOLS
A.Relative Wage								
β^H	-0.261^a	-0.255^a	-0.233^a	-0.232^a	-0.168^a	-0.176^a	-0.160^a	-0.164^a
	(-23.04)	(-25.65)	(-27.28)	(-30.59)	(-30.76)	(-33.77)	(-30.37)	(-32.12)
β^L	-0.158^a	-0.166^a	-0.163^a	-0.168^a	-0.116^a	-0.113^a	-0.107^a	-0.108^a
	(-16.34)	(-19.14)	(-9.32)	(-11.23)	(-11.63)	(-7.74)	(-10.08)	(-6.72)
$t(\hat{\beta}^L = \hat{\beta}^H)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
B.Relative Price								
γ^H	0.791^a	0.776^a	0.754^a	0.713^a	0.442^a	0.353^a	0.464^a	0.371^a
	(6.37)	(7.15)	(10.19)	(10.90)	(5.85)	(4.76)	(6.15)	(5.14)
γ^L	1.123^a	1.037^a	1.410^a	1.346^a	0.214	0.281^a	0.206	0.296^b
	(12.81)	(13.60)	(8.96)	(9.92)	(1.48)	(2.72)	(1.14)	(2.46)
$t(\hat{\gamma}^L = \hat{\gamma}^H)$	0.000	0.000	0.000	0.000	0.723	0.999	0.804	1.000
Time period	1970-2007		1970-2007		1985-2007		1985-2007	
Countries	17		17		18		17	
Observations	642		642		414		390	
mean LMR (high)	0.609		0.823		2.221		1.280	
mean LMR (low)	0.391		0.365		1.108		-0.964	

Notes: ^{a} and ^{b} denote significance at 1% and 5% levels. To investigate whether labor market regulation influences the responses of the relative wage, β, and the relative price, γ, to a productivity differential, we split the sample of 18 OECD countries into two subsamples and run the regressions (7a)-(7b) for the high and low-labor market regulation countries. β^H (β^L) and γ^H (γ^L) capture the responses of the relative wage and the relative price, respectively, in countries with high (low) labor market regulation. The row $t(\hat{\beta}^L = \hat{\beta}^H)$ ($t(\hat{\gamma}^L = \hat{\gamma}^H)$) reports the p-value of the test of $H_0 : \hat{\beta}^L = \hat{\beta}^H$ ($\hat{\gamma}^L = \hat{\gamma}^H$). '$\varrho$' is the unemployment benefits replacement rate, 'EPL$_{adj}$' the strictness of employment protection against dismissals adjusted with the share of permanent workers, 'BargCov' the bargaining coverage and 'LMR' the labor market regulation index obtained by using a principal component analysis.

in percentage point); we construct the productivity differential by taking growth rates in order to remove the time trend, i.e., $\hat{a}^T_{i,t} - \hat{a}^N_{i,t}$, since $a^T_{it} - a^N_{it}$ displays a unit root process, see section B.2.

Since sectoral unemployment rates can be directly affected by labor market regulation, we add a control LMR_{it} which varies over time. Since bargaining coverage is available on a yearly basis for four countries only, whilst data availability is erratic for the rest of countries, we do not include this indicator in our analysis. On the contrary, the adjusted employment protection legislation index, ELP_{adj}, and the unemployment benefit replacement rate, ϱ, are available on a yearly basis since 1985, except Korea. While in the baseline regression, we add EPL_{adj} as a control variable in the baseline regression, we conducted a robustness check and replaced it with the unemployment benefit replacement rate. Our results are merely quantitatively affected.

Turning to the implications of labor market regulation, we perform a split-sample analysis on the basis of the labor market regulation index, LMR_{it}, shown in the last column of Table 20 which is an overall indicator reflecting all the dimensions of labor market institutions obtained by running a principal component analysis. We explore the following relationship empirically for each sub-sample:

$$du^T_{i,t} - du^N_{i,t} = \delta_i + \sigma^k \ . \left(\hat{a}^T_{i,t} - \hat{a}^N_{i,t}\right) + \lambda^k \ .\mathrm{LMR}_{i,t} + z_{i,t}, \quad k = H, L, \tag{80}$$

where σ^H (σ^L) captures the response of the relative unemployment rate of tradables to a rise in the productivity differential in countries where the labor market regulation index, LMR_{it}, is above (below) the mean.

Results are shown in Table 22 which reports both estimated values for σ and λ. In accordance with our model's predictions, estimated values of σ in eq. (79) are negative across all specifications, i.e., higher productivity of tradables relative to non tradables lowers more the unemployment rate of tradables than tthat of non tradables. When we run the regression (80), we also find empirically that the unemployment rate of tradables falls more relative to the unemployment rate of non tradables in countries where labor market regulation is more pronounced, i.e., $\sigma^H < \sigma^L$.

C First-Order Conditions

It is worthwhile noticing that we employ below in the formal analysis the term "short-run static solutions". This terminology refers to solutions of static optimality conditions which are inserted in dynamic optimality conditions in order to analyze the equilibrium dynamics. The term "short-run" refers to first-order conditions, and the term "static" indicates that the solution holds at each instant of time, and thus in the long-run.

Table 22: Panel OLS Estimates of σ for the Whole and Sub-Samples (eqs. (79)-(80))

	Unemployment differential eqs. (79)-(80)			
	Without Control (1)	with EPL_{adj} (2)	with ϱ (3)	with EPL_{adj} and ϱ (4)
σ	-0.034^{a} (-2.58)	-0.034^{b} (-2.57)	-0.037^{a} (-2.76)	-0.037^{a} (-2.75)
λ_{EPL}		-0.001 (-0.05)		-0.001 (-0.24)
λ_{ϱ}			-0.016 (-1.60)	-0.016 (-1.61)
σ^{H}	-0.036^{c} (-1.77)	-0.036^{c} (-1.71)	-0.040^{c} (-1.90)	-0.041^{c} (-1.95)
λ^{H}_{EPL}		0.001 (0.01)		-0.001 (-0.20)
λ^{H}_{ϱ}			-0.016 (-1.23)	-0.016 (-1.23)
σ^{L}	-0.033^{c} (-1.86)	-0.031^{c} (-1.72)	-0.034^{c} (-1.89)	-0.032^{c} (-1.68)
λ^{L}_{EPL}		-0.004 (-0.40)		-0.005 (-0.56)
λ^{L}_{ϱ}			-0.015 (-0.94)	-0.016 (-1.00)
Number of observations	164	164	164	164
Number of countries	14	14	14	14

Notes: all regressions include country fixed effects. ^{a} (^{c}) denotes significance at 1% (10%) level. We split the sample of 14 OECD countries into two subsamples on the basis of the mean sample of the labor market regulation ('LMR') index obtained by using a principal component analysis. The number of observations of the sub-sample of countries with high (low) labor market regulation is 94 (70). We estimate the regression (8) for the high and low-labor market regulation countries without (column 1) or with one (columns 2 and 3) or two (column 4) labor market control variable. σ^{H} (σ^{L}) capture the responses of the unemployment rate differential between tradables and non tradables, respectively, in countries with high (low) labor market regulation. 'EPL_{adj}' is the strictness of employment protection against dismissals adjusted with the share of permanent workers, 'ϱ' is the unemployment benefits replacement rate.

C.1 Households

We set

$$\rho(t) \equiv \frac{1}{1-\frac{1}{\sigma_C}} C(t)^{1-\frac{1}{\sigma_C}} + v^T\left(L^T(t)+U^T(t)\right) + v^N\left(L^N(t)+U^N(t)\right), \tag{81}$$

where $v^j\left(L^j(t)+U^j(t)\right)$ is the disutility function from working and searching efforts. We drop the time index when it is obvious. The current-value Hamiltonian for the representative household's optimization problem is:

$$\begin{aligned}\mathcal{H}^H &= \rho + \lambda\left[r^{\star}A + W^T L^T + W^N L^N + R^T U^T + R^N U^N - P_C C - T\right] \\ &+ \xi^{T,\prime}\left[m^T U^T - s^T L^T\right] + \xi^{N,\prime}\left[m^N U^N - s^N L^N\right],\end{aligned} \tag{82}$$

where A, L^j $(j = T, N)$ are state variables; λ, $\xi^{j,\prime}$ (with $j = T, N$) are the corresponding co-state variables; C and U^j are the control variables.

Assuming that the representative agent takes m as given, first-order conditions for households are:

$$C = (P_C\lambda)^{-\sigma_C}, \tag{83a}$$

$$-v_F^T\left(L^T+U^T\right) = m^T\xi^{T,\prime} + R^T\lambda, \tag{83b}$$

$$-v_F^N\left(L^N+U^N\right) = m^N\xi^{N,\prime} + R^N\lambda, \tag{83c}$$

$$\dot{\lambda} = \lambda\left(\beta - r^{\star}\right), \tag{83d}$$

$$\dot{\xi}^{T,\prime} = \left(s^T+\beta\right)\xi^{T,\prime} - \left[\lambda W^T + v_F^T\left(L^T+U^T\right)\right], \tag{83e}$$

$$\dot{\xi}^{N,\prime} = \left(s^N+\beta\right)\xi^{N,\prime} - \left[\lambda W^N + v_F^N\left(L^N+U^N\right)\right], \tag{83f}$$

where $\xi^{j,\prime}$ (with $j = T, N$) is the utility value of the marginal job and λ the marginal utility of wealth.

Since $\xi^{j,\prime}$ represents the utility value from an additional job and $\bar{\lambda}$ corresponds to the marginal utility of wealth, the pecuniary value of the marginal job is $\xi^j(\tau) \equiv \frac{\xi^{j,\prime}(\tau)}{\bar{\lambda}}$ for $\tau \in [t, \infty)$. Using this definition, we can rewrite (83d) as follows:

$$\dot{\xi}^j = \left(s^j + r^{\star}\right)\xi^j - \left(W^j + \frac{v_F^j}{\bar{\lambda}}\right). \tag{84}$$

Abstracting from search costs implies that the marginal rate of substitution between labor and consumption, $-\frac{v_F^j}{\lambda}$, has to be equal to the wage rate W^j. In this case, the shadow price of employment ξ^j is null. As long as agents face search costs, the real wage rate must exceed the disutility from entering the labor force $-\frac{v_F^j}{\lambda}$. Since the quantity $-\frac{v_F^j}{\lambda}$ can be viewed as being the worker's reservation wage, we will refer to $W^j + \frac{v_F^j}{\lambda}$ as the worker's surplus (by keeping in mind that $v_F^j < 0$).

Solving (84) forward and using the transversality condition $\lim_{t\to\infty} \xi^j L^j \exp\left(-\left(r^\star + s^j\right)t\right) = 0$, we get:

$$\xi^j(t) = \int_t^\infty \left[W^j(\tau) - W_R^j(\tau)\right] e^{\left(s^j + r^\star\right)(t-\tau)} \mathrm{d}\tau, \tag{85}$$

where W_R^j is the reservation wage given by

$$W_R^j \equiv -\frac{v_F^j}{\bar{\lambda}} = m^j\left(\theta^j\right)\xi^j + R^j. \tag{86}$$

Differentiating $\xi^j(t)L^j(t)$ w. r. t. time and substituting the law of motion for employment $\dot{L}^j(t)$ (12) and the dynamic optimality condition (84) yields:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left(\xi^j L^j\right) &= \dot{\xi}^j L^j + \xi^j \dot{L}^j = \left(s^j + r^\star\right)\xi^j L^j - \left(W^j + \frac{v_F^j}{\bar{\lambda}}\right) L^j + \xi^j\left(m^j U^j - s^j L^j\right), \\
&= r^\star \xi^j L^j - \left[\left(W^j + \frac{v_F^j}{\bar{\lambda}}\right) L^j - \xi^j m^j U^j\right], \\
&= r^\star \xi^j L^j - \left(W^j L^j + R^j U^j + \frac{v_F^j}{\bar{\lambda}} F^j\right),
\end{aligned}$$

where $F^j \equiv L^j + U^j$ is the labor force and we have inserted eqs. (83b)-(83c), i.e., we used the fact that $m^j\xi^j = -\frac{v_F^j}{\lambda} - R^j$. Solving forward, making use of the transversality condition, we get:

$$\xi^j(t)L^j(t) = \int_t^\infty \left[\left(W^j L^j + R^j U^j\right) + \frac{v_F^j}{\bar{\lambda}} F^j\right] e^{-r^\star(\tau-t)} \mathrm{d}\tau. \tag{87}$$

Differentiating $\frac{v_F^j\left(U^j + L^j\right)}{\lambda} = m^j\left(\theta^j\right)\xi^j + R^j$ w. r. t. time and inserting (84), we can derive the dynamic equation for job seekers in sector j:

$$\begin{aligned}
-\frac{v_{FF}^j}{\bar{\lambda}}\dot{U}^j &= m^j\left(\theta^j\right)\dot{\xi}^j + \alpha_V^j m^j\left(\theta^j\right)\xi^j \frac{\dot{\theta}^j}{\theta^j} + \frac{v_{FF}^j}{\bar{\lambda}}\dot{L}^j, \\
&= \left[\left(s^j + r^\star\right) + \alpha_V^j \frac{\dot{\theta}^j}{\theta^j}\right] m^j\left(\theta^j\right)\xi^j - m^j\left(\theta^j\right)\left(W^j + \frac{v_F^j}{\bar{\lambda}}\right) + \frac{v_{FF}^j}{\bar{\lambda}}\dot{L}^j.
\end{aligned}$$

where we used the fact that $\frac{\left(m^j\right)'\theta^j}{m^j} = \alpha_V^j$. Substituting $m^j\xi^j = -\frac{v_F^j}{\lambda} - R^j$, we get:

$$\frac{v_{FF}^j}{\bar{\lambda}}\dot{U}^j = \left(\frac{v_F^j}{\bar{\lambda}} + R^j\right)\left[\left(s + r^\star\right) + \alpha_V^j \frac{\dot{\theta}^j}{\theta^j}\right] + m^j\left(\theta^j\right)\left(W^j + \frac{v_F^j}{\bar{\lambda}}\right) - \frac{v_{FF}^j}{\bar{\lambda}}\dot{L}^j. \tag{88}$$

C.2 Firms

We consider a traded sector which produces a good denoted by the superscript T that can be exported or consumed domestically. We also consider a non traded sector which produces a good denoted by the superscript N that can be consumed only domestically. Each sector consists of a large number of identical firms. Both the traded and non-traded sectors use labor, L^T and L^N, according to constant returns to scale production functions:

$$Y^T = A^T L^T, \quad \text{and} \quad Y^N = A^N L^N. \tag{89}$$

Firms post job vacancies V^j to hire workers and face a cost per job vacancy κ^j which is assumed to be constant and measured in terms of the traded good. Firms pay the wage W^j decided by the generalized Nash bargaining solution. We also consider that firms must pay a firing tax x^j per job loss which captures the extent of employment protection legislation (see e.g., Heijdra and Ligthart [2002], Veracierto [2008]).

As producers face a labor cost W^j per employee, a cost per hiring of κ^j, the profit function of the representative firm in the traded sector is:

$$\pi^T = A^T L^T - W^T L^T - \kappa^T V^T - x^T \, . \max\left\{0, -\dot{L}^T\right\}, \tag{90}$$

where x^T is a firing tax in the traded sector when $\dot{L}^T < 0$ otherwise $x^T = 0$.

Symmetrically, denoting by P the price of non traded goods in terms of traded goods, the profit function of the representative firm in the non traded sector is:

$$\pi^N = PA^N L^N - W^N L^N - \kappa^N V^N - x^N \, . \max\left\{0, -\dot{L}^N\right\}, \tag{91}$$

where x^N is a firing tax in the non traded sector when $\dot{L}^N < 0$ otherwise $x^N = 0$.

Denoting by f^j the rate at which a vacancy is matched with unemployed agents, the law of motion for labor is given by:

$$\dot{L}^j = f^j\left(\theta^j\right) - s^j L^j, \tag{92}$$

where $f^j V^j$ represents the flow of job vacancies which are fulfilled; note that f^j decreases with labor tightness θ^j.

The current-value Hamiltonian for the sector j's representative firm optimization problem is:

$$\mathcal{H}^j = \Xi^j L^j - W^j L^j - \kappa^j V^j + \left(\gamma^j + x^j\right)\left(f^j V^j - s^j L^j\right), \tag{93}$$

where Ξ^j is the marginal revenue of labor with $\Xi^T \equiv A^T$ and $\Xi^N \equiv PA^N$ and γ^j is the co-state variable associated to the labor motion equation (92).

First-order conditions can be written as follows:

$$\gamma^j + x^j = \frac{\kappa^j}{f^j\left(\theta^j\right)}, \tag{94a}$$

$$\dot{\gamma}^j = \gamma^j\left(r^\star + s^j\right) - \left(\Xi^j - x^j s^j - W^j\right), \tag{94b}$$

where γ^j represents the pecuniary value of an additional job to the representative firm of sector $j = T, N$. This can be seen more formally by solving (94b) forward and using the appropriate transversality condition. This yields:

$$\gamma^j(t) = \int_t^\infty \left[\Xi^j(\tau) - W^j(\tau) - x^j s^j\right] e^{\left(s^j + r^\star\right)(t-\tau)} \mathrm{d}\tau. \tag{95}$$

Differentiating $\gamma^j(t)L^j(t)$ w. r. t. time and inserting the law of motion for employment $\dot{L}^j(t)$ together with the dynamic optimality condition (94b), we obtain:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left(\gamma^j L^j\right) &= \dot{\gamma}^j L^j + \gamma^j \dot{L}^j = \gamma^j\left(r^\star + s^j\right)L^j + x^j s^j L^j - \left(\Xi^j - W^j\right)L^j + \gamma^j\left(f^j V^j - s^j L^j\right), \\ &= r^\star \gamma^j L^j - \left[\Xi^j L^j - W^j L^j - \gamma^j f^j V^j - x^j s^j L^j\right] = r^\star \gamma^j L^j - \pi^j, \end{aligned}$$

where we used the fact that $\gamma^j = \kappa^j / f^j - x^j$, $\pi^j = \Xi^j L^j - W^j L^j + x^j \dot{L}^j - \kappa^j V^j$ and $\dot{L}^j = f^j \theta^j - s^j L^j$. Using the first-order condition (94a) and solving forward, making use of the transversality condition, we get:

$$\begin{aligned} \gamma^j(t)L^j(t) &= \int_t^\infty \left[\Xi^j L^j - W^j L^j - \kappa^j V^j - x^j \, . \max\left\{0, -\dot{L}^j\right\}\right] e^{-r^\star(\tau - t)} \mathrm{d}\tau, \\ &= \int_t^\infty \pi^j e^{-r^\star(\tau - t)} \mathrm{d}\tau. \end{aligned} \tag{96}$$

D Matching and Wage Determination

In each sector, there are job-seeking workers U^j and firms with job vacancies V^j which are matched in a random fashion. Assuming a constant returns to scale matching function, the number of labor contracts M^j concluded per job seeker U^j gives the job finding rate m^j which is increasing in the labor market tightness θ^j:

$$m^j = \frac{M^j}{U^j} = X^j\left(\frac{V^j}{U^j}\right)^{\alpha_V^j} = X^j\left(\theta^j\right)^{\alpha_V}, \quad \alpha_V^j \in (0,1), \tag{97}$$

where α_V^j represents the elasticity of vacancies in job matches and X^j corresponds to the matching efficiency.[59] The number of matches M^j per job vacancy gives the worker-finding rate for the firm:

$$f^j = \frac{M^j}{V^j} = X^j\left(\theta^j\right)^{\alpha_V^j - 1}. \tag{98}$$

[59] Note that the flows of workers in and out of employment are equal to each other in any symmetric equilibrium, i.e., $m^j U^j = f^j V^j$. Hence equations $\dot{L}^j = f^j V^j - s^j L^j$ and $\dot{L}^j = m^j U^j - s^j L^j$ indicate that the demand for labor indeed equates the supply.

Eq. (98) shows that the instantaneous probability of the firm finding a worker is higher the lower the labor market tightness θ^j.

The representative firm of sector j posts job vacancies in order to hire workers. We assume that the wage rate is derived from a bargaining between the firm and the worker. The wage rate W^j is set so as to maximize the following expression:

$$W^j(t) = \text{argmax } \mathcal{H}_W^j = \text{argmax } \left(\xi^j(t)\right)^{\alpha_W^j} \left(\gamma^j(t) + x^j\right)^{1-\alpha_W^j}, \quad 0 \leq \alpha_W^j \leq 1, \tag{99}$$

where α_W^j and $1-\alpha_W^j$ correspond to the bargaining power of the worker and the firm, respectively. The first-order condition determining the current wage, $w(t)$ writes as follows:

$$\frac{\partial \mathcal{H}_W^j}{\partial W^j(t)} = \frac{\alpha_W^j \mathcal{H}_W^j}{\xi^j(t)} \frac{\partial \xi^j(t)}{\partial W^j(t)} + \frac{\left(1-\alpha_W^j\right) \mathcal{H}_W^j}{\gamma^j(t) + x^j} \frac{\partial \gamma^j(t)}{\partial W^j(t)} = 0. \tag{100}$$

Differentiating (85) and (95) w.r.t. the wage rate W^j, we get: $\frac{\partial \xi^j(t)}{\partial W^j(t)} = 1$ and $\frac{\partial \gamma^j(t)}{\partial W^j(t)} = -1$; inserting these into (100):

$$\alpha_W^j \left(\gamma^j(t) + x^j\right) = \left(1-\alpha_W^j\right) \xi^j(t). \tag{101}$$

By differentiating (101) w. r. t. time, inserting the dynamic equations for ξ^j given by (84) and for γ^j given by (94b), bearing in mind that $\gamma^j + x^j = \frac{1-\alpha_W^j}{\alpha_W^j}\xi^j$ (see eq. (100)), rearranging terms, leads to the wage rate:

$$W^j = \alpha_W^j \left(\Xi^j + r^\star x^j\right) + \left(1-\alpha_W^j\right) W_R^j, \tag{102}$$

where $W_R^j = -v_F^j/\bar{\lambda}$ represents the reservation wage.

An alternative expression for the reservation wage W_R^j which is equal to $-v_F^j/\bar{\lambda} = m^j\left(\theta^j\right)\xi^j + R^j$ can be derived as follows. Eliminating ξ^j from (100) by making use of (116a), i.e., $\xi^j = \frac{\alpha_W^j}{1-\alpha_W^j}\left(\gamma^j + x^j\right)$, inserting (94a), i.e., $\gamma^j + x^j = \kappa^j/f^j$, and using the fact that $m^j/f^j = \theta^j$, the reservation wage can be rewritten as follows:

$$\begin{aligned} W_R^j &= m^j\left(\theta^j\right)\xi^j + R^j, \\ &= m^j \frac{\alpha_W^j}{1-\alpha_W^j}\frac{\kappa^j}{f^j} + R^j, \\ &= \frac{\alpha_W^j}{1-\alpha_W^j}\kappa^j\theta^j + R^j. \end{aligned} \tag{103}$$

E Solving the Model

E.1 Short-Run Static Solutions

In this subsection, we compute short-run static solutions for consumption and the relative price of non tradables. Static efficiency condition (83a) can be solved for consumption which of course must hold at any point of time:

$$C = C\left(\bar{\lambda}, P\right), \tag{104}$$

with

$$C_{\bar{\lambda}} = \frac{\partial C}{\partial \bar{\lambda}} = -\sigma_C \frac{C}{\bar{\lambda}} < 0, \tag{105a}$$

$$C_P = \frac{\partial C}{\partial P} = -\alpha_C \sigma_C \frac{C}{P} < 0, \tag{105b}$$

$$\tag{105c}$$

where σ_C corresponds to the intertemporal elasticity of substitution for consumption.

Denoting by ϕ the intratemporal elasticity of substitution between the tradable and the non tradable good and inserting short-run solution for consumption (83a) into intra-temporal allocations between non tradable and tradable goods, i.e., $C^N = P_C' C$ and $C^T = \left[P^C - PP_C'\right] C$, allows us to solve for C^T and C^N:

$$C^T = C^T\left(\bar{\lambda}, P\right), \quad C^N = C^N\left(\bar{\lambda}, P\right), \tag{106}$$

where the partial derivatives are:

$$
\begin{aligned}
C^T_{\bar{\lambda}} &= -\sigma_C \frac{C^T}{\bar{\lambda}} < 0, && (107a)\\
C^T_P &= \alpha_C \frac{C^T}{P} (\phi - \sigma_C) \lessgtr 0, && (107b)\\
C^N_{\bar{\lambda}} &= -\sigma_C \frac{C^N}{\bar{\lambda}} < 0, && (107c)\\
C^N_P &= -\frac{C^N}{P} \left[(1-\alpha_C)\phi + \alpha_C \sigma_C \right] < 0, && (107d)
\end{aligned}
$$

where we use the fact that $-\frac{P''_C P}{P'_C} = \phi(1-\alpha_C) > 0$ and $P'_C C = C^N$.

Inserting the short-run static solution for consumption in non tradables $C^N\left(\bar{\lambda}, P\right)$ given by (106) into the market clearing condition for non tradables (24) allows us to solve for the relative price of non tradables:

$$
P = P\left(L^N, \bar{\lambda}, A^N\right), \tag{108}
$$

where

$$
\begin{aligned}
P_{L^N} &= \frac{\partial P}{\partial L^N} = \frac{A^N}{C^N_P} < 0, && (109a)\\
P_{\bar{\lambda}} &= \frac{\partial P}{\partial \bar{\lambda}} = -\frac{C^N_{\bar{\lambda}}}{C^N_P} < 0, && (109b)\\
P_{A^N} &= \frac{\partial P}{\partial A^N} = \frac{L^N}{C^N_P} < 0. && (109c)
\end{aligned}
$$

Inserting (109) into (106), the short-run static solutions for C^T and C^N become:

$$
C^T = C^T\left(L^N, \bar{\lambda}, A^N\right), \quad C^N = C^N\left(L^N, \bar{\lambda}, A^N\right), \tag{110}
$$

where the partial derivatives are:

$$
\begin{aligned}
\frac{\hat{C}^T}{\hat{\bar{\lambda}}} &= -\frac{\sigma_C \phi}{\left[(1-\alpha_C)\phi + \alpha_C\sigma_C\right]} < 0, && (111a)\\
\frac{\hat{C}^T}{\hat{L^N}} &= \frac{\hat{C}^T}{\hat{A^N}} = -\frac{(\phi - \sigma_C)}{\left[(1-\alpha_C)\phi + \alpha_C\sigma_C\right]} \frac{\omega_N}{\omega_C} \lessgtr 0, && (111b)\\
\frac{\hat{C}^N}{\hat{\bar{\lambda}}} &= -\sigma_C + \sigma_C = 0, && (111c)\\
\frac{\hat{C}^N}{\hat{L^N}} &= \frac{\hat{C}^N}{\hat{A^N}} = \frac{\omega_N}{\omega_C} > 0. && (111d)
\end{aligned}
$$

We denote by a hat the rate of change of the variable and rewrite $\frac{C^N}{A^N L^N} = \frac{PC^N}{P_C C} \frac{P_C C}{Y} \frac{Y}{PA^N L^N} = \frac{\alpha_C \omega_C}{\omega_N}$ with α_C the non tradable content of consumption expenditure, ω_C the GDP share of consumption expenditure and ω_N the non tradable content of GDP.

E.2 Derivation of the Dynamic Equation of the Current Account

Using the fact that $A \equiv B + \gamma^T L^T + \gamma^N L^N$, differentiating with respect to time, noticing that $\dot{(\gamma^j L^j)} = r^\star \gamma^j L^j - \pi^j$, the accumulation equation of traded bonds is given by:

$$
\begin{aligned}
\dot{B} &= \dot{A} - \dot{\gamma}^T L^T - \gamma^T \dot{L}^T - \dot{\gamma}^N L^N - \gamma^N \dot{L}^N,\\
&= r^\star\left(A - \gamma^T L^T - \gamma^N L^N\right) + \pi^T + \pi^N + W^T L^T + W^N L^N + R^T U^T + R^N U^N - T - P_C C.
\end{aligned}
$$

Remembering that $\pi^j = \Xi^j - W^j L^j - \kappa^j V^j - x^j \,.\max\left\{0, -\dot{L}^j\right\}$, inserting the market clearing condition for the non traded good (24) and the balanced government budget (23), the current account equation reduces to:

$$
\dot{B}(t) = r^\star B(t) + A^T L^T(t) - C^T(t) - G^T - \kappa^T V^T(t) - \kappa^N V^N(t). \tag{112}
$$

E.3 Equilibrium Dynamics and Formal Solutions

E.3.1 Dynamic System

Differentiating (94a) w. r. t. time, using (94b) yields

$$\frac{\dot{\theta}^j}{\theta^j} = \frac{1}{1-\alpha_V^j}\frac{\dot{\gamma}^j}{\gamma^j + x^j}.$$

Eliminating $\gamma^j + x^j$ by using (94a), leads to the dynamic equation for labor market tightness θ^j:

$$\dot{\theta}^j(t) = \frac{\theta^j(t)}{\left(1-\alpha_V^j\right)}\left\{\left(s^j + r^\star\right) - \frac{f^j\left(\theta^j(t)\right)}{\kappa^j}\left[\left(\Xi^j + r^\star x^j\right) - W^j\right]\right\}.$$

Setting the overall surplus from an additional job in sector j:

$$\Psi^j(t) = \left(\Xi^j(t) + r^\star x^j\right) + \frac{v_F^j(t)}{\bar{\lambda}}. \tag{113}$$

Inserting the Nash bargaining wage W^j given by (102) into $\left[\left(\Xi^j + r^\star x^j\right) - W^j\right]$, the dynamic equation for labor market tightness θ^j can be rewritten as follows:

$$\dot{\theta}^j(t) = \frac{\theta^j(t)}{\left(1-\alpha_V^j\right)}\left\{\left(s^j + r^\star\right) - \frac{f^j\left(\theta^j(t)\right)\left(1-\alpha_W^j\right)\Psi^j(t)}{\kappa^j}\right\}. \tag{114}$$

The overall surplus from an additional job in the traded and the non traded sector, respectively, is given by:

$$\Psi^T = \left(A^T + r^\star x^T\right) + \frac{v_F^T}{\bar{\lambda}}, \quad \Psi^N = \left[P\left(L^N, \bar{\lambda}, A^N\right)A^N + r^\star x^N\right] + \frac{v_F^N}{\bar{\lambda}}, \tag{115}$$

where the short-run static solution for the relative price of non tradables (108) has been inserted into the overall surplus from a match into the non traded sector. Partial derivatives are given by:

$$\Psi^T_{L^T} = \Psi^T_{U^T} = \frac{v^T_{FF}}{\bar{\lambda}} < 0, \tag{116a}$$

$$\Psi^N_{L^N} = P_{L^N}A^N + \frac{v^N_{FF}}{\bar{\lambda}} < 0, \tag{116b}$$

$$\Psi^N_{U^N} = \frac{v^N_{FF}}{\bar{\lambda}} < 0, \tag{116c}$$

$$\begin{aligned}\Psi^N_{A^N} &= P_{A^N}A^N + P = \frac{A^N L^N}{C_P^N} + P,\\ &= \frac{A^N L^N}{C_P^N}\left\{1 - \left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right]\frac{\alpha_C\omega_C}{\omega_N}\right\} < 0,\end{aligned} \tag{116d}$$

$$\begin{aligned}\Psi^N_{\bar{\lambda}} &= P_{\bar{\lambda}}A^N - \frac{v_F^N}{\left(\bar{\lambda}\right)^2},\\ &= -\frac{1}{\bar{\lambda}}\left\{\frac{\sigma_C P A^N}{\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right]} + \frac{v_F^N}{\bar{\lambda}}\right\} < 0,\end{aligned} \tag{116e}$$

where $P_{L^N} < 0$, $C_P^N < 0$, and we use the fact that $\frac{C^N}{A^N L^N} = \frac{PC^N}{P_C C}\frac{P_C C}{Y}\frac{Y}{PA^N L^N} = \frac{\alpha_C\omega_C}{\omega_N}$.

The adjustment of the open economy towards the steady-state is described by a dynamic system which comprises six equations. We consider that the utility function is additively separable in the disutility received by working and searching in the two sectors. Such a specification makes it impossible to switch from one sector to another instantaneously without going through a spell of search unemployment, as in Alvarez and Shimer [2011]. Because workers must search for a job to switch from one sector to another, i.e., cannot relocate hours worked from one sector to another instantaneously, the dynamic system is block recursive. The first (second) dynamic system consists of the law of motion of employment in the traded (non traded) sector described by (12), the dynamic equations for labor tightness and job seekers given by (114) and (88), respectively. We denote the steady-state value with a tilde.

Traded Sector

Linearizing the accumulation equation for traded labor (12) by setting $j = T$ and the dynamic equations for labor market tightness (114) and job seekers (88) in the traded sector, we get in matrix form:

$$\left(\dot{L}^T, \dot{\theta}^T, \dot{U}^T\right)^T = J^T\left(L^T(t) - \tilde{L}^T, \theta^T(t) - \tilde{\theta}^T, U^T(t) - \tilde{U}^T\right)^T \tag{117}$$

where J^T is given by

$$J^T \equiv \begin{pmatrix} -s^T & \left(m^T\right)' \tilde{U}^T & m^T \left(\tilde{\theta}^T\right) \\ -\frac{1-\alpha_W^T}{1-\alpha_V^T} \frac{\tilde{m}^T}{\kappa^T} \frac{v_{FF}^T}{\bar{\lambda}} & \left(s^T + r^\star\right) & -\frac{1-\alpha_W^T}{1-\alpha_V^T} \frac{\tilde{m}^T}{\kappa^T} \frac{v_{FF}^T}{\bar{\lambda}} \\ \left(2s^T + r^\star\right) + \frac{\alpha_W^T \tilde{m}^T}{1-\alpha_V^T} & -\left(m^T\right)' \tilde{U}^T & \left(s^T + r^\star\right) - \tilde{m}^T + \frac{\alpha_W^T}{1-\alpha_V^T} \tilde{m}^T \end{pmatrix}, \tag{118}$$

and where we used the fact that:

$$\begin{aligned} \frac{\tilde{f}^T \left(1 - \alpha_W^T\right) \tilde{\Psi}^T}{s^T + r^\star} &= \kappa^T, \\ \frac{v_F^T}{\bar{\lambda}} + R^T = -\tilde{m}^T \tilde{\xi}^T &= -\frac{\tilde{m}^T \alpha_W^T \tilde{\Psi}^T}{s^T + r^\star}, \\ 1 + \frac{\alpha_V^T}{1 - \alpha_V^T} \frac{\tilde{f}^T \left(1 - \alpha_W^T\right) \tilde{\Psi}^T}{\kappa^T \left(s^T + r^\star\right)} &= \frac{1}{1 - \alpha_V^T}. \end{aligned}$$

The trace denoted by Tr of the linearized 3×3 matrix (118) is given by:

$$\mathrm{Tr}J^T = \left(s^T + r^\star\right) + r^\star + \frac{\tilde{m}^T}{1 - \alpha_V^T} \left[\alpha_W^T - \left(1 - \alpha_V^T\right)\right]. \tag{119}$$

The determinant denoted by Det of the linearized 3×3 matrix (118) is unambiguously negative:

$$\mathrm{Det}J^T = -\left(s^T + r^\star\right)\left(s^T + \tilde{m}^T\right)\left[\left(s^T + r^\star\right) + \frac{\alpha_W^T}{1 - \alpha_V^T} \tilde{m}^T\right] < 0. \tag{120}$$

Assuming that the Hosios condition holds, i.e., setting $\alpha_W^T = 1 - \alpha_V^T$, the trace reduces to:

$$\mathrm{Tr}J^T = \left(s^T + r^\star\right) + r^\star, \tag{121}$$

while the determinant is given by:

$$\mathrm{Det}J^T = -\left(s^T + r^\star\right)\left(s^T + r^\star + \tilde{m}^T\right)\left(s^T + \tilde{m}^T\right) < 0. \tag{122}$$

From now on, for clarity purpose, we impose the Hosios condition in order to avoid unnecessary complications. We relax this assumption when analyzing steady-state effects and conducting a quantitative exploration of the effects of higher productivity of tradables relative to non tradables. Note that all conclusions related to the analysis of equilibrium dynamics hold whether the Hosios conditions is imposed or not.

Denoting by ν^T the eigenvalue in the traded sector, the characteristic equation for the matrix J (118) of the linearized system writes as follows:

$$\left(s^T + r^\star - \nu_i^T\right)\left\{\left(\nu_i^T\right)^2 - r^\star \nu_i^T + \frac{\mathrm{Det}J^T}{s^T + r^\star}\right\} = 0. \tag{123}$$

The characteristic roots obtained from the characteristic polynomial of degree two can be written as follows:

$$\nu_i^T \equiv \frac{1}{2}\left\{r^\star \pm \sqrt{\left(r^\star\right)^2 - 4\frac{\mathrm{Det}J^T}{s^T + r^\star}}\right\} \gtrless 0, \quad i = 1, 2. \tag{124}$$

We denote by $\nu_1^T < 0$ and $\nu_2^T > 0$ the stable and unstable eigenvalues respectively which satisfy:

$$\nu_1^T < 0 < r^\star < \nu_2^T. \tag{125}$$

Let ν_3^T be the second unstable characteristic root which writes as:

$$\nu_3^T = s^T + r^\star > 0. \tag{126}$$

Since the system features one state variable, L^T, and one negative eigenvalue, two jump variables, θ^T and U^T, and two positive eigenvalues, the equilibrium yields a unique one-dimensional saddle-path. Inserting (119) and (120) into (124), the stable and unstable eigenvalues reduce to:

$$\nu_1^T = -\left(s^T + \tilde{m}^T\right), \quad \nu_2^T = \left(s^T + r^\star + \tilde{m}^T\right). \tag{127}$$

Non Traded Sector

Linearizing the accumulation equation for non traded labor (12) by setting $j = N$ and the dynamic equations for labor market tightness (114) and job seekers (88) in the non traded sector, we get in matrix form:

$$\left(\dot{L}^N, \dot{\theta}^N, \dot{U}^N\right)^T = J^N \left(L^N(t) - \tilde{L}^N, \theta^N(t) - \tilde{\theta}^N, U^N(t) - \tilde{U}^N\right)^T, \tag{128}$$

where J^N is given by

$$J^N \equiv \begin{pmatrix} -s^N & \left(m^N\right)' \tilde{U}^N & m^N\left(\tilde{\theta}^N\right) \\ -\frac{1-\alpha_W^N}{1-\alpha_V^N}\frac{\tilde{m}^N}{\kappa^N}\left(P_{L^N}A^N + \frac{v_{FF}^N}{\bar{\lambda}}\right) & \left(s^N + r^\star\right) & -\frac{1-\alpha_W^N}{1-\alpha_V^N}\frac{\tilde{m}^N}{\kappa^N}\frac{v_{FF}^N}{\bar{\lambda}} \\ \left(2s^N + r^\star\right) + \frac{\alpha_W^N \tilde{m}^N}{1-\alpha_V^N}\left(P_{L^N}A^N\frac{\bar{\lambda}}{v_{FF}^N} + 1\right) & -\left(m^N\right)'\tilde{U}^N & \left(s^N + r^\star\right) - \tilde{m}^N + \frac{\alpha_W^N}{1-\alpha_V^N}\tilde{m}^N \end{pmatrix}, \tag{129}$$

and where we used the fact that:

$$\begin{aligned} \frac{\tilde{f}^N\left(1-\alpha_W^N\right)\tilde{\Psi}^N}{s^N + r^\star} &= \kappa^N, \\ \frac{v_F^N}{\bar{\lambda}} + R^N = -\tilde{m}^N\tilde{\xi}^N &= -\frac{\tilde{m}^N\alpha_W^N\tilde{\Psi}^N}{s^N + r^\star}, \\ 1 + \frac{\alpha_V^N}{1-\alpha_V^N}\frac{\tilde{f}^N\left(1-\alpha_W^N\right)\tilde{\Psi}^N}{\kappa^N\left(s^N + r^\star\right)} &= \frac{1}{1-\alpha_V^N}. \end{aligned}$$

The trace denoted by Tr of the linearized 3×3 matrix (129) is given by:

$$\text{Tr}J^N = \left(s^N + r^\star\right) + r^\star + \frac{\tilde{m}^N}{1-\alpha_V^N}\left[\alpha_W^N - \left(1-\alpha_V\right)\right]. \tag{130}$$

The determinant denoted by Det of the linearized 3×3 matrix (129) is unambiguously negative:

$$\begin{aligned} \text{Det}J^N &= -\left(s^N + r^\star\right)\left\{\left(s^N + \tilde{m}^N\right)\left[\left(s^N + r^\star\right) + \frac{\alpha_W^N}{1-\alpha_V^N}\tilde{m}^N\right]\right. & (131) \\ &+ \left.\frac{1-\alpha_W^N}{1-\alpha_V^N}\frac{\tilde{m}^N}{\kappa^N}P_{L^N}A^N\frac{\tilde{m}^N}{\theta^N}\left(\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N\tilde{\theta}^N\frac{\bar{\lambda}}{v_{FF}^N} - \alpha_V\tilde{U}^N\right)\right\} < 0, & (132) \end{aligned}$$

where $P_{L^N} < 0$.

Assuming that the Hosios condition holds, i.e., setting $\alpha_W^N = 1 - \alpha_V^N$, the trace reduces to:

$$\text{Tr}J^N = \left(s^N + r^\star\right) + r^\star, \tag{133}$$

while the determinant is given by:

$$\text{Det}J^N = -\left(s^N + r^\star\right)^2\left(s^N + \tilde{m}^N\right)\left\{\frac{\left(s^N + r^\star + \tilde{m}^N\right)}{\left(s^N + r^\star\right)} - \frac{P_{L^N}\tilde{L}^N}{\tilde{P}}\frac{\tilde{P}A^N}{\left(1-\alpha_V^N\right)\tilde{\Psi}^N}\left(\tilde{\chi}^N\sigma_L^N + \alpha_V\tilde{u}^N\right)\right\} < 0, \tag{134}$$

where we have rewritten the last term as follows:

$$\begin{aligned} &\frac{1-\alpha_W^N}{1-\alpha_V^N}\frac{\tilde{m}^N}{\kappa^N}P_{L^N}A^N\frac{\tilde{m}^N}{\theta^N}\left(\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N\tilde{\theta}^N\frac{\bar{\lambda}}{v_{FF}^N} - \alpha_V\tilde{U}^N\right) \\ = \; & -\frac{1-\alpha_W^N}{1-\alpha_V^N}\frac{\tilde{m}^N}{\kappa^N}P_{L^N}A^N\tilde{f}^N\tilde{F}^N\left(\tilde{\chi}^N\sigma_L^N + \alpha_V\tilde{u}^N\right), \\ = \; & -\frac{s^N}{\tilde{u}^N\left(1-\alpha_V^N\right)}P_{L^N}\tilde{L}^N A^N\frac{\left(s^N + r^\star\right)}{\tilde{\Psi}^N}\left(\tilde{\chi}^N\sigma_L^N + \alpha_V\tilde{u}^N\right), \\ = \; & -\left(s^N + r^\star\right)\left(s^N + \tilde{m}^N\right)\frac{P_{L^N}\tilde{L}^N}{\tilde{P}}\frac{\tilde{P}A^N}{\left(1-\alpha_V^N\right)\tilde{\Psi}^N} < 0, \end{aligned}$$

and where we used the fact that $\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N\tilde{\theta}^N = -\tilde{\chi}^N\frac{v_F^N}{\bar{\lambda}}$, $\tilde{f}^N = \tilde{m}^N/\tilde{\theta}^N$, and $\frac{v_F^N}{v_{FF}^N\tilde{F}^N} = \sigma_L^N$ to get the second line, $\frac{\tilde{f}^N\left(1-\alpha_W^N\right)}{\kappa^N} = \frac{\left(s^N+r^\star\right)}{\tilde{\Psi}^N}$, $\tilde{m}^N\tilde{U}^N = s^N\tilde{L}^N$, and $\tilde{U}^N/\tilde{F}^N = \tilde{u}^N$ to get the third line, $\tilde{u}^N = \frac{s^N}{s^N+\tilde{m}^N}$, multiplying the numerator and the denominator by $\tilde{P}$ and rearranging terms to get the last line.

We impose the Hosios condition in order to avoid unnecessary complications. Denoting by ν^N the eigenvalue, the characteristic equation for the matrix J (129) of the linearized system writes as follows:

$$\left(s^N + r^\star - \nu_i^N\right)\left\{\left(\nu_i^N\right)^2 - r^\star\nu_i^N + \frac{\text{Det}J^N}{s^N + r^\star}\right\} = 0. \tag{135}$$

The characteristic roots obtained from the characteristic polynomial of degree two write as follows:

$$\nu_i^N \equiv \frac{1}{2}\left\{r^\star \pm \sqrt{\left(r^\star\right)^2 - 4\frac{\text{Det}J^N}{s^N + r^\star}}\right\} \gtrless 0, \quad i = 1, 2. \tag{136}$$

We denote by $\nu_1^N < 0$ and $\nu_2^N > 0$ the stable and unstable eigenvalues respectively which satisfy:

$$\nu_1^N < 0 < r^\star < \nu_2^N. \tag{137}$$

As it will become useful later, $\nu_1^N\left(r^\star - \nu_1^N\right) = \frac{\text{Det}J^N}{s^N + r^\star}$ which can be rewritten as follows

$$\begin{aligned}
\frac{\text{Det}J^N}{s^N + r^\star} &= -\left(s^N + r^\star\right)\left(s^N + \tilde{m}^N\right)\left\{\frac{\left(s^N + r^\star + \tilde{m}^N\right)}{\left(s^N + r^\star\right)} + \frac{\omega_N}{\alpha_C \omega_C\left[\left(1 - \alpha_C\right)\phi + \alpha_C \sigma_C\right]}\right. \\
&\times \left.\frac{\tilde{P}A^N}{\left(1 - \alpha_V^N\right)\tilde{\Psi}^N}\left(\tilde{\chi}^N \sigma_L^N + \alpha_V^N \tilde{u}^N\right)\right\} < 0.
\end{aligned} \tag{138}$$

where we used the fact that $\frac{C^N}{A^N L^N} = \frac{\alpha_C \omega_C}{\omega_N}$ and $P_{L^N} = \frac{A^N}{C_P^N} < 0$.

Let ν_3^N be the second unstable characteristic root which writes as:

$$\nu_3^N = s^N + r^\star > 0. \tag{139}$$

Since the system features one state variable, L^N, and one negative eigenvalue, two jump variables, θ^N and U^N, and two positive eigenvalues, the equilibrium yields a unique one-dimensional saddle-path.

E.4 Formal Solutions for $\theta^T(t)$ and $U^T(t)$

Setting the constant $D_2^T = 0$ to insure a converging adjustment for all macroeconomic aggregates, the stable paths are given by :

$$L^T(t) - \tilde{L}^T = D_1^T e^{\nu_1^T t}, \tag{140a}$$

$$\theta^T(t) - \tilde{\theta}^T = \omega_{21}^T D_1^T e^{\nu_1^T t}, \tag{140b}$$

$$U^T(t) - \tilde{U}^T = \omega_{31}^T D_1^T e^{\nu_1^T t}, \tag{140c}$$

where $D_1^T = L_0^T - \tilde{L}^T$, and elements ω_{21}^T and ω_{31}^T of the eigenvector (associated with the stable eigenvalue ν_1^T) are given by:

$$\omega_{21}^T = \frac{\frac{1-\alpha_W^T}{1-\alpha_V^T}\frac{\tilde{m}^T}{\kappa^T}\frac{v_{FF}^T}{\lambda}\left(\tilde{m}^T + s^T + \nu_1^T\right)}{\tilde{m}^T\left(s^T + r^\star - \nu_1^T\right) + \frac{1-\alpha_W^T}{1-\alpha_V^T}\frac{\tilde{m}^T}{\kappa^T}\frac{v_{FF}^T}{\lambda}\left(m^T\right)'\tilde{U}^T} \lesseqgtr 0, \tag{141a}$$

$$\omega_{31}^T = \left(\frac{s^T + \nu_1^T}{\tilde{m}^T}\right) - \frac{\left(m^T\right)'\tilde{U}^T}{\tilde{m}^T}\omega_{21}^T \lesseqgtr 0. \tag{141b}$$

We have normalized ω_{11}^T to unity. Inserting $\nu_1^T = s^T + \tilde{m}^T$ (see (127)) into (141a) and (141b), eigenvectors reduce to:

$$\omega_{21}^T = 0, \quad \omega_{31}^T = -1. \tag{142}$$

From (142), the dynamics for labor market tightness θ^T degenerate while job seekers are negatively correlated with employment along a stable transitional path.

E.5 Formal Solutions for $\theta^N(t)$ and $U^N(t)$

Setting the constant $D_2^N = 0$ to insure a converging adjustment for all macroeconomic aggregates, the stable paths are given by:

$$L^N(t) - \tilde{L}^N = D_1^N e^{\nu_1^N t}, \tag{143a}$$

$$\theta^N(t) - \tilde{\theta}^N = \omega_{21}^N D_1^N e^{\nu_1^N t}, \tag{143b}$$

$$U^N(t) - \tilde{U}^N = \omega_{31}^N D_1^N e^{\nu_1^N t}, \tag{143c}$$

where $D_1^N = L_0^N - \tilde{L}^N$, and elements ω_{21}^N and ω_{31}^N of the eigenvector (associated with the stable eigenvalue ν_1^N) are given by:

$$\omega_{21}^N = \frac{\frac{1-\alpha_W^N}{1-\alpha_V^N}\frac{\tilde{m}^N}{\kappa^N}\left[\tilde{m}^N\left(P_{L^N}A^N + \frac{v_{FF}^N}{\lambda}\right) + \left(s^N + \nu_1^N\right)\frac{v_{FF}^N}{\lambda}\right]}{\tilde{m}^N\left(s^N + r^\star - \nu_1^N\right) + \frac{1-\alpha_W^N}{1-\alpha_V^N}\frac{\tilde{m}^N}{\kappa^N}\frac{v_{FF}^N}{\lambda}\left(m^N\right)'\tilde{U}^N} \lesseqgtr 0, \tag{144a}$$

$$\omega_{31}^N = \left(\frac{s^N + \nu_1^N}{\tilde{m}^N}\right) - \frac{\left(m^N\right)'\tilde{U}^N}{\tilde{m}^N}\omega_{21}^N \lesseqgtr 0. \tag{144b}$$

We have normalized ω_{11}^N to unity. The signs of (144a) and (144b) will be determined later.

E.6 Formal Solution for the Stock of Foreign Bonds $B(t)$

Substituting first the short-run static solutions for consumption in tradables given by (110), and using the fact that $V^j = U^j\theta^j$, the accumulation equation for traded bonds (112) can be written as follows:

$$\dot{B}(t) = r^\star B(t) + A^T L^T(t) - C^T\left(L^N(t), \bar{\lambda}, A^N\right) - G^T - \kappa^T\theta^T(t)U^T(t) - \kappa^N\theta^N(t)U^N(t). \quad (145)$$

Linearizing (145) in the neighborhood of the steady-state and inserting stable solutions given by (140) and (143) yields:

$$\dot{B}(t) = r^\star\left(B(t) - \tilde{B}\right) + \Lambda^T\left(L^T(t) - \tilde{L}^T\right) + \Lambda^N\left(L^N(t) - \tilde{L}^N\right), \quad (146)$$

where we set:

$$\Lambda^T = A^T - \kappa^T\tilde{U}^T\omega_{21}^T - \kappa^T\tilde{\theta}^T\omega_{31}^T = A^T + \kappa^T\tilde{\theta}^T > 0, \quad (147a)$$

$$\begin{aligned}\Lambda^N &= -C^T_{L^N} - \kappa^N\tilde{U}^N\omega_{21}^N - \kappa^N\tilde{\theta}^N\omega_{31}^N,\\ &= -C^T_{L^N} - \kappa^N\tilde{U}^N\left(1-\alpha_V^N\right)\omega_{21}^N - \frac{\kappa^N\tilde{\theta}^N\left(s^N+\nu_1^N\right)}{\tilde{m}^N} > 0, \end{aligned} \quad (147b)$$

where we have inserted (144b) and used the fact that $\left(m^N\right)'\theta^N/m^N = \alpha_V^N$ to get (147b); note that $C^T_{L^N} \simeq 0$ because our estimates of ϕ average about 1 while we set σ_C to one. The sign of (147b) follows from the fact that $\omega_{21}^N < 0$ (see (189)) and $s^N + \nu_1^N < 0$; the latter result stems from the fact that $\nu_1^T = -(s^T + \tilde{m}^T)$; because we have the following set of inequalities $\frac{\text{Det}J^N}{s^N+r^N} < \frac{\text{Det}J^T}{s^T+r^\star} < 0$, $\nu_1^N < -(s^N+\tilde{m}^N) < 0$ and thereby $s^N + \nu_1^N < 0$.

Solving the differential equation (146) yields:

$$B(t) = \tilde{B} + \left[\left(B_0 - \tilde{B}\right) - \frac{\Lambda^T D_1^T}{\nu_1^T - r^\star} - \frac{\Lambda^N D_1^N}{\nu_1^N - r^\star}\right]e^{r^\star t} + \frac{\Lambda^T D_1^T}{\nu_1^T - r^\star}e^{\nu_1^T t} + \frac{\Lambda^N D_1^N}{\nu_1^N - r^\star}e^{\nu_1^N t}. \quad (148)$$

Invoking the transversality condition for intertemporal solvency, and using the fact that $D_1^T = L_0^T - \tilde{L}^T$ and $D_1^N = L_0^N - \tilde{L}^N$, we obtain the linearized version of the nation's intertemporal budget constraint:

$$\tilde{B} - B_0 = \Phi^T\left(\tilde{L}^T - L_0^T\right) + \Phi^T\left(\tilde{L}^N - L_0^N\right), \quad (149)$$

where we set

$$\Phi^T \equiv \frac{\Lambda^T}{\nu_1^T - r^\star} = -\frac{\left(A^T + \kappa^T\tilde{\theta}^T\right)}{\left(s^T + \tilde{m}^T + r^\star\right)} < 0, \quad \Phi^N \equiv \frac{\Lambda^N}{\nu_1^N - r^\star} < 0. \quad (150)$$

Equation (150) can be solved for the stock of foreign bonds:

$$\tilde{B} = B\left(\tilde{L}^T, \tilde{L}^N\right), \quad B_{L^T} = \Phi^T < 0, \quad B_{L^N} = \Phi^N < 0. \quad (151)$$

For the national intertemporal solvency to hold, the terms in brackets of equation (148) must be zero so that the stable solution for net foreign assets finally reduces to:

$$B(t) - \tilde{B} = \Phi^T\left(L^T(t) - \tilde{L}^T\right) + \Phi^N\left(L^N(t) - \tilde{L}^N\right). \quad (152)$$

F Revisiting the Theory Developed by Balassa [1964] and Samuelson [1964]: Derivation of Equations in Section 2.1

This Appendix presents the formal analysis underlying the results described in section 2.1. For simplicity purposes, we abstract from firing costs. Additionally, we assume that the worker bargaining power α_W^j is symmetric across sectors.

As defined by eq. (113) that we repeat for convenience, the overall surplus from hiring in sector j, Ψ^j, is defined as the difference between the marginal product of labor (Ξ^j) and the reservation wage (W_R^j):

$$\Psi^j = \Xi^j - W_R^j. \quad (153)$$

Eq. (153) corresponds to eq. (1) in the text. The reservation wage, W_R^j, is equal to the expected value of a job, i.e., $m^j\xi^j$ with m^j the probability of finding a job, plus the unemployment benefit R^j:

$$W_R^j = \frac{\alpha_W}{1-\alpha_W}\kappa^j\theta^j + R^j, \quad (154)$$

where we used the fact that $m^j \xi^j = \frac{\alpha_W}{1-\alpha_W} \kappa^j \theta^j$. Totally differentiating eq. (154), the change of the reservation wage in percentage is proportional to the labor market tightness:

$$\hat{w}_R^j = \chi^j \hat{\theta}^j, \tag{155}$$

where $\chi^j \equiv \frac{m^j \xi^j}{W_R^j}$ corresponds to the share of the surplus associated with a labor contract; the share χ^j is smaller than one as long as job seekers receive unemployment benefits, R^j, from the State since $W_R^j = m^j \xi^j + R^j$.

The product wage W^j paid to the worker in sector j is equal to the reservation wage W_R^j plus a share α_W of the overall surplus Ψ^j:

$$W^j = \alpha_W \Psi^j + W_R^j. \tag{156}$$

Eq. (156) corresponds to eq. (2) in the text. Totally differentiating (156), the change in the product wage in percentage is proportional to the changes in the labor market tightness and the overall surplus from an additional job:

$$\begin{aligned} \hat{w}^j &= \frac{\alpha_W \Psi^j}{W^j} \hat{\Psi}^j + \frac{W_R^j}{W^j} \hat{W}_R^j, \\ &= \frac{\alpha_W \Psi^j}{W^j} \hat{\Psi}^j + \frac{W_R^j \chi^j}{W^j} \hat{\theta}^j, \end{aligned} \tag{157}$$

where we substituted (155) to get the last line. Subtracting $\hat{w}^T$ from $\hat{w}^N$ yields the wage differential between the non traded and the traded sector:

$$\begin{aligned} \hat{w}^N - \hat{w}^T &= \frac{\alpha_W \Psi^N}{W^N} \hat{\Psi}^N + \frac{W_R^N}{W^N} \hat{W}_R^N - \frac{\alpha_W \Psi^T}{W^T} \hat{\Psi}^T + \frac{W_R^T}{W^T} \hat{W}_R^T, \\ &= -\frac{\chi W_R}{W} \left(\hat{\theta}^T - \hat{\theta}^N \right) - \frac{\alpha_W \Psi}{W} \left(\hat{\Psi}^T - \hat{\Psi}^N \right), \end{aligned} \tag{158}$$

where we assume that initially, sectoral wages, W^j, the share of the surplus associated with a labor contract, χ^j, reservation wages, W_R^j, and overall surpluses, Ψ^j, are similar across sectors, i.e., $W^j \simeq W$, $\chi^j W_R^j \simeq \chi W_R$ and $\Psi^j \simeq \Psi$. **Eq. (158) corresponds to eq. (3) in the text**.

Denoting the job destruction rate by s^j and the job finding rate by m^j, and using the fact at the steady-state, the flow of unemployed workers who find a job is equalized with the flow of employed workers who lose their job, the unemployment rate u^j in sector j reads as:

$$u^j = \frac{s^j}{s^j + m^j \left(\theta^j\right)}. \tag{159}$$

Totally differentiating (159) and assuming that the elasticity of vacancies in job matches, denoted by α^V, is symmetric across sectors, the change in unemployment rate in sector j reads as:

$$\begin{aligned} du^j &= -\alpha_V u^j \frac{m^j}{s^j + m^j \left(\theta^j\right)} \hat{\theta}^j, \\ &= -\alpha_V u^j \left(1 - u^j\right) \hat{\theta}^j, \end{aligned} \tag{160}$$

where we used the fact that $1 - u^j = \frac{m^j}{s^j + m^j(\theta^j)}$. Subtracting du^T from du^N yields negative relationship between the unemployment rate differential between tradables and non tradables and the percentage change in labor market tightness in the traded relative to the non traded sector:

$$du^T - du^N = -\alpha_V u \left(1 - u\right) \left(\hat{\theta}^T - \hat{\theta}^N \right), \tag{161}$$

where we assume that at the initial steady-state, search parameters are such that $u^j \equiv u$. **Eq. (161) corresponds to eq. (4) in the text**.

When a labor contract is concluded, a surplus Ψ^j is created. The firm obtains a share $1 - \alpha_W$ of the surplus which is equal to the difference between the marginal product of labor and the Nash bargaining wage W^j:

$$\left(1 - \alpha_W\right) \Psi^j = \Xi^j - W^j.$$

The equation above can be rewritten as follows:

$$\Xi^j = \left(1 - \alpha_W\right) \Psi^j + W^j. \tag{162}$$

Eq. (162) corresponds to eq. (5) in the text. According to the definition of the representative firm's profit (16), i.e., $\pi^j = \Xi^j L^j - W^j L^j - \kappa^j V^j$ (we set $x^j = 0$ since we abstract from the firing cost in this section for simplicity purposes), the share of the surplus obtained by the firm is equal to the dividend plus the hiring cost per worker:

$$(1-\alpha_W)\,\Psi^j = \frac{\pi^j + \kappa^j V^j}{L^j}. \tag{163}$$

Totally differentiating (162) yields the change of the marginal revenue of labor in percentage:

$$\hat{\Xi}^j = \frac{(1-\alpha_W)\,\Psi^j}{\Xi^j}\hat{\Psi}^j + \frac{W^j}{\Xi^j}\hat{w}^j. \tag{164}$$

Subtracting $\hat{\Xi}^T$ from $\hat{\Xi}^N$ while assuming that initially $W^j \simeq W$, $\Xi^j \simeq \Xi$, $\Psi^j \simeq \Psi$, leads to:

$$\hat{\Xi}^N - \hat{\Xi}^T = -\frac{(1-\alpha_W)\,\Psi}{\Xi}\left(\hat{\Psi}^T - \hat{\Psi}^N\right) + \frac{W}{\Xi}\left(\hat{w}^N - \hat{w}^T\right). \tag{165}$$

Using the fact that $\hat{\Xi}^N = \hat{p} + \hat{a}^N$ and $\hat{\Xi}^T = \hat{a}^T$, one obtains a relationship between the relative price growth and both the productivity and the wage differential:

$$\hat{p} = \hat{a}^T - \hat{a}^N - \frac{(1-\alpha_W)\,\Psi}{\Xi}\left(\hat{\Psi}^T - \hat{\Psi}^N\right) + \frac{W}{\Xi}\left(\hat{w}^N - \hat{w}^T\right). \tag{166}$$

Eq. (166) corresponds to eq. (6) in the text.

G Graphical Apparatus

Before turning to the derivation of steady-state effects, we investigate graphically the long-run effects of a productivity differential.

G.1 Steady-State

Using (103), the steady-state of the open economy is described by the following set of equations:

$$\tilde{C} = \left[P_C\left(\tilde{P}\right)\bar{\lambda}\right]^{-\sigma_C}, \tag{167a}$$

$$s^T\tilde{L}^T = m^T\left(\tilde{\theta}^T\right)\tilde{U}^T, \tag{167b}$$

$$s^N\tilde{L}^N = m^N\left(\tilde{\theta}^N\right)\tilde{U}^N, \tag{167c}$$

$$\left(\tilde{L}^T + \tilde{U}^T\right) = \left[\bar{\lambda}\left(\frac{\alpha_W^T}{1-\alpha_W^T}\kappa^T\tilde{\theta}^T + R^T\right)\right]^{\sigma_L^T}, \tag{167d}$$

$$\left(\tilde{L}^N + \tilde{U}^N\right) = \left[\bar{\lambda}\left(\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N\tilde{\theta}^N + R^N\right)\right]^{\sigma_L^N}, \tag{167e}$$

$$\frac{\kappa^T}{f^T\left(\tilde{\theta}^T\right)} = \frac{\left(1-\alpha_W^T\right)\tilde{\Psi}^T}{s^T + r^\star}, \tag{167f}$$

$$\frac{\kappa^N}{f^N\left(\tilde{\theta}^N\right)} = \frac{\left(1-\alpha_W^N\right)\tilde{\Psi}^N}{s^N + r^\star}, \tag{167g}$$

$$A^N\tilde{L}^N = \tilde{C}^N, \tag{167h}$$

$$r^\star\tilde{B} + A^T\tilde{L}^T - \tilde{C}^T - \kappa^T\tilde{\theta}^T\tilde{U}^T - \kappa^N\tilde{\theta}^N\tilde{U}^N, \tag{167i}$$

and the intertemporal solvency condition

$$\tilde{B} - B_0 = \Phi^T\left(\tilde{L}^T - L_0^T\right) + \Phi^T\left(\tilde{L}^N - L_0^N\right), \tag{167j}$$

where $C^N = P_C'C$ and $C^T = (1-\alpha_C)\,P_C C$ and we used the fact that $V^j = U^j\theta^j$. The steady-state equilibrium defined by ten equations jointly determines $\tilde{C}$, $\tilde{L}^T$, $\tilde{L}^N$ $\tilde{U}^T$, $\tilde{U}^N$, $\tilde{\theta}^T$, $\tilde{\theta}^N$, $\tilde{P}$, $\tilde{B}$, $\bar{\lambda}$.

G.2 Isoclines and Stable Path in the (θ^T, L^T)-space

The labor market in the traded sector can be summarized graphically by Figure 6(a) that traces out two schedules in the (θ^T, L^T)-space. More precisely, eliminating $\tilde{U}^T$ from eq. (167d) by using

(167b), i.e., $\tilde{U}^T = \frac{s^T \tilde{L}^T}{\tilde{m}^T}$, the system which comprises eqs. (167b), (167d) and (167f) can be reduced to two equations:

$$\tilde{L}^T = \frac{\tilde{m}^T}{\tilde{m}^T + s^T}\left[\bar{\lambda}\left(\frac{\alpha_W^T}{1-\alpha_W^T}\kappa^T\tilde{\theta}^T + R^T\right)\right]^{\sigma_L^T}, \tag{168a}$$

$$\frac{\kappa^T}{f^T\left(\tilde{\theta}^T\right)} = \frac{\left(1-\alpha_W^T\right)}{\left(s^T + r^\star\right)}\tilde{\Psi}^T, \tag{168b}$$

where $\tilde{m}^T = m^T\left(\tilde{\theta}^T\right)$ and $\tilde{f}^T = f^T\left(\tilde{\theta}^T\right)$; using the fact the reservation wage $W_R^T = -\frac{v_F^T}{\bar{\lambda}}$ is equal to $\left(\frac{\alpha_W^T}{1-\alpha_W^T}\kappa^T\tilde{\theta}^T + R^T\right)$ (see eq. (103)), the overall surplus from hiring in the traded sector is given by:

$$\tilde{\Psi}^T \equiv \left(A^T + r^\star x^T\right) - \left(\frac{\alpha_W^T}{1-\alpha_W^T}\kappa^T\tilde{\theta}^T + R^T\right). \tag{169}$$

Totally differentiating eq. (168a) yields

$$\hat{\tilde{L}}^T = \sigma_L^T\hat{\bar{\lambda}} + \left[\alpha_V^T\tilde{u}^T + \sigma_L^T\tilde{\chi}^T\right]\hat{\tilde{\theta}}^T, \tag{170}$$

where $\tilde{u}^T = \frac{s^T}{s^T+\tilde{m}^T}$ and $0 < \tilde{\chi}^T = \frac{\frac{\alpha_W^T}{1-\alpha_W^T}\kappa^T\tilde{\theta}^T}{W_R^T} < 1$. The slope of the $\dot{L}^T = 0$ schedule in the (θ^T, L^T)-space writes as:

$$\left.\frac{\hat{\tilde{L}}^T}{\hat{\tilde{\theta}}^T}\right|_{\dot{L}^T=0} = \left[\alpha_V^T\tilde{u}^T + \sigma_L^T\tilde{\chi}^T\right] > 0. \tag{171}$$

Hence the decision of search (henceforth labelled DST) schedule is upward-sloping in the (θ^T, L^T)-space. According to (170), a fall in the marginal utility of wealth $\bar{\lambda}$ shifts downward the DST-schedule.

Totally differentiating eq. (168b) yields

$$\hat{\tilde{\theta}}^T\left[\left(1-\alpha_V^T\right)\tilde{\Psi}^T + \tilde{\chi}^T\tilde{W}_R^T\right] = A^T\hat{a}^T, \tag{172}$$

where we used (167f) and the fact that $-\left(f^T\right)'\theta^T/f^T = \left(1-\alpha_V^T\right)$. The slope of the $\dot{\theta}^T = 0$ schedule in the (θ^T, L^T)-space can be written as:

$$\left.\frac{\hat{\tilde{L}}^T}{\hat{\tilde{\theta}}^T}\right|_{\dot{\theta}^T=0} = +\infty. \tag{173}$$

Hence the vacancy creation (henceforth labelled VCT) schedule is a vertical line in the (θ^T, L^T)-space. According to (172), a rise in labor productivity in the traded sector A^T shifts to the right the VCT-schedule.

Having determined the patterns of isoclines in the (θ^T, L^T)-space, we now analyze the slope of the stable path. To determine the pattern of the stable path, we have to estimate:

$$\frac{\frac{L^T(t)-\tilde{L}^T}{\tilde{L}^T}}{\frac{\theta^T(t)-\tilde{\theta}^T}{\tilde{\theta}^T}} = \frac{1}{\omega_{21}^T}\frac{\tilde{\theta}^T}{\tilde{L}^T}. \tag{174}$$

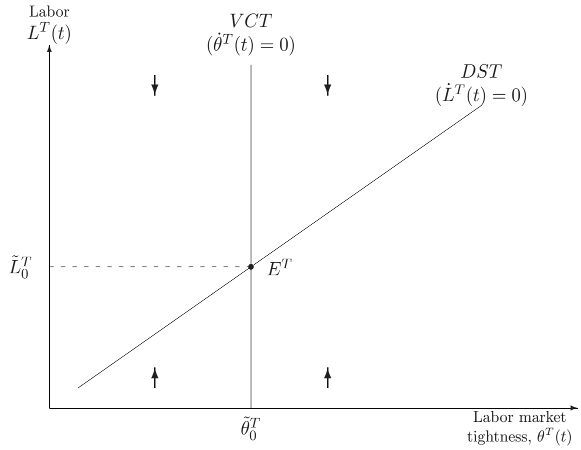
Using the fact that $\omega_{21}^T = 0$ (see (142)), the slope of the stable branch labelled SS^T in the (θ, L)-space rewrites as:

$$\left.\frac{\hat{\tilde{L}}^T}{\hat{\tilde{\theta}}^T}\right|_{SS^T} = +\infty. \tag{175}$$

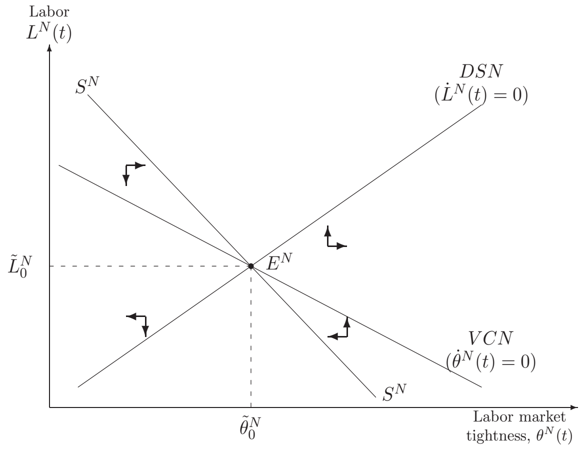
According to (175), the stable branch coincides with the VCT-schedule (see Figure 6(a)) as the dynamics for θ^T degenerate.

G.3 Isoclines and Stable Path in the (θ^N, L^N)-space

The labor market in the non traded sector can be summarized graphically by Figure 6(b) that traces out two schedules in the (θ^N, L^N)-space. More precisely, eliminating $\tilde{U}^N$ from eq. (167e) by using (167c), i.e., $\tilde{U}^N = \frac{s^N\tilde{L}^N}{\tilde{m}^N}$, and inserting the short-run static solution for the relative price of non



(a) (θ^T, L^T)-space



(b) (θ^N, L^N)-space

Figure 6: Phase Diagrams in the (θ^j, L^j)-space

tradables given by (108) implies that the system which comprises eqs. (167c), (167e), (167g), and (167h) can be reduced to two equations:

$$\tilde{L}^N = \frac{\tilde{m}^N}{\tilde{m}^N + s^N}\left[\bar{\lambda}\left(\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N\tilde{\theta}^N + R^N\right)\right]^{\sigma_L^N}, \tag{176a}$$

$$\frac{\kappa^N}{f^N\left(\tilde{\theta}^N\right)} = \frac{\left(1-\alpha_W^N\right)}{\left(s^N + r^\star\right)}\tilde{\Psi}^N, \tag{176b}$$

where $\tilde{m}^N = m^N\left(\tilde{\theta}^N\right)$ and $\tilde{f}^N = f^N\left(\tilde{\theta}^N\right)$; using the fact the reservation wage $W_R^N = -\frac{v_F^N}{\bar{\lambda}}$ is equal to $\left(\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N\tilde{\theta}^N + R^N\right)$ (see eq. (103)), the overall surplus from hiring in the non traded sector is given by:

$$\tilde{\Psi}^N \equiv \left[\left(P\left(\bar{\lambda}, L^N, A^N\right)A^N + r^\star x^N\right)\right] - \left(\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N\tilde{\theta}^N + R^N\right). \tag{177}$$

Totally differentiating eq. (176a) yields

$$\hat{\tilde{L}}^N = \sigma_L^N\hat{\bar{\lambda}} + \left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]\hat{\tilde{\theta}}^N, \tag{178}$$

where $\tilde{u}^N = \frac{s^N}{s^N+\tilde{m}^N}$ and $0 < \tilde{\chi}^N = \frac{\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N\tilde{\theta}^N}{W_R^N} < 1$. The slope of the $\dot{L}^N = 0$ schedule in the (θ^N, L^N)-space writes as:

$$\left.\frac{\hat{\tilde{L}}^N}{\hat{\tilde{\theta}}^N}\right|_{\dot{L}^N=0} = \left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right] > 0. \tag{179}$$

Hence the decision of search (henceforth labelled DSN) schedule is upward-sloping in the (θ^N, L^N)-space. According to (178), a fall in the marginal utility of wealth $\bar{\lambda}$ shifts downward the DSN-schedule.

Totally differentiating eq. (176b) yields

$$\begin{aligned}&\hat{\tilde{\theta}}^N\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\\ = & -\frac{\tilde{P}A^N\left\{\omega_N\hat{\tilde{L}}^N + \sigma_C\alpha_C\omega_C\hat{\bar{\lambda}} + \left[\omega_N - \omega_C\alpha_C\left(\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right)\right]\hat{a}^N +\right\}}{\alpha_C\omega_C\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right]},\end{aligned} \tag{180}$$

where we used (167g) and the fact that $-\left(f^N\right)'\theta^N/f^N = \left(1-\alpha_V^N\right)$. The slope of the $\dot{\theta}^N = 0$ schedule in the (θ^N, L^N)-space is:

$$\left.\frac{\hat{\tilde{L}}^N}{\hat{\tilde{\theta}}^N}\right|_{\dot{\theta}^N=0} = -\frac{\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]}{PA^N}\frac{\alpha_C\omega_C\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right]}{\omega_N} < 0. \tag{181}$$

Hence the vacancy creation (henceforth labelled VCN) schedule is downward-sloping in the (θ^N, L^N)-space. According to (181), since $\left[\omega_N - \omega_C\alpha_C\left(\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right)\right] \gtreqless 0$, a rise in labor productivity in the non traded sector A^N may shift to the left or to the right the VCN-schedule depending on whether ϕ takes high or low values; it is worthwhile mentioning that higher productivity in tradables relative to non tradables shifts to the right the VCN-schedule by appreciating the relative price and thus by raising the marginal revenue of labor in the non traded sector, i.e., by increasing $\Xi^N \equiv PA^N$. Moreover, a fall in the marginal utility of wealth $\bar{\lambda}$ shifts to the right the VCN-schedule by appreciating the relative price of non tradables.

Having determined the patterns of isoclines in the (θ^N, L^N)-space, we now analyze the slope of the stable path. To do so, we use the third line of the Jacobian matrix (129) to rewrite the element ω_{2i}^N of the eigenvector:

$$\omega_{2i}^N = \frac{\left(2s^N + r^\star\right) + \left(s^N + r^\star - \nu_i^N\right)\left(\frac{s^N+\nu_i^N}{\tilde{m}^N}\right) + \tilde{m}^N\left(P_{L^N}A^N\frac{\bar{\lambda}}{v_{FF}^N} + 1\right)}{\frac{(m^N)'\tilde{U}^N}{\tilde{m}^N}\left(s^N + \tilde{m}^N + r^\star - \nu_i^N\right)}. \tag{182}$$

The first two terms in the numerator of (182) can be rewritten as follows:

$$\left(2s^N + r^\star\right) + \left(s^N + r^\star - \nu_i^N\right)\left(\frac{s^N + \nu_i^N}{s^N}\right) = s^N + \frac{\left(s^N + r^\star\right)\left(s^N + \tilde{m}^N\right) + \nu_i^N\left(r^\star - \nu_i^N\right)}{\tilde{m}^N}, \tag{183}$$

where $\nu_i^N\left(r^\star-\nu_i^N\right)$ is equal to the determinant of the Jacobian matrix (129) given by (134). To determine the pattern of the stable path in the (θ^N,L^N)-space, we have to estimate:

$$\frac{\frac{L^N(t)-\tilde{L}^N}{\tilde{L}^N}}{\frac{\theta^N(t)-\tilde{\theta}^N}{\tilde{\theta}^N}}=\frac{1}{\omega_{21}^N}\frac{\tilde{\theta}^N}{\tilde{L}^N}. \tag{184}$$

Inserting (138) into (184), the slope of the stable branch labelled S^NS^N in the (θ^N,L^N)-space can be rewritten as follows:

$$\left.\frac{\hat{\tilde{L}}^N}{\hat{\tilde{\theta}}^N}\right|_{S^NS^N}=\frac{1}{\omega_{21}^N}\frac{\tilde{\theta}^N}{\tilde{L}^N}=-\frac{\left(s^N+\tilde{m}^N+r^\star-\nu_1^N\right)}{\left(s^N+r^\star\right)}\frac{\left(1-\alpha_V^N\right)\tilde{\Psi}^N}{\tilde{P}A^N}\frac{\alpha_C\omega_C\left[\left(1-\alpha_C\right)\phi+\alpha_C\sigma_C\right]}{\omega_N}<0, \tag{185}$$

where we denote by a hat the rate of change relative to initial steady-state. According to (185), the stable branch SS^N is downward-sloping in the (θ^N,L^N)-space.

To get (185), we proceed as follows. We first have rewritten the numerator of eigenvector ω_{21}^N given by (182) (set $i=1$) by using (183) and by inserting $\frac{\text{Det}J^N}{s^N+r^\star}$ (which is equal to $\nu_1^N\left(r^\star-\nu_1^N\right)$) given by (138):

$$\begin{aligned}
& s^N+\frac{\left(s^N+r^\star\right)\left(s^N+\tilde{m}^N\right)-\left(s^N+r^\star+\tilde{m}^N\right)\left(s^N+\tilde{m}^N\right)}{\tilde{m}^N}+\tilde{m}^N\left(P_{L^N}A^N\frac{\bar{\lambda}}{v_{FF}^N}+1\right)\\
-& \frac{\omega_N\tilde{P}A^N}{\alpha_C\omega_C\left[\left(1-\alpha_C\right)\phi+\alpha_C\sigma_C\right]}\frac{\left(s^N+r^\star\right)\left(s^N+\tilde{m}^N\right)\left(\tilde{\chi}^N\sigma_L^N+\alpha_V^N\tilde{u}^N\right)}{\left(1-\alpha_V^N\right)\tilde{\Psi}^N\tilde{m}^N}, &(186)\\
=& -\frac{\omega_N\tilde{P}A^N}{\alpha_C\omega_C\left[\left(1-\alpha_C\right)\phi+\alpha_C\sigma_C\right]}\frac{\left(s^N+r^\star\right)\left(s^N+\tilde{m}^N\right)\alpha_V^N\tilde{u}^N}{\left(1-\alpha_V^N\right)\tilde{\Psi}^N\tilde{m}^N}. &(187)
\end{aligned}$$

To get the last line, we computed the following term $\tilde{m}^N\left(P_{L^N}A^N\frac{\bar{\lambda}}{v_{FF}^N}+1\right)$ as follows:

$$\begin{aligned}
\tilde{m}^N\left(P_{L^N}A^N\frac{\bar{\lambda}}{v_{FF}^N}+1\right)&=\tilde{m}^N\left(\frac{P_{L^N}\tilde{L}^N}{\tilde{P}}\frac{\tilde{P}A^N}{\tilde{L}^N}\tilde{F}^N\sigma_L^N\frac{\bar{\lambda}}{v_F^N}+1\right),\\
=\ \tilde{m}^N&\left\{\frac{\omega_N\tilde{P}A^N}{\alpha_C\omega_C\left[\left(1-\alpha_C\right)\phi+\alpha_C\sigma_C\right]}\frac{s^N+\tilde{m}^N}{\tilde{m}^N}\frac{\left(s^N+r^\star\right)\sigma_L^N\tilde{\chi}^N}{\alpha_W^N\tilde{\Psi}^N\tilde{m}^N}+1\right\}, \quad (188)
\end{aligned}$$

where we used the fact that $\frac{v_F^N}{v_{FF}^N\tilde{F}^N}=\sigma_L^N$ to get the first line, $\frac{\tilde{L}^N}{\tilde{F}^N}=\frac{\tilde{m}^N}{s^N+\tilde{m}^N}$ and $\frac{P_{L^N}\tilde{L}^N}{\tilde{P}}=\frac{\omega_N\tilde{P}A^N}{\alpha_C\omega_C[(1-\alpha_C)\phi+\alpha_C\sigma_C]}$ to get the second line, $\tilde{m}^N\tilde{\xi}^N=\tilde{m}^N\frac{\alpha_W^N\tilde{\Psi}^N}{(s^N+r^\star)}=-\tilde{\chi}^N\frac{v_F^N}{\bar{\lambda}}$ to get (188). Inserting (188) into (186), rearranging terms, we get (187).

Inserting first (188), and multiplying ω_{21}^N (setting setting $i=1$ into (182)) by $\tilde{L}^N/\tilde{\theta}^N$, we get:

$$\begin{aligned}
\omega_{21}^N\frac{\tilde{L}^N}{\tilde{\theta}^N}&=-\frac{\frac{\omega_N\tilde{P}A^N}{\alpha_C\omega_C[(1-\alpha_C)\phi+\alpha_C\sigma_C]}\frac{\left(s^N+r^\star\right)\left(s^N+\tilde{m}^N\right)}{\left(1-\alpha_V^N\right)\tilde{\Psi}^N\tilde{m}^N}\frac{\tilde{L}^N}{\tilde{F}^N}}{\left(s^N+\tilde{m}^N+r^\star-\nu_1^N\right)},\\
&=-\frac{\frac{\omega_N\tilde{P}A^N}{\alpha_C\omega_C[(1-\alpha_C)\phi+\alpha_C\sigma_C]}\frac{\left(s^N+r^\star\right)}{\left(1-\alpha_V^N\right)\tilde{\Psi}^N}}{\left(s^N+\tilde{m}^N+r^\star-\nu_1^N\right)}<0, \quad (189)
\end{aligned}$$

where we used the fact that $\left(m^N\right)'\theta^N/m^N=\alpha_V^N$ and $\tilde{u}^N=\tilde{U}^N/\tilde{F}^N$ to get the first line, $\frac{\tilde{L}^N}{\tilde{F}^N}=\frac{\tilde{m}^N}{s^N+\tilde{m}^N}$ to get (189).

Because both the VCN-schedule and the stable branch S^NS^N are downward sloping, we have now to determine whether the stable branch S^NS^N is steeper or flatter than the VCN-schedule. To do so, we compute the following term which shows up in eq. (181):

$$\left(1-\alpha_V^N\right)\tilde{\Psi}^N+\tilde{\chi}^NW_R^N=\left(1-\alpha_V^N\right)\tilde{\Psi}^N\frac{\left(s^N+\tilde{m}^N+r^\star\right)}{\left(s^N+r^\star\right)}, \tag{190}$$

where we used the fact that $\tilde{\chi}^NW_R^N=\frac{\tilde{m}^N\alpha_W^N\tilde{\Psi}^N}{s^N+r^\star}=\frac{\tilde{m}^N\left(1-\alpha_V^N\right)\tilde{\Psi}^N}{s^N+r^\star}$. Since $\frac{\left(s^N+\tilde{m}^N+r^\star-\nu_1^N\right)}{\left(s^N+r^\star\right)}>\frac{\left(s^N+\tilde{m}^N+r^\star\right)}{\left(s+r^\star\right)}$, inspection of (181) and (185) implies that the S^NS^N-schedule is steeper than the VCN-schedule (see Figure 6(b)).

We turn now to the transitional adjustment along the stable path in the (L^N,U^N)-space by making use of (144b):

$$U^N(t)-\tilde{U}^N=\omega_{31}^N\left(L^N(t)-\tilde{L}^N\right), \tag{191}$$

where ω_{31}^N is given by eq. (144b). To sign the slope of the transitional path in the (L^N, U^N)-space, we use the third line of the Jacobian matrix (129) to rewrite the element ω_{21}^N of the eigenvector:

$$\omega_{21}^N = \frac{\left(2s^N + r^\star\right) + \left(s^N + r^\star - \nu_1^N\right)\left(\frac{s^N+\nu_1^N}{\tilde{m}^N}\right) + \frac{\tilde{m}^N \tilde{\Psi}_{L^N}}{\tilde{\Psi}_{U^N}}}{\frac{(m^N)'\tilde{U}^N}{\tilde{m}^N}\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)}. \tag{192}$$

where $\tilde{\Psi}_{L^N}$ and $\tilde{\Psi}_{U^N}$ and the partial derivatives (evaluated at the steady-state) of the overall surplus from an additional job Ψ^N in the non traded sector:

$$\Psi^N_{L^N} = \frac{\partial \Psi^N}{\partial L^N} = P_{L^N} A^N + \frac{v^N_{FF}}{\bar{\lambda}} < 0, \tag{193a}$$

$$\Psi^N_{U^N} = \frac{\partial \Psi^N}{\partial U^N} = \frac{v^N_{FF}}{\bar{\lambda}} < 0. \tag{193b}$$

Inserting (192) into (144b) allows to rewrite ω_{31}^N as follows:

$$\begin{aligned}
\omega_{31}^N &= \left(\frac{s^N + \nu_1^N}{\tilde{m}^N}\right) - \frac{\left(m^N\right)'\tilde{U}^N}{\tilde{m}^N}\omega_{21}^N, \\
&= \left(\frac{s^N + \nu_1^N}{\tilde{m}^N}\right) - \frac{\left(2s^N + r^\star\right) + \left(s^N + r^\star - \nu_1^N\right)\left(\frac{s^N+\nu_1^N}{\tilde{m}^N}\right) + \frac{\tilde{m}^N \tilde{\Psi}_{L^N}}{\tilde{\Psi}_{U^N}}}{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)}, \\
&= \frac{\left(s^N + \nu_1^N\right) - \left(2s^N + r^\star\right) - \frac{\tilde{m}^N \tilde{\Psi}_{L^N}}{\tilde{\Psi}_{U^N}}}{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)}, \\
&= -\frac{\left[\left(s^N + r^\star - \nu_1^N\right) + \frac{\tilde{m}^N \tilde{\Psi}_{L^N}}{\tilde{\Psi}_{U^N}}\right]}{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)} < 0,
\end{aligned} \tag{194}$$

where $\nu_1^N < 0$ is the stable root for the non traded labor market. Since according to (193), $\tilde{\Psi}_{L^N} < 0$ and $\tilde{\Psi}_{U^N} < 0$, we have $\omega_{31}^N < 0$. Hence, as employment declines in the non traded sector, job seekers increase in this sector.

G.4 Isoclines and Stable Path in the (u^T, L^T)-space

One can alternatively analyze the transitional adjustment in the (u^T, L^T)-space. To do so, we first determine the slopes of the isoclines $\dot{L}^T = 0$ and $\dot{\theta}^T = 0$ in the (u^T, L^T)-space. Hence, we first determine the relationship between labor market tightness and the unemployment rate by using the definition of the latter, i.e. $\tilde{u}^T = \frac{s^T}{s^T + m^T\left(\tilde{\theta}^T\right)}$. To alleviate the notation, we assume:

$$\alpha_V = \alpha_V^j, \quad \sigma_L = \sigma_L^j. \tag{195}$$

Totally differentiating the equation that describes the steady-state level of the unemployment rate, we have:

$$\hat{\tilde{\theta}}^T = -\frac{1}{\alpha_V}\left(\frac{s^T + \tilde{m}^T}{\tilde{m}^T}\right)\hat{\tilde{u}}^T. \tag{196}$$

The slope of the $\dot{L}^T = 0$ schedule in the (u^T, L^T)-space writes as:

$$\left.\frac{\hat{\tilde{L}}^T}{\hat{\tilde{u}}^T}\right|_{\dot{L}^T=0} = -\left[\alpha_V \tilde{u}^T + \sigma_L \tilde{\chi}^T\right]\frac{1}{\alpha_V}\left(\frac{s^T + \tilde{m}^T}{\tilde{m}^T}\right) < 0. \tag{197}$$

Hence the DST-schedule is downward-sloping in the (u^T, L^T)-space, as displayed in Figure 7(a).

Using eq. (172) together with eq. (196), we have:

$$-\frac{1}{\alpha_V}\left(\frac{s^T + \tilde{m}^T}{\tilde{m}^T}\right)\hat{\tilde{u}}^T\left[\left(1 - \alpha_V^T\right)\tilde{\Psi}^T + \tilde{\chi}^T \tilde{W}_R^T\right] = A^T \hat{a}^T.$$

The slope of the $\dot{\theta}^T = 0$ schedule in the (u^T, L^T)-space thus reads as:

$$\left.\frac{\hat{\tilde{L}}^T}{\hat{\tilde{u}}^T}\right|_{\dot{\theta}^T=0} = +\infty \tag{198}$$

As a result, the VCT-schedule is a vertical line in the (u^T, L^T)-space, as displayed in Figure 7(a).

Having determined that the patterns of isoclines, we turn now to the transitional adjustment along the stable path labelled XX^T. We begin by linearizing $u^j(t) = \frac{s^j}{s^j + m^j(\theta^j(t))}$ in the neighborhood of the steady-state which leads to:

$$
\begin{aligned}
u^j(t) - \tilde{u}^j &= \frac{1}{\tilde{F}^j}\left[\left(1-\tilde{u}^j\right)\left(U^j(t) - \tilde{U}^j\right) - \tilde{u}^j\left(L^j(t) - \tilde{L}^j\right)\right], \\
&= \frac{1}{\tilde{F}^j}\left[\left(1-\tilde{u}^j\right)\omega_{31}^j - \tilde{u}^j\right] D_1^j e^{\nu_1^j t}. \qquad (199)
\end{aligned}
$$

where we used the stable paths for $L^j(t)$ and $U^j(t)$. Using (199) and the fact that $\left(L^j(t) - \tilde{L}^j\right) = D_1^j e^{\nu_1^j t}$, the slope of the stable path in the (u^j, L^j)-space,

$$
\begin{aligned}
\left.\frac{\frac{L^j(t)-\tilde{L}^j}{\tilde{L}^j}}{\frac{u^j(t)-\tilde{u}^j}{\tilde{u}^j}}\right|_{XX^j} &= -\tilde{F}^j \frac{\tilde{u}^j}{\tilde{L}^j} \frac{1}{\left[\left(1-\tilde{u}^j\right)\omega_{31}^j - \tilde{u}^j\right]}, \\
&= \frac{s^j}{\tilde{m}^j} \frac{1}{\left[\left(1-\tilde{u}^j\right)\omega_{31}^j - \tilde{u}^j\right]}, \qquad (200)
\end{aligned}
$$

where we used the fact that:

$$
\begin{aligned}
\tilde{F}^j \frac{\tilde{u}^j}{\tilde{L}^j} &= \frac{\tilde{U}^j}{\tilde{L}^j}, \\
&= \frac{\frac{s^j}{s^j+\tilde{m}^j}}{\frac{\tilde{m}^j}{s^j+\tilde{m}^j}}, \\
&= \frac{s^j + \tilde{m}^j}{\tilde{m}^j}\tilde{u}^j = \frac{s^j}{\tilde{m}^j},
\end{aligned}
$$

since $\tilde{U}^j/\tilde{L}^j = s^j/\tilde{m}^j$.

Focusing on the traded sector, inserting the stable path (see section E.4) for job seekers, i.e., $U^T(t) - \tilde{U}^T = \omega_{31}^T D_1 e^{\nu_1^T t}$ with $\omega_{31}^T = -1$ (see eq. (142)), the stable path XX^T shown in Figure 7(a) is described by:

$$
\begin{aligned}
\left.\frac{\hat{L}^T(t)}{\hat{u}^T(t)}\right|_{XX^T} &= -\frac{s^T + \tilde{m}^T}{\tilde{m}^T}\tilde{u}^T < 0, \qquad (201) \\
&= -\frac{\tilde{u}^T}{1-\tilde{u}^T}. \qquad (202)
\end{aligned}
$$

Eq. (202) reveals that in countries where the unemployment benefit scheme is more generous (i.e., ϱ takes higher values) or worker bargaining power is greater (i.e., α_W takes higher values), the stable path becomes steeper since labor market tightness is initially low and thus the unemployment rate u^T is high.

We now demonstrate that the slope of the eigenvector (202) in the (u^T, L^T)-space is larger (i.e., less negative) than the slope of the DST-schedule described by eq. (197):

$$
\begin{aligned}
0 > &-\frac{s^T + \tilde{m}^T}{\tilde{m}^T}\tilde{u}^T > -\left[\alpha_V \tilde{u}^T + \sigma_L \tilde{\chi}^T\right]\frac{1}{\alpha_V}\left(\frac{s^T + \tilde{m}^T}{\tilde{m}^T}\right), \\
0 > &-\frac{\sigma_L \tilde{\chi}^T\left(s^T + \tilde{m}^T\right)}{\alpha_V}. \qquad (203)
\end{aligned}
$$

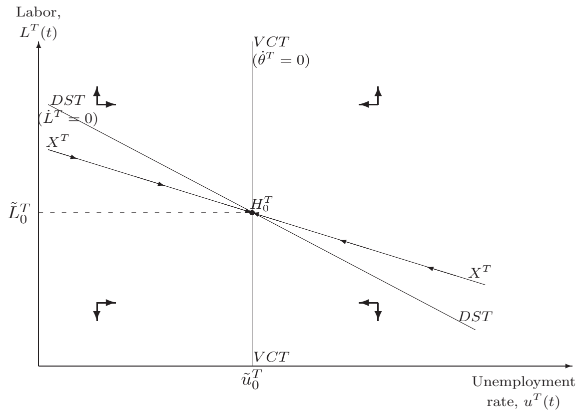
Since the term on the RHS of inequality is unambiguously negative, the stable branch which corresponds to the XX^T-schedule is flatter than the DST-schedule.

The adjustment of labor and unemployment rate in the traded sector is depicted in Figure 2(a). Following an increase in productivity of tradables relative to non tradables, the decision of search-schedule shifts (slightly) to the left as a result of the positive wealth effect (captured by a decline in $\bar{\lambda}$, see eq. (170)); at the same time, the vacancy creation-schedule which is vertical also shifts to the left (see eq. (172)) as a result of the rise in A^T which encourages firms to post more job vacancies; as a result, θ^T increases which raises the probability of finding a job and thus lowers unemployment. The unemployment rate declines on impact. Along the stable path, u^T falls while employment builds up.

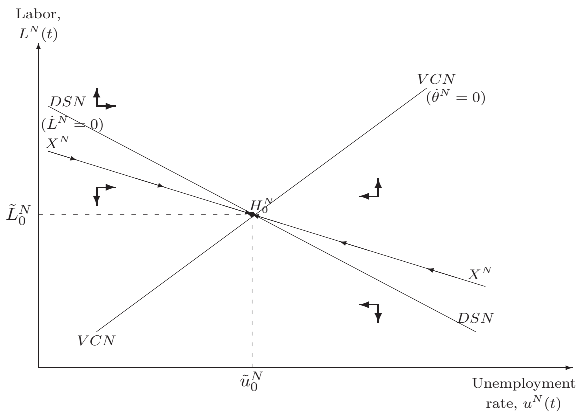
G.5 Isoclines and Stable Path in the (u^N, L^N)-space

The steady-state level of the non traded sector is described by:

$$
\tilde{u}^N = \frac{s^N}{s^N + m^N\left(\tilde{\theta}^N\right)} \qquad (204)
$$



(a) (u^T, L^T)-space



(b) (u^N, L^N)-space

Figure 7: Phase Diagrams in the (u^j, L^j)-space

Totally differentiating eq. (204) leads to:

$$\hat{\tilde{\theta}}^N = -\frac{1}{\alpha_V}\left(\frac{s^N + \tilde{m}^N}{\tilde{m}^N}\right)\hat{\tilde{u}}^N. \tag{205}$$

The slope of the $\dot{L}^N = 0$ schedule in the (u^N, L^N)-space reads as:

$$\left.\frac{\hat{\tilde{L}}^N}{\hat{\tilde{u}}^N}\right|_{\dot{L}^N=0} = -\left[\alpha_V \tilde{u}^N + \sigma_L \tilde{\chi}^N\right]\frac{1}{\alpha_V}\left(\frac{s^N + \tilde{m}^N}{\tilde{m}^N}\right) < 0. \tag{206}$$

Hence the DSN-schedule is downward-sloping in the (u^N, L^N)-space, as displayed in Figure 7(b).

Inserting first (108) and totally differentiating eq. (176b) leads to:

$$\left[(1-\alpha_V)\,\tilde{\Psi}^N + \chi^N \tilde{W}_R^N\right]\hat{\tilde{\theta}}^N = P_{\bar{\lambda}} A^N d\bar{\lambda} + P_{L^N} A^N dL^N + \left(P_{A^N} A^N + \tilde{P}\right) dA^N, \tag{207}$$

where $P_{L^N} < 0$.

Inserting eq. (205) into eq. (207) gives us the slope of of the $\dot{\theta}^N = 0$ schedule in the (u^N, L^N)-space:

$$\left.\frac{\hat{\tilde{L}}^N}{\hat{\tilde{u}}^N}\right|_{\dot{\theta}^N=0} = -\frac{\left[(1-\alpha_V)\,\tilde{\Psi}^N + \chi^N \tilde{W}_R^N\right]}{\alpha_V P_{L^N} A^N L^N}\left(\frac{s^N + \tilde{m}^N}{\tilde{m}^N}\right) > 0. \tag{208}$$

where the positive sign of eq. (208) follows from eq. (110) indicating that $P_{L^N} < 0$. As a result, the VCN-schedule is an upward-sloping line in the (u^N, L^N)-space, as displayed in Figure 7(b).

Having determined the patterns of isoclines, we turn now to the transitional adjustment along the stable path labelled XX^N by making use of (200):

$$\left.\frac{\frac{L^N(t)-\tilde{L}^N}{\tilde{L}^N}}{\frac{u^N(t)-\tilde{u}^N}{\tilde{u}^N}}\right|_{XX^N} = \frac{\tilde{U}^N}{\tilde{L}^N}\frac{1}{\left[(1-\tilde{u}^N)\,\omega_{31}^N - \tilde{u}^N\right]}. \tag{209}$$

As will be useful, we first determine the expression of eigenvector ω_{31}^N by inserting eq. (182) into (144b):

$$\omega_{31}^N = -\frac{\left(s^N + r^\star - \nu_1^N\right) + \tilde{m}^N\left(P_{L^N} A^N \frac{\bar{\lambda}}{v_{FF}^N} + 1\right)}{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)}. \tag{210}$$

Then, we use (210) to derive an expression for $\left(1-\tilde{u}^N\right)\omega_{31}^N - \tilde{u}^N$:

$$\left(1-\tilde{u}^N\right)\omega_{31}^N - \tilde{u}^N = -\frac{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right) + \left(1-\tilde{u}^N\right)\tilde{m}^N P_{L^N} A^N \frac{\bar{\lambda}}{v_{FF}^N}}{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)}. \tag{211}$$

Inserting (211) into eq. (209) gives us the slope of the stable path XX^N in the (u^N, L^N)-space:

$$\left.\frac{\hat{L}^N(t)}{\hat{u}^N(t)}\right|_{XX^N} = -\frac{s^N}{\tilde{m}^N}\frac{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)}{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right) + (1-\tilde{u}^N)\,\tilde{m}^N P_{L^N} A^N \frac{\bar{\lambda}}{v_{FF}^N}} < 0. \tag{212}$$

Since $v_{FF}^N < 0$ and $P_{L^N} < 0$, the stable branch XX^N is downward-sloping in the (u^N, L^N)-space.

We now demonstrate that the slope of the stable branch (212) in the (u^N, L^N)-space is larger (i.e., less negative) than the slope of the DSN-schedule described by eq. (206):

$$\begin{aligned}
0 > \quad & -\frac{s^N}{\tilde{m}^N}\frac{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)}{\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right) + (1-\tilde{u}^N)\,\tilde{m}^N P_{L^N} A^N \frac{\bar{\lambda}}{v_{FF}^N}} > -\left[\alpha_V \tilde{u}^N + \sigma_L \tilde{\chi}^N\right]\frac{1}{\alpha_V}\left(\frac{s^N + \tilde{m}^N}{\tilde{m}^N}\right), \\
& \left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right)\alpha_V \tilde{u}^N < \left[\alpha_V \tilde{u}^N + \sigma_L \tilde{\chi}^N\right]\left[\left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right) + \left(1-\tilde{u}^N\right)\tilde{m}^N P_{L^N} A^N \frac{\bar{\lambda}}{v_{FF}^N}\right], \\
& 0 < \sigma_L \chi^N \left(s^N + \tilde{m}^N + r^\star - \nu_1^N\right) + \left[\alpha_V \tilde{u}^N + \sigma_L \tilde{\chi}^N\right]\left(1-\tilde{u}^N\right)\tilde{m}^N P_{L^N} A^N \frac{\bar{\lambda}}{v_{FF}^N}.
\end{aligned} \tag{213}$$

Since the term on the RHS of inequality is unambiguously positive, the stable branch which corresponds to the XX^N-schedule is flatter than the DSN-schedule, as can be seen in Figure 7(b).

The adjustment of labor and unemployment rate in the non traded sector is depicted in Figure 2(b). Following an increase in productivity of tradables relative to non tradables, the decision of search-schedule shifts to the left as a result of the positive wealth effect (captured by a decline in $\bar{\lambda}$); at the same time, the vacancy creation-schedule which is upward-sloping also shifts to the left (see eq. (207)) as a result of the rise in A^N which encourages firms to post more job vacancies. More specifically, a rise in A^N has an ambiguous effect on PA^N. Assuming $\sigma_C = \phi = 1$, A^N has no impact whilst the positive wealth effect stimulates consumption in non tradables and thus appreciates the relative price of non tradables which increases the surplus from an additional job. Consequently, θ^N increases which raises the probability of finding a job and thus lowers u^N in the long-run. The unemployment rate declines significantly on impact and overshoots its new steady-state level. Along the stable path, u^N increases while employment declines. Intuitively, as L^N falls along XX^N, the relative price appreciates which induces non traded firms to post more job vacancies. The rise in the labor market tightness θ^N leads agents to search for a job and thus increases the number of job seekers. The decline in employment L^N triggered by the positive wealth effect and the rise in the number of job seekers U^N produces an increase in u^N along the stable path.

H Steady-State and Short-Run Effects of Higher Relative Productivity

In this section, we first solve the steady-state and derive the long-term changes following a rise in the productivity differential between tradables and non tradables, $\hat{a}^T - \hat{a}^N$. Steady-state values are denoted with a tilde while the rate of change relative to initial steady-state is denoted by a hat. Then, we analyze the dynamic adjustment toward the long-run equilibrium following higher productivity in tradables relative to non tradables.[60]

H.1 Steady-State

We now describe the steady-state of the economy consisting of six equations which can be solved for sectoral employment and labor market tightness, i.e., $\tilde{L}^j = L^j\left(A^T, A^N\right)$ and $\tilde{\theta}^j = L^j\left(A^T, A^N\right)$ with $j = T, N$, the stock of foreign assets, $\tilde{B} = B\left(A^T, A^N\right)$, and the shadow value of wealth, $\bar{\lambda}$.

First, setting $\dot{\theta}^j = 0$ into eq. (114), we obtain the vacancy creation equation (which holds for the traded sector and non traded sector):

$$\frac{\kappa^j}{f^j\left(\tilde{\theta}^j\right)} = \frac{\left(1-\alpha_W^j\right)}{s^j + r^\star}\tilde{\Psi}^j, \quad \tilde{\Psi}^j \equiv \left(\Xi^j + r^\star x^j\right) - \tilde{W}_R^j, \quad j = T, N, \tag{214}$$

where $\Xi^N = P(.)\,A^N$ with $P(.)$ given by eq. (108). The LHS term of eq. (214) represents the expected marginal cost of recruiting in sector $j = T, N$. The RHS term represents the marginal benefit of an additional worker which is equal to the share, received by the firm, of the rent created by the encounter between a vacancy and a job-seeking worker. A rise in labor productivity raises the surplus from hiring $\tilde{\Psi}^j$; as a result, firms post more job vacancies which increases the labor market tightness $\tilde{\theta}^j$.

Second, setting $\dot{\xi}^j = 0$ into eq. (84) and using the fact that $\tilde{W}^j - \tilde{W}_R^j = \alpha_W \tilde{\Psi}^j$ leads to $\tilde{\xi}^j = \frac{\alpha_W \tilde{\Psi}^j}{s^j + r^\star}$. Rewriting the latter equation by inserting the vacancy creation equation (214) for sector j to eliminate $\tilde{\Psi}^j$ gives the expected value of finding a job, i.e., $\tilde{m}^j\tilde{\xi}^j = \frac{\alpha_W}{1-\alpha_W}\kappa^j\tilde{\theta}^j$. Plugging this equation into (14b) leads to the equality between the utility loss from participating the labor market in sector j and the marginal benefit from search, i.e., $\frac{\zeta^j\left(\tilde{F}^j\right)^{\frac{1}{\sigma_L}}}{\bar{\lambda}} = \frac{\alpha_W}{1-\alpha_W}\kappa^j\tilde{\theta}^j + R^j$. Setting $\dot{L}^j = 0$ into eq. (12) to eliminate $\tilde{U}^j$ so that $\tilde{F}^j = \left(\frac{s^j + \tilde{m}^j}{\tilde{m}^j}\right)\tilde{L}^j$, the decision of search equation reads as (which holds for the traded sector and non traded sector):

$$\tilde{L}^j = \frac{\tilde{m}^j}{\tilde{m}^j + s^j}\left[\frac{\bar{\lambda}}{\zeta^j}\left(\frac{\alpha_W^j}{1-\alpha_W^j}\kappa^j\tilde{\theta}^j + R^j\right)\right]^{\sigma_L^j}, \quad j = T, N, \tag{215}$$

where $\left(\frac{\alpha_W^j}{1-\alpha_W^j}\kappa^j\tilde{\theta}^j + R^j\right)$ corresponds to the reservation wage, $\tilde{W}_R^j$, reflecting the marginal benefit from search. According to (215), higher labor market tightness increases labor $\tilde{L}^j$ by raising the job-finding rate for the worker and thus the employment rate $\frac{\tilde{m}^j}{\tilde{m}^j + s^j}$. Moreover, for given $\bar{\lambda}$, the rise in the reservation wage $\frac{\alpha_W^j}{1-\alpha_W^j}\kappa^j\tilde{\theta}^j + R^j$ induces agents to supply more labor.

Third, setting $\dot{B} = 0$ into eq. (25), we obtain the market clearing condition for the traded good:

$$r^\star\tilde{B} + A^T\tilde{L}^T - \tilde{C}^T - \kappa^T\tilde{U}^T\tilde{\theta}^T - \kappa^N\tilde{U}^N\tilde{\theta}^N = 0, \tag{216}$$

[60] While we calibrate the model to data by considering government spending, we set $G^j = 0$ to derive analytical results in order avoid unnecessary complications.

where $\tilde{C}^T = C^T\left(\tilde{L}^N, \bar{\lambda}, A^N\right)$.

The system comprising eqs. (214)-(216) can be solved for the steady-state sectoral labor market tightness and employment, and traded bonds. All these variables can be expressed in terms of the labor productivity index A^j and the marginal utility of wealth, i.e., $\tilde{\theta} = \theta\left(A^T\right)$, $\tilde{L}^T = L^T\left(\bar{\lambda}, A^T\right)$, $\tilde{\theta}^N = \theta^N\left(\bar{\lambda}, A^N\right)$, $\tilde{L}^N = L^N\left(\bar{\lambda}, A^N\right)$, and $\tilde{B} = B\left(\bar{\lambda}, A^T, A^N\right)$. Inserting first $\tilde{B} = B\left(\bar{\lambda}, A^T, A^N\right)$, and $\tilde{L}^j = L^j\left(\bar{\lambda}, A^N\right)$, the intertemporal solvency condition (149) can be solved for the equilibrium value of the marginal utility of wealth:

$$\bar{\lambda} = \lambda\left(A^T, A^N\right). \tag{217}$$

Setting first $\dot{L}^j = 0$ into (12), inserting $\tilde{L}^j = L^j\left(\bar{\lambda}, A^j\right)$, one can solve for U^j; then the relationship $V^j = \theta^j U^j$ can be solved for the steady-state job vacancy in sector j. Using the fact that $\tilde{C}^T = C^T\left(\tilde{L}^N, \bar{\lambda}, A^N\right)$, inserting $L^N\left(\bar{\lambda}, A^N\right)$ and using the fact that $Y^T = A^T L^T$ with $\tilde{L}^T = L^T\left(\bar{\lambda}, A^T\right)$, allows us to solve for ratio $\upsilon_{NX} = \frac{Y^T - C^T}{Y^T}$:

$$\upsilon_{NX} = \upsilon_{NX}\left(A^T, A^N\right), \tag{218}$$

where we have eliminated $\bar{\lambda}$ by using (217).

H.2 Steady-State Effects of Productivity Shocks

Eliminating $\tilde{U}^T$ from eq. (167d) by using (167b), i.e., $\tilde{U}^T = \frac{s^T \tilde{L}^T}{\tilde{m}^T}$, eliminating $\tilde{U}^N$ from eq. (167e) by using (167c), i.e., $\tilde{U}^N = \frac{s^N \tilde{L}^N}{\tilde{m}^N}$, and inserting the short-run static solution for the relative price of non tradables given by (108), inserting the short-run static solution for consumption in tradables given by (110) into the market clearing condition for traded goods (167i), the steady-state can be reduced to a system which comprises six equations:

$$\tilde{L}^T = \frac{\tilde{m}^T}{\tilde{m}^T + s^T}\left[\frac{\bar{\lambda}}{\zeta^T}\left(\frac{\alpha_W^T}{1-\alpha_W^T}\kappa^T \tilde{\theta}^T + R^T\right)\right]^{\sigma_L^T}, \tag{219a}$$

$$\frac{\kappa^T}{f^T\left(\tilde{\theta}^T\right)} = \frac{\left(1-\alpha_W^T\right)}{\left(s^T + r^\star\right)}\left[\left(A^T + r^\star x^T\right) - \left(\frac{\alpha_W^T}{1-\alpha_W^T}\kappa^T \tilde{\theta}^T + R^T\right)\right], \tag{219b}$$

$$\tilde{L}^N = \frac{\tilde{m}^N}{\tilde{m}^N + s^N}\left[\frac{\bar{\lambda}}{\zeta^N}\left(\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N \tilde{\theta}^N + R^N\right)\right]^{\sigma_L^N}, \tag{219c}$$

$$\frac{\kappa^N}{f^N\left(\tilde{\theta}^N\right)} = \frac{\left(1-\alpha_W^N\right)}{\left(s^N + r^\star\right)}\left\{\left[\left(P\left(\tilde{L}^N, \bar{\lambda}, A^N\right)A^N + r^\star x^N\right)\right] - \left(\frac{\alpha_W^N}{1-\alpha_W^N}\kappa^N \tilde{\theta}^N + R^N\right)\right\}, \tag{219d}$$

$$r^\star \tilde{B} + A^T \tilde{L}^T - C^T\left(\tilde{L}^N, \bar{\lambda}, A^N\right) - \kappa^T \frac{s^T \tilde{L}^T}{\tilde{f}^T} - \kappa^N \frac{s^N \tilde{L}^N}{\tilde{f}^N}, \tag{219e}$$

and the intertemporal solvency condition

$$\tilde{B} - B_0 = \Phi^T\left(\tilde{L}^T - L_0^T\right) + \Phi^N\left(\tilde{L}^N - L_0^N\right), \tag{219f}$$

where we abstract from government spending on tradables and non tradables, $\Phi^T = -\frac{\left(A^T + \kappa^T \tilde{\theta}^T\right)}{\left(s^T + \tilde{m}^T + r^\star\right)} < 0$ and $\Phi^N \equiv \frac{\Lambda^N}{\nu_1^N - r^\star} < 0$ (see (150)); to get (219e), we use the fact that $\tilde{U}^j = \frac{s^j \tilde{L}^j}{\tilde{m}^j}$ and $f^j = m^j/\theta^j$. Note that the market clearing condition for non tradables (167h) can be solved for the relative price of non tradables. To avoid unnecessary complications, we set $G^N = 0$ so that eq. (167h) reduces to $Y^N = C^N$. The solution for the relative price of non tradables is $P = P\left(L^N, \bar{\lambda}, A^N\right)$. Totally differentiating the market clearing condition for non tradables, we get:

$$\hat{p} = \frac{-\hat{a}^N - \hat{l}^N - \sigma_C \hat{\bar{\lambda}}}{\left[\left(1-\alpha_C\right)\phi + \alpha_C \sigma_C\right]} \tag{220}$$

Inserting (220) into the short-run static solution for consumption in tradables (106), we get:

$$\hat{C}^T = -\frac{\left[\sigma_C \hat{\bar{\lambda}} + \alpha_C\left(\phi - \sigma_C\right)\left(\hat{a}^N + \hat{l}^N\right)\right]}{\left[\left(1-\alpha_C\right)\phi + \alpha_C \sigma_C\right]}. \tag{221}$$

As will become clear later, it is convenient to first solve the steady-state without the intertemporal solvency condition (219f), i.e., to solve the system comprising (219a)-(219e), which allows us to express the steady-state values in terms of the stock of traded bonds, the marginal utility of wealth and labor productivity indices A^j (with $j = T, N$). Totally differentiating the system of equations (219a)-(219e), using both (220) and (221), yields in matrix form:

$$\begin{pmatrix} 1 & -\left[\alpha_V^T \tilde{u}^T + \sigma_L^T \tilde{\chi}^T\right] & 0 & 0 & 0 \\ 0 & \left[\left(1-\alpha_V^T\right)\tilde{\Psi}^T + \tilde{\chi}^T W_R^T\right] & 0 & 0 & 0 \\ 0 & 0 & 1 & -\left[\alpha_V^N \tilde{u}^N + \sigma_L^N \tilde{\chi}^N\right] & 0 \\ 0 & 0 & \tilde{P}A^N & a_{44} & 0 \\ a_{51} & -\omega_V^T\left(1-\alpha_V^T\right) & a_{53} & -\omega_V^N\left(1-\alpha_V^N\right) & \end{pmatrix} \begin{pmatrix} \hat{\tilde{L}}^T \\ \hat{\tilde{\theta}}^T \\ \hat{\tilde{L}}^N \\ \hat{\tilde{\theta}}^N \\ \mathrm{d}\tilde{B}/\tilde{Y} \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_L^T \hat{\bar{\lambda}} \\ A^T \hat{a}^T \\ \sigma_L^N \hat{\bar{\lambda}} \\ -\tilde{P}A^N \sigma_C \hat{\bar{\lambda}} + \tilde{P}A^N \left\{\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] - 1\right\}\hat{a}^N \\ -\left(1-\omega_N\right)\hat{a}^T - \frac{\left(1-\alpha_C\right)\omega_C\alpha_C\left(\phi-\sigma_C\right)}{\left[\left(1-\alpha_C\right)\phi+\alpha_C\sigma_C\right]}\hat{a}^N - \frac{\left(1-\alpha_C\right)\omega_C\sigma_C\phi}{\left[\left(1-\alpha_C\right)\phi+\alpha_C\sigma_C\right]}\hat{\bar{\lambda}} \end{pmatrix}, \tag{222}$$

where we used the fact that $\frac{C^T}{Y} = \frac{C^T}{P_C C}\frac{P_C C}{Y} = \left(1-\alpha_C\right)\omega_C$, and $\frac{Y^T}{Y} = \left(1-\omega_N\right)$, we set $\omega_V^j = \frac{\kappa V}{Y}$; the terms a_{44}, a_{51}, a_{53} are given by:

$$a_{44} = \left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N \tilde{W}_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right], \tag{223a}$$

$$a_{51} = \left[\left(1-\omega_N\right) - \omega_V^T\right], \tag{223b}$$

$$a_{53} = \left\{\frac{\left(1-\alpha_C\right)\omega_C\alpha_C\left(\phi-\sigma_C\right)}{\left[\left(1-\alpha_C\right)\phi+\alpha_C\sigma_C\right]} - \omega_V^N\right\}. \tag{223c}$$

System (219a)-(219e) can be solved for steady-state employment and labor market tightness in the traded and non traded sectors, and the stock of foreign assets as follows:

$$\tilde{L}^T = L^T\left(\bar{\lambda}, A^T\right), \tag{224a}$$

$$\tilde{\theta}^T = \theta^T\left(A^T\right), \tag{224b}$$

$$\tilde{L}^N = L^N\left(\bar{\lambda}, A^N\right), \tag{224c}$$

$$\tilde{\theta}^N = \theta^N\left(\bar{\lambda}, A^N\right), \tag{224d}$$

$$\tilde{B} = B\left(\bar{\lambda}, A^T, A^N\right), \tag{224e}$$

where partial derivatives are given by

$$\frac{\hat{\tilde{\theta}}^T}{\hat{a}^T} = \frac{A^T}{\left[\left(1-\alpha_V^T\right)\tilde{\Psi}^T + \tilde{\chi}^T W_R^T\right]} > 0, \tag{225a}$$

$$\frac{\hat{\tilde{L}}^T}{\hat{\bar{\lambda}}} = \sigma_L^T > 0, \tag{225b}$$

$$\frac{\hat{\tilde{L}}^T}{\hat{a}^T} = \frac{\left[\alpha_V^T \tilde{u}^T + \sigma_L^T \tilde{\chi}^T\right] A^T}{\left[\left(1-\alpha_V^T\right)\tilde{\Psi}^T + \tilde{\chi}^T W_R^T\right]} > 0, \tag{225c}$$

$$\frac{\hat{\tilde{\theta}}^N}{\hat{\bar{\lambda}}} = -\frac{\tilde{P}A^N\left(\sigma_L^N + \sigma_C\right)}{\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] + \tilde{P}A^N\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]} < 0, \tag{225d}$$

$$\frac{\hat{\tilde{\theta}}^N}{\hat{a}^N} = \frac{\tilde{P}A^N\left\{\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] - 1\right\}}{\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] + \tilde{P}A^N\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]} > 0, \tag{225e}$$

$$\frac{\hat{\tilde{L}}^N}{\hat{\bar{\lambda}}} = \frac{\sigma_L^N\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] - \sigma_C\tilde{P}A^N\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]}{\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] + \tilde{P}A^N\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]} \gtrless 0, \tag{225f}$$

$$\frac{\hat{\tilde{L}}^N}{\hat{a}^N} = \frac{\tilde{P}A^N\left\{\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] - 1\right\}\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]}{\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] + \tilde{P}A^N\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]} > 0, \tag{225g}$$

$$\frac{\mathrm{d}\tilde{B}/\tilde{Y}}{\hat{a}^T} = -\left\{\left(1-\omega_N\right) + \frac{\left\{\left[\left(1-\omega_N\right) - \omega_V^T\right]\left[\alpha_V^T\tilde{u}^T + \sigma_L^T\tilde{\chi}^T\right] - \omega_V^T\left(1-\alpha_V^T\right)\right\}A^T}{\left[\left(1-\alpha_V^T\right)\tilde{\Psi}^T + \tilde{\chi}^T W_R^T\right]}\right\} < 0, \tag{225h}$$

$$\begin{aligned}\frac{\mathrm{d}\tilde{B}/\tilde{Y}}{\hat{a}^N} &= -\left\{\frac{\tilde{P}A^N\left\{\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] - 1\right\}\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]\left[\frac{(1-\alpha_C)\omega_C\alpha_C(\phi-\sigma_C)}{(1-\alpha_C)\phi+\alpha_C\sigma_C} - \omega_V^N\right]}{\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] + \tilde{P}A^N\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]}\right. \\ &\quad + \left.\frac{\left(1-\alpha_C\right)\omega_C\alpha_C\left(\phi-\sigma_C\right)}{\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right]}\right\} < 0,\end{aligned} \tag{225i}$$

$$\begin{aligned}\frac{\mathrm{d}\tilde{B}/\tilde{Y}}{\hat{\bar{\lambda}}} &= -\left\{\left[\left(1-\omega_N\right) - \omega_V^T\right]\sigma_L^T + \frac{\left(1-\alpha_C\right)\omega_C\sigma_C\phi}{\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right]}\right. \\ &\quad + \frac{\sigma_L^N\left[\frac{(1-\alpha_C)\omega_C\alpha_C(\phi-\sigma_C)}{(1-\alpha_C)\phi+\alpha_C\sigma_C} - \omega_V^N\right]\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right]}{\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] + \tilde{P}A^N\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]} \\ &\quad - \left.\frac{\tilde{P}A^N\sigma_C\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]\left[\frac{(1-\alpha_C)\omega_C\alpha_C(\phi-\sigma_C)}{(1-\alpha_C)\phi+\alpha_C\sigma_C} - \omega_V^N\right]}{\left[\left(1-\alpha_V^N\right)\tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right]\left[\left(1-\alpha_C\right)\phi + \alpha_C\sigma_C\right] + \tilde{P}A^N\left[\alpha_V^N\tilde{u}^N + \sigma_L^N\tilde{\chi}^N\right]}\right\} \lesseqgtr 0.\end{aligned} \tag{225j}$$

H.3 The Dynamic Adjustment

The tilde is suppressed below for the purposes of clarity. We now explore effects of higher productivity in tradables relative to non tradables by focusing on the labor market. Figure 8(a) depicts the labor market equilibrium in the traded sector which can be summarized by two schedules:[61]

$$L^T = \frac{m^T}{m^T + s^T}\left(\bar{\lambda}W_R^T\right)^{\sigma_L^T}, \tag{226a}$$

$$\frac{\kappa^T}{f^T} = \frac{\left(1-\alpha_W^T\right)\left[\left(A^T + r^\star x^T\right) - W_R^T\right]}{s^T + r^\star}, \tag{226b}$$

where $W_R^T \equiv \left(\frac{\alpha_W^T}{1-\alpha_W^T}\kappa^T\theta^T + R^T\right)$ is the reservation wage in the traded sector. The first equation (226a) represents the decision of search schedule in the traded sector (henceforth DST) which

[61] Totally differentiating the DST- and VCT-schedule yields:

$$\hat{l}^T = \sigma_L^T\hat{\bar{\lambda}} + \left[\alpha_V^T u^T + \sigma_L^T\chi^T\right]\hat{\theta}^T, \quad \hat{\theta}^T = \frac{A^T}{\left[\left(1-\alpha_V^T\right)\tilde{\Psi}^T + \tilde{\chi}^T W_R^T\right]}\hat{a}^T.$$

The slope of the DST-schedule in the (θ^T, L^T)-space is given by $\left[\alpha_V^T u^T + \sigma_L^T\chi^T\right] > 0$.

is upward-sloping in the (θ^T, L^T)-space. The reason is that a rise in the labor market tightness raises the probability of finding a job and thus increases employment L^T by reducing the number of job seekers. Moreover, because we consider an endogenous labor force participation decision, the consecutive increase in the reservation wage induces agents to supply more labor. The second equation (226b) represents the vacancy creation schedule (henceforth VCT) which is a vertical line in the (θ^T, L)-space. Note that Figure 8(a) depicts the logarithm form of the system (226).

By raising the surplus from hiring, a rise in labor productivity in the traded sector A^T shifts to the right the VCT-schedule from VCT_0 to VCT_1. Because traded firms post more job vacancies, the labor market tightness θ_1^T exceeds its initial level θ_0^T. Note that θ^T jumps immediately to its new higher steady-state level while traded employment builds up over time along the isocline $\dot{\theta}^T = 0$ until the economy reaches the new steady-state. While increased labor market tightness raises traded employment by pushing up the reservation wage and reducing unemployment, the positive wealth effect moderates the expansionary effect on labor supply. Graphically, the fall in $\bar{\lambda}$ shifts to the right the DST-schedule. The new steady state is E_1^T.

Since we are interested in the movement of sectoral wages, it is useful to explore the long-run adjustment in the traded wage following a rise in labor productivity A^T. The labor market in the traded sector can alternatively be summarized graphically in the (θ^T, W^T)-space as shown in Figure 9(a). Using the fact $\left(1-\alpha_W^T\right)\Psi^T = A^T - W^T$, the VCT-schedule is downward sloping and convex toward the origin, reflecting diminishing returns in vacancy creation. The slope of the VCT-schedule in the (θ^T, W^T)-space is:

$$\left.\frac{\mathrm{d}W^T}{\mathrm{d}\theta^T}\right|^{VCT} = -\frac{\left(s^T + r^\star\right)\kappa^T\left(1-\alpha_V^T\right)}{f^T\theta^T} = -\frac{\left(1-\alpha_W^T\right)\Psi^T\left(1-\alpha_V^T\right)}{\theta^T} < 0. \tag{227}$$

The wage setting-schedule (WST henceforth) is upward sloping (see eq. (22)). Using the fact that $\left(F^T\right)^{1/\sigma_L^T}/\bar{\lambda} = W_R^T$, the WST-schedule is $W^T = \alpha_W^T\left(A^T + r^\star x^T\right) + \left(1-\alpha_W^T\right)W_R^T$ with a slope in the (θ^T, W^T)-space given by:

$$\left.\frac{\mathrm{d}W^T}{\mathrm{d}\theta^T}\right|^{WST} = \frac{\left(1-\alpha_W^T\right)\chi^T W_R^T}{\theta^T} = \alpha_W^T\kappa^T > 0. \tag{228}$$

A rise in A^T shifts to the right the VCT-schedule by stimulating labor demand which exerts an upward pressure on the the traded wage. Because workers get a fraction α_W^T of the increased surplus, the productivity shock shifts to the left the WST-schedule.[62] Hence, the new steady-state at F_1^T is associated with a higher traded wage. The higher the worker bargaining power, the larger the shift of the WST curve and thereby the more W^T increases. To see it formally, totally differentiating the Nash bargaining traded wage and eliminating θ^T by using the vacancy creation schedule (i.e., eq. (226b)) yields the deviation in percentage of the traded wage from its initial steady state:[63]

$$\hat{w}^T = \Omega^T\hat{a}^T > 0, \quad \Omega^T = \frac{\alpha_W^T\left[\left(1-\alpha_V^T\right)\left(s^T + r^\star\right) + m^T\right]}{\left[\left(1-\alpha_V^T\right)\left(s^T + r^\star\right) + \alpha_W^T m^T\right]}\frac{A^T}{W^T}, \tag{229}$$

where $\Omega^T > 0$ represents the sensitivity of the traded wage to a change in the labor productivity index A^T.

We now turn to the non traded labor market equilibrium depicted in Figure 8(b) which is summarized by two schedules:

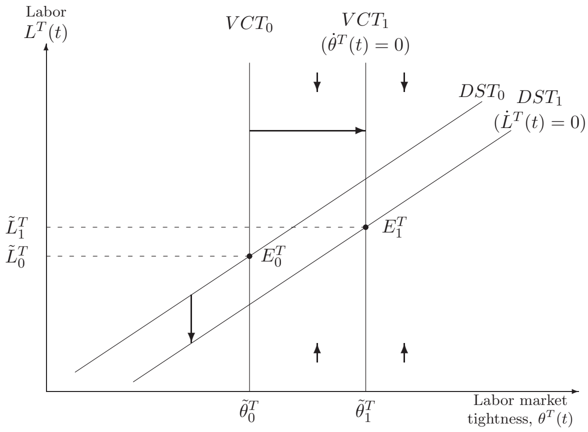
$$L^N = \frac{m^N}{m^N + s^N}\left(\bar{\lambda}W_R^N\right)^{\sigma_L^N}, \tag{230a}$$

$$\frac{\kappa^N}{f^N} = \frac{\left(1-\alpha_W^N\right)\left[\left(P\left(L^N, \bar{\lambda}, A^N\right)A^N + r^\star x^N\right) - W_R^N\right]}{s^N + r^\star}, \tag{230b}$$

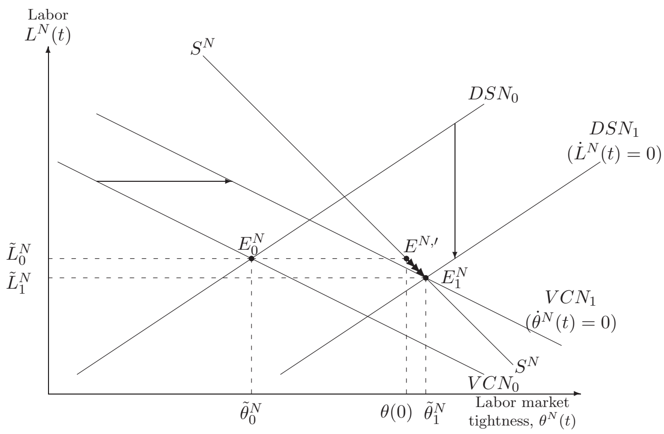
where we have inserted the short-run static solution for the relative price of non tradables (108) and $W_R^N \equiv \left(\frac{\alpha_W}{1-\alpha_W}\kappa^N\theta^N + R^N\right)$ is the reservation wage in the non traded sector. While eq. (230a) represents the decision of search schedule (henceforth DSN) which is upward-sloping in the (θ^N, L^N)-space, eq. (230b) corresponds to the vacancy creation schedule in the non traded sector (henceforth VCN). Note that whether we consider the traded or the non traded sector, the same logic applies to explain the positive relationship between the employment and the labor market tightness along the DSj-schedule (with $j = T, N$).

[62] Note that the shift in the VCT-schedule dominates the shift in the WST-schedule because workers and firms have to share the surplus, i.e., $0 < \alpha_W^T < 1$.

[63] To get (229), we used the fact that $\chi^T W_R^T = m^T\frac{\alpha_W^T\Psi^T}{s^T + r^\star}$.

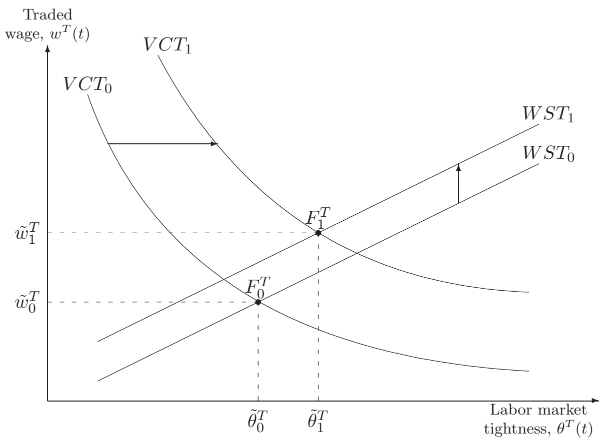


(a) (θ^T, L^T)-space

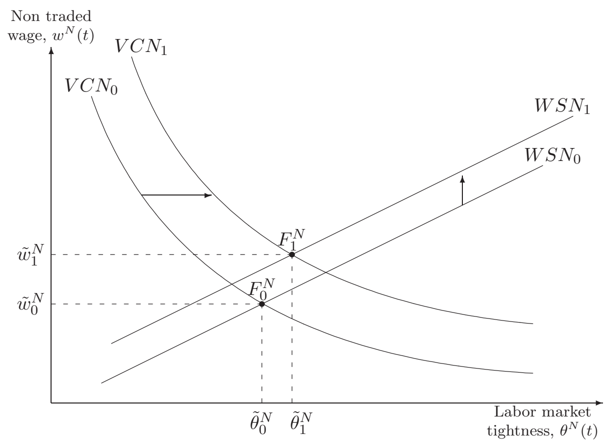


(b) (θ^N, L^N)-space

Figure 8: Effects of a Productivity Differential and the Stable Adjustment



(a) (θ^T, W^T)-space



(b) (θ^N, W^N)-space

Figure 9: Long-Run Sectoral Wage Effects of a Productivity Differential

Totally differentiating eq. (230a) gives the slope of the DSN-schedule:

$$\hat{l}^N = \sigma_L^N \hat{\bar{\lambda}} + \left[\alpha_V^N u^N + \sigma_L^N \chi^N\right] \hat{\theta}^N.$$

The slope of the DSN-schedule in the (θ^N, L^N)-space is given by $\left[\alpha_V^N u^N + \sigma_L^N \chi^N\right] > 0$. Totally differentiating (230b) gives the slope of the VCN-schedule:

$$\begin{aligned} & \hat{\theta}^N \left[\left(1-\alpha_V^N\right) \tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right] \left[\left(1-\alpha_C\right)\phi + \alpha_C \sigma_C\right] \\ = \; & -PA^N \left\{\hat{l}^N + \sigma_C \hat{\bar{\lambda}} + \left\{1 - \left[\left(1-\alpha_C\right)\phi + \alpha_C \sigma_C\right]\right\} \hat{a}^N\right\}. \end{aligned}$$

The slope of the VCN-schedule is negative and given by:

$$\left.\frac{\hat{L}^N}{\hat{\theta}^N}\right|^{VCN} = -\frac{PA^N}{\left[\left(1-\alpha_V^N\right) \tilde{\Psi}^N + \tilde{\chi}^N W_R^N\right] \left[\left(1-\alpha_C\right)\phi + \alpha_C \sigma_C\right]} < 0.$$

As depicted in Figure 8(b), the VCN-schedule is downward-sloping in the (θ^N, L^N)-space. The reason is as follows. Because an increase in non traded labor raises output of this sector, the relative price of non tradables must depreciate for the market clearing condition (24) to hold. The fall in P drives down the surplus from hiring an additional worker in the non traded sector which results in a decline in labor market tightness θ^N as firms post less job vacancies.

Imposing $\sigma_C = 1$, a rise in A^N raises the surplus from hiring if and only if the elasticity of substitution ϕ between traded and non traded goods is larger than one. The reason is that only in this case, the share of non tradables in total expenditure rises which results in an expansionary effect on labor demand in the non traded sector. In Figure 8(b), we assume that $\sigma_C = \phi = 1$, so that the productivity shock does not impinge on the vacancy creation decision because the share of non tradables remains unchanged. Yet, by producing a positive wealth effect, higher labor productivity of non tradables shifts the VCN-schedule to the right by inducing agents to consume more which in turn raises P and thereby the surplus from hiring. The fall of the shadow value of wealth also shifts the DSN-schedule to the right as agents are induced to supply less labor. While θ^N is unambiguously higher at the new steady-state E_1^N, the positive wealth effect exerts two conflicting effects on L^N. In Figure 8(b), non traded employment falls in line with our numerical results.[64]

We now explore the long-run adjustment in the non traded wage which is depicted in Figure 9(b). As for the traded sector, the WSN-schedule is upward sloping while the VCN-schedule is downward sloping. Formally, using the fact $\left(1-\alpha_W^N\right)\Psi^N = PA^N + r^\star x^N - W^N$, the wage setting and vacancy creation decisions are described by the following equalities:

$$W^N = \alpha_W^N \left(PA^N + r^\star x^N\right) + \left(1-\alpha_W^N\right) W_R^N, \tag{231a}$$

$$W^N = \left(PA^N + r^\star x^N\right) - \frac{\kappa^N \left(s^N + r^\star\right)}{f^N}. \tag{231b}$$

Before analyzing in more details the effects of a productivity shock on the non traded wage, it is convenient to determine analytically the long-run response of W^N. Totally differentiating the wage setting decision in the non traded sector allows us to solve for the change in the labor market tightness $\hat{\theta}^N = \frac{PA^N\left(\hat{p}+\hat{a}^N\right)}{\left[\left(1-\alpha_V^N\right)\Psi^N + \chi^N W_R^N\right]}$. Totally differentiating the Nash bargaining non traded wage and plugging $\hat{\theta}^N$, yields the deviation in percentage of the non traded wage from its initial steady state:

$$\hat{w}^N = \Omega^N \left(\hat{p} + \hat{a}^N\right), \quad \Omega^N = \frac{\alpha_W^N \left[\left(1-\alpha_V^N\right)\left(s^N + r^\star\right) + m^N\right]}{\left[\left(1-\alpha_V^N\right)\left(s^N + r^\star\right) + \alpha_W^N m^N\right]} \frac{PA^N}{W^N}. \tag{232}$$

According to (232), the combined effects of higher labor productivity A^N and the appreciation of the relative price of non tradables pushes up the non traded wage in the long-run. Inserting the long-run change in the equilibrium value of the relative price of non tradables, i.e., $\hat{p} = \frac{(1+\Theta)\left(\hat{a}^T - \hat{a}^N\right)}{(\phi+\Theta)} - \frac{\mathrm{d}\upsilon_{NX}}{(\phi+\Theta)}$ (see eq. (31)), and using the fact that $\chi^N W_R^N = m^N \frac{\alpha_W^N \Psi^N}{s^N + r^\star}$ allows us to rewrite (232) as follows:

$$\hat{w}^N = \frac{\Omega^N}{\left(\phi + \Theta^N\right)} \left[\hat{a}^T \left(1 + \Theta^T\right) + \hat{a}^N (\phi - 1) - \mathrm{d}\upsilon_{NX}\right]. \tag{233}$$

Imposing the elasticity of substitution ϕ to be equal to one, labor productivity in the non traded sector does no longer impinge on W^N. In this case, the change in the non traded wage is only driven by $\hat{a}^T > 0$ which appreciates the relative price of non traded goods and thereby stimulates

[64] In all scenarios, we numerically find that L^N declines.

labor demand in that sector. Further assuming that labor market parameters are similar across sectors so that $\Theta^j \simeq \Theta$ and $\Omega^j \simeq \Omega$ (with $j = T, N$), we find that the non traded wage is equal to $\Omega \left[\hat{a}^T - \frac{\mathrm{d}\upsilon_{NX}}{(1+\Theta)}\right]$. By producing a long-run improvement in the trade balance NX and thereby stimulating the demand for tradables, a productivity shock exerts a negative impact on the relative wage W^N/W^T. As depicted in Figure 9(b), due to the *labor accumulation effect*, a productivity shock biased toward the traded sector induces smaller shifts in the VCN- and the WSN-schedule. Note that, as for the traded labor market, the shift in the VCN-schedule dominates the shift in the WSN-schedule because the worker bargaining power α_W^N is smaller than one.

I Solving Graphically for the Steady-State: Graphical Apparatus

The steady-state can be described by considering alternatively the goods market or the labor market. Due to the lack of empirical estimates at a sectoral level, and to avoid unnecessary complications, we impose $\alpha_V^j = \alpha_V$, $\alpha_W^j = \alpha_W$ from now on.

I.1 The Goods Market: Graphical Apparatus

To build intuition about steady-state changes, we investigate graphically the long-run effects of a rise in A^T/A^N. To do so, it is convenient to rewrite the steady-state (219) as follows:

$$\frac{C^T}{C^N} = \frac{\varphi}{1-\varphi}\tilde{P}^{\phi}, \tag{234a}$$

$$\frac{L^T}{L^N} = \frac{m^T}{m^N}\frac{\left(s^N + m^N\right)}{\left(s^T + m^T\right)}\frac{\left[\bar{\lambda}W_R^T/\zeta^T\right]^{\sigma_L^T}}{\left[\bar{\lambda}W_R^N/\zeta^N\right]^{\sigma_L^N}}, \tag{234b}$$

$$\frac{\kappa^T}{f^T\left(\theta^T\right)} = \frac{\left(1-\alpha_W\right)\Psi^T}{\left(s^T + r^\star\right)}, \tag{234c}$$

$$\frac{\kappa^N}{f^N\left(\theta^N\right)} = \frac{\left(1-\alpha_W\right)\Psi^N}{\left(s^N + r^\star\right)}, \tag{234d}$$

$$\frac{Y^T\left(1 + \upsilon_B - \upsilon_V^T - \upsilon_V^N\right)}{Y^N} = \frac{C^T}{C^N}. \tag{234e}$$

We denote by $\upsilon_B \equiv \frac{r^\star B}{Y^T}$ the ratio of interest receipts to traded output, by $\upsilon_V^j \equiv \frac{\kappa^j V^j}{Y^T}$ the share of hiring cost in sector $j = T, N$ in traded output. Remembering that $Y^T = A^T L^T$ and $Y^N = A^N L^N$, the system (234) can be solved for C^T/C^N, L^T/L^N, θ^T, θ^N, and P, as functions of $A^T, A^N, \left(1 + \upsilon_B - \upsilon_V^T - \upsilon_V^N\right)$. Inserting these functions into $Y^N = C^N$ (see eq. (167h)), and $B - B_0 = \Phi^T\left(L^T - L_0^T\right) + \Phi^N\left(L^N - L_0^N\right)$ (see eq. (167j)), the system can be solved for B and $\bar{\lambda}$ as functions of A^T and A^N. Hence, when solving the system (234), we assume that the stock of foreign bonds and the marginal utility of wealth are exogenous which allows us to separate intratemporal reallocation effects triggered by the change in the share of tradables from the dynamic (or intertemporal) effects stemming from the accelerated hiring process that increases the demand for tradables in the long-run.

When focusing on the goods market, the equilibrium can be characterized by two schedules in the $(y^T - y^N, p)$-space where we denote the logarithm in lower case. The steady state is summarized graphically in Figure 10(b).

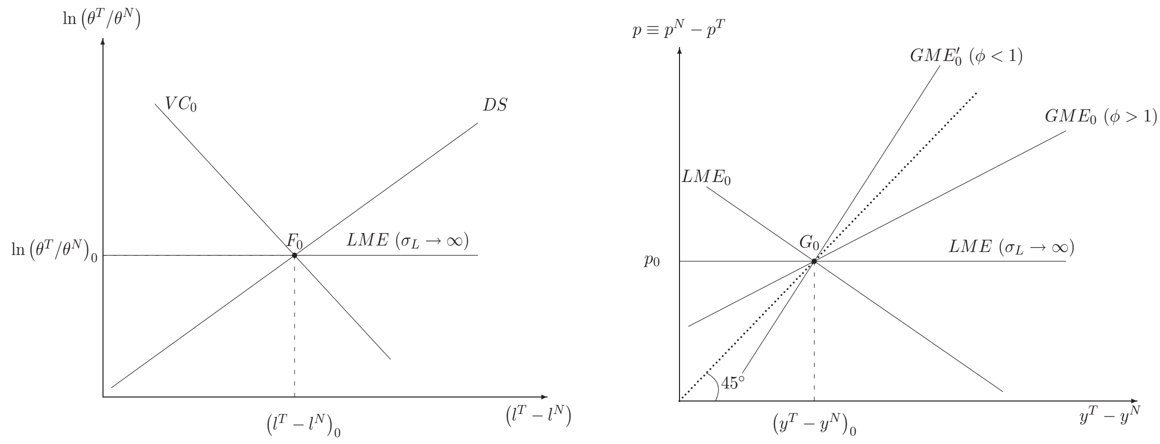
Denoting by $\upsilon_{NX} \equiv NX/Y^T$ the ratio of net exports to traded output, with $\upsilon_{NX} \equiv -\left(\upsilon_B - \upsilon_V^T - \upsilon_V^N\right)$, and inserting (234a) into the market clearing condition (234e) leads to

$$\frac{Y^T}{Y^N} = \frac{\varphi}{1-\varphi}\frac{1}{\left(1-\upsilon_{NX}\right)}P^{\phi}. \tag{235}$$

Eq. (235) corresponds to eq. (26) in the text. Totally differentiating (235) and denoting the percentage deviation from its initial steady-state by a hat yields the goods market equilibrium-schedule (GME henceforth):

$$\left(\hat{y}^T - \hat{y}^N\right)\Big|^{GME} = \phi\hat{p} - d\ln\left(1-\upsilon_{NX}\right). \tag{236}$$

According to (236), the GME-schedule is upward-sloping in the $(y^T - y^N, p)$-space with a slope equal to $1/\phi$. Following a rise in traded output relative to non traded output, the relative price of non tradables must appreciate to clear the goods market, and all the more so as the elasticity of



(a) $(l^T - l^N, \ln\left(\theta^T/\theta^N\right))$-space

(b) $(y^T - y^N, p)$-space

Figure 10: Steady-State

substitution ϕ is smaller. The 45° dotted line allows us to consider two cases. When $\phi > 1$ ($\phi < 1$), the GME-schedule is flatter (steeper) than the 45° dotted line.

We now characterize the labor market equilibrium. Totally differentiating (214) gives the deviation in percentage of the sectoral labor market tightness from its initial steady-state, i.e., $\hat{\theta}^j = \frac{\Xi^j}{\left[\left(1-\alpha_V^j\right)\Psi^j + \chi^j W_R^j\right]} \hat{\Xi}^j$. Totally differentiating (215) gives the deviation in percentage of sectoral labor from its initial steady-state, i.e., $\hat{l}^j = \left[\alpha_V^j u^j + \sigma_L \chi^j\right] \hat{\theta}^j$. Substituting the former into the latter, differentiating the production function $Y^j = A^j L^j$ to eliminate $\hat{l}^j$, and using the fact that $\chi^j W_R^j = \frac{\alpha_W^j \Psi^j}{s^j + r^\star}$ at the steady-state, one obtains the labor market equilibrium (LME henceforth) schedule:

$$\left(\hat{y}^T - \hat{y}^N\right)\Big|^{LME} = -\Theta^N \hat{p} + \left(1 + \Theta^T\right)\hat{a}^T - \left(1 + \Theta^N\right)\hat{a}^N, \tag{237}$$

where we set

$$\Theta^j \equiv \frac{\Xi^j \left(s^j + r^\star\right)\left[\alpha_V u^j + \sigma_L \chi^j\right]}{\Psi^j \left[\left(1 - \alpha_V\right)\left(s^j + r^\star\right) + \alpha_W^j m^j\right]}, \tag{238}$$

in order to write formal solutions in a compact form. As depicted in Figure 10(b), the LME-schedule is downward-sloping in the $(y^T - y^N, p)$-space with a slope equal to $-1/\Theta^N$ (see eq. (237)). An appreciation in the relative price of non tradables raises the surplus from hiring which induces non traded firms to post more job vacancies. By raising the expected value of a job, the consecutive rise in the labor market tightness induces agents to increase the search intensity for a job in the non traded sector but less so as the elasticity of labor supply σ_L is lower. More precisely, lower values of σ_L indicate that workers experience a larger switching cost from one sector to another; in this configuration, the term Θ^j is smaller so that the LME-schedule is steeper. Conversely, when we let σ_L tend toward infinity, the case of perfect mobility of labor across sectors is obtained; in this configuration, the LME-schedule becomes a horizontal line.

I.2 The Labor Market: Graphical Apparatus

When focusing on the labor market, the model can be summarized graphically by two schedules in the $(l^T - l^N, \ln\left(\frac{\theta^T}{\theta^N}\right))$-space, as shown in Figure 10(a).

As will be useful later, we first solve for the relative price of non tradables by using the goods market clearing condition (235). Using production functions, i.e., $Y^j = A^j L^j$, solving (235) for the relative price yields:

$$P = \left[\left(\frac{1-\varphi}{1-\varphi}\right)\left(1 - \upsilon_{NX}\right)\left(\frac{A^T}{A^N}\right)\left(\frac{L^T}{L^N}\right)\right]^{\frac{1}{\phi}}. \tag{239}$$

Applying the implicit function theorem, we have:

$$P = P\left[\left(\frac{L^T}{L^N}\right), \left(1 - \upsilon_{NX}\right), \left(\frac{A^T}{A^N}\right)\right], \tag{240}$$

where

$$\hat{p} = \frac{1}{\phi}\left[d\ln\left(\frac{L^T}{L^N}\right) + d\ln\left(\frac{A^T}{A^N}\right) + d\ln\left(1 - \upsilon_{NX}\right)\right]. \tag{241}$$

I.2.1 The Decision of Search Schedule in the $(l^T - l^N, \ln\left(\frac{\theta^T}{\theta^N}\right))$-space

Imposing $\sigma_L^j = \sigma_L$ into (234b), which implies that the marginal utility of wealth does not impinge relative labor supply, the decision of search equation reduces to:

$$\frac{L^T}{L^N} = \frac{m^T}{m^N}\frac{m^N + s^N}{m^T + s^T}\left(\frac{W_R^T}{W_R^N}\frac{\zeta^N}{\zeta^T}\right)^{\sigma_L}, \tag{242}$$

where $W_R^j \equiv \frac{\alpha_W^j}{1-\alpha_W^j}\kappa^j\tilde{\theta}^j + R^j$ is the reservation wage. **Eq. (242) corresponds to eq. (28) in the text**. Taking logarithm and differentiating eq. (239) yields:

$$\hat{l}^T - \hat{l}^N = \left[\alpha_V u^T + \sigma_L\chi^T\right]\hat{\theta}^T - \left[\alpha_V u^N + \sigma_L\chi^N\right]\hat{\theta}^N, \tag{243}$$

where we used the fact that $d\ln\left(\frac{m^j}{m^j+s^j}\right) = \alpha_V u^j\hat{\theta}^j$ and $\hat{w}_R^j = \chi^j\hat{\theta}^j$ with $\chi^j = \frac{\frac{\alpha_W}{1-\alpha_W}\kappa^j\theta^j}{W_R^j}$. Assuming that the labor markets display initially similar features across sectors, i.e., $u^j \simeq u$, $\chi^j \simeq \chi$, eq. (243) reduces to:

$$\left(\hat{\theta}^T - \hat{\theta}^N\right)\Big|^{DS} = \frac{1}{[\alpha_V u + \sigma_L\chi]}\left(\hat{l}^T - \hat{l}^N\right). \tag{244}$$

Inspection of (244) reveals that the DS-schedule:

- is upward-sloping in the $(l^T - l^N, \ln\left(\frac{\theta^T}{\theta^N}\right))$-space;
- is steeper as the workers are more reluctant to shift hours worked across sectors (i.e., the elasticity of labor supply σ_L is smaller), the unemployment benefit scheme is more generous or the worker bargaining power α_W is lower (because higher unemployment benefits R or a lower worker bargaining power both reduce the share of the surplus associated with a labor contract in the marginal benefit of search χ).

I.2.2 The Vacancy-Creation Schedule in the $(l^T - l^N, \ln\left(\frac{\theta^T}{\theta^N}\right))$-space

Dividing (234c) by (234d) and using (98) leads to the vacancy creation equation:

$$\frac{\kappa^T}{\kappa^N}\frac{\left(s^T + r^\star\right)}{\left(s^N + r^\star\right)}\frac{X^N}{X^T}\left(\frac{\theta^T}{\theta^N}\right)^{1-\alpha_V} = \frac{\Xi^T + r^\star x^T - W_R^T}{\Xi^N + r^\star x^N - W_R^N}, \tag{245}$$

where $\frac{\Xi^T + r^\star x^T - W_R^T}{\Xi^N + r^\star x^N - W_R^N} = \frac{\Psi^T}{\Psi^N}$. **Eq. (245) corresponds to eq. (27) in the text**. Totally differentiating (245) by sing the fact that the change in overall surplus Ψ^j in percentage is given by

$$\hat{\Psi}^j = \frac{\Xi^j\hat{\Xi}^j - \chi^j W_R^j\hat{\theta}^j}{\Psi^j}, \tag{246}$$

yields:

$$\left(\hat{\theta}^T - \hat{\theta}^N\right)\Big|^{VC} = \frac{\Xi^T\hat{a}^T}{\left[(1-\alpha_V)\tilde{\Psi}^T + \chi^T W_R^T\right]} - \frac{\Xi^N\left(\hat{p} + \hat{a}^N\right)}{\left[(1-\alpha_V)\tilde{\Psi}^N + \chi^N W_R^N\right]}. \tag{247}$$

Eliminating the relative price by using (241), collecting terms, assuming that initially $\Xi^j \simeq \Xi$, $\Psi^j \simeq \Psi$, $W_R^j \simeq W_R$, $\chi^j \simeq \chi$, eq. (247) can be rewritten as follows:

$$\begin{aligned}\left(\hat{\theta}^T - \hat{\theta}^N\right)\Big|^{VC} &= -\frac{\Xi}{\phi\left[(1-\alpha_V)\Psi + \chi W_R\right]}\left(\hat{l}^T - \hat{l}^N\right) \\ &+ \frac{\Xi\left[(\phi-1)\left(\hat{a}^T - \hat{a}^N\right) - d\ln\left(1-\upsilon_{NX}\right)\right]}{\phi\left[(1-\alpha_V)\Psi + \chi W_R\right]}.\end{aligned} \tag{248}$$

Inspection of (248) reveals that the VC-schedule:

- is downward-sloping in the $(l^T - l^N, \ln\left(\frac{\theta^T}{\theta^N}\right))$-space with a slope equal to $-\frac{\Xi}{\phi[(1-\alpha_V)\Psi+\chi W_R]}$;
- is steeper as the elasticity of substitution between traded and non traded goods ϕ is smaller or the worker bargaining power is lower (because it reduces χW_R);
- shifts to the right following higher productivity of tradables relative to non tradables (i.e., $\left(\hat{a}^T - \hat{a}^N\right) > 0$) as long as $\phi > 1$ or when the country experiences a higher steady-state trade balance surplus, i.e., if $-d\ln\left(1-\upsilon_{NX}\right) \simeq d\upsilon_{NX} > 0$;

J Long-Run Relative Price and Relative Wage Effects of Higher Relative Productivity of Tradables

This section analyzes analytically the consequences on the relative wage and the relative price of an increase in relative sectoral productivity A^T/A^N. It compares the steady-state of the model before and after the productivity shock biased towards the traded sector. To shed some light on the transmission mechanism, we analytically break down the relative wage and relative price effects in two components: a labor market frictions effect and a labor accumulation effect.

Equating demand for tradables in terms of non tradables given by eq. (236) and supply (237) yields

$$\begin{aligned}\left(\hat{y}^T - \hat{y}^N\right) &= \phi\hat{p} - d\ln\left(1-\upsilon_{NX}\right),\\ &= -\Theta^N\hat{p} + \left(1+\Theta^T\right)\hat{a}^T - \left(1+\Theta^N\right)\hat{a}^N.\end{aligned}$$

Collecting terms leads to the deviation in percentage of the relative price from its initial steady-state:

$$\hat{p} = \frac{\left(1+\Theta^T\right)\hat{a}^T - \left(1+\Theta^N\right)\hat{a}^N}{\left(\phi+\Theta^N\right)} + \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta^N\right)}. \tag{249}$$

Eq. (249) corresponds to eq. (29) in the text. It is worthwhile noticing that $\hat{p}$ given by eq. (249) is determined by the system which comprises the goods market equilibrium (235), the decision of search equation (242), and the vacancy creation equation (245). This implies that $P = P\left(A^T, A^N, \upsilon_{NX}\right)$. Invoking the intertemporal solvency condition (149) allows us to solve for $\upsilon_{NX} = \upsilon_{NX}\left(A^T, A^N\right)$.

To determine the long-run adjustment in the relative wage, $\Omega \equiv W^N/W^T$, we first derive the deviation in percentage of the sectoral wage. To do so, we totally differentiate the vacancy creation equation for sector j given by eq. (214):

$$\hat{\theta}^j = \frac{\Xi^j}{\left[\left(1-\alpha_V\right)\Psi^j + \chi^j W_R^j\right]}\hat{\Xi}^j. \tag{250}$$

We repeat the Nash bargaining wage given by eq. (22) for convenience by imposing $\alpha_W^j = \alpha_W$:

$$W^j = \alpha_W\left(\Xi^j + r^\star x^j\right) + \left(1-\alpha_W\right)W_R^j. \tag{251}$$

Totally differentiating (251) and plugging the change in the labor market tightness leads to:

$$\begin{aligned}\hat{w}^j &= \frac{\alpha_W\Xi^j}{W^j}\hat{\Xi}^j + \frac{\left(1-\alpha_W\right)\chi^j W_R^j}{W^j}\hat{\theta}^j,\\ &= \frac{\Xi^j}{W^j}\frac{\left[\alpha_W\left(1-\alpha_V\right)\Psi^j + \chi^j W_R^j\right]}{\left[\left(1-\alpha_V\right)\Psi^j + \chi^j W_R^j\right]}.\end{aligned} \tag{252}$$

Using the fact that at the steady-state, we have $\chi^j W_R^j = m^j\xi^j = \frac{m^j\alpha_W\Psi^j}{s^j+r^\star}$, eq. (252) can be rewritten as follows:

$$\begin{aligned}\hat{w}^j &= \frac{\Xi^j}{W^j}\frac{\left[\alpha_W\left(1-\alpha_V\right)\Psi^j + \frac{m^j\alpha_W\Psi^j}{s^j+r^\star}\right]}{\left[\left(1-\alpha_V\right)\Psi^j + \frac{m^j\alpha_W\Psi^j}{s^j+r^\star}\right]},\\ &= \frac{\Xi^j}{W^j}\frac{\alpha_W\left[\left(1-\alpha_V\right)\left(s^j+r^\star\right) + m^j\right]}{\left[\left(1-\alpha_V\right)\left(s^j+r^\star\right) + \alpha_W m^j\right]}\hat{\Xi}^j.\end{aligned} \tag{253}$$

Eq. (253) corresponds to eq. (32) in the text. In order to write formal solutions in a compact form, we set:

$$\Omega^j \equiv \frac{\Xi^j}{W^j}\frac{\alpha_W\left[\left(1-\alpha_V\right)\left(s^j+r^\star\right) + m^j\right]}{\left[\left(1-\alpha_V\right)\left(s^j+r^\star\right) + \alpha_W m^j\right]}. \tag{254}$$

Using the fact that $\hat{\Xi}^N = \hat{p} + \hat{a}^N$ and $\hat{\Xi}^T = \hat{a}^T$, subtracting $\hat{w}^T$ from $\hat{w}^N$ by combining (253) and (254) and inserting (249) leads to the deviation in percentage of the relative wage:

$$\begin{aligned}\hat{\omega} &= \hat{w}^N - \hat{w}^T,\\ &= \Omega^N\left(\hat{p} + \hat{a}^N\right) - \Omega^T\hat{a}^T,\\ &= \left\{\Omega^N\left[\frac{\left(1+\Theta^T\right)\hat{a}^T + \left(\phi-1\right)\hat{a}^N}{\left(\phi+\Theta^N\right)}\right] - \Omega^T\hat{a}^T\right\} - \Omega^N\frac{\mathrm{d}\upsilon_{NX}}{\phi+\Theta^N}.\end{aligned} \tag{255}$$

Eq. (255) corresponds to eq. (33) in the text.

K Analyzing Graphically the Long-Run Effects of Technological Change Biased toward the Traded Sector

This section analyzes graphically the consequences on the relative wage and the relative price of an increase in relative sectoral productivity A^T/A^N, by breaking down the relative wage and relative price effects in a labor market frictions effect and a labor accumulation effect.

K.1 Effects of Higher Productivity in Tradables Relative to Non Tradables

In order to facilitate the discussion, we assume that $\Theta^j \simeq \Theta$. Under this assumption, eq. (249) reduces to:

$$\hat{p} = \frac{(1+\Theta)\left(\hat{a}^T - \hat{a}^N\right)}{(\phi+\Theta)} + \frac{\mathrm{d}\ln(1-\upsilon_{NX})}{(\phi+\Theta)}, \tag{256}$$

where $\mathrm{d}\ln(1-\upsilon_{NX}) \simeq -d\upsilon_{NX}$ by using a first-order Taylor approximation.

Eq. (256) breaks down the relative price response into two components: a labor market frictions effect and a labor accumulation effect. The first term on the RHS of eq. (256) corresponds to the labor market frictions effect. When we let σ_L tend toward infinity, we have $\lim_{\sigma_L\to\infty}\frac{(1+\Theta)}{(\phi+\Theta)} = 1$; in this configuration, a productivity differential between tradables and non tradables by 1% appreciates the relative price by 1% as well, in line with the prediction of the standard BS model. Graphically, as shown in Figure 11(a), the LME-schedule is a horizontal line because the allocation of the labor force across sectors is perfectly elastic to the ratio of sectoral reservation wages. A productivity shock biased toward the traded sector shifts higher the LME-schedule which results in a relative price appreciation, from p_0 to p_{BS}, i.e., by the same amount as the productivity differential. The LME-schedule intercepts the 45° line at point BS'.

As long as $\sigma_L < \infty$, workers experience a mobility cost when moving from one sector to another; hence, the term Θ takes finite values while graphically, the LME-schedule is downward sloping in the $(y^T - y^N, p)$-space. Graphically, higher productivity in tradables relative to non tradables shifts to the right the LME-schedule from LME_0 to LME_1: this shift corresponds to the labor market frictions effect. If $\phi > 1$, the GME-schedule is flatter than the 45° line so that the intersection is at G'; since $p' < p_{BS}$, the relative price appreciates by less than the productivity differential between tradables and non tradables, in line with our empirical findings. Conversely, if $\phi < 1$, the relative price must appreciate more than proportionately (i.e., by more than 1%) following higher productivity of tradables relative to non tradables (by 1 percentage point). In this configuration, the GME-schedule is steeper that the 45° line so that the LME_1-schedule intercepts the GME-schedule at a point which lies to the north west of BS'. Hence, through the labor market frictions channel, a productivity differential between tradables and non tradables by 1% appreciates the relative price of non tradables by less (more) than 1% if traded and non traded goods are substitutes (complements).

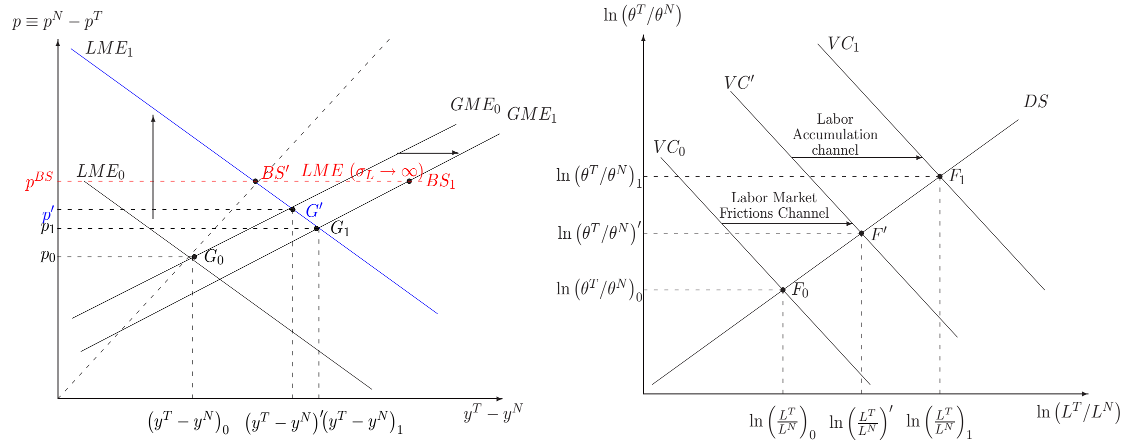
The second term on the RHS of eq. (256) reveals that a productivity differential between tradables and non tradables also impinges on the relative price of non tradables by affecting net exports and hiring expenditure expressed as a share of traded output, as summarized by $d\upsilon_{NX}$. The combined effect of the improvement in the trade balance and permanently increased hiring expenditure has an expansionary effect on the demand for tradables which drives down the relative price of non tradables, as captured by $d\upsilon_{NX} > 0$. In terms of Figure 11(a), the labor accumulation channel shifts the GME-schedule to the right, regardless of the value of the elasticity of substitution between traded and non traded goods. It is worthwhile noticing that a change in υ_{NX} no longer impinges on the relative price p and thus the labor accumulation channel vanishes when we let σ_L tend toward infinity, i.e., if agents are not subject to switching costs from one sector to another. Formally, we have $\lim_{\sigma_L\to\infty}\frac{1}{\phi+\Theta} = 0$. In this case, the GME_1-schedule intercepts the LME_1-schedule at BS_1. Unlike, when $\sigma_L < \infty$, the intercept is at G_1 if $\phi > 1$.

We turn to the relative response. To facilitate the discussion, we assume that $\Theta^j \simeq \Theta$ and $\Omega^j \simeq \Omega$ so that eq. (255) reduces to:

$$\hat{\omega} = -\Omega\left[\frac{(\phi-1)}{\phi+\Theta}\left(\hat{a}^T - \hat{a}^N\right) + \frac{\mathrm{d}\upsilon_{NX}}{\phi+\Theta}\right]. \tag{257}$$

Through the labor market frictions channel, captured by the first term in brackets in the RHS of eq. (257), higher productivity growth in tradables relative to non tradables lowers the relative wage ω only if $\phi > 1$. In terms of Figure 11(b), technological change biased toward the traded sector shifts to the right the VC-schedule from VC_0 to VC'. Unlike, with an elasticity ϕ smaller than one, the VC-schedule would shift to the left because the share of non tradables rises which has an expansionary effect on recruitment in the non traded sector.

As captured by the second term on the RHS of eq. (257), a productivity differential between tradables and non tradables also impinges on the relative wage through a labor accumulation chan-



(a) $(y^T - y^N, p)$-space

(b) $(l^T - l^N, \ln(\theta^T/\theta^N))$-space

Figure 11: Long-Run Relative Price and Relative Wage Effects of Technological Change Biased toward the Traded Sector

nel. Graphically, as depicted in Figure 11(b), higher productivity in tradables relative to non tradables shifts further to the right the VC-schedule from VC' to VC_1. Hence, while ω unambiguously declines if the elasticity of substitution is larger than one, when $\phi < 1$, the relative wage response to a productivity differential is ambiguous. In the latter case, a productivity differential between tradables and non tradables drives down ω through the labor accumulation channel while it increases the relative wage through the labor market frictions channel.

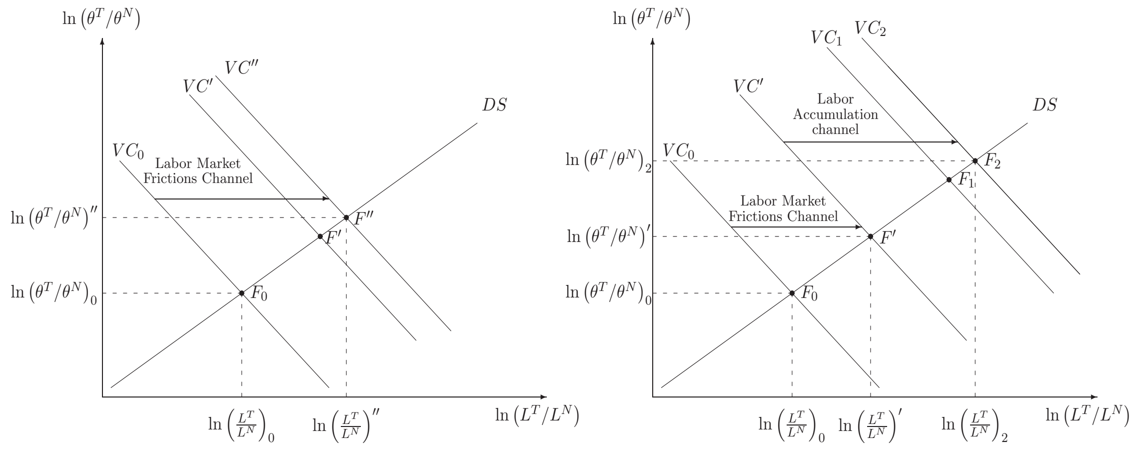
K.2 Implications of Labor Market Institutions

In this subsection, we analyze graphically the implications of labor markets institutions for the relative wage response to technological change biased toward the traded sector. In our framework, the strictness of legal protection against dismissals is captured by a firing tax denoted by x^j paid to the State by the representative firm in the sector which reduces employment. The generosity of the unemployment benefit scheme is captured by the level of R^j; unemployment benefits are assumed to be a fixed proportion ϱ of the wage rate W^j, i.e., $R^j = \varrho W^j$. Additionally, a higher worker bargaining power measured empirically by the bargaining coverage is captured by the parameter α_W. Because the transmission mechanism varies according the type of labor market institution, we differentiate between the firing cost on the one hand, the generosity of the unemployment benefit scheme and the worker bargaining power on the other.

The implications of a higher firing tax is depicted in Figure 12(a) where we assume an elasticity between traded and non traded goods in consumption ϕ larger than one. In this configuration, as mentioned previously, technological change biased toward the traded sector shifts to the right the VC-schedule. As highlighted in Figure 12(a), higher productivity in tradables relative to non tradables shifts further to the right the VC-schedule from VC' to VC'', thus resulting in a larger increase in θ^T/θ^N because hiring in the non traded sector which decumulates employment is limited by the firing tax. Consequently, the relative wage ω declines more, in line with our empirical findings, through a stronger labor market frictions effect. However, a higher firing tax also moderates the decline in the relative wage since net exports increase less. Intuitively, as recruiting expenditure are curbed by the firing tax, the productivity differential leads to a smaller current account deficit, thus moderating the necessary trade balance improvement.

In contrast to a firing tax, raising the unemployment benefit replacement rate or the worker bargaining power leads to a larger long-run rise in net exports and thus amplifies the decline in the relative wage through the labor accumulation channel. The implication of a higher replacement rate ϱ or a larger worker bargaining power α_W is depicted in Figure 12(b) where we consider an elasticity of substitution ϕ larger than one. Figure 12(b) shows that technological change biased toward the traded sector shifts further to the right the VC-schedule from VC_1 to VC_2 in countries where the replacement rate ϱ is higher or the worker bargaining power α_W larger. As mentioned above, the larger increase in net exports amplifies the expansionary effect on hiring in the traded sector which pushes up further the ratio of labor market tightness θ^T/θ^N. Hence, the relative wage of non tradables falls more through a stronger labor accumulation effect. Raising ϱ or α_W also modifies the labor market frictions channel by increasing the mobility of labor across sectors.[65]

[65] In countries with a higher worker bargaining power α_W, firms are willing to recruit more (because it



(a) Consequences of a higher firing tax

(b) Consequences of a higher replacement rate or worker bargaining power

Figure 12: Implications for the Relative Wage Response of Labor Market Regulation in the $(l^T - l^N, \ln(\theta^T/\theta^N))$-space

Because we find numerically that raising ϱ or α_W merely modifies the relative wage response to a productivity differential between tradables and non tradables through the labor market frictions channel, we restrict our attention to the labor accumulation channel in Figure 11(b).

L Effects of Higher Relative Productivity of Tradables on Unemployment Rate Differential

In this section, we investigate the effects of higher productivity in tradables relative to non tradables on the unemployment rate of tradables relative to non tradables. To alleviate the notation, we drop the superscript $\tilde{x}$ to denote steady-state values since we focus on steady-state changes.

To write analytical expression in a compact form, it is useful to set:

$$\Sigma^j = \frac{\Xi^j}{(1-\alpha_V)\,\Psi^j + \chi^j W_R^j}. \tag{258}$$

which implies (see eq (172) for the traded sector and eq. (180) for the non traded sector):

$$\hat{\theta}^j = \Sigma^j \hat{\Xi}^j. \tag{259}$$

Differentiating the definition of the steady-state level for the sectoral unemployment rate described by:

$$u^j = \frac{s^j}{s^j + m^j(\theta^j)}, \tag{260}$$

one obtains the standard negative relationship between u^j and the labor market tightness in sector j:

$$\hat{u}^j = -\alpha_V \frac{m^j}{s^j + m^j}\hat{\theta}^j. \tag{261}$$

Using the fact that $\hat{\Xi}^T = \hat{a}^T$ and $\hat{\Xi}^N = \hat{p} + \hat{a}^N$, subtracting $\hat{u}^N$ from $\hat{u}^T$ by using (259) and (261), one obtains:

$$\begin{aligned} \hat{u}^T - \hat{u}^N &= -\alpha_V\left[\frac{m^T}{s^T+m^T}\Sigma^T\hat{a}^T - \frac{m^N}{s^N+m^N}\Sigma^N\left(\hat{p}+\hat{a}^N\right)\right], \\ &= -\alpha_V\left\{\left[\frac{m^T}{s^T+m^T}\Sigma^T - \frac{m^N}{s^N+m^N}\Sigma^N\left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right]\hat{a}^T - \frac{m^N}{s^N+m^N}\Sigma^N\left(\frac{\phi-1}{\phi+\Theta^N}\right)\hat{a}^N\right\} \\ &+ \alpha_V\frac{m^N}{s^N+m^N}\Sigma^N\frac{\mathrm{d}\ln(1-\upsilon_{NX})}{(\phi+\Theta^N)}, \end{aligned} \tag{262}$$

is relatively less costly due to a higher probability to fill a job vacancy) while workers are less reluctant to move from one sector to another (since they receive a larger share χ of the surplus associated with a labor contract in the marginal benefit of search). In economies with a more generous unemployment benefit scheme, while workers are more reluctant to move from one sector to another (because χ falls), the vacancy creation is more elastic to technological change. Since the latter effect predominates, the labor mobility rises.

where we have inserted the decomposition of the steady-state change of the relative price of non tradables given by eq. (249) to determine the percentage change in the labor market tightness in the non traded sector:

$$\begin{aligned}
\hat{\theta}^N &= \Sigma^N \left(\hat{p} + \hat{a}^N\right), \\
&= \Sigma^N \left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\hat{a}^T + \Sigma^N \left[1 - \frac{1+\Theta^N}{\phi+\Theta^N}\right]\hat{a}^N + \Sigma^N \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta^N\right)}, \\
&= \Sigma^N \left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\hat{a}^T + \Sigma^N \left(\frac{\phi-1}{\phi+\Theta^N}\right)\hat{a}^N + \Sigma^N \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta^N\right)}. \qquad (263)
\end{aligned}$$

Using the fact that at the steady-state, $\frac{m^j}{s^j+m^j} = \left(1-u^j\right)$, eq. (262) can be rewritten as follows:

$$\begin{aligned}
\hat{u}^T - \hat{u}^N &= -\alpha_V \left\{ \left[\left(1-u^T\right)\Sigma^T - \left(1-u^N\right)\Sigma^N \left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right]\hat{a}^T - \left(1-u^N\right)\Sigma^N \left(\frac{\phi-1}{\phi+\Theta^N}\right)\hat{a}^N \right\} \\
&+ \alpha_V \left(1-u^N\right)\Sigma^N \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta^N\right)}. \qquad (264)
\end{aligned}$$

To facilitate the discussion of the effect of a productivity differential on the unemployment rate in the traded relative to the non traded sector, we assume that at the initial steady-state, we have $\Theta^j \simeq \Theta$, $u^j \simeq u$, $\Sigma^j \simeq \Sigma$, and we multiply both sides of eq. (264) by u in order to express the unemployment differential in percentage point so that eq. (264) reduces to:

$$du^T - du^N = -\alpha_V u\left(1-u\right)\Sigma\left[\left(\frac{\phi-1}{\phi+\Theta}\right)\left(\hat{a}^T - \hat{a}^N\right) - \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta\right)}\right]. \qquad (265)$$

Eq. (265) corresponds to equation (37) in the main text. Eq. (265) breaks down the response of the unemployment differential to a productivity differential into two components: a labor market frictions effect and a labor accumulation effect. The first term on the RHS of (265) corresponds to the labor market frictions effect. Through this channel, higher productivity gains in tradables relative to non tradables lower or increase the unemployment rate in the traded sector relative to the non traded sector depending on whether the elasticity of substitution between tradables and non tradables ϕ is smaller or higher than one. If $\phi < 1$, as our evidence suggest, a productivity differential between tradables and non tradables appreciates the relative price of non tradables more than proportionately. Because the share of non tradables increases, non traded firms recruit more which result in a larger decline in u^N relative to u^T. The second term on the RHS corresponds to the labor accumulation effect. Through this channel, the long-run increase in net exports raises the demand for tradables and thus encourages firms to recruit more. When $\phi < 1$, the labor market frictions effect and the labor accumulation effect have conflicting effects on the unemployment differential between tradables and non tradables. If the labor accumulation effect predominates, a productivity differential lowers the unemployment rate in the traded sector by a larger amount than that in the non traded sector. When $\phi > 1$, higher productivity in tradables relative to non tradables unambiguously drives down the unemployment differential between tradables and non tradables.

M The Role of Endogenous Sectoral Labor Force Participation Decision

In this section, we look at a special case of the model for which the sectoral labor force is inelastic, i.e., $\sigma_L = 0$ (reflecting the situation of labor immobility across sectors), in order to highlight the role of an endogenous sectoral labor force participation decision in driving the long-run effects of a productivity differential between tradables and non tradables. Then, we analyze the implications of $\sigma_L \to \infty$ (reflecting the situation of perfect mobility of labor across sectors).

M.1 Equilibrium Dynamics when $\sigma_L = 0$

To begin with, we determine the dynamic system. Denoting by W_R^j the reservation wage in sector j, the first-order conditions for the traded and the non traded sector described by eqs. (83b)-(83c) respectively, implies that $F^j \equiv L^j + U^j = \left(\bar{\lambda} W_R^j / \zeta^j\right)^{\sigma_L}$ with $W_R^j \equiv R^j + m^j\left(\theta^j\right)\xi^j$. Using the fact that $U^j = \left(\bar{\lambda} W_R^j / \zeta^j\right)^{\sigma_L} - L^j$, the dynamic equation for employment (12) can be rewritten as follows:

$$\dot{L}^j = m^j\left(\theta^j\right)\left(\bar{\lambda} W_R^j / \zeta^j\right)^{\sigma_L} - \left[s^j + m^j\left(\theta^j\right)\right] L^j.$$

Assuming that labor force is fixed, i.e., setting $\sigma_L = 0$, then the equation above reads as:

$$\dot{L}^j = m^j\left(\theta^j\right) - \left[s^j + m^j\left(\theta^j\right)\right] L^j. \tag{266}$$

Imposing $\alpha_W^j = \alpha_W$ and using the fact that $m^j\left(\theta^j\right)\xi^j = \frac{\alpha_W}{1-\alpha_W}\kappa^j\theta^j$ together with $-\frac{v_F^j}{\bar{\lambda}} = W_R^j$ and $W_R^j \equiv R^j + m^j\left(\theta^j\right)\xi^j$, the Nash bargaining wage can be rewritten as follows:

$$\begin{aligned} W^j &= \alpha_W\left(\Xi^j + r^\star x^j\right) - \left(1-\alpha_W\right)\frac{v_F^j}{\bar{\lambda}}, \\ &= \alpha_W\left(\Xi^j + r^\star x^j + \kappa^j\theta^j\right) + \left(1-\alpha_W\right)R^j. \end{aligned} \tag{267}$$

We now determine the dynamic equation for the labor market tightness. Plugging (267) into (114) yields:

$$\begin{aligned} \dot{\theta}^j(t) &= \frac{\theta^j(t)}{\left(1-\alpha_V^j\right)}\left\{\left(s^j + r^\star\right) - \frac{f^j\left(\theta^j(t)\right)}{\kappa^j}\left[\left(\Xi^j + r^\star x^j\right) - W^j\right]\right\}, \\ &= \frac{\theta^j(t)}{\left(1-\alpha_V^j\right)}\left\{\left(s^j + r^\star\right) - \frac{f^j\left(\theta^j(t)\right)\left(1-\alpha_W\right)}{\kappa^j}\Psi^j\right\}, \end{aligned} \tag{268}$$

where the overall surplus from an additional job Ψ^j is:

$$\Psi^j \equiv \Xi^j + r^\star x^j - \frac{\alpha_W}{1-\alpha_W}\kappa^j\theta^j - R^j, \tag{269}$$

with $\Xi^T = A^T$ and $\Xi^N = PA^N$.

Traded Sector

Linearizing the accumulation equation for labor (266) and the dynamic equation for labor market tightness (268) in the traded sector, we get in matrix form:

$$\left(\dot{L}^T, \dot{\theta}^T\right)^T = J^T\left(L^T(t) - \tilde{L}^T, \theta^T(t) - \tilde{\theta}^T\right)^T \tag{270}$$

where J^T is given by

$$J^T \equiv \begin{pmatrix} -\left(s^T + \tilde{m}^T\right) & \left(\tilde{m}^T\right)'\left(1-\tilde{L}^T\right) \\ 0 & \left[\left(s^T + r^\star\right) + \tilde{m}^T\frac{\alpha_W}{1-\alpha_V}\right] \end{pmatrix}, \tag{271}$$

with $\tilde{m}^T = m^T\left(\tilde{\theta}\right)$.

The trace denoted by Tr of the linearized 2×2 matrix (270) is given by:

$$\mathrm{Tr}J^T = r^\star + \frac{\tilde{m}^T}{1-\alpha_V}\left[\alpha_W - \left(1-\alpha_V\right)\right]. \tag{272}$$

The determinant denoted by Det of the linearized 2×2 matrix (118) is unambiguously negative:

$$\mathrm{Det}J^T = -\left(s^T + \tilde{m}^T\right)\left[\left(s^T + r^\star\right) + \frac{\alpha_W}{1-\alpha_V}\tilde{m}^T\right] < 0. \tag{273}$$

From now on, for clarity purpose, we impose the Hosios condition in order to avoid unnecessary complications:

$$\alpha_W = \left(1-\alpha_V\right). \tag{274}$$

Denoting by ν^T the eigenvalue, the characteristic equation for the matrix J (271) of the linearized system writes as follows:

$$\left(\nu_i^T\right)^2 - r^\star\nu_i^T + \mathrm{Det}J^T = 0. \tag{275}$$

The characteristic roots obtained from the characteristic polynomial of degree two can be written as follows:

$$\begin{aligned} \nu_i^T &\equiv \frac{1}{2}\left\{r^\star \pm \sqrt{\left(r^\star\right)^2 - 4\mathrm{Det}J^T}\right\} \gtrless 0, \quad i = 1,2, \\ &\equiv \frac{1}{2}\left\{r^\star \pm \sqrt{\left(r^\star\right)^2 + 4\left(s^T + \tilde{m}^T\right)^2 + 4r^\star\left(s^T + \tilde{m}^T\right)}\right\}, \\ &\equiv \frac{1}{2}\left\{r^\star \pm \left[r^\star + 2\left(s^T + \tilde{m}^T\right)\right]\right\}, \end{aligned} \tag{276}$$

where we used the fact that $\text{Det}J^T = -\left(s^T + \tilde{m}^T\right)\left(s^T + r^\star + \tilde{m}^T\right)$.

We denote by $\nu_1^T < 0$ and $\nu_2^T > 0$ the stable and unstable eigenvalues respectively which satisfy:

$$\nu_1^T = -\left(s^T + \tilde{m}^T\right) < 0 < r^\star < \nu_2^T = \left(s^T + r^\star + \tilde{m}^T\right). \tag{277}$$

Non Traded Sector

Linearizing the accumulation equation for non traded labor (266) by setting $j = N$ and the dynamic equation for labor market tightness (268) in the non traded sector by inserting first the solution for the relative price of non tradables (108), i.e., $P = P\left(L^N, \bar{\lambda}, A^N\right)$, we get in matrix form:

$$\left(\dot{L}^N, \dot{\theta}^N\right)^T = J^N\left(L^N(t) - \tilde{L}^N, \theta^N(t) - \tilde{\theta}^N\right)^T \tag{278}$$

where J^N is given by

$$J^N \equiv \begin{pmatrix} -\left(s^N + \tilde{m}^N\right) & \left(m^N\right)'\left(1 - \tilde{L}^N\right) \\ -\frac{1-\alpha_W}{1-\alpha_V}\frac{\tilde{m}^N}{\kappa^N}P_{L^N}A^N & \left[\left(s^N + r^\star\right) + \tilde{m}^N\frac{\alpha_W}{1-\alpha_V}\right] \end{pmatrix}, \tag{279}$$

with $P_{L^N} = \frac{\partial P}{\partial L^N} = \frac{A^N}{C_P^N} < 0$.

The trace is:

$$\text{Tr}J^N = r^\star + \frac{\tilde{m}^N}{1-\alpha_V}\left[\alpha_W - (1-\alpha_V)\right]. \tag{280}$$

The determinant denoted by Det of the linearized 2×2 matrix (279) is unambiguously negative:

$$\text{Det}J^N = -\left(s^N + \tilde{m}^N\right)\left[\left(s^N + r^\star\right) + \frac{\alpha_W}{1-\alpha_V}\tilde{m}^N\right] + \frac{1-\alpha_W}{1-\alpha_V}\frac{\tilde{m}^N}{\kappa^N}P_{L^N}A^N\left(m^N\right)'\left(1 - \tilde{L}^N\right) < 0. \tag{281}$$

Assuming that the Hosios condition (274) holds, the determinant (281) can be rewritten as follows:

$$\begin{aligned} \text{Det}J^N &= -\left(s^N + \tilde{m}^N\right)\left(s^N + r^\star\right)\left[\left(\frac{s^N + r^\star\tilde{m}^N}{s^N + r^\star}\right) - \frac{1-\alpha_W}{1-\alpha_V}\frac{\tilde{m}^N}{\kappa^N}\frac{P_{L^N}A^N m^{N,\prime}}{\left(s^N + r^\star\right)}\frac{\left(1 - \tilde{L}^N\right)}{\left(s^N + \tilde{m}^N\right)}\right], \\ &= -\left(s^N + \tilde{m}^N\right)\left(s^N + r^\star\right)\left[\left(\frac{s^N + r^\star\tilde{m}^N}{s^N + r^\star}\right) - \tilde{P}A^N\frac{P_{L^N}L^N}{\tilde{P}}\frac{\alpha_V\tilde{u}^N}{\left(1-\alpha_V\right)\tilde{\Psi}^N}\right] < 0, \end{aligned} \tag{282}$$

where we computed the following term:

$$\begin{aligned} & \frac{1-\alpha_W}{1-\alpha_V}\frac{\tilde{m}^N}{\kappa^N}\frac{P_{L^N}A^N m^{N,\prime}}{\left(s^N + r^\star\right)}\frac{\left(1 - \tilde{L}^N\right)}{\left(s^N + \tilde{m}^N\right)} \\ =\; & \frac{\left(1-\alpha_W\right)}{\left(s^N + r^\star\right)}\frac{\tilde{m}^N}{\tilde{\theta}^N\kappa^N}\frac{m^{N,\prime}\tilde{\theta}^N}{\tilde{m}^N}\frac{\tilde{m}^N\tilde{U}^N}{\left(1-\alpha_V\right)}\frac{P_{L^N}A^N}{\left(s^N + \tilde{m}^N\right)}, \\ =\; & \frac{\alpha_V}{\tilde{\Psi}^N}\frac{s^N\tilde{L}^N}{\left(1-\alpha_V\right)}\frac{P_{L^N}A^N}{\left(s^N + \tilde{m}^N\right)}, \\ =\; & \left(\frac{\alpha_V}{1-\alpha_V}\right)\frac{\tilde{u}^N}{\tilde{\Psi}^N}\frac{P_{L^N}L^N}{\tilde{P}}\tilde{P}A^N. \end{aligned} \tag{283}$$

To get (283), we used the fact that $\frac{(1-\alpha_W)\tilde{f}^N}{\kappa^N(s^N+r^\star)} = \frac{1}{\tilde{\Psi}^N}$, $1 - \tilde{L}^N = \tilde{U}^N$, $\tilde{m}^N\tilde{U}^N = s^N\tilde{L}^N$, and $\tilde{u}^N = \frac{s^N}{s^N+\tilde{m}^N}$.

We denote by $\nu_1^N < 0$ and $\nu_2^N > 0$ the stable and unstable eigenvalues respectively which satisfy:

$$\nu_1^N < 0 < r^\star < \nu_2^N. \tag{284}$$

M.2 Formal Solutions for $L^T(t)$ and $\theta^T(t)$

The stable paths for the labor market in the traded sector are given by :

$$L^T(t) - \tilde{L}^T = D_1^T e^{\nu_1^T t}, \tag{285a}$$

$$\theta^T(t) - \tilde{\theta}^T = \omega_{21}^T D_1^T e^{\nu_1^T t}, \tag{285b}$$

where $D_1^T = L_0^T - \tilde{L}^T$, and element ω_{21}^T of the eigenvector (associated with the stable eigenvalue ν_1^T) is given by:

$$\omega_{21}^T = \frac{\left(s^T + \tilde{m}^T + \nu_1^T\right)}{m^{\prime,T}\left(1 - \tilde{L}^T\right)} = 0. \tag{286}$$

where we used the fact that $\nu_1^T = -\left(s^T + \tilde{m}^T\right)$ (see eq. (277)). From (285a), the dynamics for labor market tightness θ^T degenerate.

M.3 Formal Solutions for $L^N(t)$ and $\theta^N(t)$

The stable paths for the labor market in the non traded sector are given by :

$$
\begin{aligned}
L^N(t) - \tilde{L}^N &= D_1^N e^{\nu_1^N t}, && (287a) \\
\theta^N(t) - \tilde{\theta}^N &= \omega_{21}^N D_1^N e^{\nu_1^N t}, && (287b)
\end{aligned}
$$

where $D_1^N = L_0^N - \tilde{L}^N$, and element ω_{21}^N of the eigenvector (associated with the stable eigenvalue ν_1^N) is given by:

$$
\begin{aligned}
\omega_{21}^N &= \frac{\left(s^N + \tilde{m}^N + \nu_1^N\right)}{m'^{,N}\left(1 - \tilde{L}^N\right)}, \\
&= \frac{\frac{1-\alpha_W}{1-\alpha_V}\frac{\tilde{m}^N}{\kappa^N} P_{L^N} A^N}{\left(s^N + r^\star + \tilde{m}^N - \nu_1^N\right)} < 0. && (288)
\end{aligned}
$$

M.4 Formal Solution for the Stock of Foreign Bonds $B(t)$

Substituting first the short-run static solutions for consumption in tradables given by (110), and using the fact that $V^j = U^j \theta^j$, the accumulation equation for traded bonds (112) can be written as follows:

$$
\dot{B}(t) = r^\star B(t) + A^T L^T(t) - C^T\left(L^N(t), \bar{\lambda}, A^N\right) - \kappa^T \theta^T(t)\left(1 - L^T(t)\right) - \kappa^N \theta^N(t)\left(1 - L^N(t)\right), \quad (289)
$$

where we used the fact that $U^j = 1 - L^j$ when $\sigma_L = 0$.

Linearizing (289) in the neighborhood of the steady-state and inserting stable solutions given by (285) and (287) yields:

$$
\dot{B}(t) = r^\star\left(B(t) - \tilde{B}\right) + \Lambda^T\left(L^T(t) - \tilde{L}^T\right) + \Lambda^N\left(L^N(t) - \tilde{L}^N\right), \quad (290)
$$

where we set:

$$
\begin{aligned}
\Lambda^T &= A^T + \kappa^T \tilde{\theta}^T > 0, && (291a) \\
\Lambda^N &= -C^T_{L^N} - \kappa^N \tilde{U}^N \omega_{21}^N - \kappa^N \tilde{\theta}^N \omega_{31}^N, \\
&= -C^T_{L^N} + \kappa^N \tilde{\theta}^N\left[1 - \frac{\left(s^N + \tilde{m}^N + \nu_1^N\right)}{\alpha_V \tilde{m}^N}\right] > 0, && (291b)
\end{aligned}
$$

where we have inserted (144b) and used the fact that $\left(m^N\right)' \theta^N / m^N = \alpha_V$ to get (291b); note that $C^T_{L^N} \simeq 0$ as long as $\phi \simeq \sigma_C$ in line with evidence for a typical OECD economy. The sign of (291b) follows from the fact that $\omega_{21}^N < 0$ (see (288)).

Solving the differential equation (290) yields:

$$
B(t) = \tilde{B} + \left[\left(B_0 - \tilde{B}\right) - \frac{\Lambda^T D_1^T}{\nu_1^T - r^\star} - \frac{\Lambda^N D_1^N}{\nu_1^N - r^\star}\right] e^{r^\star t} + \frac{\Lambda^T D_1^T}{\nu_1^T - r^\star} e^{\nu_1^T t} + \frac{\Lambda^N D_1^N}{\nu_1^N - r^\star} e^{\nu_1^N t}. \quad (292)
$$

Invoking the transversality condition for intertemporal solvency, and using the fact that $D_1^T = L_0^T - \tilde{L}^T$ and $D_1^N = L_0^N - \tilde{L}^N$, we obtain the linearized version of the nation's intertemporal budget constraint:

$$
\tilde{B} - B_0 = \Phi^T\left(\tilde{L}^T - L_0^T\right) + \Phi^N\left(\tilde{L}^N - L_0^N\right), \quad (293)
$$

where we set

$$
\Phi^T \equiv \frac{\Lambda^T}{\nu_1^T - r^\star} = -\frac{\left(A^T + \kappa^T \tilde{\theta}^T\right)}{\left(s^T + \tilde{m}^T + r^\star\right)} < 0, \quad \Phi^N \equiv \frac{\Lambda^N}{\nu_1^N - r^\star} < 0. \quad (294)
$$

Equation (294) can be solved for the stock of foreign bonds:

$$
\tilde{B} = B\left(\tilde{L}^T, \tilde{L}^N\right), \quad B_{L^T} = \Phi^T < 0, \quad B_{L^N} = \Phi^N < 0. \quad (295)
$$

For the national intertemporal solvency to hold, the terms in brackets of equation (292) must be zero so that the stable solution for net foreign assets finally reduces to:

$$
B(t) - \tilde{B} = \Phi^T\left(L^T(t) - \tilde{L}^T\right) + \Phi^N\left(L^N(t) - \tilde{L}^N\right). \quad (296)
$$

M.5 Solving Graphically for the Steady-State

We investigate graphically the long-run effects of a rise in the the ratio of sectoral productivity. Assuming $\alpha_W^j = \alpha_W$ and setting $\sigma_L = 0$, the steady-state (234) reduces to the following system which comprises five equations:

$$\frac{\tilde{C}^T}{\tilde{C}^N} = \frac{\varphi}{1-\varphi}\tilde{P}^{\phi}, \tag{297a}$$

$$\frac{\tilde{L}^T}{\tilde{L}^N} = \frac{\tilde{m}^T}{\tilde{m}^N}\frac{\left(s^N+\tilde{m}^N\right)}{\left(s^T+\tilde{m}^T\right)}\frac{\zeta^N}{\zeta^T}, \tag{297b}$$

$$\frac{\kappa^T}{f^T\left(\tilde{\theta}^T\right)} = \frac{\left(1-\alpha_W^T\right)\tilde{\Psi}^T}{\left(s^T+r^\star\right)}, \tag{297c}$$

$$\frac{\kappa^N}{f^N\left(\tilde{\theta}^N\right)} = \frac{\left(1-\alpha_W^N\right)\tilde{\Psi}^N}{\left(s^N+r^\star\right)}, \tag{297d}$$

$$\frac{\tilde{Y}^T\left(1-\upsilon_{NX}\right)}{\tilde{Y}^N} = \frac{\tilde{C}^T}{\tilde{C}^N}, \tag{297e}$$

where $-\upsilon_{NX} = \upsilon_B - \upsilon_V^T - \upsilon_V^N$.

Goods Market

Because we restrict ourselves to the analysis of the long-run effects, the tilde is suppressed for the purposes of clarity. To characterize the steady-state, we focus on the goods market which can be summarized graphically by two schedules in the $(y^T - y^N, p)$-space, where we denote the logarithm of variables with lower-case letters.

The goods market equilibrium (GME)-schedule that we repeat for convenience is identical to (236):

$$\left(\hat{y}^T - \hat{y}^N\right)\Big|^{GME} = \phi\hat{p} - d\ln\left(1-\upsilon_{NX}\right). \tag{298}$$

The GME-schedule is upward-sloping in the $(y^T - y^N, p)$-space and the slope of the GME-schedule is equal to $1/\phi$.

The labor market equilibrium (LME)-schedule that we repeat for convenience is identical to (244),

$$\hat{y}^T - \hat{y}^N\Big|^{LME} = -\Theta^N\hat{p} + \left(1+\Theta^T\right)\hat{a}^T - \left(1+\Theta^N\right)\hat{a}^N, \tag{299}$$

except for the elasticity Θ^j of employment to the marginal revenue of labor which reduces to:

$$\Theta^T \equiv \frac{A^T\alpha_V^T u^T}{\left[\left(1-\alpha_V\right)\Psi^T + \tilde{\chi}^T W_R^T\right]} > 0, \tag{300a}$$

$$\Theta^N \equiv \frac{PA^N\alpha_V^N u^N}{\left[\left(1-\alpha_V\right)\Psi^N + \chi^N W_R^N\right]} > 0. \tag{300b}$$

The LME-schedule is downward-sloping in the $(y^T - y^N, p)$-space and the slope of the LME-schedule is equal to $-\frac{1}{\Theta^N}$. When $\sigma_L = 0$, Θ^j is smaller so that the LME-schedule is steeper.

Labor Market

Imposing $\sigma_L = 0$ into eq. (234b), the decision of search (DS)-schedule reduces to:

$$\frac{L^T}{L^N} = \frac{m^T}{m^N}\frac{m^N+s^N}{m^T+s^T}\frac{\zeta^N}{\zeta^T}. \tag{301}$$

Taking logarithm and differentiating eq. (301) yields:

$$\hat{l}^T - \hat{l}^N = \alpha_V u^T\hat{\theta}^T - \alpha_V u^N\hat{\theta}^N. \tag{302}$$

Assuming that the labor markets display similar features across sectors, i.e., $u^j \simeq u$, eq. (302) reduces to:

$$\left(\hat{\theta}^T - \hat{\theta}^N\right)\Big|_{\sigma_L=0}^{DS} = \frac{1}{\alpha_V u}\left(\hat{l}^T - \hat{l}^N\right). \tag{303}$$

The DS-schedule is upward-sloping in the $(l^T - l^N, \ln\left(\frac{\theta^T}{\theta^N}\right))$-space. Comparing (303) with (244), it is straightforward to show that the DS-schedule becomes steeper when $\sigma_L = 0$. The VC-schedule is downward-sloping and identical to (248).

M.6 Effects of Higher Relative Productivity of Tradables when $\sigma_L = 0$

Equating demand for tradables in terms of non tradables given by eq. (298) and supply (299) yields the deviation in percentage of the relative price from its initial steady-state (249). When assuming $\Theta^{j,\prime} \simeq \Theta'$, eq. (249) reduces to:

$$\hat{p} = \frac{(1+\Theta')\left(\hat{a}^T - \hat{a}^N\right)}{(\phi + \Theta')} + \frac{\mathrm{d}\ln\left(1 - \upsilon_{NX}\right)}{(\phi + \Theta')}, \tag{304}$$

where

$$\Theta' \equiv \frac{\Xi \alpha_V u}{\left[(1-\alpha_V)\Psi + \tilde{\chi} W_R\right]} < \Theta \equiv \frac{\Xi\left[\alpha_V u + \sigma_L \chi\right]}{\left[(1-\alpha_V)\Psi + \tilde{\chi} W_R\right]}, \tag{305}$$

with Θ given by (238). Assuming $\sigma_L = 0$ lowers the elasticity Θ of sectoral employment w.r.t. marginal revenue of labor. Intuitively, increased productivity induce firms to post more job vacancies which raises the labor market tightness and thus the probability of finding a job. When $\sigma_L > 0$, higher θ^j increases L^j through two channels: i) by triggering an outflow from unemployment, and ii) by inducing agents to increase the search intensity for a job. Because the latter effect vanishes if $\sigma_L = 0$, employment becomes less responsive to productivity gains, as captured by a lower Θ, i.e., $\Theta' < \Theta$ (see inequality (305)). Since $\Theta' < \Theta$, comparing eq. (304) with eq. (31) shows that when setting $\sigma_L = 0$, the labor market frictions effect captured by the first term on the RHS of eq. (304) is moderated or amplified depending on whether ϕ is larger or smaller than one. In the former case, traded output increases less so that the relative price of non tradables must appreciate by a smaller amount to clear the goods market. If $\phi < 1$, a productivity differential between tradables and non tradables raises the share of non tradables and thus has an expansionary effect on labor demand in the non traded sector. When $\sigma_L = 0$, as detailed below, firms must increase wages by a larger amount. To compensate for the higher unit labor cost, non traded firms set higher prices so that p increases more. Irrespective of whether ϕ is larger or smaller than one, a productivity differential between tradables and non tradables exerts a larger negative impact on p when $\sigma_L = 0$ through the labor accumulation effect. The reason is that following higher net exports, because the reallocation of labor across sectors is absent, traded output increases less which in turn triggers a greater excess of demand for tradables, thus leading to a larger depreciation in the relative price of non tradables (i.e., a larger decline in p).

Equating labor supply (303) with labor demand (248) while assuming $\Theta^j \simeq \Theta$ and $\Omega^j \simeq \Omega$ leads to the deviation in percentage of the relative wage from its initial steady-state:

$$\hat{\omega} = -\frac{\Omega}{\phi + \Theta'}\left[(\phi - 1)\left(\hat{a}^T - \hat{a}^N\right) + \mathrm{d}\upsilon_{NX}\right]. \tag{306}$$

Eq. (306) shows that assuming a fixed labor force by setting $\sigma_L = 0$ amplifies both the labor market frictions effect (captured by the first term on the RHS of eq. (306)) and the labor market accumulation effect (captured by the second term on the RHS of eq. (306)). Intuitively, higher productivity shifts the VC-schedule along a steeper DS-schedule, thus resulting in larger changes in the ratio θ^T/θ^N and in the relative wage ω. As discussed in section 5.2, across all scenarios, even if the labor market frictions effect raises the relative wage (when setting $\phi < 1$), the labor market accumulation effect predominates. Setting $\sigma_L = 0$ amplifies the negative impact of the labor accumulation effect on the relative wage by such an amount that the model cannot account quantitatively for the size of decline in the relative wage (i.e., tends to overstate the decline in ω) found in the data.

M.7 Effects of Higher Relative Productivity of Tradables when $\sigma_L \to \infty$

In this subsection, we investigate the relative price and relative wage effects of higher productivity of tradables relative to non tradables when we let σ_L tend toward infinity. In this configuration, the case of perfect mobility of labor emerges.

As mentioned in section I, the steady-state can be characterized graphically by considering alternatively the goods market or the labor market. When we let σ_L tend toward infinity, eq. (238) implies that Θ, which captures the elasticity of sectoral employment w.r.t. the marginal revenue of labor, tends toward infinity. Inspection of (236) and (237) indicates that when $\sigma_L \to \infty$, the slope of the GME-schedule (equal to $1/\phi$) is unaffected while the LME-schedule (whose slope is equal to $1/\Theta^N$) becomes a horizontal line. Applying l'Hôpital's rule, eq. (249) reduces to:

$$\begin{aligned}\lim_{\sigma_L \to \infty} \hat{p} &= \frac{1+\Theta^T}{1+\Theta^N}\hat{a}^T - \hat{a}^N, \\ &= \frac{\Xi^T \chi^T \left[\left(1-\alpha_V^N\right)\Psi^N + \chi^N W_R^N\right]}{\Xi^N \chi^N \left[\left(1-\alpha_V^T\right)\Psi^T + \chi^T W_R^T\right]}\hat{a}^T - \hat{a}^N. \end{aligned} \tag{307}$$

According to our quantitative analysis, while labor market parameters are allowed to vary across sectors, the term in front of $\hat{a}^T$ is close to one for the baseline calibration . As a result, a 1 percentage point increase in the productivity differential between tradables and non tradables appreciates the relative price of non tradables by 1% approximately. Assuming that $\Theta^j \simeq \Theta$ and applying l'Hôpital's rule, the rate of change of the relative price described by eq. (31) reduces to:

$$\lim_{\sigma_L \to \infty} \hat{p} = \hat{a}^T - \hat{a}^N. \tag{308}$$

Consequently, a model with labor market frictions reaches the same conclusion as the standard neoclassical model with a competitive labor market as long as the elasticity of labor supply at the extensive margin tends toward infinity.

Inspection of (244) and (248) indicates that when $\sigma_L \to \infty$, the DS-schedule (whose slope is equal to $\frac{1}{[\alpha_V u + \sigma_L \chi]}$) becomes a horizontal line while the VC-schedule (whose slope is equal to $-\frac{\Xi}{\phi[(1-\alpha_V)\Psi + \chi W_R]}$) is unaffected. Applying l'Hôpital's rule, eq. (255) reduces to:

$$\begin{aligned}
\lim_{\sigma_L \to \infty} \hat{\omega} &= \left[\Omega^N \frac{1+\Theta^T}{1+\Theta^N} - \Omega^T\right] \hat{a}^T, \\
&= \left\{\Omega^N \frac{\Xi^T \chi^T \left[\left(1-\alpha_V^N\right)\Psi^N + \chi^N W_R^N\right]}{\Xi^N \chi^N \left[\left(1-\alpha_V^T\right)\Psi^T + \chi^T W_R^T\right]} - \Omega^T\right\} \hat{a}^T. \qquad (309)
\end{aligned}$$

Assuming that $\Theta^j \simeq \Theta$ and applying l'Hôpital's rule, the rate of change of the relative wage described by eq. (34) reduces to:

$$\lim_{\sigma_L \to \infty} \hat{\omega} = \left(\Omega^N - \Omega^T\right) \hat{a}^T, \tag{310}$$

where Ω^j captures the elasticity of the sectoral wage w.r.t the marginal revenue of labor; according to (310), the effect of higher productivity in tradables relative to non tradables on the relative wage is proportional to $\Omega^N - \Omega^T$. More precisely, when we let $\sigma_L \to \infty$, while the ratio of labor market tightness remains unaffected if $\Theta^j \simeq \Theta$, technological change biased toward the traded sector may influence the relative wage as long as the elasticity of sectoral wage w.r.t. the marginal revenue of labor Ω^j varies across sectors. For our benchmark parametrization, we have $\Omega^j \simeq \Omega$ so that the relative wage is (almost) unaffected by a productivity differential.

When we let search parameters vary across sectors and σ_L tend toward infinity in eq. (264), we have:

$$\lim_{\sigma_L \to \infty} \hat{\theta}^N = \Sigma^N \left(\lim_{\sigma_L \to \infty} \hat{p} + \hat{a}^N\right) = \Sigma^N \hat{a}^T, \tag{311}$$

where we inserted (308). Making use of (265), the unemployment rate differential reduces to:

$$\lim_{\sigma_L \to \infty} \left(du^T - du^N\right) = -\alpha_V \left[u^T\left(1-u^T\right)\Sigma^T - u^N\left(1-u^N\right)\Sigma^N\right] \hat{a}^T. \tag{312}$$

In conclusion, a model with labor market frictions reaches the same conclusions as the standard neoclassical model with a competitive labor market as long as the elasticity of labor supply at the extensive margin tends toward infinity.

N Calibration Procedure

In this section, we provide more details about the calibration to a representative OECD economy and to data from 18 OECD countries. Section A.2 and section A.3 present the source and construction of data.

N.1 Initial Steady-State

Assuming that the elasticity of labor supply at the extensive margin (σ_L^j), the elasticity of vacancies in job matches (α_V^j), and the worker bargaining power (α_W^j) are symmetric across sectors, i.e., $\sigma_L^j = \sigma_L$, $\alpha_V^j = \alpha_V$ and $\alpha_W^j = \alpha_W$, and normalizing to 1 the parameters ζ^T and A^N that correspond to the disutility from working and searching for a job in the traded sector and the productivity of labor in the non traded sector, respectively, the calibration reduces to 20 parameters: $r^\star$, β, σ_C, σ_L, ϕ, φ, ζ^N, ω_G $(= \frac{G}{Y})$ ω_{G^N} $(= \frac{PG^N}{G})$, A^T, s^T, s^N, $1-\alpha_V$, α_W, κ^T, κ^N, X^T, X^N, x^N, ϱ, and initial conditions B_0, L_0^T, L_0^N.

Since we focus on the long-run equilibrium, the tilde is suppressed for the purposes of clarity. The steady-state of the open economy comprises 14 equations:

$$C = \left(P_C\bar{\lambda}\right)^{-\sigma_C}, \tag{313a}$$

$$U^T = \frac{s^T L^T}{m^T}, \tag{313b}$$

$$U^N = \frac{s^N L^N}{m^N}, \tag{313c}$$

$$m^T = X^T \left(\theta^T\right)^{\alpha_V}, \tag{313d}$$

$$m^N = X^N \left(\theta^N\right)^{\alpha_V}, \tag{313e}$$

$$L^T = \frac{m^T}{s^T + m^T}\left[\frac{\bar{\lambda}W_R^T}{\zeta^T}\right]^{\sigma_L}, \tag{313f}$$

$$L^N = \frac{m^N}{s^N + m^N}\left[\frac{\bar{\lambda}W_R^N}{\zeta^N}\right]^{\sigma_L}, \tag{313g}$$

$$\frac{\kappa^T}{f^T} = \frac{\left(1-\alpha_W\right)\Psi^T}{s^T + r^\star}, \quad \Psi^T \equiv A^T - W_R^T, \tag{313h}$$

$$\frac{\kappa^N}{f^N} = \frac{\left(1-\alpha_W\right)\Psi^N}{s^N + r^\star}, \quad \Psi^N \equiv PA^N + r^\star x^N - W_R^N, \tag{313i}$$

$$V^T = \theta^T U^T, \tag{313j}$$

$$V^N = \theta^N U^N, \tag{313k}$$

$$A^N L^N = C^N + G^N, \tag{313l}$$

$$r^\star B + A^T L^T = C^T + G^T + \kappa^T\theta^T U^T + \kappa^N\theta^N U^N, \tag{313m}$$

and the intertemporal solvency condition

$$B - B_0 = \Phi^T\left(L^T - L_0^T\right) + \Phi^N\left(L^N - L_0^N\right), \tag{313n}$$

where the system jointly determines C, U^T, U^N, m^T, m^N, L^T, L^N, θ^T, θ^N, V^T, V^N, P, B, $\bar{\lambda}$.

Some of the values of parameters can be taken directly from data, but others need to be endogenously calibrated to fit a set of an average OECD economy features. Among the 20 parameters, 6 parameters, i.e., κ^T, κ^N, X^T, X^N, ζ^N, φ, together with initial conditions (B_0, L_0^T, L_0^N) must be set in order to match key properties of a typical OECD economy. More precisely, the parameters κ^T, κ^N, X^T, X^N, ζ^N, φ, together with the set of initial conditions are set to target θ^T, θ^N, m^T, m^N, L^N/L, α_C, υ_{NX}. Denoting by υ_{G^N} the ratio of government spending in non tradables, G^N, to the non traded output, Y^N, the steady-state can be reduced to the following seven equations:

$$\frac{\kappa^T}{f^T} = \frac{\left(1-\alpha_W\right)\Psi^T}{s^T + r^\star}, \tag{314a}$$

$$\frac{\kappa^N}{f^N} = \frac{\left(1-\alpha_W\right)\Psi^N}{s^N + r^\star}, \tag{314b}$$

$$m^T = X^T \left(\theta^T\right)^{\alpha_V}, \tag{314c}$$

$$m^N = X^N \left(\theta^N\right)^{\alpha_V}, \tag{314d}$$

$$\frac{A^T L^T\left(1-\upsilon_{NX}\right)}{A^N L^N\left(1-\upsilon_{G^N}\right)} = \frac{\varphi}{1-\varphi}P^{\phi}, \tag{314e}$$

$$\frac{L^T}{L^N} = \frac{m^T}{m^N}\frac{m^N + s^N}{m^T + s^T}\left(\frac{W_R^T}{W_R^N}\zeta^N\right)^{\sigma_L}, \tag{314f}$$

$$B - B_0 = \Phi^T\left(L^T - L_0^T\right) + \Phi^N\left(L^N - L_0^N\right), \tag{314g}$$

which jointly determine θ^T, θ^N, m^T, m^N, L^T/L^N, P, B. The ratio L^T/L^N implicitly determines L^N/L:

$$\frac{L^N}{L} = \frac{L^N}{L^T + L^N} = \frac{1}{\frac{L^T}{L^N} + 1}. \tag{315}$$

The relative price of non tradables P implicitly determines the non tradable content of consumption expenditure:

$$\alpha_C = \frac{\left(1-\varphi\right)P^{1-\phi}}{\varphi + \left(1-\varphi\right)P^{1-\phi}}. \tag{316}$$

The net foreign asset position B implicitly determines $\upsilon_{NX} = \frac{NX}{Y^T}$ with $NX = Y^T - C^T - G^T$ and $-\upsilon_{NX} = \upsilon_B - \upsilon_{V^T} - \upsilon_{V^N}$ with $\upsilon_B \equiv \frac{r^\star B}{Y^T}$, $\upsilon_{V^T} = \frac{\kappa^T V^T}{Y^T}$ and $\upsilon_{V^N} = \frac{\kappa^N V^N}{Y^T}$. To see it, multiply both sides of eq. (314g) by $\frac{r^\star}{Y^T}$:

$$\upsilon_B = \upsilon_{B_0} + r^\star \Phi^T \left(\frac{1}{A^T} - \upsilon_{L_0^T} \right) + r^\star \Phi^N \left(\frac{L^N}{A^T L^T} - \upsilon_{L_0^N} \right), \tag{317}$$

where $\upsilon_{B_0} \equiv \frac{r^\star B_0}{Y^T}$, $\upsilon_{L_0^T} \equiv \frac{L_0^T}{Y^T}$, $\upsilon_{L_0^N} \equiv \frac{L_0^N}{Y^T}$. Since we have

$$\upsilon_{V^T} = \frac{\kappa^T \theta^T s^T}{A^T m^T}, \tag{318a}$$

$$\upsilon_{V^N} = \frac{\kappa^N \theta^N s^N}{A^T m^N} \frac{L^N}{L^T}, \tag{318b}$$

where we used the fact that $V^j = \theta^j U^j$ and $U^j = \frac{s^j L^j}{m^j}$ at the steady-state; according to (318) the ratios $\upsilon_{V^T} = \frac{\kappa^T V^T}{Y^T}$ and $\upsilon_{V^N} = \frac{\kappa^N V^N}{Y^T}$ are pinned down by θ^T, θ^N, m^T, m^N, L^N/L^T which are endogenously determined by system (314). Eqs (317) and (318) determine the ratio of net exports to traded output (i.e., υ_{NX}):

$$\upsilon_B - \upsilon_{V^T} - \upsilon_{V^N} \equiv -\upsilon_{NX}. \tag{319}$$

In order to finish the proof that system (314) can be solved for θ^T, θ^N, m^T, m^N, L^T/L^N, P, B, we have to determine analytical expressions of W_R^T, W_R^N, Ψ^T, Ψ^N. The reservation wage in sector j, W_R^j, is defined as the sum of the expected value of a job $m^j \xi^j = \frac{\alpha_W}{1-\alpha_W} \kappa^j \theta^j$ and the unemployment benefit $R^j = \varrho W^j$. The Nash bargaining wage in sector j, W^j, can be rewritten as follows:

$$\begin{aligned} W^j &= \alpha_W \left(\Xi^j + r^\star x^j \right) + (1-\alpha_W) \left(\frac{\alpha_W}{1-\alpha_W} \kappa^j \theta^j + \varrho W^j \right), \\ &= \frac{\alpha_W \left(\Xi^j + r^\star x^j + \kappa^j \theta^j \right)}{1 - (1-\alpha_W) \varrho}. \end{aligned} \tag{320}$$

Plugging (320) into the definition of the reservation wage in sector j, we have:

$$\begin{aligned} W_R^j &= \frac{\alpha_W}{1-\alpha_W} \kappa^j \theta^j + \varrho W^j, \\ &= \frac{\alpha_W}{1-\alpha_W} \kappa^j \theta^j + \varrho \frac{\alpha_W \left(\Xi^j + r^\star x^j + \kappa^j \theta^j \right)}{1 - (1-\alpha_W) \varrho}. \end{aligned} \tag{321}$$

Since $\Xi^T = A^T$ and $\Xi^N = P A^N$, the reservation wage in the traded sector, W_R^T, is a function of θ^T, while the reservation wage in the non traded sector, W_R^N, is a function of θ^N and P. Since $\Psi^j = \Xi^j - W_R^j$, the overall surplus from an additional job in the traded sector, Ψ^T, is a function of θ^T, while the overall surplus from an additional job in the non traded sector, Ψ^N, is a function of θ^N and P.

To begin with, labor market parameters of the traded sector, i.e., the matching efficiency X^T and the recruiting cost κ^T, can be set to target the monthly job finding rate m^T and the labor market tightness θ^T. To show it more formally, we first compute the share of the overall surplus from an additional worker obtained by the firm, $(1-\alpha_W) \Psi^T$, which is equal to the excess of labor productivity over the Nash bargaining wage, $A^T - W^T$; inserting (320), one obtains:

$$\begin{aligned} (1-\alpha_W) \Psi^T &= A^T - \frac{\alpha_W \left(A^T + \kappa^T \theta^T \right)}{1 - (1-\alpha_W) \varrho}, \\ &= \frac{(1-\alpha_W)(1-\varrho) A^T - \alpha_W \kappa^T \theta^T}{1 - (1-\alpha_W) \varrho}. \end{aligned} \tag{322}$$

Plugging (322) into (314a) and using the fact that $f^T = \frac{m^T}{\theta^T}$ allows us to rewrite the vacancy-creation equation in the traded sector as follows:

$$\frac{\kappa^T \theta^T}{m^T} \left(s^T + r^\star \right) = \frac{(1-\alpha_W)(1-\varrho) A^T - \alpha_W \kappa^T \theta^T}{1 - (1-\alpha_W) \varrho}. \tag{323}$$

Equations (314c) and (323) form a separate subsystem which jointly determine θ^T and m^T; parameters κ^T and X^T are set in order to target θ^T and m^T shown in Table 6. It is worthwhile mentioning that while theoretically κ^T and X^T jointly determine θ^T and m^T, we find numerically that θ^T is mostly affected by κ^T while m^T is mostly determined by X^T.

The remaining equations (314b), (314d)-(314g) form a separate subsystem which jointly determine m^N, θ^N, P, L^T/L^N, and υ_{NX}:

$$\frac{\kappa^N\theta^N}{m^N}\left(s^N + r^\star\right) = \frac{\left(1-\alpha_W\right)\left(1-\varrho\right)\left(PA^N + r^\star x^N\right) - \alpha_W \kappa^N\theta^N}{1-\left(1-\alpha_W\right)\varrho}, \tag{324a}$$

$$m^N = X^N\left(\theta^N\right)^{\alpha_V}, \tag{324b}$$

$$\frac{A^T L^T\left(1-\upsilon_{NX}\right)}{A^N L^N\left(1-\upsilon_{G^N}\right)} = \frac{\varphi}{1-\varphi}P^\phi, \tag{324c}$$

$$\frac{L^T}{L^N} = \frac{m^T}{m^N}\frac{m^N+s^N}{m^T+s^T}\left(\frac{W_R^T}{W_R^N}\zeta^N\right)^{\sigma_L}, \tag{324d}$$

$$\upsilon_{NX} = -\left(\upsilon_B - \upsilon_{V^T} - \upsilon_{V^N}\right), \tag{324e}$$

where υ_B, υ_{V^T}, υ_{V^N} are given by eqs. (317), (319), (320), respectively; to rewrite (314b) as (324a), we used the fact that $\left(1-\alpha_W\right)\Psi^N = \frac{(1-\alpha_W)(1-\varrho)\left(PA^N+r^\star x^N\right)-\alpha_W\kappa^N\theta^N}{1-(1-\alpha_W)\varrho}$. Remembering that P determines α_C and LT/LN determines LN/L, parameters κ^N, X^N, φ, ζ^N and initial conditions (B_0, L_0^T, L_0^N) are set in order to target θ^N and m^N (see columns 11 and 7 in Table 6), α_C and L^N/L (see columns 2 and 1 in Table 5), $\upsilon_{NX} \simeq 0$ as we assume that at the initial steady-state, the balance of trade is nil. While theoretically the four parameters and initial conditions are endogenously determined to target θ^N, m^N, α_C, L^N/L and υ_X, we find numerically that θ^N is mostly affected by κ^N, m^T by X^N, α_C by φ, L^N/L by ζ^N, and υ_{NX} by initial conditions.

N.2 Calibration to a Representative OECD Economy

In order to assess the ability of our model to account for the evidence, we proceed in two stages. Since we find analytically that a productivity differential between tradables and non tradables exerts two opposite effects on the relative wage if the elasticity of substitution in consumption between tradables and non tradables is smaller than one, we have to investigate whether the model can generate a decline in the relative wage that is similar to that in the data for the whole sample. To do so, we first calibrate our model to a representative OECD economy. The quantitative exploration of a productivity shock biased toward the traded sector allows us to investigate whether our model can produce:

- a fall in the relative wage regardless of the value of the elasticity of substitution which displays a large dispersion across countries;
- a larger decline in the relative wage in countries where labor markets are mode regulated.

This section provides more details about how we calibrate the model to match the key empirical properties of a representative OECD economy. Our reference period for the calibration of the non tradable share given in Table 5 is running from 1990 to 2007 while labor market parameters have been computed over various periods. Due to the availability of data, we were able to estimate sectoral unemployment rates for 10 European countries and 5 OECD economies as ILO does not provide series for sectoral employment and unemployment for France, the Netherlands, and Norway at a sectoral level. Regarding Korea, while ILO provides data necessary for the computation of sectoral unemployment rates, the OECD does not provide unemployment by duration for this country which prevents the computation of job finding and job destruction rates. Data for the labor markets are described in Table 6.[66]

We first describe the parameters that are taken directly from the data; we start with the preference parameters shown in panel A of Table 23:

- One period in the model is a month.
- The world interest rate, $r^\star$, equal to the subjective time discount rate, β, is set to 0.4%.
- We assume that utility for consumption is logarithmic and thus set the intertemporal elasticity of substitution for consumption, σ_C, to 1.
- We set the elasticity of substitution (in consumption) between traded and non traded goods to 1 in the baseline calibration.[67]

[66]For sectoral unemployment rates, and monthly job finding and job destruction rates, we take the EU-10 unweighed average due to data availability.

[67]Excluding estimates of ϕ for Italy which are negative (see Table 9), column 1 of Table 8 reports consistent estimates for the elasticity of substitution ϕ between traded and non traded goods which average to 0.9. The advantage of setting ϕ to 1 in the baseline scenario is twofold. First, the share of non traded goods in consumption expenditure α_C coincides with the weight of the non traded good in the overall consumption bundle $1-\varphi$ if $\phi = 1$. Second, setting $\phi = 1$ implies that only the labor accumulation channel is (mostly) in effect as the labor market frictions channel almost totally vanish which allows us to highlight the intertemporal effect trigged by the hiring boom.

- Next, we turn to the elasticity of labor supply at the extensive margin which is assumed to be symmetric across sectors. We choose σ_L to be 0.6 in our baseline setting but conduct a sensitivity analysis with respect to this parameter.[68]

We pursue with the non-tradable content of consumption expenditure, employment, government spending displayed in panel B:

- The weight of consumption in non tradables $1-\varphi$ is set to 0.42 to target a non-tradable content in total consumption expenditure (i.e. α_C) of 42%, in line with the average of our estimates shown in the last line of Table 5.
- In order to target a non tradable content of labor of 66% which corresponds to the 18 OECD countries' unweighted average shown in the last line of Table 5, we set ζ^N to 0.18 (see eq. (11)) while ζ^T has been normalized to 1.
- Government spending as a percentage of GDP is set to 20% and we set the non tradable content of government expenditure, i.e., $\omega_{G^N} = \frac{PG^N}{G}$, to 90%.[69]
- We assume that traded firms are 28 percent more productive than non traded firms in line with our estimates; we thus normalize A^N to 1 and set A^T to 1.28;

We describe below the choice of parameters characterizing the labor markets of a typical OECD economy in panel C:

- In line with our estimates shown in the last line of Table 6, we set the rates of separation in the traded (i.e., s^T) and the non traded (i.e., s^N) sector to 1.48% and 1.54% respectively. To capture the U.S. (EU-12) sectoral labor markets, we set s^T and s^N to 2.24% ($s^T = 1.18\%$) and 2.46% ($s^N = 1.25\%$), respectively.
- We set $1-\alpha_V$ to 0.6 in line with the estimates documented by Barnichon [2012] who reports an elasticity of the matching function with respect to unemployed workers of about 0.6.
- As it is common in the literature, we impose the Hosios [1990] condition, and set the worker bargaining power α_W to 0.6 in the baseline scenario.
- To target the labor market tightness for a representative OECD economy in the traded sector, $\theta^T = 0.24$, and in the non traded sector, $\theta^N = 0.34$, we set the recruiting cost to $\kappa^T = 1.482$ and $\kappa^N = 0.575$ in the traded and the non traded sector respectively. To target the sectoral labor market tightness for the US (EU-12), i.e., $\theta^T = 0.43$ ($\theta^T = 0.21$) and $\theta^N = 0.65$ ($\theta^N = 0.30$), respectively, we choose $\kappa^T = 1.333$ ($\kappa^T = 1.535$) and $\kappa^N = 0.476$ ($\kappa^N = 0.597$).
- When calibrating to a representative OECD economy, we set the matching efficiency in the traded (non traded) sector X^T (X^N) to 0.307 (0.262) to target a monthly job finding rate m^T (m^N) of 17.4% (17.0%). A job destruction rate in the traded (non traded) sector s^T (s^N) of 1.48% (1.54%) together with a monthly job finding rate of 17.4% (17.0%) leads to an unemployment rate u^T (u^N) of 7.9% (8.3%) in the traded (non traded) sector. To target a monthly job finding rate m^T for the US (EU-12) in the traded sector and in the non traded sector m^N of 44.4% (12.4%) and 44.0% (12.2%), respectively, in line with the data shown in Table 6, we set the matching efficiency parameters X^T and X^N to 0.620 (0.231) and 0.521 (0.197), respectively. The job destruction rates s^T and s^N are set to 2.2% (1.2%) and 2.4% (1.2%) which leads to an unemployment rate in the traded sector u^T and in the non traded sector u^N of 4.8% (8.7%) and 5.3% (9.3%), respectively.

Finally, we present the parameters that capture the labor market institutions shown in panel D:

- Since the advance notice and the severance payment are both expressed in monthly salary equivalents, we have $x^j = \tau W^j$ with $\tau \geq 0$. Values of τ are shown in the last column of Table 6. For the baseline calibration, we set the firing tax τ to 4.2. When calibrating to the US (EU-12) economy, we set $\tau = 0$ ($\tau = 4.3$) in line with estimates shown in the last column of Table 6. When conducting the sensitivity analysis, we set τ to 13 which corresponds to the highest value for the firing cost.

[68] Using data from the Panel Study of Income Dynamics, Fiorito and Zanella [2012] find that aggregate time-series results deliver an extensive margin elasticity in the range 0.8-1.4, which is substantially larger than the corresponding estimate (0.2-0.3) reported by Chetty, Friedman, Manoli, and Weber [2011]. Using Japanese data, Kuroda and Yamamoto [2008] report a Frisch elasticity on the extensive margin which falls in the range of 0.6 to 0.8 for both sexes. By calibrating a model with endogenous participation decision, Haefke and Reiter [2011] find labor supply elasticities for the baseline case of 0.4 and 0.65 for men and women, respectively.

[69] The market clearing condition for the traded good and the non traded good at the steady-state are $r^\star B + Y^T = C^T + G^T + \kappa^T V^T + \kappa^N V^N$ and $Y^N = C^N + G^N$, respectively.

- Assuming that unemployment benefits are a fixed proportion of the wage rate, i.e., $R^j = \varrho W^j$, with ϱ the replacement rate, we choose a value for ϱ of 52.4%, in line with our estimates shown in Table 6. When calibrating to the US (EU-12) economy, we set $\varrho = 26.1\%$ ($\varrho = 55.9\%$) in line with estimates shown in column 14 of Table 6. When conducting the sensitivity analysis, we set ϱ to 78.2% which corresponds to the highest value for the unemployment benefit replacement rate.

Finally, we choose values for B_0, L_0^T, L_0^N for the ratio of net exports to traded output to be nil at the initial steady-state, i.e., $\upsilon_{NX} \simeq 0$.

N.3 Calibration to Each OECD Economy

In a second stage, we move a step further and compare the predicted values with estimates for each country and the whole sample as well. The initial steady-state of each OECD economy is described by the system (314) that comprises seven equations. To calibrate our model to each OECD economy in our sample, we use the same baseline calibration for each country, except for the elasticity of substitution ϕ between traded and non-traded goods, and labor market parameters which are allowed to vary across economies. More specifically, the elasticity of substitution ϕ between traded and non traded goods is set in accordance with its estimates shown in the first column of Table 8.[70] The parameters which capture the degree of labor market regulation such as the firing cost x, and the replacement rate ϱ are set to their values shown in the last two columns of Table 6. The matching efficiency X^j in sector j is set to target the job finding rate m^j summarized in columns 5 and 7 of Table 6. The job destruction rate s^j is set in accordance to its value reported in columns 6 and 8 of Table 6. Ideally, the recruiting cost κ^j would be set in order to target θ^j; however, the series for job vacancies by economic activity are available for a maximum of seven years and for a limited number of countries. On the contrary, the OECD provides data for job openings (for the whole economy) over the period 1980-2007 allowing us to calculate the labor market tightness, i.e., $\theta = V/U$, for several countries that we target along with the ratio θ^T/θ^N by choosing κ^T and κ^N. Thus, when calibrating the model to each OECD economy, the costs per job vacancy κ^T and κ^N are chosen to target the aggregate labor market tightness θ shown in column 13 and the ratio of sectoral labor market tightness θ^T/θ^N obtained by dividing column 10 by column 11.

When data for sectoral labor market tightness are not available, we target the average value θ^T/θ^N for EU-12 if the country is a member of the European Union, the average value for the US for English-speaking countries (excluding European economies), and average value for the OECD otherwise. When data for job openings are not available at an aggregate level, we first calibrate the model to EU-12 (US, OECD), in particular choosing κ^T and κ^N to target an aggregate labor market tightness θ of 0.12 (0.59, 0.18) and a ratio θ^T/θ^N of 0.75 (0.66, 0.77); then, we set κ^T and κ^N chosen for EU-12 if the country is a member of the European Union, chosen for the US for Canada, and chosen for the OECD otherwise. Finally, because labor market parameters cannot be calculated at a sectoral level for France, the Netherlands and Norway, we assume that the job destruction rate s and the matching efficiency X are identical across sectors and are chosen in accordance with estimates shown in column 6 (or alternatively in column 8) of Table 6 for the former and to target m^j shown in column 5 (or alternatively in column 7) of Table 6 for the latter.

N.4 Correction of the bias to map theoretical results into elasticities estimated empirically

In this section, we compute the bias originating from search frictions varying across sectors which must be accounted for in order to map theoretical results for relative price and relative responses to a productivity differential into elasticities estimated empirically.

The long-run change of the relative price (29) can be rewritten as follows:

$$\begin{aligned}\hat{p} &= \frac{\left(1+\Theta^T\right)\hat{a}^T - \left(1+\Theta^N\right)\hat{a}^N}{\left(\phi+\Theta^N\right)} + \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta^N\right)}, \\ &= \left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\left\{\left(\hat{a}^T-\hat{a}^N\right)+\hat{a}^N\left[1-\left(\frac{1+\Theta^N}{1+\Theta^T}\right)\right]\right\} + \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta^N\right)}. \end{aligned} \tag{325}$$

Because empirically we consider a productivity differential $\hat{a}^T - \hat{a}^N$, to make our estimates comparable with our numerical results, we have to adjust the long-run change in the relative price computed numerically with the following term:

$$\text{bias } \hat{p} = \left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\left[1-\left(\frac{1+\Theta^N}{1+\Theta^T}\right)\right]\hat{a}^N. \tag{326}$$

[70] We also choose the weight of consumption in non tradables $1-\varphi$ to target a non-tradable content in total consumption expenditure (i.e., α_C) for each country in line with our estimates shown in column 2 of Table 5.

Table 23: Baseline Parameters (Representative OECD Economy)

Definition	Value		Reference
	OECD	Sensitivity	
Period of time	month	month	standard
$\hat{a}^T - \hat{a}^N$	1%	1%	shock
A.Preferences			
Subjective time discount rate, β	0.4%	0.4%	equal to the world interest rate
Intertemporal elasticity of substitution for consumption, σ_C	1	1	standard
Elasticity of labor supply at the extensive margin, σ_L	0.6	0.2-1	Fiorito and Zanella [2012]
Elasticity of substitution, ϕ	1	0.6-1.5	our estimates (KLEMS [2011], OECD Economic Outlook)
B.Non Tradable Share			
Weight of consumption in non traded goods, $1-\varphi$	0.42	0.42	set to target $\alpha_C = 42\%$ (United Nations [2011])
Disutility from working and searching for a job, ζ^N	0.15	0.15	set to target $L^N/L = 66\%$ (KLEMS [2011])
Non Tradable content of government expenditure, ω_{G^N}	0.90	0.90	our estimates (OECD [2012b], IMF [2011])
Labor productivity index for the traded sector, A^T	1.28	1.28	our estimates (KLEMS [2011])
C.Labor Market			
Job destruction rate in sector $j = T$, s^T	1.48%	1.18-2.24%	our estimates (ILO)
Job destruction rate in sector $j = N$, s^N	1.54%	1.25-2.46%	our estimates (ILO)
Elasticity of matches w.r.t. U, $1-\alpha_V$	0.6	0.6	Barnichon [2012]
Worker bargaining power, α_W	0.6	0.6	Hosios [1990] condition
Hiring cost in sector $j = T$, κ^T	1.482	1.333-1.535	set to target θ^T
Hiring cost in sector $j = N$, κ^N	0.575	0.476-0.597	set to target θ^N
Matching efficiency in sector $j = T$, X^T	0.307	0.231-0.620	set to target m^T
Matching efficiency in sector $j = N$, X^N	0.262	0.197-0.521	set to target m^N
D.Labor Market Institutions			
Firing cost, x^N	4.2	0-13	our estimates (Fondazione De Benedetti)
Replacement rate, ϱ	52.4%	26.1-55.9%	our estimates (OECD, Benefits and Wages Database)

Subtracting (326) from (325) leads to:

$$
\begin{aligned}
\hat{p}' &= \hat{p} - \text{bias } \hat{p}, &(327)\\
&= \left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\left(\hat{a}^T - \hat{a}^N\right) + \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta^N\right)}, &(328)
\end{aligned}
$$

where we denote by $\hat{p}'$ the value of $\hat{p}$ which has been adjusted with the bias originating from the presence of search frictions which vary across sectors and thus make the elasticity Θ^j of sectoral employment L^j w.r.t. the marginal revenue of labor, Ξ^j, slightly different between sectors. Once the value of $\hat{p}$ has been adjusted with, we can map the deviation in percentage of the relative price of non tradables from its initial steady-state derived analytically into the elasticity of the relative price, γ, estimated empirically:

$$
\begin{aligned}
\gamma &= \frac{\hat{p}'}{\hat{a}^T - \hat{a}^N},\\
&= \left(\frac{1+\Theta^T}{\phi+\Theta^N}\right) + \frac{1}{\left(\phi+\Theta^N\right)}\frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\hat{a}^T - \hat{a}^N}. &(329)
\end{aligned}
$$

Eq. (329) corresponds to eq. (38a) in the main text. The first term on the RHS of eq. (329) corresponds to the effect of a productivity differential $\hat{a}^T - \hat{a}^N$ of 1% on the relative price keeping net exports fixed while the second term captures the impact of the long-run adjustment in net exports caused by rise in productivity of tradables relative to non tradables of 1%.

The same logic applies to the relative wage. The long-run reaction of the relative wage described by (167j) can be rewritten as follows:

$$
\begin{aligned}
\hat{\omega} &= -\left\{\left[\Omega^T - \Omega^N\left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right]\hat{a}^T - \left[\Omega^N - \Omega^N\left(\frac{1+\Theta^N}{\phi+\Theta^N}\right)\right]\hat{a}^N\right\} + \Omega^N\frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\phi+\Theta^N},\\
&= -\left[\Omega^T - \Omega^N\left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right]\left\{\left(\hat{a}^T - \hat{a}^N\right) + \left\{1 - \frac{\left[\Omega^N - \Omega^N\left(\frac{1+\Theta^N}{\phi+\Theta^N}\right)\right]}{\left[\Omega^T - \Omega^N\left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right]}\right\}\hat{a}^N\right\}\\
&\quad + \Omega^N\frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\phi+\Theta^N}. &(330)
\end{aligned}
$$

We have to adjust the long-run change in the relative wage computed numerically with the following term:

$$
\text{bias } \hat{\omega} = -\left[\Omega^T - \Omega^N\left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right]\left\{1 - \frac{\left[\Omega^N - \Omega^N\left(\frac{1+\Theta^N}{\phi+\Theta^N}\right)\right]}{\left[\Omega^T - \Omega^N\left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right]}\right\}\hat{a}^N. \qquad (331)
$$

Subtracting (331) from (330) leads to:

$$
\begin{aligned}
\hat{\omega}' &= \hat{\omega} - \text{bias } \hat{\omega}, &(332)\\
&= -\left[\Omega^T - \Omega^N\left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right]\left(\hat{a}^T - \hat{a}^N\right) + \Omega^N\frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\phi+\Theta^N}, &(333)
\end{aligned}
$$

where we denote by $\hat{\omega}'$ the value of $\hat{\omega}$ which has been adjusted with the bias originating from the presence of search frictions which vary across sectors and thus make Θ^j along with Ω^j slightly different between sectors. Once the value of $\hat{\omega}$ has been adjusted with, we can map the deviation in percentage of the relative wage from its initial steady-state derived analytically into the elasticity of the relative wage, β, estimated empirically:

$$
\begin{aligned}
\beta &= \frac{\hat{\omega}'}{\hat{a}^T - \hat{a}^N},\\
&= -\left[\Omega^T - \Omega^N\left(\frac{1+\Theta^T}{\phi+\Theta^N}\right)\right] + \frac{\Omega^N}{\phi+\Theta^N}\frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\hat{a}^T - \hat{a}^N}. &(334)
\end{aligned}
$$

Eq. (334) corresponds to eq. (38b) in the main text. The first term on the RHS of eq. (334) corresponds to the effect of a productivity differential $\hat{a}^T - \hat{a}^N$ of 1% on the relative wage keeping net exports fixed while the second term captures the impact of the long-run adjustment in net exports caused by rise in productivity of tradables relative to non tradables of 1%. It is worthwhile mentioning that the rise in net exports exerts a negative impact on both $\hat{p}'$ and $\hat{\omega}'$ and thus the term $\frac{\mathrm{d}\ln(1-\upsilon_{NX})}{\hat{a}^T - \hat{a}^N}$ which shows up in eqs. (329) and (334) is negative.

Table 24 gives a sense of the correction term in columns 3 and 6 and compares $\hat{\omega}$ with $\hat{\omega}'$, and $\hat{p}$ with $\hat{p}'$.

Table 24: Comparison of Computed Numerically Responses Before and After Bias Correction

Country	Relative wage response			Relative price response		
	(1)	(2)	(3)	(4)	(5)	(6)
	$\hat{\omega}$	$\hat{\omega}'$	bias $\hat{\omega}$	$\hat{p}$	$\hat{p}'$	bias $\hat{p}$
AUS	0.179	0.172	0.007	1.179	1.166	0.013
AUT	-0.337	-0.318	-0.019	0.691	0.684	0.007
BEL	-0.294	-0.281	-0.013	0.724	0.715	0.009
CAN	0.009	0.015	-0.006	1.017	1.011	0.006
DEU	-0.423	-0.420	-0.003	0.572	0.562	0.010
DNK	-0.527	-0.515	-0.012	0.473	0.468	0.005
ESP	-0.286	-0.261	-0.025	0.760	0.750	0.010
FIN	-0.384	-0.355	-0.029	0.628	0.638	-0.010
FRA	-0.355	-0.346	-0.009	0.650	0.645	0.005
GBR	-0.049	-0.050	0.001	0.956	0.944	0.012
IRL	-0.171	-0.148	-0.023	0.831	0.844	-0.013
ITA	-0.272	-0.266	-0.006	0.729	0.729	0.000
JPN	-0.152	-0.145	-0.007	0.860	0.853	0.007
KOR	-0.685	-0.640	-0.045	0.379	0.373	0.006
NLD	-0.286	-0.280	-0.006	0.711	0.706	0.005
NOR	-0.292	-0.286	-0.006	0.705	0.703	0.002
SWE	0.134	0.144	-0.010	1.161	1.152	0.009
USA	-0.037	-0.035	-0.002	0.972	0.974	-0.002
EU-12	-0.160	-0.149	-0.011	0.855	0.849	0.006
Whole sample	-0.229	-0.218	-0.011	0.783	0.778	0.005

Notes: $\hat{p}$ and $\hat{\omega}$ correspond to deviations in percentage of the relative price and the relative wage from their initial steady-state which are computed numerically following a productivity differential of 1%; we denote by $\hat{p}'$ and $\hat{\omega}'$ the steady-state changes in the relative price and relative wage computed numerically once their values have been adjusted with the bias originating from the presence of search frictions which vary across sectors. Columns 3 and 6 show that magnitude of bias for the relative wage and the relative price which must be subtracted from $\hat{p}$ and $\hat{\omega}$ in order to make elasticities computed numerically directly comparable with β and γ which are estimated empirically.

The numerical computation of the unemployment rate differential is subject to the same bias the relative price and the relative wage. The long-run reaction of the unemployment differential between tradables and non tradables described by (264) can be rewritten as follows:

$$
\begin{aligned}
du^T - du^N &= -\alpha_V \left\{ \left[u^T \left(1-u^T\right) \Sigma^T - u^N \left(1-u^N\right) \Sigma^N \left(\frac{1+\Theta^T}{\phi + \Theta^N} \right) \right] \hat{a}^T - u^N \left(1-u^N\right) \Sigma^N \left(\frac{\phi - 1}{\phi + \Theta^N} \right) \hat{a}^N \right\} \\
&+ \alpha_V u^N \left(1-u^N\right) \Sigma^N \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi + \Theta^N\right)}, \\
&= -\alpha_V \Delta^T \left\{ \hat{a}^T - \hat{a}^N + \hat{a}^N \left[1 - \frac{u^N\left(1-u^N\right)\Sigma^N}{\Delta^T} \left(\frac{\phi-1}{\phi+\Theta^N} \right) \right] \right\} \\
&+ \alpha_V u^N \left(1-u^N\right) \Sigma^N \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi + \Theta^N\right)}, \qquad (335)
\end{aligned}
$$

where we set

$$
\Delta^T = \left[u^T \left(1-u^T\right) \Sigma^T - u^N \left(1-u^N\right) \Sigma^N \left(\frac{1+\Theta^T}{\phi+\Theta^N} \right) \right]. \qquad (336)
$$

We have to adjust the long-run change in the relative wage computed numerically with the following term:

$$
\text{bias}\left(du^T - du^N\right) = -\alpha_V \Delta^T \hat{a}^N \left[1 - \frac{u^N\left(1-u^N\right)\Sigma^N}{\Delta^T} \left(\frac{\phi-1}{\phi+\Theta^N} \right) \right]. \qquad (337)
$$

Subtracting (337) from (335) leads to:

$$
\begin{aligned}
\left(du^T - du^N\right)' &= \left(du^T - du^N\right) - \text{bias}\left(du^T - du^N\right), \qquad (338) \\
&= -\alpha_V \Delta^T \left(\hat{a}^T - \hat{a}^N\right) + \alpha_V u^N \left(1-u^N\right) \Sigma^N \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\left(\phi+\Theta^N\right)}, \qquad (339)
\end{aligned}
$$

where we denote by $\left(du^T - du^N\right)'$ the value of $du^T - du^N$ which has been adjusted with the bias originating from the presence of search frictions which vary across sectors and thus make Θ^j along with Σ^j slightly different between sectors. Once the value of $du^T - du^N$ has been adjusted with, we can map the unemployment rate differential derived analytically into its response, σ, estimated empirically:

$$
\begin{aligned}
\sigma &= \frac{\left(du^T - du^N\right)'}{\hat{a}^T - \hat{a}^N}, \\
&= -\alpha_V \Delta^T + \alpha_V u^N \left(1-u^N\right) \frac{\Sigma^N}{\phi+\Theta^N} \frac{\mathrm{d}\ln\left(1-\upsilon_{NX}\right)}{\hat{a}^T - \hat{a}^N}. \qquad (340)
\end{aligned}
$$

Eq. (340) corresponds to eq. (39) in the main text. Eq. (340) is used to compute numerically the response of the unemployment rate differential to higher relative productivity of tradables by 1%, as reported in column 8 of Table 8. Column 9 of Table 8 shows results when we let σ_L tend toward infinity. When we abstract from labor mobility costs, the unemployment rate differential reduces to eq. (312). In this case, changes in u^T relative to u^N are only driven by differences in search frictions between sectors.

References

Bai, Jushan, and Serena Ng (2002) Determining the Number of Factors in Approximate Factor Models. *Econometrica*, 70(1), pp. 191-221.

Breitung Jörg (2000) The Local Power of Some Unit Root Tests for Panel Data, in B. Baltagi (ed.), Advances in Econometrics, vol. 15: Nonstationary Panels, Panel Cointegration, and Dynamic Panels, Amsterdam: JAI Press, 161-178.

Cashin, Paul and John C. McDermott (2003) Intertemporal Substitution and Terms-of-Trade Shocks. *Review of International Economics*, vol. 11(4), 604-618.

Chang, Yoosoon (2002) Nonlinear IV Unit Root Tests in Panels with Cross-Sectional Dependency. *Journal of Econometrics*, 110, pp. 261-292.

Choi In (2001) Unit Root Tests for Panel Data. *Journal of International Money and Finance*, 20, pp. 249-272.

Cordoba (de), Gonzalo Fernandez and Timothy J. Kehoe (2000) Capital Flows and Real Exchange Rate Fluctuations Following Spain's Entry into the European Community. *Journal of International Economics*, 51(1), 49-78.

Elsby, Michael W. L., Bart Hobijn and Aysegül Sahin (2013) Unemployment Dynamics in the OECD. *The Review of Economics and Statistics*, vol. 95(2), pp. 530-548.

Hadri, Kaddour (2000) Testing for Unit Roots in Heterogeneous Panel Data. *Econometrics Journal*, 3, pp. 148-161.

Haefke, Christian and Michael Reiter (2011) What Do Participation Fluctuations Tell Us About Labor Supply Elasticities? *IZA Discussion Paper*, No. 6039.

Hosios, Arthur J. (1990) On the Efficiency of Matching and Related Models of Search and Unemployment. *The Review of Economic Studies*, 57(2), pp. 279-98.

Im, Kyung So, Hashem M. Pesaran and Yongcheol Shin (2003) Testing for Unit Roots in Heterogeneous Panels. *Journal of Econometrics*, 115, pp. 53-74.

Kuroda, Sachiko and Isamu Yamamoto (2008) Estimating Frisch Labor Supply Elasticity in Japan. *Journal of the Japanese and International Economies*, 22(4), pp. 566-585.

Levin Andrew, Chien-Fu Lin and Chia-Shang James Chu (2002) Unit Root Test in Panel Data: Asymptotic and Finite Sample Properties. *Journal of Econometrics*, 108, pp. 1-24.

Maddala, Gangadharrao S. and Shaowen Wu (1999) A Comparative Study of Unit Root Tests with Panel Data and a New Test. *Oxford Bulletin of Economics and Statistics*, 61, pp. 631-652.

Mark, Nelson C. and Donggyu Sul (2003) Vector Estimation by Panel DOLS and Long-run Money Demand. *Oxford Bulletin of Economics and Statistics*, 65(5), pp. 655-680.

Mendoza, Enrique, G. (1995) The Terms of Trade, Real Exchange Rate and Economic Fluctuations. *International Economic Review*, 36, pp. 101-137.

Morshed, Mahbub A. K. M., and Stephen J. Turnovsky (2004) Sectoral Adjustment Costs and Real Exchange Rate Dynamics in a Two-sector Dependent Economy. *Journal of International Economics*, 63, pp. 147-177.

Nickell, Stephen, Luca Nunziata and Wolfgang Ochel (2005) Unemployment in the OECD since the 1960s: What do we Know? *Economic Journal*, 115, pp. 1-27.

Ostry, Jonathan, and Carmen M. Reinhart (1992) *Private Saving and Terms of Trade Shocks: Evidence from Developing Countries*, IMF Staff Papers 39(3), pp. 495-517.

Pedroni, Peter (1999) Critical Values for Cointegration Tests in Heterogeneous Panels with Multiple Regressors. *Oxford Bulletin of Economics and Statistics*, 61, pp. 653-670.

Pedroni, Peter (2004) Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests with an Application to the PPP Hypothesis. *Econometric Theory*, 20, pp. 597-625.

Pesaran, Hashem M. (2007) A Simple Panel Unit Root Test In The Presence Of Cross Section Dependence. *Journal of Applied Econometrics*, 22(2), pp. 265-312.

Pesaran, Hashem M. and Ron P. Smith (1995) Estimating Long-run Relationships from Dynamic Heterogeneous Panels. *Journal of Econometrics*, 68, pp. 79-113.

Pesaran, Hashem M., Yongcheol Shin and Ron P. Smith (1999) Pooled Mean Group Estimation of Dynamic Heterogeneous Panels.*Journal of the American Statistical Association*, 94, pp. 621-634.

Shimer, Robert (2005) The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review* 95(1), pp. 25-49.

Stockman Alan C. and Linda L. Tesar (1995) Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements. *American Economic Review* 85(1), pp. 168-185.