An alternative model of International Migration: Endogenous Two Sided Borders and Optimal Legal Systems

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Abstract

In a 2-country and 3-period OLG model with education, we study the impact on international migration of the two sided characteristics of borders. Individuals must first "leave" their home country before "entering" the destination country. Indeed, each social planner chooses the static welfare optimal level of education, consumption, labor and capital. A unique migration flow is compatible with the market steady-state equilibrium and the maximizing social welfare solution. Difference in education generates differences in steady-state capital per capita. Consequently, both price differentials and incentives for illegal migration exist. Application to real world cases is provided[1].

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Key words: International Migration, Overlapping Generations Models, Immigration Law and Legal Systems.

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1 INTRODUCTION

Australia, Canada, the USA and New Zealand have all been founded by migration. However, nowadays, they have in common the implementation of migration programs to determine who is eligible to migrate. In this paper an optimal legal system of migration represents the optimal migration policy based on economic criteria a given country implements. There are various legal systems of migration: those which are explicit and those which are implicit. It is well understood that most countries do not allow every immigrant to enter their country (by explicit selection devices), but is not well understood that many country refrain emigrants from leaving their country (by implicit stay-home incentives), even if borders are legally open.

The objective of the paper is to provide a rationale to these optimal legal systems of international migration. Since countries adopt various migration criteria, among which education is an important one for the social planner, this paper proposes a 3-period overlapping generations model in which individuals train when they are young and work when they are adults. Finally, when old, they optimally choose their retirement date. Such a model allows conclusions on both growth and welfare prior to migration (in autarky) and post-migration (when borders are open on their two sides).

The motivation for such a framework directly comes from empirical facts. Australia and Canada have organized an explicit optimal legal system of migration based on points to be accumulated by any would-be migrant. If they succeed in overpassing the threshold, would-be migrants are allowed to settle. A key feature of these legal systems of migration is that the threshold figure is not permanent. It is optimally and legally set by governments given the specific economic needs of the country.

Many developing countries (especially in Africa) have chosen an implicit legal system of migration. For instance they prefer to invite foreign professors to teach in the country rather than to let students migrate to be educated abroad. Some others have chosen to finance home PhD programs with a very high research allowance for students, or some other charge very low interest rates on education loans. The main reason for such incentives to stay home is to refrain as much as possible student migration. It costs less to pay foreign professors to teach (and students to learn) than to loose human capital in the future. Indeed, if a student is successful on the foreign labor market, he never return-migrates.In this paper, we describe the two-sided border by the fact that individuals must first "leave" their home country, before "entering" the destination country.

The literature can broadly be classified between empirical and theoretical studies. Withers (1987), for example, empirically shows that the skill level of migrants arriving in Australia has tended to increase in the postwar period at a more rapid rate than that of the resident population as a whole. In other words, the effectiveness of the points system in raising the mean skill level of immigrants depends on the existence of a large demand for visas to enter Australia. A study of the worldwide market for skilled immigrants by Cobb-Clark and Connolly (1997) suggests that the skills of those wanting to enter Australia are influenced by a range of factors, some of which are internal to Australia (e.g., economic conditions), while others are external (e.g., immigration policies of other countries). These factors are likely to have more impact on immigrant quality than the points system. The points system used in a number of the components of the immigration program in Australia offers a mechanism of selecting immigrants who will adjust rapidly to the circumstances of the Australian labor market and who will bring benefits to Australia. Variations in immigrant quality in Australia are likely to be affected more by conditions in the world-wide market for skilled immigrants than by the Australian points system. Understanding the worldwide market for skilled immigrants and determining the net benefits to Australia of different types of immigrants are important issues for consideration.

Theoretical literature concentrates on endogenous quotas of migrants through a voting system which allows governments to implement immigration policies. Epstein and Nitzan (2005) analyze the endogenous determination of a migration quota, viewing it as an outcome of a two-stage political struggle between two interest groups: those in favor and those against the proposed migration quota. Theoretical effects of the government policy depend on whether there is lobbying between those natives who agree and those who disagree with the proposal of a quota of migrants. Mayrs (2010) derives a general equilibrium model with overlapping generations where natives require a compensating wage differential for working in one sector rather than in another. Price and wage effects of immigration are analyzed on natives: the young, working in one of two sectors, and the old. The outcome of a majority voting on immigration is determined bt a given sector as well as the social optimum. The main findings are: i) the old determine any majority voting outcome of non-zero immigration into both sectors; ii) socially optimal immigration is smaller than or equal to the majority voting outcome,;and iii) immigration is not necessarily a substitute for native mobility across sectors. Candau (2011) analyzes how trade liberalization and immigration can potentially affect the welfare of native skilled and unskilled workers and how this expected impact plays on immigration policy. The novelty resides in the attempt to set up endogenous immigration restrictions by integrating swing voters in a model of geographical economics with two kinds of immobile workers (skilled and unskilled). It is shown that trade liberalization can lead the winner candidate to increase the quota on immigration.

Mayr (2012) determines occupation-specific immigration quotas in a political economy framework with endogenous prices and compares them to the social optimum. It shows that positive quotas for specific occupations can be the political outcome, even when total welfare effects of immigration are negative. Two of the main findings are that the (unique) voting outcome on immigration quotas is i) positive if workers are immobile across occupations and ii) negative (positive) for occupations where the native labor supply is sufficiently large (small), if workers are mobile across occupations.

Contrary to the brain drain literature, which states that high skilled immigrants benefit the destination country since they generate higher earning profiles, this paper suggests that a legal system of migration that refrains permanent migrations in order to educate young individuals in their home country benefits the welfare of the local country. Our model departs from the literature relative to endogenous quotas. Indeed, we propose an alternative way to obtain the optimal flow of migrants a country is willing to accept. Each social planner chooses the number of migrants that maximizes the welfare of his own country.

Since some countries adopt implicit legal systems of migration to avoid brain waste, we choose to build a model where only young individuals are allowed to permanently migrate. The reason is that sending countries loose their human capital throughout the migrant's life-cycle, as mentioned above. Subsection 8.4 discusses the possibility of return migrations. For now, immigrants will obtain a high education degree in the destination country and get the return to education in this country. The post-migration steady-state equilibrium is a function of the flow of migrants, which is an instrument for the domestic migration policy. Since countries differ with respect to their return to education, incentives for migration exist. The way migration ceases is not a pure market mechanism but the result of the social planner's decision. The social planner chooses the level of migrants that leads his country to the post-migration static welfare optimum. Due to differences in the return to education, social planners choose different migration flows. Closing borders also means preventing market equalization of prices in the long run, contrary to Galor (1986).

Most of the time, the two-sided nature of border crossing is not theoretically analyzed, but empirically, legal migration systems take into account this double reality. In that case, even if countries have the same way of selecting migrants, they do not select the same level of migration flow. The emergence of the asymmetry of borders across countries is due to differences in the return to education. When one of the two countries elicits a higher return to education than the other, the flow of migrants optimally chosen by this country is not equal to the one chosen by the other country. In the case where one of the two countries wants to send more migrants than the other one is ready to accept, incentives for illegal migration exist in post-migration equilibrium.

Section 2 presents the model, Section 3 the temporary equilibrium of the economy in autarky, and Section 4 the autarkic perfect-foresight inter-temporal equilibrium. Section 5 is devoted to international migration. Section 6 makes the link between theory and applications. Section 7 presents the explicit real optimal legal system of Canada and Australia. Possible extensions of the model are presented in Section 8 prior to Section 9, which concludes.

2 THE MODEL

Consider a perfectly competitive international world with no uncertainty, with two countries, $i = 1, 2$, where economic activity in each country is operated over infinite discrete time, such that $t = 0, 1, 2, \dots, \infty$. In every period, a new generation of individuals N_t^i is born and is supposed to be constant over time. Consequently, in autarky $N_{t+1}^i = N_t^i = N^i$, where $N^i = 1 > 0$ for simplicity[2]. In each country, a single tradable good is produced using three factors of production: the capital, the adult efficient labor, and the old efficient labor.Capital depreciates fully after one period. Individuals and firms make rational decisions under perfect foresights.

2.1 THE INDIVIDUAL

Individuals are identical within as well as across generations. Individuals born in country $i = 1, 2$ live three periods, each of them being normalized to unity. In the first period when young, they borrow E_{t-1}^i on their future savings S_t^i when they are an adult in order to train at the total cost ae_{t-1}^i, where a is the price of one unit of education e_{t-1}^i in country i. Education is an individual's choice. In the second period, adult supply ℓ_t^i subunits of labor are paid at the given competitive wage w_t^i so that the total earning of an adult is $w_t^i \ell_t^i (e_{t-1}^i)^{\varepsilon^i}$, where $0 < \varepsilon^i < 1$ is the country specific return to education. They consume c_t^i, and the rest $S_t^i = s_t^i + R_t^i E_{t-1}^i$ is saved, where $R_t^i = 1 + r_t^i$ is the given competitive factor of interest, and r_t^i is the competitive interest rate in country i during period t. The total saving S_t^i is devoted to s_t^i for the second period, and $R_t^i a e_{t-1}^i$ is used for reimbursing

[2]Section 8 relaxes this assumption.

the first period training. In the third period when old, individuals consume d^i_{t+1}. This consumption is financed through the return on the second period savings $R^i_{t+1}s^i_t$ and their third period labor supply. Old labor supply is paid at the given competitive wage p^i_{t+1} during $\ ^i_{t+1}$ subunits of time, where R^i_{t+1} and p^i_{t+1} are perfectly anticipated. Note that the third period consumption is a function of the level of education via the total savings S^i_t. Rational individuals maximize their log-linear utility function and solve the following program where   is the time preference, and   is the preference for leisure:

$$\max_{c^i_t, e^i_{t-1}, \ell^i_t, d^i_{t+1}, \theta^i_{t+1}} \log c^i_t + \quad \log(1 - \ell^i_t) + \quad \log d^i_{t+1}(e^i_{t-1}; \ ) + \quad \log(1 - \ ^i_{t+1})$$

subject to:

$$\begin{cases} ae^i_{t-1} & = & E^i_{t-1}; \\ c^i_t + s^i_t + R^i_t ae^i_{t-1} & = & w^i_t \ell^i_t (e^i_{t-1})^{\varepsilon^i}; \\ d^i_{t+1} & = & R^i_{t+1}s^i_t + p^i_{t+1} \ ^i_{t+1}. \end{cases} \tag{1}$$

2.2 THE FIRM

In each country $i = 1; 2$, production occurs within a period according to a constant return to scale production technology, which is stationary over time. The output Q^i_t of the single goods is produced by a representative competitive firm at time t with three factors of production, capital K^i_t, young efficient labor $N^i_t = \ell^i_t(e^i_{t-1})^{\varepsilon^i}$, and old efficient labor $\Theta^i_t = N^i_{t-1} \ ^i_t = \ ^i_t$. The production technology is given by the following Cobb-Douglas production function $Q^i_t = K^{i\,1-\sigma-\nu}_t \left[\ell^i_t(e^i_{t-1})^{\varepsilon^i}\right]^\sigma \ ^{i\nu}_t$, where $0 < \ < 1$ is the elasticity of young efficient labor and $0 < \ < 1$ is the elasticity of old efficient labor. The rational representative competitive firm maximizes its profit

$$\max_{K^i_t, \ell^i_t, \theta^i_t} \ ^i_t = K^{i\,1-\sigma-\nu}_t \left[\ell^i_t(e^i_{t-1})^{\varepsilon^i}\right]^\sigma \ ^{i\nu}_t - w^i_t \ell^i_t (e^i_{t-1})^{\varepsilon^i} - p^i_t \ ^i_t - R^i_t K^i_t. \tag{2}$$

We now turn to the study of the temporary equilibrium, which is the solution of the two previous problems, the one of the individual and the one of the firm.

3 TEMPORARY EQUILIBRIUM OF THE ECONOMY IN AUTARKY

The objective of this section is to determine the temporary equilibrium of the economy in autarky. For doing this, let us recall the definition.

DEFINITION **1** *In country i, the temporary equilibrium of period t is a competitive equilibrium given perfect anticipations on prices, R^i_{t+1} and p^i_{t+1}, and given past variables, s^i_{t-1} and $I^i_{t-1} = N^i_{t-1}s^i_{t-1}$, or equivalently $K_t = s_{t-1}$.*

Consider the individual's problem 1. Solving the first period budget constraint for s^i_t and replacing its new expression into the second period budget constraint gives:

$$d^i_{t+1} = R^i_{t+1}\left[w^i_t \ell^i_t (e^i_{t-1})^{\varepsilon^i} - R^i_t ae^i_{t-1} - c^i_t\right] + p^i_{t+1} \ ^i_{t+1}. \tag{3}$$

Replacing (3) into the objective function, individuals solve the following program:

$$\max_{c^i_t, e^i_t, \ell^i_t, \theta^i_{t+1}} \log c^i_t + \quad \log(1 - \ell^i_t) + \quad \log\left[R^i_{t+1}\left[w^i_t \ell^i_t (e^i_{t-1})^{\varepsilon^i} - R^i_t ae^i_{t-1} - c^i_t\right] + p^i_{t+1} \ ^i_{t+1}\right]$$

$$+ \quad \log(1 - \ {}^{i}_{t+1}):$$

The first order condition gives the following relations:

$$\frac{1}{c_t^i} = \frac{R_{t+1}^i}{d_{t+1}^i}; \tag{4}$$

$$"^i w_t^i \ell_t^i (e_{t-1}^i)^{\varepsilon^i - 1} = R_t^i a \iff (e_{t-1}^i)^{\varepsilon^i} = \frac{R_t^i a e_{t-1}^i}{"^i w_t^i \ell_t^i}; \tag{5}$$

$$\frac{}{1 - \ell_t^i} = \frac{R_{t+1}^i w_t^i (e_{t-1}^i)^{\varepsilon^i}}{d_{t+1}^i}; \tag{6}$$

$$\frac{p_{t+1}^i}{d_{t+1}^i} = \frac{}{1 - \ {}^{i}_{t+1}}: \tag{7}$$

A rational competitive firm solves problem 2

$$\max_{K_t^i, \ell_t^i, \theta_t^i} \ {}^{i}_{t} = K_t^{i\,1-\sigma-\nu} \left[\ell_t^i (e_{t-1}^i)^{\varepsilon^i}\right]^{\sigma} \ {}^{i\nu}_{t} - w_t^i \ell_t^i (e_{t-1}^i)^{\varepsilon^i} - p_t^i \ {}^{i}_{t} - R_t^i K_t^i:$$

The first order condition is:

$$(1 - \quad - \quad) Q_t^i = R_t^i K_t^i; \tag{8}$$

$$Q_t^i = w_t^i \ell_t^i (e_{t-1}^i)^{\varepsilon^i}; \tag{9}$$

$$Q_t^i = p_t^i \ {}^{i}_{t}: \tag{10}$$

LEMMA **1** *In temporary equilibrium, the adult efficient labor supply is constant, and the old efficient labor supply is also constant. We have* $\ell_{t+1}^i = \ell_t^i = \ell^i$ *and* ${}^{i}_{t+1} = {}^{i}_{t} = {}^{i}$*, where*

$$\ell^i = \frac{1 - \quad + \quad (1 - \quad - \quad)}{(1 + \quad (1 - "^i))(1 - \quad) + \quad (1 - \quad - \quad)}; \tag{11}$$

$${}^{i} = \frac{}{(1 - \quad) + }: \tag{12}$$

The proof of Lemma 1 is given in Appendix A.

Using (5), (8), and (9), we have

$$e_{t-1}^i = \frac{"^i \quad K_t^i}{(1 - \quad - \quad) a} \iff e_t^i = \frac{"^i \quad K_{t+1}^i}{(1 - \quad - \quad) a}: \tag{13}$$

PROPERTY **1** *The level of education is an increasing linear function of capital and of the returns to education, as well as a decreasing function of the education cost,* a*.*

4 THE AUTARKIC PERFECT-FORESIGHT INTERTEMPORAL EQUILIBRIUM

The perfect-foresight inter-temporal equilibrium with constant population growth is obtained with the capital dynamics $K^i_{t+1} = s^i_t$.

LEMMA **2** *The dynamics of the economy are convergent*

$$K^i_{t+1} = \frac{(1- \quad - \quad)}{1- \quad + \quad (1- \quad - \quad)} \quad (1 - ''^i)^{`i\sigma} \left[\frac{''^i}{(1- \quad - \quad)a}\right]^{\varepsilon^i\sigma} \quad {}^{i\nu} K_t^{i\,1-(1-\varepsilon^i)\sigma-\nu}:$$

The steady-state equilibrium is unique

$$\overline{K}^i = \left[\frac{(1- \quad - \quad) \ (1 - ''^i)}{1- \quad + \quad (1- \quad - \quad)} \left[\frac{''^i}{a(1- \quad - \quad)}\right]^{\varepsilon^i\sigma} \quad {}^{`i\sigma} \quad {}^{i\nu}\right]^{\frac{1}{(1-\varepsilon^i)\sigma+\nu}}:$$

Proof. By Lemma 1, whatever the generation, efficient labor is constant over time so that the production of the current period t is

$$Q_t = K_t^{i\,1-\sigma-\nu}({`^i} e^i_{t-1}{}^{\varepsilon^i})^{\sigma} \quad {}^{i\nu}:$$

Using $K^i_{t+1} = s^i_t$ the dynamics of the economy are

$$K^i_{t+1} = w^i_t {`^i_t} (e^i_{t-1})^{\varepsilon^i} - R^i_t a e^i_{t-1} - c^i_t:$$

Using the first order condition of the firm (9) and the first order condition of the individuals ((5) and (32)) (see Appendix A), we have

$$\frac{1- \quad + \quad (1- \quad - \quad)}{(1- \quad - \quad)} K^i_{t+1} = \quad (1 - ''^i) Q^i_t;$$

$$K^i_{t+1} = \frac{(1- \quad - \quad)}{1- \quad + \quad (1- \quad - \quad)} \quad (1 - ''^i) Q^i_t:$$

Replacing the production by its expression, we have

$$K^i_{t+1} = \frac{(1- \quad - \quad)}{1- \quad + \quad (1- \quad - \quad)} \quad (1 - ''^i)({`^i} e^i_{t-1}{}^{\varepsilon^i})^{\sigma} \quad {}^{i\nu} K_t^{i\,1-\sigma-\nu}:$$

Using (13) in $(e^i_{t-1})^{\varepsilon^i}$, we have:

$$K^i_{t+1} = \frac{(1- \quad - \quad)}{1- \quad + \quad (1- \quad - \quad)} \quad (1 - ''^i)^{`i\sigma} \left[\frac{''^i \quad K^i_t}{(1- \quad - \quad)a}\right]^{\varepsilon^i\sigma} \quad {}^{i\nu} K_t^{i\,1-\sigma-\nu}:$$

Isolating K^i_t, the dynamics of the economy are convergent

$$K^i_{t+1} = \frac{(1- \quad - \quad)}{1- \quad + \quad (1- \quad - \quad)} \quad (1 - ''^i)^{`i\sigma} \left[\frac{''^i}{(1- \quad - \quad)a}\right]^{\varepsilon^i\sigma} \quad {}^{i\nu} K_t^{i\,1-(1-\varepsilon^i)\sigma-\nu}: \qquad (14)$$

The steady-state equilibrium is unique

$$\overline{K}^i = \left[\frac{(1- \quad - \quad) \ (1 - ''^i)}{(1- \quad) + \quad (1- \quad - \quad)} \left[\frac{''^i}{a(1- \quad - \quad)}\right]^{\varepsilon^i\sigma} \quad {}^{`i\sigma} \quad {}^{i\nu}\right]^{\frac{1}{(1-\varepsilon^i)\sigma+\nu}}: \qquad (15)$$

Note that the steady-state capital per worker is a quasi-concave function of $''^i$. This will be important for the next section. □

5 International Migration

Let us now consider that there are two countries, $i = 1; 2$. Countries are solely characterized by a difference in the return to education in the production function. We assume that the following inequality $\varepsilon^1 > \varepsilon^2$ holds for the rest of the theoretical analysis. There are no other differences between countries. In country 2 the productivity of education is higher than in country 1, since $\varepsilon^i \in [0; 1]$.

5.1 Incentives for Permanent International Migration

Suppose that labor is permitted to migrate internationally. Let us assume that only the young can permanently migrate. Migrants spend their education time, their working time as well as their leisure or their retirement time over the three periods in the immigration country. The borders between countries are supposed to be opened at time $t - 1 = 0$.

Proposition **1** *As long as* $\log \left[\frac{\varepsilon^2 e_1^1}{\varepsilon^1 e_1^2}\right] < \log \left[\frac{Q_2^2}{Q_2^1}\right]$, *international migration is unilateral. Rational individuals born in country i have an incentive for permanent migration in country j, where $i \neq j$.*

Proposition 1 shows that permanent migration occurs if the utility of the ratio of the marginal returns to education of country 1 over country 2 is less than the discounted utility of the ratio of the production of country 2 over country 1[3]

Proof. Rational individuals born in country 1 have an incentive for permanent migration in country 2 if their indirect utility evaluated at the steady-state price system of country 2 over their life-cycle is higher than their indirect utility evaluated at the steady-state prices of country 1. The condition is:

$$\log c_1^1 + \log(1 - \ell_1^1) + \log d_2^1 < \log c_1^2 + \log(1 - \ell_1^2) + \log d_2^2:$$

Note that we know from the previous sections that the labor supply is an increasing function of the return to education, see (33) in Appendix A, so that we have the following relationship

$$\log(1 - \ell_1^1) < \log(1 - \ell_1^2):$$

We now prove that

$$\log c_1^1 + \log d_2^1 < \log c_1^2 + \log d_2^2;$$

$$\log \left[\frac{c_1^1}{c_1^2}\right] < \log \left[\frac{d_2^2}{d_2^1}\right]:$$

Using relation (32) in Appendix A

$$c_1^i = \frac{1 -}{(1 - \; - \;)} K_2^i;$$

and using (13), we have

$$K_2^i = \frac{a(1 - \; - \;)}{\varepsilon_i} e_1^i;$$

which we put into the previous consumption relation. We now have

$$c_1^i = \left[\frac{1 -}{}\right] \left[\frac{a}{\varepsilon_i}\right] e_1^i:$$

[3]From Proposition 1, one can view education as a second order individual' motive of migration. In other words, those who are less educated prefer to migrate in order to eat more.

Replace these expressions into the condition relative to the incentives for permanent migration

$$\log\left[\frac{\varepsilon^2 e_1^1}{\varepsilon^1 e_1^2}\right] < \log\left[\frac{d_2^2}{d_2^1}\right].$$

Using relation (31) in Appendix A, we have

$$\log\left[\frac{\varepsilon^2 e_1^1}{\varepsilon^1 e_1^2}\right] < \log\left[\frac{Q_2^2}{Q_2^1}\right].$$

As long as $\varepsilon^1 e^2 > \varepsilon^2 e^1$, the left hand side is always negative so that the condition is satisfied, considering that in the right hand side, the ratio of production is greater than one[4]. □

5.2 Dynamics with permanent International Migration

Subsection 5.2 is devoted to the study of the dynamics of capital in country 2 and country 1. Without loss of generality, we consider that incentives for migration are directed from country 1 to country 2. In this model, only the young are permitted to permanently migrate from country 1 to country 2. In steady-state equilibrium, period $t-1=0$, borders are open. A fraction m^i of the young is allowed to migrate. As it will be shown, m^i may be positive or negative depending on the direction of the incentives for international migration. Consequently, according to the previous Subsection 5.1, $m^1 < 0$ characterizes the fact that individuals emigrate from country 1, while $m^2 > 0$ characterizes the fact that individuals immigrate in country 2.

Since after migration individuals are identical in each country — they train in the home country if they do not migrate, or they train abroad if they migrate — in a given period $t \geq 2$, the population in country 2 is $L_t^2 = \ell_t^2 + m^2\ell_t^1 = (1+m^2)\ell_t^2$, while the population in country 1 is $L_t^1 = (1-m^1)\ell_t^1$. Consequently, in each country, efficient labor is defined as $L_t^2 e_{t-1}^{\varepsilon^2} = (1+m^2)\ell_t^2 e_{t-1}^{\varepsilon^2}$ and $L_t^1 e_{t-1}^{\varepsilon^1} = (1-m^1)\ell_t^1 e_{t-1}^{\varepsilon^1}$. The production function of country 2 is

$$\begin{aligned} Q_t^2 &= (K_t^2)^{1-\sigma-\nu}(1+m^2)^{\sigma}(\ell_t^2 e_{t-1}^{\varepsilon^2})^{\sigma}(1+m^2)^{\nu}\;{}_t^{\nu} \\ \iff\; Q_t^2 &= (1+m^2)^{\sigma+\nu}(K_t^2)^{1-\sigma-\nu}(l_t^2 e_{t-1}^{\varepsilon^2})^{\sigma}\;{}_t^{\nu}. \end{aligned}$$

The production function of country 1 is

$$\begin{aligned} Q_t^1 &= (K_t^1)^{1-\sigma-\nu}(1-m^1)^{\sigma}(\ell_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}(1-m^1)^{\nu}\;{}_t^{\nu} \\ \iff\; Q_t^1 &= (1-m^1)^{\sigma+\nu}(K_t^1)^{1-\sigma-\nu}(l_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}\;{}_t^{\nu}. \end{aligned}$$

[4] We can easily prove that such a situation exists. Indeed, suppose that $\varepsilon^1 > \varepsilon^2$ and that at the same time $\partial K/\partial\varepsilon^i < 0$ which occurs for high ε^2 since the steady-state capital per worker is a quasi concave function of ε^i. Using (36), the level of education e^i is a concave function of ε^i so that we have $e^2 > e^1$. Consequently $\varepsilon^1 e^2 > \varepsilon^2 e^1$ is satisfied. It is sufficient to note that the production is also a concave function of ε^i so that $\varepsilon^2 > \varepsilon^1$ is equivalent to $Q_2^2 > Q_2^1$, and the right hand side is positive. The inequality holds. One can also redo the same reasoning in the increasing part of the steady-state capital per worker by assuming $\varepsilon^2 > \varepsilon^1$ so that $Q_2^2 > Q_2^1$. Moreover, there exists many cases for which $\varepsilon^1 e^2 > \varepsilon^2 e^1$ is possible, especially when the difference in the return in education is high enough, $\varepsilon^2 - \varepsilon^1 > \alpha$ a positive number. Consequently, the same type of result arises in the increasing part of the steady-state capital per worker.

Note that there are no indexes on the old efficient labor since whatever the country, old efficient labor supply is the same. A rational firm in country $i = 1; 2$ maximizes its profit,

$$\max_{K_t^2,\ell_t^2,\theta_t^2} (1+m^2)^{\sigma+\nu}(K_t^2)^{1-\sigma-\nu}(\ell_t^2 e_{t-1}^{\varepsilon^2})^{\sigma}\theta_t^{\nu} - w_t^2(1+m^2)\ell_t^2 e_{t-1}^{\varepsilon^2} - p_t^2(1+m^2)\theta_t^2 - R_t^2 K_t^2;$$

$$\max_{K_t^1,\ell_t^1,\theta_t^1} (1-m^1)^{\sigma+\nu}(K_t^1)^{1-\sigma-\nu}(\ell_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}\theta_t^{\nu} - w_t^1(1-m^1)\ell_t^1 e_{t-1}^{\varepsilon^1} - p_t^1(1-m^1)\theta_t^1 - R_t^1 K_t^1.$$

The first order condition for country $i = 1; 2$ where m^i is positive for $i = 2$ or negative for $i = 1$

$$(1 - \; - \;)\frac{Q_t^i}{1+m^i} = R_t^i \frac{K_t^i}{1+m^i}; \tag{16}$$

$$\frac{Q_t^i}{1+m^i} = w_t^i \ell_t^i e_{t-1}^{\varepsilon^i}; \tag{17}$$

$$\frac{Q_t^i}{1+m^i} = p_t^i \theta_t^i. \tag{18}$$

Note that the following relations are unchanged compared with autarkic equilibrium, but now, due to migration flows, the population can no longer be normalized to unity as was the case in autarky. The dynamics of country 2 and country 1 are

$$K_{t+1}^2 = (1+m^2)s_t^2;$$
$$K_{t+1}^1 = (1-m^1)s_t^1.$$

Consequently, considering that $m^2 > 0$ and $m^1 < 0$, the individual's first and second period budget constraints are modified as follows

$$\begin{cases} ae_{t-1}^i &= E_{t-1}^i; \\ c_t^i + \frac{k_{t+1}^i}{1+m^i} + R_t^i ae_{t-1}^i &= w_t^i \ell_t^i (e_{t-1}^i)^{\varepsilon^i}; \\ d_{t+1}^i &= R_{t+1}^i \frac{k_{t+1}^i}{1+m^i} + p_{t+1}^i \theta_{t+1}^i. \end{cases}$$

Using exactly the same procedure as in autarky, we obtain the new expressions of the consumption of the old

$$d_{t+1}^i = (1 - \;)\frac{Q_{t+1}^i}{1+m^i};$$

the consumption of the young

$$c_t^i = \left[\frac{1 - \;}{(1 - \; - \;)}\right]\frac{K_{t+1}^i}{1+m^i};$$

the adult and old labor are unchanged, and finally

$$e_{t-1}^i = \frac{\epsilon^i}{(a(1 - \; - \;))}\frac{K_t^i}{1+m^i}.$$

PROPERTY **2** *The old efficient labor supply is independent of the returns to education,* ϵ^i*, i.e., there is labor market integration of migrants when old.*

Using the second period budget constraint, we can easily compute the steady-state capital per worker in each country.

$$\hat{K}^2 = \left[\frac{(1- \quad - \quad)\;(1- {}''^{i})(1+m^2)^{\nu+\sigma(1-\varepsilon^i)}}{(1- \quad) + \quad (1- \quad - \quad)} \left[\frac{''^{i}}{a(1- \quad - \quad)} \right]^{\varepsilon^i \sigma} \grave{}^{i\sigma} \quad {}^{i\nu} \right]^{\frac{1}{\nu+\sigma(1-\varepsilon^i)}} ; \quad (19)$$

$$\hat{K}^1 = \left[\frac{(1- \quad - \quad)\;(1- {}''^{i})(1-m^1)^{\nu+\sigma(1-\varepsilon^i)}}{(1- \quad) + \quad (1- \quad - \quad)} \left[\frac{''^{i}}{a(1- \quad - \quad)} \right]^{\varepsilon^i \sigma} \grave{}^{i\sigma} \quad {}^{i\nu} \right]^{\frac{1}{\nu+\sigma(1-\varepsilon^i)}} : \quad (20)$$

Since both post-migration economies converge to a market steady-state equilibrium, we now investigate by which migration policy the social planner can guide the economy towards a first-best static welfare optimum. In standard overlapping generations models, this is designated as the Golden Rule, and the government would calculate a tax system that leads the static capital per capita to maximize total consumption in that static state. Our problem is not exactly the same for two reasons. The first reason is that there is no tax system in our economy, and the second reason is that our problem is multidimensional. Since there is no tax system, the government uses the migration rate as a policy instrument in order to choose the static welfare maximizing level of education, adult and old labor, consumption, as well as the capital per worker ratio. Consequently, we must reformulate the social planner's problem, and this is the objective of the next subsection.

5.3 The Static Welfare Optimum with permanent International Migration

We define the static welfare optimum of the economy and examine how it can be reached. It is defined as the stationary state that a social planner would select to maximize welfare under the feasibility constraint. The welfare criterion a collectivity must choose in order to rank all possible steady states has usually been described — following Samuelson (1958) — as the one that maximizes aggregate consumption. In standard models, this is called the Golden Rule, and the government would calculate the static capital per capita that achieves this. Our problem is slightly different in the sense that now the social planner of each country $i = 1; 2$ maximizes the static welfare, and by doing this, he chooses the optimal levels of education e^i_w (where the subscript w captures the welfare maximizing solution of each variable), adult labor $\grave{}^i_w$ and old labor $\;^i_w$, adult and old consumptions c^i_w and d^i_w, as well as the capital per worker k^i_w. He uses the level of migration m^i as an instrument to guide the economy toward the static welfare optimum, taking into account the macroeconomic equilibrium constraint of his country.

In the integrated world economy, the benevolent social planner in each country $i = 1; 2$ solves the following problem

$$\max_{K^i_w, \ell^i_w, \theta^i_w, e^i_w, c^i_w, d^i_w} \log[c^i_w] + \quad \log(1 - \grave{}^i_w) + \quad \log[d^i_w] + \quad \log(1 - \;^i_w);$$

subject to the macroeconomic equilibrium constraint

$$ae^i_w + c^i_w + d^i_w + K^i_w = K^i_w{}^{1-\sigma-\nu} (\grave{}^i_w e^{\varepsilon^i}_w)^{\sigma} \;{}^{\nu}_w :$$

In each country $i = 1; 2$ the first order condition is

$$(1 - \quad - \quad) Q^i_w = K^i_w; \quad (21)$$

$$\frac{Q_w^i}{c_w^i \ell_w^i} = \frac{}{1 - \ell_w^i}; \qquad (22)$$

$$ae_w^i = \varepsilon^i \; Q_w^i; \qquad (23)$$

$$\frac{Q_w^i}{c_w^i \;^i} = \frac{}{1 - \;_w^i}; \qquad (24)$$

$$d_w^i = \; c_w^i: \qquad (25)$$

The post migration macroeconomic constraint of the country 2 is as follows

$$c_w^i = Q_w^i - ae_w^i - d_w^i - K_w^i:$$

Using (21), (22), and (25) and isolating $\frac{Q^i}{c^i}$ gives

$$\frac{Q_w^i}{c_w^i} = \frac{(1 + \;)}{\; + (1 - \varepsilon^i)}: \qquad (26)$$

Putting the last expression into (22) and isolating ℓ_w^i gives the optimal adult labor ℓ_w^i in each country $i = 1; 2$

$$\ell_w^i = \frac{(1 + \;)}{(1 + \;) + \;[\; + \;(1 - \varepsilon^i)]}: \qquad (27)$$

Also, putting (26) in (24) and isolating $\;_w^i$ gives the optimal old labor in each country $i = 1; 2$

$$\;_w^i = \frac{(1 + \;)}{[\; + \;(1 - \varepsilon^i)] + \;(1 + \;)}: \qquad (28)$$

Using (21) into (23) and isolating e we find the expression of the chosen level in education in country i

$$e_w^i = \frac{\varepsilon^i \; K_w^i}{(1 - \; - \;)a}: \qquad (29)$$

From relation (21) we deduce the optimal capital per worker that maximizes the welfare in each country

$$K_w^i = [(1 - \; - \;)(\frac{\varepsilon^i}{(1 - \; - \;)a})^{\varepsilon^i \sigma} \ell_w^\sigma \;_w^\nu]^{\frac{1}{\nu + \sigma(1 - \varepsilon^i)}}: \qquad (30)$$

PROPOSITION **2**

1. *If the return to education is lower in country 2 than in country 1, the level of migration the social planner of country 2 implements is less than the one chosen by the social planner of country 1.*

2. *There are always incentives for illegal migration from country 1 toward country 2.*

Proof. To find the optimal level of migrants, we equalize $\hat{K}^i(m^i) = K^i_w$ so that $m^{i\star} = \Psi^{-1}(K^i_w)$: This leads to the expression of the welfare maximizing level of migrants for each country

$$m^{2\star} = \left[\left[\frac{1 - \quad + \quad (1 - \quad - \quad)}{(1 - \varepsilon^2)}\right] \frac{\ell^{2\sigma}_w}{\ell^{2\sigma}} \frac{{}^{2\nu}_w}{{}^{2\nu}}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^2)}} - 1;$$

$$m^{1\star} = 1 - \left[\left[\frac{1 - \quad + \quad (1 - \quad - \quad)}{(1 - \varepsilon^1)}\right] \frac{\ell^{1\sigma}_w}{\ell^{1\sigma}} \frac{{}^{1\nu}_w}{{}^{1\nu}}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^1)}} :$$

□

LEMMA **3** *Since $\ell^i{}_w; \ell^i; {}^i_w$ and $1 = \left[(1 - \varepsilon^i)\ {}^{i\nu}\right]$ are increasing functions of the return to education $\varepsilon^i; i = 1; 2;$ $m^{2\star}$ is an increasing convex function of ε^2, and $m^{1\star}$ is a decreasing concave function of ε^1.*

The proof is given in Appendix B.

PROPOSITION **3** *There are incentives for illegal migration.*

Proof. Incentive for migration are directed from country 1 to country 2, if and only if $\mid m^{2\star} \mid < \mid m^{1\star} \mid$, and in the remaining of the paper, we will assume that this condition holds. If not, international migration is in the opposite direction. □

In what follows, since we study post-migration perfect foresight equilibria, the post-migration flow is defined $m = \min\{m^1; m^2\}$, which is exactly anticipated by each country.

5.4 INCENTIVE FOR ILLEGAL MIGRATION

Each social planner maximizes the utility of his own country; consequently, all education, consumption, labor, and capital are set at their welfare maximizing levels. When borders are open, there exists $m^{i\star}$ so that $\hat{K}^i(m^{i\star}) = K^i_w$ is satisfied.

PROPOSITION **4** *In post-migration steady-state equilibrium, there are incentives for illegal migration.*

Proof. Let us consider the case were the optimal desired flows of migrants differ across countries, since $\varepsilon^2 < \varepsilon^1$. In that case and under the unilateral migration condition, the two migration flows satisfy the following inequality $\mid m^{1\star} \mid \geq \mid m^{2\star} \mid$. Consequently, there are incentives for country 1 to support illegal migration flows in the direction of country 2. □

5.5 THE EMERGENCE OF AN OPTIMAL PRICE DIFFERENTIAL BETWEEN COUNTRIES

PROPOSITION **5** *In post-migration steady-state equilibrium, there is no prices equalization across countries.*

Proof. Since the returns to education differ across countries, the optimal migration policies lead the economies to different steady-state equilibria. Indeed, we have two main cases

1. The first case is such that $m^{1\star} \geq m^{2\star}$, so that country 2 reaches the optimal level "before" country 1. In such a case, $\hat{K}^1(m^{2\star}) < K_w^1$ and $\hat{K}^2(m^{2\star}) = K_w^2$. Consequently, by assumption on the returns to education, $"^1 > "^2$, we necessarily have $\hat{K}^1(m^{2\star}) < \hat{K}^2(m^{2\star})$.

2. The second case is such that $m^{1\star} < m^{2\star}$, so that country 1 reaches the optimal level "before" country 2. In such a case, $\hat{K}^1(m^{1\star}) = K_w^1$ and $\hat{K}^2(m^{1\star}) > K_w^2$ according to our assumptions. Consequently, $\hat{K}^1(m^{1\star}) > \hat{K}^2(m^{1\star})$.

A natural consequence of such differences in steady-state capital is that there is no prices equalization across countries. It always remains a wage differential $\overline{w}^1 \neq \overline{w}^2$, $\overline{p}^1 \neq \overline{p}^2$, and there is an interest rate differential across countries, $\overline{R}^1 \neq \overline{R}^2$. □

Moreover, rewritting (19) and (20) to obtain the steady state capital per individual leads us to conclude that the wage of the sending country increases with migrants, and the interest rate decreases with migrants. Such a result is compatible with the implicit legal system of migration, which sets high wages and low interest rates in order to refrain individuals from migrating.

6 Theory and Applications

There are various cases. Hereafter are listed some of the cases that we have selected for their potential applicability in the real world. Some theoretical comments and empirical facts are provided. All the following figures illustrate each social planer's optimal migration rates against the return to education of each country.

6.1 First case $m^{2\star} < m^{1\star}$ with $\varepsilon^2 < \varepsilon^1$

6.1.1 Theoretical comments

Case 1 is devoted to unilateral migration from country 1 to country 2. It suggests that the less educated country chooses a much bigger optimal migration flow than the other country. In this case, the receiving country closes its borders prior the sending country. Consequently, there exists an incentive for illegal migration from country 1 to country 2.

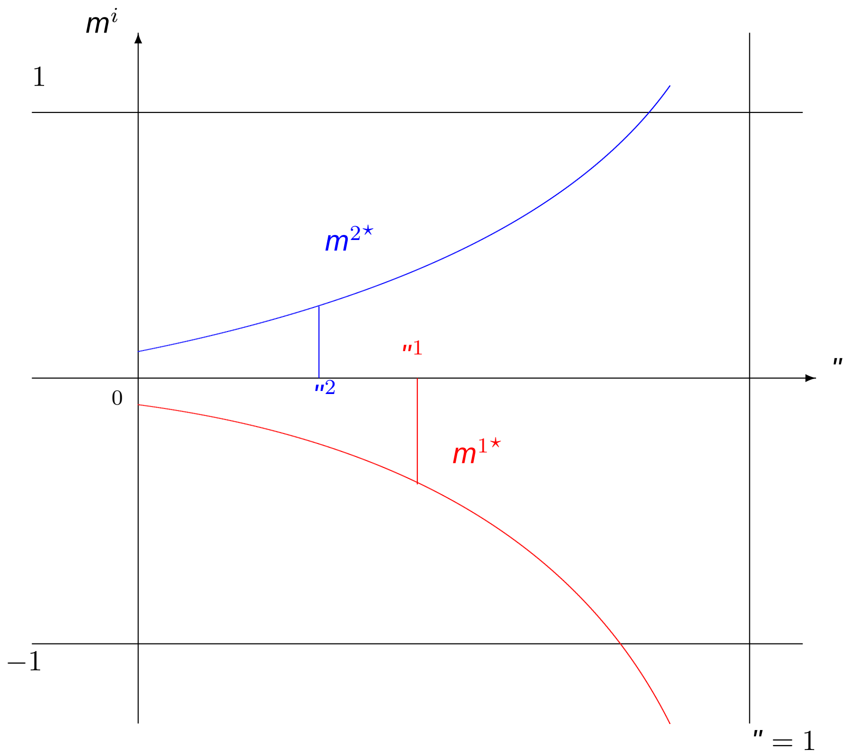


Figure 1: $\text{Elasticity}_{L_w/\varepsilon^2} + \text{Elasticity}_{\Theta_w/\varepsilon^2} \geq \Theta L\ \text{Elasticity}_{L/\varepsilon^2}$

6.1.2 Empirical fact: The Wall between USA and Mexico

Figure 1 is compatible with the wall between USA and Mexico[5]. The most important flow of migration all over the world during the period 2010-2011 is the one existing between Mexico and the USA with more than 11 million migrants. The phenomenon is not new and started during the 80s. A wall between the USA and Mexico was built in 2002. It is now a 1,300 km long barrier which helps to limit illegal migration from Mexico to the USA. However, the discontinuity of the wall is the result of strong bargaining between the governments of the two countries. Indeed, it was the interest of Mexico to let people migrate since those who migrate provided Mexico with large remittance funds (about $ 368 per migrant on average). Moreover, those who return migrate change their social status from blue collar to entrepreneur, see Mesnard (2004). The wall has limited Mexican immigration by about 25 %. The US government wanted to stop migration unilaterally, while the Mexican government wanted to let people migrate. Illegal migration involves low skilled individuals.

[5]Empirical literature includes case studies of Mexican communities that send illegal migrants to the United States and estimates of the U.S. illegal immigrant population (Hanson G.H. and A. Spilimbergo (1999)). Frank D. Bean et al. (1990), using monthly INS data for 1977-1989, find that border apprehensions declined substantially following the Immigration Reform and Control Act of 1986. Borjas et al. (1991), using annual INS data for 1967-1984, find that apprehensions by the U.S. Border Patrol are positively correlated with U.S. expenditure on border enforcement and U.S. real wages.

6.2 SECOND CASE $m^{2\star} > m^{1\star}$ WITH $\varepsilon^2 < \varepsilon^1$

6.2.1 THEORETICAL COMMENTS

Case 2 is devoted to unilateral migration from country 1 to country 2, where the less educated country chooses a much smaller migration flow than the other country. In this case, the sending country closes its borders prior the other country and does not let its individuals leave the home country.

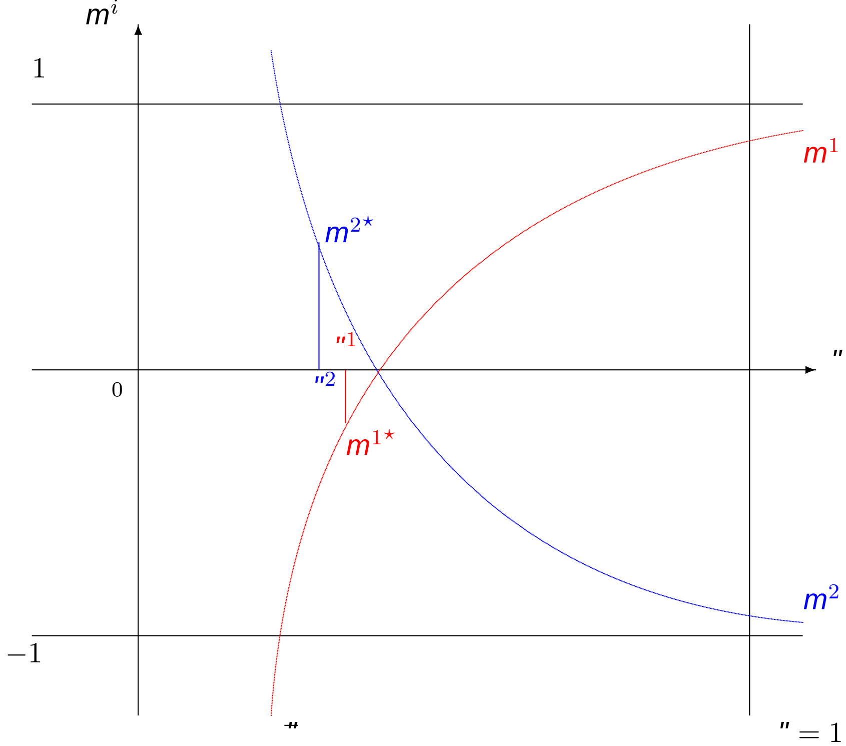


Figure 2: $\text{Elasticity}_{L_w/\varepsilon^2} + \text{Elasticity}_{\Theta_w/\varepsilon^2}$ < ΘL $\text{Elasticity}_{L/\varepsilon^2}$

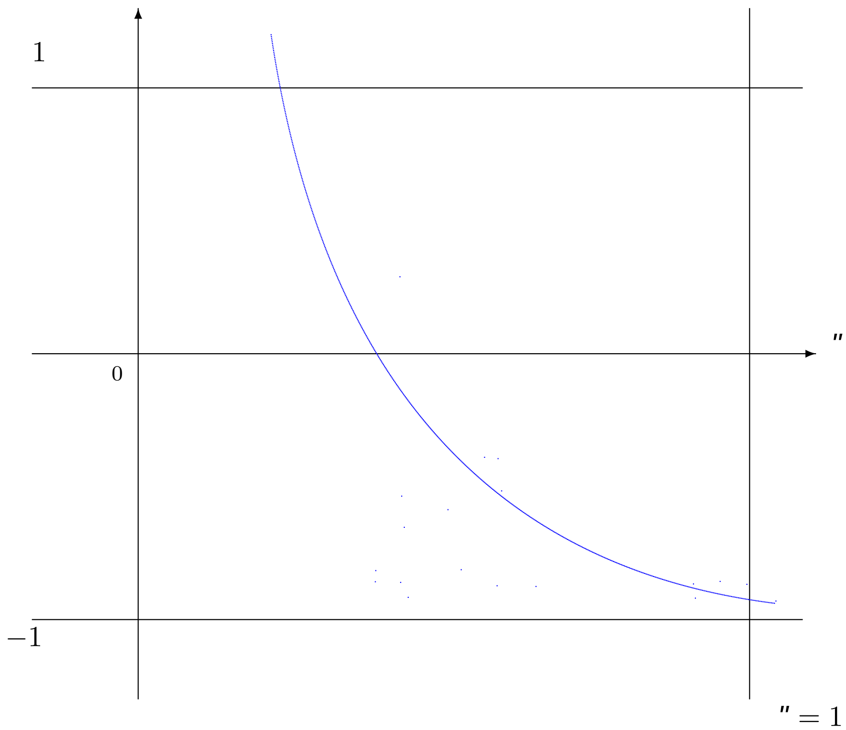
6.2.2 EMPIRICAL FACT: THE SENEGAL MIGRATION DIRECTED TO FRANCE

In 2007, the president of Senegal Abdoulaye Wade asked the president of France Nicolas Sarkosy for the right to choose who is able to migrate from Senegal to France since those who migrate never return migrate if they are successful in the French labor market. This caused a deficit of human capital in Senegal, which doesn't help the country to recover from the poverty trap in which it has fallen. Even if Senegal invested a great deal in education in proportion of its GDP, brain waste is still a major problem. Figure 2 shows that the less educated country doesn't want to send migrants to the more educated country.

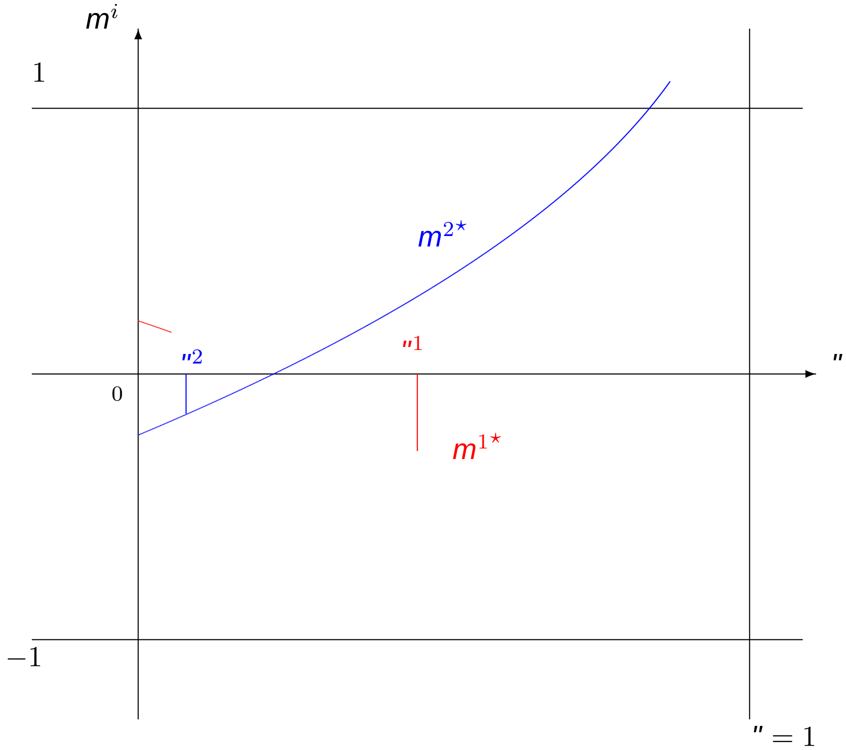
6.3 THIRD CASE $m^{2\star} > m^{1\star}$ WITH $\varepsilon^2 > \varepsilon^1$

6.3.1 THEORETICAL COMMENTS

Suppose that incentives for international migration are reversed, since $''^2 > ''^1$. Figure 3 bellow shows this particular case of unilateral migration from the country with a higher education level to the country with a lower education level. In this case, the sending country chooses a smaller optimal migration flow than the receiving country. Such a situation characterizes a continued demand of immigrants by receiving country 2.



m2m2 m1



m^i
1
$m^{2\star}$
"2
"1
"
0
$m^{1\star}$
-1
" = 1

6.5 EMPIRICAL FACT: THE BILATERAL MIGRATION IN EUROPE

In recent years, the Schengen zone authorized individuals to freely migrate from one country to another. Such a legal system of migration allows for bilateral migration flows.

7 OPTIMAL LEGAL SYSTEMS: THE CANADIAN AND AUSTRALIAN CASES

The focus of this theoretical paper is to provide a rationale for explaining how country-specific optimal legal systems emerge in order to regulate national migration flows. Double sided borders have not been theoretically modeled in the literature. It is important to have theories taking into account that migration is a two step experience. Crossing borders means leaving one country (and crossing the "exit" border) prior to entering the other one (and crossing the "entrance" border). To our knowledge, this paper is the first to attempt that. The objective of this Section is to put our theoretical results in perspective with the existing legal system for both Canada and Australia. The relevance of the previous model is supported by empirical facts. Indeed, prior to migrating to Canada or to Australia, a migrant must apply for migration and if qualified, he/she can migrate. How does such a legal system work in practice?

7.1 THE CANADIAN LEGAL SYSTEM OF MIGRATION

The Canadian Visa of Immigration is obtained according to a legal system of points, see Chaabane (2011). The law sets how many points are necessary in order to be eligible to immigration. This number of points (67 points minimum out go 100 possible in 2014) is flexible and changes depending on the economic needs of the country (73 points in 2004). The government can make migration easier or harder to obtain. The following conditions are required to be admissible:

1. to have a job offer;
2. to have been a legal resident (Landing resident) for at least one year, or to have been a foreign student;
3. to be a qualified worker with at least one year of experience in one of the admissible industries of the country during the last 10 years.

Points are given according to various categories of criteria, which are public knowledge to any applicant to migration[6]. Table 1 makes a list of them.

7.2 THE AUSTRALIAN LEGAL SYSTEM OF MIGRATION

Not only Canada has set up criteria for migration. Australia also does with the General Skilled Migration Program for individuals who are not sponsored by a "godfather" firm (or individual) but who are highly qualified in certain jobs for which there are specific Australian needs. Applicants should be more than 18 years old and not over 50 in order to accumulate points. They must speak English, have an Australian Experience, especially in the "Australia's Skilled Occupation List" or have Australian Diploma. Various Visas exists.

[6] In October 2014, a valid job offer in the region of Montréal gives 6 points, and a validated job offer outside the region of Montréal gives 10 points.

Table 1: Migration Criteria and corresponding points

Criterium	Maximum Points
Education	25
Language (French or English)	24
Experience	21
Age < 49	10
Job offer	10
Adaptability	10
Total	100

1. Onshore Visas are built for individuals who already are living in Australia and who want to be integrated in the General Skilled Migration Program.
2. The Offshore Visas are made for foreigners who apply for permanent migration to Australia. This is the most important number of demands, and these Visas are restricted to qualified workers.

The number of points in 2011 was 65 points out of 100 possible points. Those who do not reach the threshold enter a specific category called "reserve". If the number of points falls, they become immediately eligible prior to any other current applicant.

7.3 The link between the existing legal system of migration and our results

As Table 1 shows, the Canadian social planner has chosen four criteria relative to "education" in a wide sense (Education, Language, Experience and Age), as our theoretical approach does. Education is an important criterion for the social planner, even if it is not necessarily the first criterion for migrants. Most empirical studies underline the migrant motive, and not the social planner's motives. That is the reason why we build a 3-period model with education, where the social planner chooses these criteria in order to select migrants. The main reason for such a selection is that selected migrants are economically useful for the country. A welfare maximizing criteria is therefore suitable.

8 Possible extensions of the model

8.1 The complementarity or substitutability between migrants and natives workers

Our model can also be extended to the case where native workers and migrant workers are not perfect substitutes. In order to translate this reality into the model, one can consider the following production function:

$$Q_t(K_t; L_t; e_{t-1}; \ _t) = K_t^{1-\nu-\sigma}\left[a\ell_t^{\rho} + (1-a)m^{\rho}\right]^{\frac{\sigma}{\rho}} e_{t-1}^{\sigma} \ ^{\nu}$$

where   is the elasticity of substitution between native and migrant in the production function. Depending on the various possible values of parameter , such a function accounts for the possibility of complementarity among native and migrant workers.

8.2 The migration of adults

Suppose two countries are endowed with two different returns to education. Without loss of generality, one country is more efficient that the other one, referred to here as country H, i.e., the high return to education country. The other country is the L country, the low return to education country. Moreover, suppose that only adults are allowed to migrate. Such an assumption is reasonable since in most countries individuals need to reach maturity prior to making a decision to migrate. Compared with the current version of the paper, the difference in the return to education generates differences in the optimal migration flow of each country. The major difference with this paper is that the H country accepts everybody migrating from L country. The human capital is more attractive for an individual from L country since they are better off in the H country rather than staying home. The social planner of L country closes the borders when the country reaches it social welfare maximum, while the social welfare function of country H is always increasing in the migration rate. For that reason, country H never closes its borders and always accept all migrants.

8.3 The migration cases with population growth

Suppose now that we relax the assumption of a constant population over time in order to allow it to evolve through time. The law of motion population is $N_{t+1} = (1+n)N_t$. The dynamics of the economy is changed as $K_{t+1} = N_t s_t$. Define the capital per worker $k_t = K_t/N_t$ and use the law of motion of the population growth to have $(1+n)k_{t+1} = s_t$ to obtain new relations. The equilibrium on the labor market between the demand L_t and the supply becomes $L_t = N_t`_t e^{\varepsilon}_{t-1}$ for adults and $\Theta_t = N_{t-1} \;_t$ for the old. With the law of motion of the population growth, rewrite the old labor as $\Theta_t = \frac{\theta_t}{1+n}$. The production per capita is $q_t = Q_t/N_t = k^{1-\sigma-\nu}(`_t e^{\varepsilon}_{t-1})^{\sigma}(\frac{\theta_t}{1+n})^{\nu}$. The first order condition of the firm is not changed except that K_t is replaced by k_t and $\;_t$ by $\frac{\theta_t}{1+n}$. Relation (32) becomes $c_t = \frac{(1+n)(1-\sigma)}{\beta(1-\sigma-\nu)}k_t$, relation (13) becomes $e_{t-1} = \frac{(1+n)\varepsilon\sigma}{a(1-\sigma-\nu)}k_t$. Using exactly the same procedure as before, the steady-state equilibrium is

$$\bar{k} = \left[\frac{(1- \quad - \quad)\;(1-''i)}{(1+n)^{1+\nu-\sigma\varepsilon}\left[1- \quad + \quad (1- \quad - \quad)\right]} \left[\frac{''i}{a(1- \quad - \quad)}\right]^{\varepsilon^i\sigma} `_i^{\sigma} \;\; _i^{\nu} \right]^{\frac{1}{(1-\varepsilon^i)\sigma+\nu}} :$$

Note that the steady state is a decreasing function of the rate of population growth n, whatever its sign, positive or negative. One can also redo the exercise for international migration and see that the same transformation occurs on the steady-state equilibrium.

8.4 The return migration cases

The return migration case is complex to study. Indeed, the set of all possible patterns of migration is the following, where M_t means migrating during period t, and H_t live in the home country during period t.

$$\mathcal{M}_{\emptyset} = \{(M_1; M_2; M_3); (M_1; M_2; H_3); (H_1; H_2; H_3); (H_1; M_2; M_3); (H_1; H_2; M_3); (H_1; M_2; H_3)\}$$

$$\mathcal{M} = \{(M_1; H_2; H_3); (M_1; H_2; M_3)\}$$

Only set $\mathcal{M}$ is interesting to study. It occurs if and only if incentives for migration hold. To migrate abroad for education and definitely return migrate (respectively to migrate abroad for education, return migrate when adult, and migrate again when old) is possible if the life-cycle indirect utility is higher than the life-cycle indirect utilities of each other cases, including those in $\mathcal{M}_{\emptyset}$. The major difficulty in studying the convergence of the dynamics of these two cases is that prices will change over time with migration flows so that an individual must anticipate all possible movements prior to make the decision to migrate and return migrate. Nothing indicates that in an overlapping generations model with three periods, each economy reaches a post-migration steady-state equilibrium.

9 CONCLUSION

In a 3-period overlapping generations model with two countries, this paper proposed an alternative theory of international migration. Indeed, contrary to the traditional literature on international migration, in this model, international migrations cease due to the optimal legal system each social planner implements in his country. Differences in social planner's decisions are due to differences in the return to education across countries. As a consequence, each social planer does not chose the same level of migrants in each country.Thus, an optimal legal system for migration emerges and generates endogenous two sided borders across countries. Even if each country uses the same method for designing its optimal international migration policy, the optimal level of migration flows varies across countries. The first natural consequences are the non equalization of prices, and there always remains wage differentials and an interest rate differentials across countries in the post-migration steady-state equilibrium with optimal legal systems of international migration. Since migration flows are unilateral, a second natural consequence of the non equalization of the steady-states is that incentive for illegal migration always exists.

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A Appendix

Proof. Proof of Lemma 1. Using Definition 1 forward, $K^i_{t+1} = s^i_t$, rewrite the second period budget constraint forward as follows:

$$d^i_{t+1} = R^i_{t+1} K^i_{t+1} + p^i_{t+1} \quad {}^i_{t+1}$$

Using the first order condition of the firm (6) and (8)

$$d^i_{t+1} = (1 - \quad) Q^i_{t+1}; \tag{31}$$

which we put into (4) the first order condition of the individual to have

$$\frac{1}{c^i_t} = \frac{(1 - \quad - \quad) Q^i_{t+1}}{(1 - \quad) Q^i_{t+1} K^i_{t+1}} \Longleftrightarrow c^i_t = \frac{(1 - \quad)}{(1 - \quad - \quad)} K^i_{t+1}: \tag{32}$$

Put (32) into (6):

$$\frac{\quad}{1 - `^i_t} = \frac{(1 - \quad - \quad) \quad Q^i_t}{(1 - \quad) K^i_{t+1} `^i_t}: \tag{33}$$

By using (8) and (9), we have

$$\frac{Q^i_t}{K^i_{t+1}} = \frac{(1 - \quad - \quad) + 1 - \quad}{(1 - \quad - \quad)(1 - ''^i)}; \tag{34}$$

which we replace into (33) to have $`^i_{t+1} = `^i_t = `^i$ where

$$`^i = \frac{1 - \quad + \quad (1 - \quad - \quad)}{(1 + \quad (1 - ''^i))(1 - \quad) + \quad (1 - \quad - \quad)}: \tag{35}$$

Note that using (4), we can rewrite (7) as

$$\frac{c^i_t}{1 - \quad {}^i_{t+1}} = \frac{P^i_{t+1}}{R^i_{t+1}}; \tag{36}$$

and using (36) and (32), we have ${}^i_{t+1} = {}^i_t = {}^i$ where

$${}^i = \frac{\quad}{(1 - \quad) + \quad}: \tag{37}$$

□

B Appendix

In order to prove Lemma 3, one can derivate carefully the expressions $m^{i\star}$ in order to show that $m^{2\star}$ is an increasing convex function of ε^2, and $m^{1\star}$ is a decreasing concave function of ε^1.

Let us define $L_w = \ell_w^{2\ \sigma}$, $L = \ell^{2\sigma}$, $\Theta_w = \ _w^{2\ \nu}$, $\Theta = \ ^{2\nu}$,

$$\frac{\partial m^{2\star}}{\partial \varepsilon^2} = \left[\frac{1 - \ \ + \ \ (1 - \ \ - \ \ )}{(\ \ + \ \ (1 - \varepsilon^2))}\right]$$

$$\times \frac{\left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w \frac{\partial \Theta_w}{\partial \varepsilon^2}\right] L(1-\varepsilon^2)\Theta - L_w(1-\varepsilon^2)\Theta_w \left[\frac{\partial L}{\partial \varepsilon^2}\Theta + \frac{L\,\varepsilon^2\Theta}{(1-\varepsilon^2)^2\Theta^2}\right]}{L(1-\varepsilon^2)\Theta^2}$$

$$\times \left[\left[\frac{1 - \ \ + \ \ (1 - \ \ - \ \ )}{(1-\varepsilon^2)}\right] \frac{L_w}{L}\frac{\Theta_w}{\Theta}\right]^{\frac{1-\nu-\sigma(1-\varepsilon^2)}{\nu+\sigma(1-\varepsilon^2)}}.$$

Note that the previous expression is positive if and only if the following condition is satisfied:

$$\forall \varepsilon^2 \neq 1;\ \left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w \frac{\partial \Theta_w}{\partial \varepsilon^2}\right] L \geq L_w \Theta_w \left[\frac{\partial L}{\partial \varepsilon^2} + \frac{L\,\varepsilon^2}{(1-\varepsilon^2)^2\Theta^2}\right].$$

The previous inequality is a condition relative to ε^2.

$$A(\varepsilon^2)^2 - B\varepsilon^2 + C \geq 0$$

where

$$A = \Theta\left[\left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w \frac{\partial \Theta_w}{\partial \varepsilon^2}\right] L - \Theta L_w \Theta_w \frac{\partial L}{\partial \varepsilon^2}\right];$$

$$B = \Theta\left[2\Theta^2 L_w \Theta_w \frac{\partial L}{\partial \varepsilon^2} - L_w \Theta_w L - 2\Theta L\left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w \frac{\partial \Theta_w}{\partial \varepsilon^2}\right]\right];$$

$$C = \Theta\left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w \frac{\partial \Theta_w}{\partial \varepsilon^2}\right] L - \Theta^2 L_w \Theta_w \frac{\partial L}{\partial \varepsilon^2}.$$

As long as we have $A > 0$, the migration flow is an increasing convex function of ε^2, which is equivalent to the following condition, and, holds for reasonable values of parameters.

$$\text{Elasticity}_{L_w/\varepsilon^2} + \text{Elasticity}_{\Theta_w/\varepsilon^2} \geq \Theta L\ \text{Elasticity}_{L/\varepsilon^2}$$

We have shown that m^2 is an increasing and convex migration function. Consequently, for country 2, the lower the return to education ε^2 the higher the migration flows. In the same way, m^1 is a decreasing concave function of the return to education.